5. Performance Measurement

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WS 2007
An overview

- Measuring performance with asset pricing models
  - Jensen’s alpha, Treynor index, Sharpe ratio, $M^2$, APT alpha, Upmarket-downmarket beta

- Measuring performance without asset pricing models
  - Grinblatt-Titman measure, Tracking error, Information ratio, Style analysis, Attribution analysis
Performance and CAPM

- If the market is in equilibrium and no manager has superior information, then all the funds must be on one line (SML).

- The expected return should depend only on the beta factor.
Jensen’s Alpha (1)

- If a fund manager has „superior“ performance, then his fund lies above the SML.
- The distance to the SML is the alpha

\[ \alpha = E(r_P - r_f) - \beta_P \left(E(r_m - r_f)\right) \]
## Jensen Alpha (2)

- Consider Fund A with a beta of 0.8 and the data for the last 4 quarters

<table>
<thead>
<tr>
<th>Quarter</th>
<th>T-Bill Rate</th>
<th>Fund Return</th>
<th>Excess Return</th>
<th>S&amp;P 500</th>
<th>Excess Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q1</td>
<td>2.97%</td>
<td>-8.77%</td>
<td>-11.74%</td>
<td>-5.86%</td>
<td>-8.83%</td>
</tr>
<tr>
<td>Q2</td>
<td>3.06%</td>
<td>-6.03%</td>
<td>-9.09%</td>
<td>-2.94%</td>
<td>-6.00%</td>
</tr>
<tr>
<td>Q3</td>
<td>2.85%</td>
<td>14.14%</td>
<td>11.29%</td>
<td>13.77%</td>
<td>10.92%</td>
</tr>
<tr>
<td>Q4</td>
<td>1.88%</td>
<td>24.99%</td>
<td>23.08%</td>
<td>14.82%</td>
<td>12.94%</td>
</tr>
<tr>
<td>Mean</td>
<td>2.69%</td>
<td>6.08%</td>
<td>3.39%</td>
<td>4.95%</td>
<td>2.26%</td>
</tr>
<tr>
<td>SD</td>
<td>16.22%</td>
<td>16.68%</td>
<td>10.87%</td>
<td>11.26%</td>
<td></td>
</tr>
</tbody>
</table>

Asset Management

Youchang Wu
Jensen’s Alpha (3)

- What was the Alpha for Fund A?

- Suppose Fund B has a beta of 1.2 and an Alpha of 4.5%. How would you rank Fund A and Fund B?

- By how much could one lever Fund A to achieve a beta of 1.2?
Jensen‘s Alpha (4)

- What is Jensen‘s Alpha for the levered fund?

- How would we now rank Fund A versus Fund B?
Treynor Index (1)

- Measures the "excess return per unit of systematic risk (beta)".
- Considers the leverage effect.

\[
Treynor = \frac{E[r_P - r_f]}{\beta_P}
\]
Treynor Index (2)

- Treynor Index for Fund A: 3.39%/0.8=4.24%
- Treynor Index for Fund B: 7.2%/1.2=6%
  (Note that the excess rate of return of Fund B must have been 4.5%+2.26%*1.2 = 7.2%.)
- Treynor Index for the S&P 500: 2.26%
  (=average excess return of the market)
Sharpe Ratio (1)

- Divides the excess return by the volatility.
- $\sigma_p$ is frequently defined as the volatility of the excess rate of return.
- Slope of the lines of rf to A or B
- Ratio of return to risk

$$Sharpe = \frac{E[r_p - r_f]}{\sigma_p}$$
Sharpe Ratio (2)

- For the previous example, calculate the Sharpe Ratio for Fund A and the S&P 500.
- Recall that the estimation of the standard deviation of the rate or return from a sample of $T$ observations is

$$
\sigma = \left( \sum_{t=1}^{T-1} \frac{1}{T-1} (r_t - E[r_t])^2 \right)^{0.5}
$$

- and the estimation of the expected return is

$$
E[r_t] = \frac{1}{T} \sum_{t=1}^{T} r_t
$$
Sharpe Ratio (3)

- Sharpe ratio for Fund A

- Sharpe ratio for the S&P 500

- Was there „superior“ performance?
Sharpe Ratio (4)

- The Sharpe ratio should be expressed on an annual basis.
- This requires transforming a quarterly Sharpe ratio in the following way:

\[ \text{SharpeRatio}\text{Annual} = \frac{E[r_p - r_f] \cdot 4}{\sigma_p \cdot \sqrt{4}} = \text{SharpeRatio}\text{Quarterly} \sqrt{4} \]
Sharpe Ratio (5)

- The higher the Sharpe ratio the better is the portfolio performance, because either
  - the return was higher or
  - the risk was smaller
- Comparison with the Sharpe ratio of the market
- Rough guide: Sharpe ratio above 1 is very good and above 2 is extraordinary
- Note: The Sharpe ratio ignores correlations between the fund and clients’ other investments.
Modigliani-Modigliani ($M^2$) Measure (1)

- Easier to interpret than Sharpe Ratio
- Return of a fund with the same risk as the benchmark,
- Includes leverage effekt

$$M^2_p = r_f + \frac{E[r_p - r_f]}{\sigma_p} \sigma_m$$
Modigliani-Modigliani ($M^2$) Measure (2)

- Calculate $M^2$ for Fund A.

- According to $M^2$, did Fund A have superior performance?
Summarizing CAPM-based PMs

- Sharpe Ratio, $M^2$: Consider return and total risk (volatility)
  - Return per unit of total risk
  - Assumption: entire wealth is invested in the fund

- Alpha, Treynor: Consider return and systematic risk
  - The idiosyncratic risk of a stock is ignored.
  - Only systematic risk counts.
  - Assumption: only a small part of the investor’s wealth is invested in the fund.
Problems of CAPM-based PMs

- What is the appropriate risk-free rate?
- What is the appropriate market portfolio?
- Market-timing abilities may give biased results.
Upmarket and Downmarket Betas

- Estimating two regression coefficients $\beta_{up}$ and $\beta_{down}$

- Market timing expertise exists when $\beta_{up} - \beta_{down} > 0$
APT Alpha (1)

- Stock returns are related to basic factors:
  \[ r_{j,t} = a_j + \beta_{1,j} F_{1,t} + \beta_{2,j} F_{2,t} + \ldots + \beta_{n,j} F_{n,t} + \epsilon_{j,t} \]

- Expected returns are linearly related to factor exposures:
  \[ E[r_j] = r_f + \beta_{1,j} \gamma_1 + \beta_{2,j} \gamma_2 + \ldots + \beta_{n,j} \gamma_n \]

- Performance measurement using APT:
  \[ Alpha_{\text{APT}} = E[r_p] - [r_f + \beta_{1,j} \gamma_1 + \beta_{2,j} \gamma_2 + \ldots + \beta_{n,j} \gamma_n] \]
APT Alpha (2)

- A popular model used in practice is the Fama-French three factor model
  - Market factor
  - Size factor
  - Book-to-market factor
- The momentum factor is also often considered
- Problems with performance measurement using APT:
  - Factors are not theoretically specified
  - Factor structure not uniquely determined
Grinblatt-Titman Measure

- Does not require a benchmark portfolio
- Assume that portfolio weights are observable
- Manager with market timing abilities will increase asset weight in asset classes with increasing returns
- Positive correlation between
  - Changes in portfolio weights
  - Return
- Portfolio Change Measure

\[
PCM = \sum_{t=2}^{T} \sum_{j=1}^{N} \frac{r_{j,t} (w_{j,t} - w_{j,t-1})}{T - 1}
\]
Tracking Error and Information Ratio

- Relative performance = portfolio return - benchmark return
- Tracking Error = \( \sigma \) (portfolio return - benchmark return)
- Information Ratio = \( \frac{\text{Relative Performance}}{\text{Tracking Error}} \)
- Risk relative to benchmark = \( \frac{\sigma \text{(Portfolio)}}{\sigma \text{(Benchmark)}} \)
Tracking Error and Information Ratio

- See the data for Fund I and II and the benchmark
  
<table>
<thead>
<tr>
<th>Quarter</th>
<th>Benchmark</th>
<th>Fund I Return</th>
<th>Relative Performance I</th>
<th>Fund II Return</th>
<th>Relative Performance II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q1</td>
<td>8.77%</td>
<td>11.74%</td>
<td>2.97%</td>
<td>8.95%</td>
<td>0.18%</td>
</tr>
<tr>
<td>Q2</td>
<td>-6.03%</td>
<td>-9.09%</td>
<td>-3.06%</td>
<td>-5.50%</td>
<td>0.53%</td>
</tr>
<tr>
<td>Q3</td>
<td>7.14%</td>
<td>11.29%</td>
<td>4.15%</td>
<td>7.55%</td>
<td>0.41%</td>
</tr>
<tr>
<td>Q4</td>
<td>4.99%</td>
<td>23.08%</td>
<td>18.09%</td>
<td>6.00%</td>
<td>1.01%</td>
</tr>
</tbody>
</table>
  
  Mean 3.72% 9.255% 5.54% 4.25% 0.53%

- Which one has a higher information ratio?

- What is the relative risk ratio for each fund?
Style Analysis (1)

- Asset returns follow a factor model
  \[ r_{j,t} = \alpha_j + \beta_{1,j} F_{1,t} + \beta_{2,j} F_{2,t} + \ldots + \beta_{n,j} F_{n,t} + \epsilon_{j,t} \]
- Asset class factor model: each factor is the return of an asset class and \( \sum \beta_i = 1 \)
- Choice of basic asset classes
Style Analysis (2)

- Factor exposures can be estimated by
  - Regression
  - Constrained Regression
    - Sum of factor loadings = 1
  - Quadratic programming: minimize the variance of return difference with constraints
    - Sum of factor loadings = 1
    - Individual loading between 0 and 1
Estimated style profile: an example

- Both unconstrained and constrained regression give problematic results
- Quadratic programming is the recommended estimation method

<table>
<thead>
<tr>
<th></th>
<th>Bills</th>
<th>Intermedi ate Bonds</th>
<th>Long-term Bonds</th>
<th>Corporate Bonds</th>
<th>Mortgages</th>
<th>Value Stocks</th>
<th>Growth Stocks</th>
<th>Medium Stocks</th>
<th>Small Stocks</th>
<th>Foreign Bonds</th>
<th>European Stocks</th>
<th>Japanese Stocks</th>
<th>Total</th>
<th>R-squ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unconstrained</td>
<td>14.69</td>
<td>-69.51</td>
<td>-2.54</td>
<td>16.57</td>
<td>5.19</td>
<td>109.52</td>
<td>-7.86</td>
<td>-41.83</td>
<td>45.65</td>
<td>-1.85</td>
<td>6.15</td>
<td>-1.46</td>
<td>72.71</td>
<td>95.20</td>
</tr>
<tr>
<td>Constrained Regression</td>
<td>42.65</td>
<td>-68.64</td>
<td>-2.38</td>
<td>15.29</td>
<td>4.58</td>
<td>110.35</td>
<td>-8.02</td>
<td>-43.62</td>
<td>47.17</td>
<td>-1.38</td>
<td>5.77</td>
<td>-1.79</td>
<td>100.00</td>
<td>95.16</td>
</tr>
<tr>
<td>Quadratic</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>69.81</td>
<td>0</td>
<td>30.04</td>
<td>0</td>
<td>0.15</td>
<td>0</td>
<td>100.00</td>
<td>0</td>
<td>92.22</td>
</tr>
</tbody>
</table>
Style Analysis (4)

Figure 3
Trustee's Commingled - U.S. Portfolio
January 1985 - December 1989

Style based on 60 monthly returns

Selection 7.8%
Style 92.2%
Style Analysis (5)

- Consider the rates of return on „factor portfolios“ given on the next page.
- Assume that the fund’s „style“ is characterized by the regression results estimated above.
- What was the relative performance of this fund according to style analysis?
Style Analysis (6)

Rates of return on „factor portfolios“

<table>
<thead>
<tr>
<th></th>
<th>Value stocks</th>
<th>Small stocks</th>
<th>European stocks</th>
<th>Fund Return I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q1</td>
<td>8.77%</td>
<td>11.74%</td>
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<td>4.99%</td>
<td>23.08%</td>
<td>18.09%</td>
<td>18.67%</td>
</tr>
<tr>
<td>Mean</td>
<td>3.72%</td>
<td>9.255%</td>
<td>5.54%</td>
<td>8.20%</td>
</tr>
</tbody>
</table>
Style Analysis (7)

- What was the average rate of return of the benchmark?
- What was the average rate of return of the fund?
- What was the relative performance according to style analysis?
Attribution Analysis (1)

- Attribution analysis attempts to identify the sources of relative performance, i.e., the difference between portfolio return and benchmark return.

- A popular decomposition:
  Total performance
  = allocation effect + selection effect
Attribution Analysis (2)

\[ r_p - r_b = \sum_{i=1}^{N} [(w_{pi} - w_{bi})(r_{bi} - r_b)] + \sum_{i=1}^{N} w_{pi}(r_{pi} - r_{bi}) \]

\( w_{pi}, w_{bi} = \text{weights of the } i\text{th market segment (asset class, industry group) in the actual portfolio and the benchmark portfolio, respectively} \)

\( r_{pi}, r_{bi} = \text{return to the } i\text{th market segment in the actual portfolio and the benchmark portfolio, respectively} \)

\( r_b = \text{the total return to the benchmark portfolio} \)
Attribution analysis: example

The following information is given. Compute the allocation effect and selection effect.

<table>
<thead>
<tr>
<th></th>
<th>investment weights</th>
<th></th>
<th>Return</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fund</td>
<td>Benchmark</td>
<td>Difference</td>
</tr>
<tr>
<td>Stock</td>
<td>0.5</td>
<td>0.6</td>
<td>-0.1</td>
</tr>
<tr>
<td>Bond</td>
<td>0.38</td>
<td>0.3</td>
<td>0.08</td>
</tr>
<tr>
<td>Cash</td>
<td>0.12</td>
<td>0.1</td>
<td>0.02</td>
</tr>
</tbody>
</table>
Summary

- Returns must be adjusted by risk!
- Asset pricing models give some guidance on how to adjust for risks
- However we do not know exactly what is the right asset pricing model and how to implement it
- Measuring performance without asset pricing models is possible if portfolio holding data are available, or if a benchmark is pre-specified
- Style analysis is a tool to backout a suitable benchmark from the data
- Attribution analysis identifies the source of relative performance