Intermediated Investment Management

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Abstract

Investment advisers perform the role of assisting clients with their investments and distributing portfolio management services. While the vast majority of clients employ advisory services, an important issue is how well advisers perform in this capacity. Our theoretical model analyzes the impacts on portfolio performance, fund flows, fund sizes and welfare from the use of advisers. An important aspect of our analysis is the extent to which conflicts of interest such as influence activity can bias the asset allocation decisions of advisers. Interestingly advisory services are utilized to a greater extent under this circumstance. We show that investment advisers help to improve social welfare, but much of the welfare gain is extracted by the portfolio manager. When influence activity is feasible, investors welfare is adversely affected by the presence of advisers.

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1 Introduction

Distributers play an integral role in the way many products and services are brought to market. They serve as intermediaries between consumers and the marketplaces where purchases are made. Intermediation is just as important in the investment management industry. Investment advisers serve the needs of investors in the important capacity of finding suitable investment vehicles and assisting them in creating a portfolio to achieve their goals. The Investment Counsel Association of America (ICAA, 2004) has estimated the total amount of assets managed by SEC-registered investment advisers at $23.4 trillion in the year 2004. In the UK, the Association for Independent Financial Advisers asserts that their members account for 80% of all business related to investments, pensions and protection. As the nature and variety of financial assets expands at an enormous rate, the scope and breadth of advisory services has kept pace in recent years. For instance, as of 2004 23% of investment advisory firms reported expertise in hedge fund activities, an increase from just 15% the year before. Advisory services are offered through many organizational forms such as investment banks, commercial banks, partnerships and sole proprietorships. Services offered include asset allocation, financial planning, portfolio management, pension consulting, information dissemination, securities ratings, market timing and even the selection of other investment advisers. Another example is the fund of hedge funds structure that is becoming more common as an alternative investment strategy.

Although in practice the term investment adviser encompasses a wide variety of functions, our paper concerns those activities that are related to intermediation between investor clients and portfolio managers. Since advisers often advise multiple clients simultaneously and have direct relationships with portfolio managers, the possibility of real or apparent conflict of interest is important. For instance in a recent keynote address Spatt (2005) emphasizes the importance of product distribution in mutual funds and potential conflicts of interest therein. As he points out, “Yet many investors do not understand the mechanism by which they can undertake such investments and brokers and advisers play an important role in facilitating product distribution, .... Of course, the difficulty with this approach is how distorted can be the adviser’s incentives–for example, how often do advisers recommend low-cost mutual funds?”

1Investment advisers with at least $25 million in management are required to register and file documents with the SEC; others are regulated by the states. One reason why this figure is so large is due to multiple layers of advisory services.
The role of intermediaries in the investment management industry has largely been ignored by the existing literature. Most theoretical models on delegated portfolio manager consider only the bilateral relationship between investors and portfolio managers. The contribution of our paper is to model explicitly the impacts of intermediation on the equilibrium in the investment management industry and to consider the consequences of conflicts of interest.

Conflicts of interest derive from the explicit and implicit compensation arrangements in the investment advisory relationship. In the case of mutual funds, for instance brokers receive direct compensation depending on where they steer investment in the form of front end loads, back end loads and 12b-1 fees. The selling broker is typically compensated by a rebate of these fees back from the fund management company. Industry estimates state that mutual fund investors paid $3.2 billion in front end loads, $2.8 billion in back end loads and $8.8 billion in 12b-1 fees in the year 2002 alone. Other forms of compensation are less explicit. 69% of investment advisers report having the discretionary authority to direct client transactions to specific broker-dealers and reported receiving research or other products in connection with such transactions. These ‘soft dollar commitments’ can be substantial and important in the provision of incentives in the relationship. A SEC study [SEC, 1998] asserted that substantially all investment advisers surveyed had some form of soft dollar relationship. Advisers are also compensated implicitly by a host of other arrangements. Examples include CFA exam review courses, AIMR membership dues, office rent, utilities, salaries, travel expense, Bloomberg terminals and even honoraria for lectures by a prominent finance professor. The potential for bias looms large in such instances.

The purpose of this paper is to study the investment management structure of advisers, investors and portfolio managers, and to determine the effect of influence on fund flows, fund returns, fees and investor welfare. We find, surprisingly, that when asset allocation is biased through kickbacks from the portfolio manager to advisers, the usage of advisory services increases. Therefore conflicts of interest, when recognized, do not reduce the utilization of investment advisory services. We also find that even though net investment returns are lower by investing through indirect as opposed to direct channels, both can exist simultaneously in equilibrium. This indicates that empirical observations of heteroge-

\[^2\] E.g., Bhattacharya and Pfleiderer [1985] and Stoughton [1993].

\[^3\] Front end loads are extra sales charges added onto the purchase price of the fund at the purchase date, back end loads are added at the redemption date and 12b-1 fees are paid on a continuing annual basis as long as the fund is held.

\[^4\] See Bergstresser, Chalmers and Tufano [2006].
nous returns between different distribution channels can be reconciled in a rational model of investor behavior.

Our model consists of investors, a representative investment adviser, a passive (e.g., index) fund and an active fund run by a portfolio manager. Investors have heterogeneous wealth levels, and can go directly to the portfolio manager or through the indirect channel of using an adviser. In order to invest directly, investors would have to pay a cost in order to identify an active manager who can outperform a passive index. As a result of this, only high networth individuals invest directly. However they earn a surplus over their reservation opportunity, which is to invest in the passive fund. Portfolio managers have market power over the fees they charge, but as the active fund has diminishing returns to size, there is an optimal amount of assets invested actively. They are able to charge fees increasing in their expected portfolio ‘alpha’ when contrasted with the passive fund. Portfolio managers earn a positive profit. Investment advisers also charge a fee, which compensates them for their information in making optimal asset allocations on behalf of the investors who invest through them. We analyze this model and extend it to consider the impact of biased investment advisers whose asset allocation decisions are no longer made in the best interests of maximizing risk-adjusted returns. We solve for the optimal amount of bias preferred by the portfolio manager and the kickback required to sustain it. Finally we also derive the form of an equilibrium without an adviser and compare outcomes to the other two cases.

The seminal paper on the subject of investment management is the AFA Presidential address of Sharpe (1981). This paper was the first to recognize, in the context of the standard Markowitz (1952) paradigm, that there are more than two parties (the client and a portfolio manager) in the practical problem of investment management. In this paper, Sharpe distinguished between passive and active (requiring superior information) investment activities. He found that with only a single active manager, a relative performance objective would be optimal, however with more than one active manager the general first-best outcome could not be achieved, because of the covariances between any two active managers.

Empirical work on the subject of mutual funds and their distribution channels is very recent and still at an early stage. Bergstresser et al. (2006) look at the performance of mutual funds offered through the
They find that brokered funds do not offer greater diversification opportunities, nor do they offer higher risk-adjusted returns. In fact, even before fees are deducted, risk-adjusted returns are lower for funds offered through the brokerage channel as compared with those making direct offerings to investors. The paper goes on to explore other hypotheses, including behavioral control, for potential benefits that investment advisers might provide. No evidence for any hypothesis that investment advisers offer positive benefits to investors was found.

A second important paper in this same empirical area is that of Christoffersen, Evans and Musto (2005). They use a new database of fund filings in which inflows and outflows are characterized by the category of intermediary that is involved. Looking only at the subset of funds that exclusively deal only with captive or unaffiliated brokers, they find that average (as opposed to contractual maximum) loads rebated to brokers are significantly lower for the captive brokers as compared to the unaffiliated brokers. They find mixed performance benefits for captive v. unaffiliated brokers. The former add more value in the case of inflows; the latter in the case of outflows. More pertinent to our paper, though, they find some evidence that the higher rebates to unaffiliated brokers buys favorable treatment in attracting inflows.

A recent paper, Chen, Yao and Yu (forthcoming), shows that mutual funds sold through insurance agents underperform those sold by non-insurance counterparts by more than 1%. The authors find that their evidence is consistent with cross-selling activities for other products offered by the fund family. This evidence is consistent with our model of influence activities and the impacts therein.

Grundy (2005) develops a model in which it is optimal to employ an intermediary, the investment bank, in order to achieve the optimal fund size. The bank can do this when its advantage in terms of resolving information asymmetry outweighs the additional contracting imperfections. By contrast in our model, a competitive adviser resolves the information asymmetry problem completely given a known benchmark passive asset.

An interesting paper exploring the role of kickbacks in the medical field is that of Pauly (1979). He considers a medical practitioner who is able to engage in ‘fee-splitting’ practices with a specialist. He finds that there is no point in prohibiting such practices in a fully competitive environment because

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5 Some examples of mutual fund companies that utilize direct channels include Fidelity, Vanguard and Janus. Examples of companies that offer their products through brokers include Investment Company of America, Washington Mutual and Putnam.
services are provided at marginal cost. However, when there are market imperfections such as monopoly or incomplete pricing of insurance, fee-splitting can actually improve client welfare.

Several other papers have examined the structure of investment management industry. Mamaysky and Spiegel (2002) and Gervais, Lynch and Musto (2005) provide a theory for the existence of fund families. Massa (1997) presents a model to explain the market segmentation and fund proliferation in the mutual fund industry. Chen, Hong and Kubik (2006) document that many fund families farm out a sizeable fraction of their funds to unaffiliated advisory firms and that funds managed externally significantly underperform those ran internally. Furthermore, the subject of agency issues within mutual fund families is one of considerable interest (Massa, Gaspar, and Matos, 2006).

In the next section 2 we set up the basic model, with behavioral assumptions on the three participants in the game: the investors, the investment adviser and the portfolio manager. In section 3 we derive the form of the equilibrium without the possibility of influence activity by portfolio managers. Section 4 derives the impact of biased asset allocation decisions and kickbacks from the portfolio manager to the advisers. Section 5 considers the form of the equilibrium in a situation where advisory services are not available. Section 6 compares the outcome and welfare in the three alternative scenarios. The section 7 concludes the paper.

2 Model Setup

In this section we describe the various agents, the model of their behavior and how they interact. There are three classes of agents in the model: (1) the active manager; (2) the set of investment advisers, modeled as a representative agent; (3) the pool of investors in the economy.

2.1 Assets and Managers

There are two types of assets in which investors can invest. First, there is a passive fund, such as an index fund, with expected gross return, $R_f$ (i.e., one plus the rate of return). If investors are risk averse and all use the same model of risk premia in pricing assets, then we could describe $R_f$ as a ‘risk-adjusted’ expected return. However to simplify description of the problem, we simply refer to this as an expected return, as if investors are risk-neutral. In order to invest in this benchmark asset directly, we assume that
a ‘small’ amount of proportional transaction costs need to be incurred, in the amount equal to $\tau$. We assume that this transaction cost can be avoided through the use of an investment adviser.

The second type of asset is an active fund, whose expected return (once again risk-adjusted) is equal to $R_p$. The active portfolio manager utilizes her expertise in managing these assets. However, because of market impact or limited applicability of the portfolio managers expertise, we consider the assumption that there is decreasing returns to scale in the amount of investment. That is we assume that

$$R_p = \alpha - \gamma A,$$

where $\alpha$ represents the initial expected return (assumed to be greater than the passive return) and $\gamma$ is a coefficient representing the rate at which returns decline with respect to the aggregate amount of funds, $A$, that are placed with the portfolio manager. For a discussion of this assumption see Berk and Green (2004) and Dangl, Wu and Zechner (2006).

In addition to investing in the passive asset and obtaining returns net of transactions costs equal to $R_f - \tau$, the investors can choose to delegate their portfolio decisions to an investment adviser who advises multiple clients, or they can decide to invest directly with the portfolio manager. Because there are a large number of active managers with inferior returns relative to the passive fund, we assume that there is a fixed ex ante learning cost, $C_0$, that must be expended by an investor in order to identify potentially valuable active managers. Therefore a direct investor pays the cost $C_0$ and then is able to identify a manager whose initial fund returns are relatively superior. Therefore, without loss of generality, we assume $\alpha > R_f$. Only by paying this cost, can investors guarantee themselves that they are dealing with a good manager. Investors decide optimally whether or not to pay this fixed cost. Alternatively they can avoid it by delegating their funds to the adviser in which case he makes the investment choice for them.

The investment adviser has access also to the same set of two investment opportunities. In this case, the adviser has an advantage over the investors themselves, in that the transaction cost of purchasing the passive fund is avoided. However the adviser incurs, at the beginning of the period, a constant marginal

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6Empirical evidence supporting this effect can be found in Chen, Hong, Huang and Kubik (2004) and Ang, Rhodes-Kropf and Zhao (2006).

7One interpretation of this cost, is that it is the cost of paying for their own 'private adviser' whose sole function is to certify that the portfolio manager is not drawn from the bad pool.
cost based on the initial amount of assets under management, representing for instance the cost of effort required to gain information about the returns of superior active portfolio managers.\footnote{Our cost structure for the advisers and the investors is similar to Holmström and Tirole [1997] who derive a theory of financial intermediation from these assumptions.} For simplicity, we assume that the constant marginal cost is

\[ c_A = \frac{\tau}{R_f}, \]  

i.e., variable costs are equal to the present value of investor transaction costs. This parametric specification implies that if the adviser's portfolio has a return equal to \( R_f \) and the adviser charges a fee equal to the marginal cost, then investors will be indifferent between investing via the adviser and investing in the passive fund directly. This not only simplifies our analysis significantly, but also makes sure that any potential welfare improvement due to the presence of the adviser does not simply come from any exogenously assumed cost advantage of the adviser.

Finally we describe the nature of the fees that are charged by the active manager and the investment adviser. We assume that the adviser charges a proportional fee, \( f_A \), based on the end-of-period value of all assets under management, including both assets invested with the passive fund and assets invested through the active manager. The portfolio manager also charges a proportional fee, \( f_P \), on the assets managed actively.

The investors therefore have to decide amongst the three investing strategies. If they invest in the passive index, they incur the transaction fee. If they invest directly with the portfolio manager, they pay their fixed information collection fee as well as the portfolio management fee. If they delegate their decision to the adviser, they can take advantage of his expertise in making the optimal asset allocation decision, but they have to pay two management fees for the actively invested portion of their holdings: both that of the adviser and that of the portfolio manager.

The active manager can receive funds directly from the investors (the direct channel) or indirectly through the investment adviser (the indirect channel).
2.2 Investors Behavior

Assume that each investor has wealth $x + C_0$, and $x$ follows a Pareto distribution with the following probability density function:

$$f(x) = \frac{kA_m^k}{x^{k+1}}, \quad k > 1,$$

(3)

where $A_m > 0$ denotes the minimum wealth level (net of the searching cost $C_0$)\(^9\). The Pareto distribution, named after the Italian economist Wilfredo Pareto, has been widely used to describe the distribution of wealth among individuals. Empirical studies have found that this distribution characterizes the real-world wealth distribution fairly well, except for an obvious failure at the lower tail of the distribution\(^10\).

An important feature of this distribution is that the probability density $f(x)$ decreases monotonically in wealth, implying that the fraction of wealthy investors is relatively small while the fraction of investors with low levels of wealth is relatively large. The parameter $k$ characterizes how equally wealth is distributed. Equality of wealth is characterized by $k \to \infty$, while $k = 1$ corresponds to complete inequality\(^11\). Pareto’s original estimate of $k$ based on income data clustered around 1.5. Later estimates of this parameter ranged from 1.6 to 2.4 for income distributions (Champernowne (1952)) and from 1.3 to 2.0 for wealth distributions (Yntema (1933)).

We standardize the population to be 1. Therefore the total wealth available for investment is\(^12\)

$$W = \int_{A_m}^{\infty} x f(x)dx + C_0 = \frac{kA_m}{k-1} + C_0, \quad k > 1.$$  

(4)

Based on their wealth level, investors can choose whether to invest directly or indirectly. Let $A_D$ denote the amount of direct investment by the investors and let $A_I$ represent the amount of indirect investment to the active fund through the investment adviser. Therefore the total amount of money under active management is $A = A_D + A_I$ and the rate of return of the portfolio manager is $R_p = \alpha - \gamma(A_D + A_I)$.

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9Our general conclusion does not depend on this specific assumption about the wealth distribution. In previous versions, we have utilized several different assumptions about the wealth distribution; our results are qualitatively similar.

10See Persky (1992) for a brief review of this literature.

11The Gini coefficient, a widely used measure of inequality of wealth distribution, for the Pareto distribution is given by $\frac{1}{2k-1}$.

12The mean of a Pareto distribution is infinite when $k \leq 1$. 
Investors will invest either directly or indirectly in the actively managed portfolio if the expected return is not less than the reservation rate of return $R_f - \tau$. Suppose that the expected return of indirect investing is equal to the reservation rate, $R_f - \tau$, which will indeed be the case given our assumption on the advisory cost and the competitive nature of the advisory service industry. Then an investor with wealth $A^* + C_0$ will be indifferent between contracting directly with the portfolio manager and getting a net rate of return $R_p(1 - f_p)$ and investing via the adviser, where $A^*$ satisfies the following condition.

$$[\alpha - \gamma(A_D + A_I)](1 - f_p)A^* = (R_f - \tau)(A^* + C_0),$$

i.e.,

$$A^* = \frac{C_0(R_f - \tau)}{(1 - f_p)[\alpha - \gamma(A_D + A_I)] - (R_f - \tau)}.$$  \hspace{1cm} (5)

Note that this relation assumes that investors behave in an ‘atomistic’ fashion; they do not take into account the diseconomy of scale for the active portfolio manager when they decide where to channel their funds.

It is obvious that all investors whose wealth is smaller than $A^* + C_0$ will prefer to invest via the adviser rather than contract directly with the portfolio manager. Those with wealth greater than $A^* + C_0$ will prefer to contract directly with the portfolio manager. We therefore refer to this latter set of investors as ‘high networth individuals’. Indeed there is evidence to this effect. In [CI 2004] it is documented that customers who purchase funds through a brokered mutual funds sales force channel as opposed to the direct channel are less wealthy (with median income of $93,800 v. $101,300). They also have lower median financial assets ($363,700 v. $447,900).

Given that all investors with wealth levels greater than $A^* + C_0$ invest directly, we can solve for the amount of money channeled directly to the portfolio manager (net of the searching cost):

$$A_D = \int_{A^*}^{\infty} x f(x) dx = \frac{kA_m^k}{(k - 1)(A^*)^{k-1}}.$$  \hspace{1cm} (6)

Note that if $A^* = A_m$, then $A_D = W - C_0$, and all investors would contract with the portfolio manager.
directly. To make our analysis interesting, we assume that

$$\tau < \frac{C_0(R_f - \tau)}{A_m}.$$ 

This implies that there are always some investors preferring the indirect channel if the return of the active portfolio after the management fee equals $R_f$.

We also assume that if all wealth net of the searching cost $C_0$ is invested in the active portfolio, the return of the active portfolio will be lower than the reservation rate of the investors, i.e.,

$$\alpha - \gamma(W - C_0) < R_f - \tau.$$ 

Combining the two conditions above, we have

$$\tau < \min(R_f - \alpha + \gamma(W - C_0), \frac{C_0(R_f - \tau)}{A_m}).$$  \hspace{1cm} (7)$$

This upper bound on the transaction costs is maintained throughout our analysis.

3 Investment Management Equilibrium

We now solve for the equilibrium levels of investment management. First we discuss the behavior of the adviser and subsequently the behavior of the portfolio manager. In this section, we assume that there are no indirect ‘kickbacks’ to the adviser so that his decision-making is unbiased and only evaluates the respective asset returns. In the next section, we allow for influence activities between the portfolio manager and the advisers.

3.1 Advisers Behavior

Our model features competitive behavior on the part of investment advisers. It is a critical assumption that they behave atomistically and do not consider the impact on the rate of return on the active portfolio in making their asset allocation decisions. Suppose that a representative fund adviser charges a proportional advisory fee $f_A$, based on the total end-of period asset value on his clients’ accounts. He
can either put the client’s money in the passive portfolio or in the portfolio actively managed by the portfolio manager. Let $w$ represent the portfolio weight on the active asset and $1 - w$ the portfolio weight for the passive asset. Then the investment adviser will solve the following problem:

$$\max_w \quad w f_A [\alpha - \gamma (A_D + A_I)] (1 - f_P) + (1 - w) f_A R_f. \quad (8)$$

Obviously the proportional fee, $f_A$, charged by the adviser will not affect his asset allocation. If the following condition representing equality of the respective rates of return is not satisfied, the adviser would put all of the funds in either the active or passive portfolio:

$$[\alpha - \gamma (A_D + A_I)] (1 - f_P) = R_f. \quad (9)$$

Notice that the adviser behaves in a ‘returns-taking’ manner, i.e., he takes the return $R_p = \alpha - \gamma (A_D + A_I)$ as given. It cannot be an equilibrium consistent with our basic assumptions if equation (9) is not satisfied. In particular if the active portfolio had a higher net return than the passive, then all funds in the economy would be invested in active management, which by assumption would depreciate the rate of return below the reservation value of the investors. On the other hand, the passive portfolio cannot have a higher expected return than the active in equilibrium because the active portfolio always has a higher marginal return to the first dollar of investment.

Since the adviser operates in a competitive environment, competition will drive down the advisory fee, $f_A$, to the marginal cost $c_A = \tau / R_f$. Note that this advisory fee satisfies the participation constraint of the indirect investors: indirect investors earn a return of $R_f$ before the advisory fee since both the active portfolio and the passive fund have a return of $R_f$; as a result, the net return after the advisory fee, $(1 - \tau / R_f) R_f$, is equal to investors’ reservation return $R_f - \tau$. The adviser just breaks even, which must be the case in a competitive market.

Substituting equation (9) into equation (5), we have

$$A^* = \frac{C_0 (R_f - \tau)}{\tau}. \quad (10)$$
Notice importantly that this threshold level of wealth does not depend on $\alpha$. Therefore the investors can optimally decide whether or not to collect information in the equilibrium without knowing the portfolio managers potential ability, $\alpha$.

Substituting equation (10) into equation (6) then gives the equilibrium amount of money invested directly:

$$A_D = \frac{kA_m^k \tau^{k-1}}{(k-1)(C_{0}(R_f - \tau))^{k-1}}.$$ (11)

The most important property of this relationship is that the amount of money invested directly is independent of the fees of the portfolio manager. This is because of the competitive nature of the adviser. If the active manager attempts to increase her fees, for the same gross rate of return, the adviser reduces his asset allocation to active investing. Therefore the same net rate of return is achieved by active investing and thus there is no effect on the marginal direct investor or the aggregate amount of money invested directly. This property is critical to understanding the model and will be exploited below.

### 3.2 Portfolio Manager Behavior

Now we describe the problem faced by the portfolio manager. From equation (9) we know that total funds under active management satisfy

$$A_D + A_I = (\alpha - \frac{R_f}{1-f_P}) \frac{1}{\gamma}.$$ (12)

That is, funds under management are related to the ‘grossed-up’ difference between the passive and active rate of return and scaled by the depreciation factor, $\gamma$. The manager now takes this into account and solves the following fee maximization problem:

$$\max_{f_P} [\alpha - \gamma(A_D + A_I)](A_D + A_I)f_P$$ (13)

subject to equation (12).
Substituting the constraint into the objective function, the manager’s problem is

$$\max_{f_P} \left[ \frac{R_f}{1 - f_P} \right] \left[ \alpha - \frac{R_f}{1 - f_P} \right] \frac{1}{\gamma} f_P.$$ 

It is easy to solve this for the optimal portfolio manager fees, which we record as a proposition.

**Proposition 1.** The optimal portfolio manager fee in the investment management equilibrium without influence is

$$f_P^* = \frac{\alpha - R_f}{\alpha + R_f}. \quad (14)$$

Now using this result, we can determine how the fee is impacted by $\alpha$. Taking the partial derivative of equation (14),

$$\frac{\partial f_P}{\partial \alpha} = \frac{2R_f}{(\alpha + R_f)^2} > 0.$$ 

Hence sensibly the fees are increasing in the managerial ability. Further we can solve for the active managers profit:

$$\Pi_P = \frac{(\alpha - R_f)^2}{4\gamma}, \quad (15)$$

and the total assets allocated to the manager:

$$A_I + A_D = \frac{\alpha - R_f}{2\gamma}. \quad (16)$$

Substituting equation (11) into equation (16), we have

$$A_I = \frac{\alpha - R_f}{2\gamma} - \frac{k A_m^{k-1}}{(k-1)(C_0(R_f - \tau))^{k-1}}. \quad (17)$$

We assume that $C_0$ is big enough, or $\gamma$ is small enough, to ensure that $A_I$ in (17) is positive, i.e., not all
investment into the active portfolio comes from the direct channel.\textsuperscript{13}

In summary the investment management equilibrium features positive profits of the portfolio manager and zero profit of the investment adviser. Returns to the active manager net of management fees are only equal to those of the passive fund. Rates of return earned by direct investors in the active fund exceeds that of indirect investors. Nevertheless only the high net worth individuals find it optimal to invest directly.

4 Influence Activities

We now extend the model to consider the important aspect of rebates or 'kickbacks' from the portfolio manager to the investment adviser. The idea here is that the portfolio manager desires to influence the decisions of the investment adviser. The purpose of this part of our model is to evaluate what happens in equilibrium and to see what the impact is on the investors and their investment performance.

Rebates associated with influence activity can take various forms, either explicit or implicit. The most common type of a rebate involves the load charge of a mutual fund. In this case the fund charges a sales fee and then rebates a large part back to the sales agent (broker). In practice actual loads are based on a declining scale based on actual investment in a fund. There are front end loads, back end loads and 12b-1 fees, which are incurred on a continuing basis.\textsuperscript{14} Bergstresser et al.\textsuperscript{[2006]} document the nature of these charges for different investment channels, and document that the charges are much higher for brokered sales as compared to direct sales. Christoffersen et al.\textsuperscript{[2005]} document the average loads paid when investing through two types of brokers: 'captive' and 'unaffiliated'.\textsuperscript{14} In addition their data allows them to identify the amount of these loads rebated. They find that a much higher percentage of the loads are rebated to unaffiliated brokers as opposed to captive brokers. In addition to these explicit payments there are 'soft dollar commitments' and other implicit subsidies as mentioned earlier.

\textsuperscript{13}If this condition would not hold, the adviser is not utilized at all. This analysis appears in section 5.
\textsuperscript{14}Captive brokers are defined as those that invest with only a single fund family, while unaffiliated brokers sell to multiple fund families.
4.1 Impact on Fund flows

We choose to model the behavior of such influence activity in the following way. The rebate or kickback is specified ex ante and paid at the end of the period by the portfolio manager to the adviser in the form of a lump sum transfer of monetary value, $K_A$. In return for this transfer committed by the portfolio manager, the adviser biases his investment decision in favor of the portfolio manager. We treat the commitment of the manager, whether explicit or implicit, as enforceable between the manager and adviser. The bias in the asset allocation decision is handled in the following way. Suppose that the expected (risk-adjusted) return of the portfolio manager is $R_p$ and the passive index returns $R_f$. Then, the adviser may still invest in the active fund as long as $R_p (1 - f_P) + \delta \geq R_f$. That is, $\delta$ represents the amount of bias in the form of a ‘ghost’ return that is effectively added to the active return when assets are allocated by the adviser. Of course this does not translate into any realized return at all on the part of the investors.

With the introduction of the bias, the equilibrium condition that will be satisfied by the adviser’s asset allocation decision will be

$$[\alpha - \gamma (A_D + A_I)](1 - f_P) = R_f - \delta. \quad (18)$$

This expression can be rewritten to provide the fund size as

$$A_D + A_I = (\alpha - \frac{R_f - \delta}{1 - f_P}) \frac{1}{\gamma}. \quad (19)$$

As in the case without the bias, we can substitute equation (18) into (5) and we find that the threshold wealth level becomes

$$A^* = \frac{C_0 (R_f - \tau)}{\tau - \delta}, \quad (20)$$

and the amount invested directly becomes

$$A_D = \frac{k A_m^k (\tau - \delta)^{k-1}}{(k-1)(C_0(R_f - \tau))^{k-1}}. \quad (21)$$
We see first by comparing respectively equations \((20)\) with \((10)\) and \((21)\) with \((11)\) that the impact of the bias increases the wealth level of the marginal investor who invests directly. Consequently the amount of funds invested directly decreases due to the bias. These impacts continue up to the point where the direct investment drops to zero, which occurs when the adviser bias converges to \(\tau\). Hence a key result is that the kickbacks shift investors from the direct investment channel to the indirect investment channel. This occurs because with biased investment advisers, the diseconomy of scale results in less attractive net returns. As investors are shifted into the indirect channel, we will see that they suffer a welfare loss because they are now pushed down to their reservation utility. As before, though, the portfolio managers fee is constrained by the asset allocation decision of the adviser and the amount of direct investment in the active portfolio is independent of the fee in equilibrium.

### 4.2 Kickback Required by the Adviser

To see what consideration must be provided, we first consider the equilibrium advisory fee in the presence of kickbacks. Since the money allocated to the passive fund earns \(R_f\), while the money allocated to the active portfolio earns only \(R_f - \delta\), an advisory fee equal to the marginal cost \(c_A = \tau / R_f\) will lead to a net return of indirect investing lower than \(R_f - \tau\). Therefore such a fee cannot satisfy investors’ participation constraint any longer.

The equilibrium advisory fee that satisfies investors’ participation constraint must be determined by setting the aggregate net return to investing through the adviser to that of investing in the passive fund:

\[
(1 - f_A) [A_I (R_f - \delta) + (W - \theta C_0 - A_D - A_I) R_f] = (W - \theta C_0 - A_D) (R_f - \tau),
\]

where \(\theta \equiv \int_{A^*}^{+\infty} f(x) dx = (A_m / A^*)^k\) denotes the fraction of investors choosing the direct channel, \(W - \theta C_0 - A_D\) represents the total amount of money handled by the adviser, of which \(A_I\) is allocated in the active fund and the rest is allocated to the passive asset.\(^{15}\)

\(^{15}\)The total wealth used up in searching by the direct investors. While we assume all wealth that is neither invested directly in the active portfolio nor spent in searching is delegated to the adviser, our result holds as long as the total wealth intermediated by the adviser is larger than \(A_I\) in equilibrium.
Solving for $f_A$ gives the following fee relationship:

$$f_A = \frac{A_I \delta - (W - \theta C_0 - A_D) \tau}{A_I \delta - (W - \theta C_0 - A_D) R_f}. \quad (23)$$

It can be seen that for fixed $A_I$, as $\delta$ increases, the advisory fee is reduced.

From equation (22), we can also see that the adviser's total fee income is $(W - \theta C_0 - A_D) \tau - A_I \delta$. Since $(W - \theta C_0 - A_D) \tau$ is just enough to cover the advisory cost, the adviser has a net loss of $A_I \delta$.

Therefore he is effectively bearing all the cost of distortions in his asset allocation. It follows that in order to be fully compensated for administering such distortions, the adviser must get a kickback, $K_A$, such that

$$K_A = A_I \delta. \quad (24)$$

Another way to understand the amount of kickback required by the adviser is as follows. For any positive $\delta$, an individual adviser can always increase his portfolio return by shifting $A_I$ from the active portfolio to the passive. The gain from such a deviation, on the condition that other advisers do not deviate, is $A_I \delta$. Since indirect investors always get their reservation rate of return, all the benefits from such a deviation are captured by the adviser. Therefore, to prevent the adviser from such a deviation, the portfolio manager must provide a kickback equal to $A_I \delta$.

### 4.3 Optimal Influence Activity

We now endogenize the amount of influence activity by allowing the portfolio manager to choose her optimal level. In this respect she must decide on the amount of the compensation that must be rebated to the adviser in order to convince him to adopt the degree of distortion requested by her.

The portfolio manager will try to maximize her profit net of the kickback payments. Therefore, her problem is

$$\max_{f_p, \delta} \Pi_p = [\alpha - \gamma(A_D + A_I)](A_D + A_I) f_p - A_I \delta$$

Note that the adviser incurs a cost of $(W - \theta C_0 - A_D) \tau / R_f$ at the beginning of the period. This is equivalent to a cost of $(W - \theta C_0 - A_D) \tau$ at the end of the period.
subject to the constraints \([18]\) and \([21]\). We can now solve for the optimal portfolio manager fees, kick-
back payments and bias imparted to the investment adviser.

**Proposition 2.** When influence activity is feasible, the optimal fee charged by the portfolio manager is

\[ f^*_P = \frac{\alpha - R_f + 2\tau/k}{\alpha + R_f}. \]  

(25)

The optimal amount of bias introduced through the portfolio managers kickback to the investment adviser
is

\[ \delta^*_P = \frac{\tau}{k}. \]  

(26)

**Proof.** Substituting the constraints into the objective function, we see that the first order condition for
the optimal fee is

\[ f_P = \frac{\alpha + 2\delta - R_f}{\alpha + R_f}. \]

Substituting this expression back into the objective function, we get

\[ \Pi_P = \frac{(\alpha - R_f)^2}{4\gamma} + A_P \delta. \]

Therefore we have

\[ \frac{\partial \Pi_P}{\partial \delta} = \frac{kA^k_m(\tau - \delta)^{k-2}}{(k-1)(C_0(R_f - \tau))^{k-1}(\tau - k\delta)}. \]

Setting this expression equal to zero implies that the first order condition for the optimal bias is

\[ \delta = \frac{\tau}{k}. \]

The second order conditions can be verified in a straightforward fashion. \(\square\)

\(^{17}\)If \(k > 2\), \(\delta = \tau\) also satisfies the first order condition. However, in this case, \(A_P = 0\), the portfolio manager’s profit is not
maximized. Therefore \(\delta = \tau\) is not optimal.
From the portfolio manager’s point of view, the distortion $\delta$, which leads to a lower return of the active portfolio, has two competing effects on her profit. First, it helps her to extract the surplus from the direct investors. After introducing the distortion, both the amount of investment through the direct channel and the surplus of the remaining direct investors become lower. Second, as we have shown, due to rational expectations and the participation constraint of the indirect investors, the amount of kickbacks that needs to be paid by the portfolio manager is increasing in the level of distortion and the amount of indirect investment. Clearly, the first effect generates an incentive to increase $\delta$, while the second effect generates the opposite. As $\delta$ becomes larger, the amount of investment remaining in the direct channel decreases, resulting in a lower marginal benefit of increasing $\delta$. At the same time, the marginal cost increases since the amount of indirect investment is larger as $\delta$ becomes larger. This leads to an interior optimal $\delta$, which maximizes the profit of the portfolio manager.

From equation (26), we can easily see that the optimal bias is increasing in the transaction cost $\tau$ and decreases in $k$, which measures the degree of equality of the wealth distribution. The transaction cost $\tau$ represents the maximum bias that can be induced before all investors leave the direct channel. Therefore, not surprisingly, the optimal bias is increasing in $\tau$.

The relation between the optimal bias and $k$ is also intuitively appealing. When $k$ is large, there are fewer high networth investors, therefore, the portfolio manager does not extract much surplus by inducing a bias. By contrast, when $k$ is close to 1, the fraction of high networth investors is relatively large. As a result, the portfolio manager has a stronger incentive to shift them to the indirect channel in order to extract their surplus.

To further analyze the impact of these sort of actions, we compute the equilibrium size of the fund. Given that the fee is determined by (25), from equation (19) the fund size in the equilibrium with the kickback is

$$A_D + A_I = \frac{a - R_f}{2\gamma}.$$  

Note that the fund size is exactly the same as the fund size in the investment management equilibrium without the kickback, equation (12). This implies that the gross return of the active fund is the same as when the portfolio manager is precluded from influencing the adviser. However the net return after
management fee is lower than in the case without influence, since \( f_P \) is greater.

Influence activity, therefore, allows the portfolio manager to effectively force the investors to go through the adviser and allows her to extract the surplus of the large investors. Indeed, substituting the optimal \( \delta^* \) in equation (26) into (21), we see that the active investment through the direct channel in the equilibrium with influence activity is

\[
A_D = \frac{k A_0^k (\tau - \tau/k)^{k-1}}{(k-1)(C_0(R_f - \tau))^{k-1}},
\]

(27)

which is strictly smaller than \( A_D \) without influence activity as embodied in equation (11).

We record the observations from this section in the form of a proposition.

**Proposition 3.** In the investment management equilibrium where advisers are subject to influence activity, the active fund size is the same as when advisers are unbiased, however, more investors utilize advisory services by investing indirectly and the net return of the active fund is lower. Portfolio managers charge a higher fee, while advisers charge a lower fee but receive a compensatory kickback from the portfolio manager.

A striking result of our analysis is that when advisers are more subject to influence, they are actually used to a greater extent in equilibrium than when they are not biased. The reason is that the portfolio manager optimally raises her fees, which makes direct investment less attractive. The adviser’s asset management business becomes larger, and he is forced to lower his fees in order to be competitive with the alternative asset. The adviser still breaks even in equilibrium. These results are in accord with the fact that ‘fee-only’ advisers advertise their independence and lack of influence, while brokers often receive their entire compensation from load rebates, rather than assets under management.

5 **Equilibrium Without Advisers**

In order to investigate the role of investment advisers in delegated portfolio management, we now examine an equilibrium in which investment advisers do not exist and then compare the outcomes with the two equilibria we have already analyzed. When there is no investment adviser, the only vehicle for
active investing is directly through the portfolio manager. As a result, \( A_f = 0 \), and the fund size is determined solely by \( A_D \). The portfolio manager maximizes her profit by choosing an optimal fee and fund size. Therefore, the manager’s problem can be written as:

\[
\max_{A_D, f_P} \Pi_P = (\alpha - \gamma A_D) A_D f_P \tag{28}
\]

subject to

\[
A_D = \frac{k A_m^k}{(k - 1)(A^*)^{k-1}}, \tag{29}
\]

\[(\alpha - \gamma A_D)(1 - f_P) A^* = (R_f - \tau)(A^* + C_0), \tag{29}
\]

where the second constraint ensures that the marginal investor is indifferent between investing with the portfolio manager and the passive fund. We can see from equation (29) that there is an inverse relation between fund size and fees charged by the portfolio manager. Therefore either variable can be optimized by the manager equivalently.

Note that the informational assumptions made here are somewhat stronger than those needed previously with the investment adviser. Recall previously that the decision whether to invest directly did not depend in equilibrium on the potential value of active management, \( \alpha \). Now we must assume that the direct investor knows in advance exactly what \( \alpha \) will obtain, after the cost, \( C_0 \), is expended. This, of course does not violate our earlier justification for the cost, since without paying it, there would be an adverse selection problem borne by the investor.

Problem (28) is now solved in the next proposition.

**Proposition 4.** In the portfolio management equilibrium without advisers, there exists a unique interior optimal fund size and management fee, which are the solutions to the following set of equations:

\[
\alpha - R_f + \tau - 2\gamma A_D - \frac{\lambda k}{k-1} A_D^{1/(k-1)} = 0, \tag{30}
\]
and

\[ f_p = \frac{\alpha - \gamma A_D - (R_f - \tau + \lambda A_D^{1/(k-1)})}{\alpha - \gamma A_D}, \]  

(31)

where

\[ \lambda \equiv C_0 (R_f - \tau) \left( \frac{k - 1}{k A_m^k} \right)^{1/(k-1)}. \]  

(32)

Proof. See Appendix A.1.

A general analytical solution to the equation system in Proposition 4 for an arbitrary \( k \) is not available. Appendix A.2 provides the solutions for two special values of \( k \) which are close to the empirically observed values: \( k = 1.5 \) and \( k = 2 \).

6 Comparison of Equilibria

In this section we compare outcomes in the three scenarios: no advisers, unbiased advisers, and advisers subject to influence activity. In particular we address whose interests investment advisers really serve: the investors or the portfolio manager. The key question is how investors are impacted by the presence or lack thereof of the adviser. We also address the consequence of influence activity on aggregate social welfare.

6.1 Outcomes

The comparison between the equilibria with unbiased and biased adviser is relatively easy to make. However, it is less obvious how equilibrium outcomes change when there is no adviser. To illustrate the differences, we construct a numerical example and solve for the equilibria in the three cases: (1) an unbiased investment adviser; (2) an investment adviser with influence; and (3) no investment adviser. The outcomes are illustrated as a function of \( k \) in figure 1.

We plot the fund size in the three equilibria in figure 1a. Here the solid line represents the fund sizes in both the biased- and unbiased-adviser equilibria since they are the same. As we can see from
This figure compares the outcomes of three different equilibria. Solid lines correspond to the equilibrium with unbiased advisers (in panel (b), it also corresponds to the equilibrium with biased advisers), dashed lines correspond to the equilibrium with biased advisers, while dotted lines correspond to the equilibrium without advisers. The values of parameters other than $k$ are as follows: $R_f = 1.05, \alpha = 1.1, \gamma = 10^{-9}, A_m = 5 \times 10^7, C_0 = 5 \times 10^7, \tau = 0.02$. 

Figure 1: Comparison between equilibria
the graph, the size of the active portfolio is substantially larger in the presence of investment advisers, especially when \( k \) is large, i.e., when the fraction of high networth investors in the economy is not so large. This is because investment advisers help to bring some small investors into the active portfolio.

Figure 1b compares the amount of investment through the direct channel. In the case without advisers, this is equivalent to the total fund size. While the fund size is important for the portfolio manager’s profit, the amount of direct-channel investment is critical for the investor welfare since only the direct investors get surplus. Not surprisingly, the direct-channel investment is smallest in the presence of biased advisers because many investors are shifted to the indirect channel via the kickback. Compared to the equilibrium with unbiased advisers, the equilibrium without advisers features a smaller amount of direct investment when \( k \) is small and a larger amount of direct investment when \( k \) is large. This is because when there are many high networth investors, the monopolist portfolio manager will find it optimal to increase the fee such that the fund size does not increase too substantially. Similarly, when there are fewer high networth investors, the monopolist portfolio manager will reduce her fee more significantly such that the fund size does not decrease too much.

Figure 1c illustrates the portfolio managers fees as a function of \( k \). In the case with unbiased advisers, the fee is again constant and is lowest among all three equilibria. In the other two cases, as the fraction of high networth individuals increases, fees are able to be increased. But it increases more if there is no adviser than if there are biased advisers. For most reasonable values of \( k \), fees are higher in the case without advisers than with advisers subject to influence. However, the opposite is true when \( k \) is large. This is exactly the situation in which high networth investors are rare and the manager wants to mitigate the shrinkage of fund size by reducing the fee.

Figure 1d compares the return of the active portfolio after the management fee. In the presence of unbiased advisers, this return always equals \( R_f \). It is lower in equilibrium with kickbacks than in the other two cases, indicating that investors are generally hurt by the presence of biased advisers. The effect of the kickback is stronger when \( k \) is smaller. The portfolio return in the absence of investment advisers is higher than \( R_f \) when \( k \) is large while lower than \( R_f \) when \( k \) is small, indicating that high networth investors may benefit from the presence of unbiased investment advisers when \( k \) is sufficiently small, but that they may suffer from it when \( k \) is very large.
6.2 Welfare Analysis

We now analyze how the aggregate social welfare is affected by the presence of advisers, as well as by influence activity. Since investment advisers, when in existence, as well as the indirect investors, always have zero surplus, we can define the social welfare as the sum of the portfolio manager’s profit and the surplus of the direct investors, where investor surplus is defined relative to the default of paying not paying the search cost, investing in the passive asset and earning net return $R_f - \tau$.

The effect of influence activity on investor welfare has been described in section 4. Recall that influence activity shifts some investors from the direct channel to the indirect channel. Investors who are forced to switch to the indirect channel lose their surplus, while those who remain in the direct channel get a lower net returns as the portfolio manager raises her fee. The total effect on investor surplus is therefore unambiguously negative.

Not surprisingly, the portfolio manager’s profit changes in an opposite direction. Substituting the optimal fee $f^*_P$ and the optimal bias $\delta^*$ in Proposition 2 into the objective function of the portfolio managers, we get the portfolio manager’s profit in equilibrium with influence activity as

$$
\Pi_P = \frac{(\alpha - R_f + 2\tau/k)(\alpha - R_f)}{4\gamma} - \left(\frac{\alpha - R_f}{2\gamma} - A_D\right)\frac{\tau}{k}
$$

$$
= \frac{(\alpha - R_f)^2}{4\gamma} + \frac{A_D\tau}{k},
$$

where $A_D$ is given by equation 27. One can see that compared to profit in the zero bias equilibrium (equation 15), the portfolio manager’s profit increases by $A_D\tau/k$.

Combining the profit for the portfolio manager with the surplus earned by the investors we compute the social welfare in the three scenarios. The details of these computations are carried out in appendix A.3. We are able to prove the following proposition:

**Proposition 5.** The levels of social welfare in equilibria with no advisers, $U^0$, unbiased advisers, $U^1$, and
biased advisers, $U^2$, are given respectively by

$$U^0 = (\alpha - \gamma A^0_D - R_f + \tau)A^0_D - \theta^0 C_0(R_f - \tau),$$  \hspace{1cm} (34)\]

$$U^1 = A^1_D \tau - \theta^1 C_0(R_f - \tau) + \Pi^1_p,$$  \hspace{1cm} (35)\]

$$U^2 = A^2_D (\tau - \delta) - \theta^2 C_0(R_f - \tau) + \Pi^2_p,$$  \hspace{1cm} (36)\]

where $\theta^i \equiv \int_{A^i}^{+\infty} f(x)dx$ denotes the fraction of investors choosing the direct channel, $i = 0$ (no adviser), $i = 1$ (unbiased adviser) and $i = 2$ (biased adviser). Furthermore, we have

$$U^1 - U^2 = (A^1_D - A^2_D) \tau - (\theta^1 - \theta^2) C_0(R_f - \tau) > 0,$$  \hspace{1cm} (37)\]

i.e., the social welfare is strictly higher in equilibrium with unbiased advisers than in equilibrium with biased advisers.

Proof. See Appendix A.3 \hfill \Box

Figure 2a plots the social welfare in the three equilibria under the same parameter values used to plot figure 1. Consistent with Proposition 5, the social welfare in the unbiased-adviser equilibrium (the solid line) is always higher than in the equilibrium with kickbacks (the dashed line). More interestingly, the figure also shows that both of these equilibria dominate the no-adviser equilibrium: the social welfare in the equilibrium without advisers (the dotted line) is lower than in the other two equilibria for all different values of $k$ we consider.

Figure 2b compares the profits of the portfolio manager in three equilibria for various values of $k$. With the presence of unbiased investment advisers, the profit of the portfolio manager is independent of the wealth distribution parameter $k$, therefore it is a horizontal line in the diagram. The portfolio manager is strictly better-off in the equilibrium with kickbacks. She benefits more from the advisers’ biased allocation when $k$ is smaller. This is because she extracts more rents from the high networth investors when there are many such investors in the economy.

For most reasonable values of $k$, the portfolio manager also benefits from the presence of investment advisers, even when kickbacks are forbidden. This is not surprising because the existence of investment
This figure compares welfare in three different equilibria. Solid lines correspond to the equilibrium with unbiased advisers, dashed lines correspond to the equilibrium with biased advisers, while dotted lines correspond to the equilibrium without advisers. The values of parameters other than $k$ are as follows: $R_f = 1.05, \alpha = 1.1, \gamma = 10^{-9}, A_m = 5 \times 10^7, C_0 = 5 \times 10^7, \tau = 0.02$. 

Figure 2: Welfare comparison across equilibria
advisers allows the portfolio manager to provide her services to small investors, who will otherwise not participate in the active portfolio. Interestingly, this is not always the case. When $k$ is small, the portfolio manager's profit is higher in the no-adviser equilibrium than in the equilibrium with unbiased advisers, indicating that when there are many wealthy investors, the portfolio manager may not mind giving up all small investors. Instead of holding the after-fee return of her portfolio to equal $R_f$ and attracting investment from both direct and indirect channels, she extracts more rents by running a small and high management fee fund that caters only to large investors. Since the after-fee return of her portfolio will then be below $R_f$, no unbiased advisers will allocate assets into it. However, the portfolio manager's profit is still higher due to higher management fee ratio.

Figure 2c plots the investor surplus in different equilibria (see Appendix A.3 for the exact expressions of investor surplus.) Consistent with our analytical results, investor surplus is higher when advisers are unbiased than when they are biased. The figure also shows that from the investors' point of view, biased advisers are always worse than no advisers at all. More interestingly, when $k$ is large, even unbiased advisers can reduce the investor welfare. Only when $k$ is small, investors are better off with unbiased advisers than without advisers. This is consistent with the shape of net return of the active portfolio plotted in figure 1d.

In summary, our analysis shows that investment advisers, even if they can be influenced by the portfolio manager, improve social welfare. However, although investment advisers claim to serve investors, their presence mainly benefits the portfolio manager. Even when investment advisers are unbiased, they improve investor welfare only if the fraction of high-networth investors in the economy is large. If they can be influenced by the portfolio manager, then investors are strictly worse off with their presence.

7 Conclusions

The market for financial products and services is expanding at an enormous pace as corporations and financial institutions package cash flows and contingent claims in ever more creative ways. As the myriad of alternatives placed before investors reaches such incredible proportions the role of an important intermediary, the investment adviser in allocating assets becomes critical. Investment advisory services are employed by many types and categories of investors. For instance, the fastest growing segment of mutual
funds inflows comes through brokers, not through direct investment (with the exception of Vanguards Index 500 fund). Investment advisers are employed by large corporate pension funds and university endowment committees. As the hedge fund industry has skyrocketed funds of hedge funds have gained popularity, as well. The purpose of this paper has been to look at the role such financial intermediaries play in the overall structure for investment management services.

In our model investment advisers facilitate the participation of small investors in an actively managed portfolio. The impact on investors’ welfare depends on the wealth distribution. If the economy is largely made up of small investors, then unbiased advisers lower welfare of large investors since investment advisors provide small investors with the access to the expertise of portfolio manager and since there is a diseconomy of scale in active portfolio management. By contrast, if the fraction of wealthy investors is relatively large, then investors benefit from the presence of unbiased advisers. The portfolio managers’ welfare is impacted in the opposite manner. Considering aggregate social welfare, we find that investment advisers benefit society, although most of the benefits are extracted by portfolio management firms.

We have shown that portfolio managers wish to expend resources to influence the advisers’ decision making. The result of these activities will unambiguously lower the aggregate welfare of investors, although the effects are concentrated at investors with higher wealth levels. We show that payments to influence the advisers allow the portfolio managers to charge higher fees. However, fund size is unaffected. As a result, the gross return of actively managed portfolios is unaffected, but the net return decreases. Thus, there is a positive relationship between the management fee and influence activities. Most importantly influence actually increases the magnitude of indirect investing. Influence activities are also positively related to the cost of investing in the passive portfolio.

The presence of investment advisers also creates an externality for investors who do not use their services since they ensure competitive returns to the actively managed portfolio. As a result, the amount of direct investment is independent of fund characteristics. Therefore, fund characteristics such as managerial skill influences the quantity sold through the advisory services. For example, the importance of indirect sales through investment advisers increases with the skill of the active portfolio manager. Ceteris paribus, large funds are sold primarily indirectly whereas small funds feature proportionally more
While our results point out that the adviser mainly serves the portfolio manager and not the investors, we are reluctant to take the position that there should be any regulation of investment advisory services other than to enhance transparency and disclosure of influence activities. With such disclosures investors should be able to select appropriate individuals to act on their behalf. Further even though investors are worse off with biased advisers, the portfolio manager and society is better off compared to not having investment advisers. It is worth keeping in mind that in the real world, the value of active managers is endogenous. If their potential profit is curtailed by regulations, they are less likely to make the significant personal and institutional investment necessary to attain these levels of expertise.

A Appendix

A.1 Proof of Proposition 4

Proof. Combining the two constraints in problem (28), we immediately obtain equation (31). Substituting this expression back into the objective function and differentiating, we have

\[
\frac{\partial \Pi_P}{\partial A_D} = (\alpha - R_f + \tau) - 2\gamma A_D - \frac{\lambda k}{k-1} A_D^{1/(k-1)}
\]

\[
\frac{\partial^2 \Pi_P}{\partial A_D^2} = -2\gamma - \frac{\lambda k}{(k-1)^2} A_D^{(2-k)/(k-1)} < 0.
\]

Equation (30) is given by setting the first order condition above equal to zero. Since \(\Pi_P\) is strictly concave when \(A_D > 0\), the first order condition is both necessary and sufficient condition for the solution to this maximization problem; furthermore, the optimal \(A_D\) is unique. To prove the existence of an interior solution, \(0 < A_D < W - C_0\), to the first order condition, note that \(\frac{\partial \Pi_P}{\partial A_D} > 0\) if \(A_D = 0\). Due to the monotonicity of the first derivative, it suffices to show this derivative becomes negative as \(A_D \to W - C_0\), i.e., as \(A_D\) converges to the aggregate wealth of the economy net of the searching cost \(C_0\). This is guaranteed by the boundary condition (7), which implies that

\[
\alpha - R_f - \gamma(W - C_0) + \tau < 0.
\]

\[\blacksquare\]
A.2 Two Special Cases

An analytical solution to the first order condition for the no-adviser equilibrium, equation (30), is not available for an arbitrary $k$. We provide the solutions for two special values of $k$ which are close to the empirically observed values: $k = 1.5$ and $k = 2$. We can show that the optimal fund size chosen by the portfolio manager (via the optimal fee) for $k = 1.5$ and $k = 2$, respectively is

\[
A_D = \begin{cases} 
\frac{\alpha - R_f + \tau}{2\gamma + C_0(R_f - \tau)/A_D^0} & \text{if } k = 2, \\
(\sqrt{\gamma^2 + C_0(R_f - \tau)(\alpha - R_f + \tau)/(3A_D^0)} - \gamma)\frac{3A_D^0}{C_0(R_f - \tau)} & \text{if } k = 1.5.
\end{cases}
\]  

(38)

Accordingly, the optimal fee is

\[
f_P = \begin{cases} 
1 - \frac{R_f - \tau + C_0(R_f - \tau)A_D(2A_D^0)}{\alpha - \gamma A_D} & \text{if } k = 2, \\
1 - \frac{R_f - \tau + C_0(R_f - \tau)A_D^2}{\alpha - \gamma A_D^2} & \text{if } k = 1.5;
\end{cases}
\]

(39)

and the return of the active portfolio net of the management fee is

\[
R_P(1 - f_P) = \begin{cases} 
R_f - \tau + \frac{C_0(R_f - \tau)A_D}{2A_D^0} & \text{if } k = 2, \\
R_f - \tau + \frac{C_0(R_f - \tau)A_D^2}{9A_D^0} & \text{if } k = 1.5.
\end{cases}
\]

(40)

The optimal fund size can be derived easily from the first order condition, while the optimal fee and the net return of the active portfolio follow from the two constraints in problem (28).

A.3 Proof of Proposition 5

Proof. In the case without investment advisers, the number of direct investors is the same as the number of investors investing in the active portfolio. Denote the total surplus of direct investors, relative to the default of passive investment, by $S_D^0$, we have

\[
S_D^0 = \int_{A_D^0}^{+\infty} [x(\alpha - \gamma A_D^0)(1 - f_P) - (x + C_0)(R_f - \tau)]f(x)dx
\]

\[
= [(\alpha - \gamma A_D^0)(1 - f_P) - R_f + \tau]A_D^0 - \theta^0 C_0(R_f - \tau),
\]

where $A_D^0$ is the threshold wealth level (net of the searching cost $C_0$) that makes the marginal investor indifferent between the passive fund and investing with the active portfolio manager, $A_D^0 = \int_{A_D^0}^{+\infty} x f(x)dx,$
\[ \theta^0 \equiv \int_{A_0^m}^{+\infty} f(x)dx = \left( \frac{A_m}{A_0^m} \right)^k. \]

Correspondingly, the total social welfare can be written as:

\[ U^0 = S_D^0 + \Pi_P^0 \]
\[ = S_D^0 + (\alpha - \gamma A_0^0)A_D^0 f_P \]
\[ = (\alpha - \gamma A_0^0 - R_f + \tau)A_D^0 - \theta^0 C_0(R_f - \tau). \]

In the equilibrium with unbiased advisers, the after-fee return of the active index is equated to \( R_f \) by the adviser's actions. Therefore the direct investor's surplus, \( S_D^1 \), is given by

\[ S_D^1 = \int_{\frac{\tau}{\tau - C_0(R_f - \tau)}}^{+\infty} [xR_f - (x + C_0)(R_f - \tau)]f(x)dx \]
\[ = A_D^1 \tau - \theta^1 C_0(R_f - \tau), \]

where \( A_D^1 \) is derived in equation [11], and \( \theta^1 \equiv \left( \frac{\tau A_m}{C_0(R_f - \tau)} \right)^k. \)

Similarly, since the after-fee return of the active portfolio in the case with kickback equals \( R_f - \delta \), the total surplus of the direct investors in the kickback equilibrium is given by

\[ S_D^2 = A_D^2 (\tau - \delta) - \theta^2 C_0(R_f - \tau), \]

where \( A_D^2 \) is derived in equation [27], and \( \theta^2 \equiv \left( \frac{\tau - \delta A_m}{C_0(R_f - \tau)} \right)^k. \)

Adding the manager's profit to the investor surplus, we get the social welfare \( U^1 \) and \( U^2 \) in Proposition 5.

To compare the social welfare in equilibria with and without kickback, first note that allowing kickbacks reduces the investors' surplus by

\[ S_D^1 - S_D^2 = A_D^2 \delta + (A_D^1 - A_D^2)\tau - (\theta^1 - \theta^2)C_0(R_f - \tau). \]

Furthermore, recall that from equation [33], we know that allowing kickbacks increases the portfolio manager's profit by \( A_D^2 \delta \). Combining these two welfare effects, we have

\[ U^1 - U^2 = (A_D^1 - A_D^2)\tau - (\theta^1 - \theta^2)C_0(R_f - \tau) \]
\[ = \int_{\frac{\tau - \delta}{\tau - C_0(R_f - \tau)}}^{\frac{\tau}{\tau - C_0(R_f - \tau)}} x f(x)[\tau - \frac{C_0(R_f - \tau)}{x}]dx > 0. \]
The above expression is positive for any $\delta > 0$, since $\tau - C_0(R_f - \tau)/x$ is positive for any $x$ not less than $C_0(R_f - \tau)/\tau$. Therefore kickbacks always decrease social welfare. Our proof also makes it clear that in the equilibrium with kickbacks, the welfare loss of the investors staying in the direct channel is exactly offset by the gain of the portfolio manager. The loss of investors who would originally choose the direct channel, but are forced to switch to the indirect channel because of the kickback, is the deadweight loss of social welfare. \hfill \Box
References


