


# Fuzzy Logic and GIS

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**M**any phenomena show a degree of vagueness or uncertainty that cannot be properly expressed with crisp sets of class boundaries. Spatial features often do not have clearly defined boundaries, and concepts like “steep”, “close”, or “suitable” can better be expressed with degrees of membership to a fuzzy set than with a binary yes/no classification. This chapter introduces the basic principles of fuzzy logic, a mathematical theory that has found many applications in various domains. It can be applied whenever vague phenomena are involved.



## 1.1 Fuzziness

In human thinking and language we often use uncertain or vague concepts. Our thinking and language is not binary, i.e., black and white, zero or one, yes or no. In real life we add much more variation to our judgments and classifications. These vague or uncertain concepts are said to be fuzzy. We encounter *fuzziness* almost everywhere in our everyday lives.

### 1.8.1 Motivation

When we talk about tall people, the concept of “tall” will be depending on the context. In a society where the average height of a person is 160cm, somebody will be considered to be tall differently from a population with an average height of 180cm. In land cover analysis we are not able to draw crisp boundaries of, for instance, forest areas or grassland. Where does the grassland end and the forest start? The boundaries will be vague or fuzzy.

In real life applications we might look for a suitable site to build a house. The criteria for the area that we are looking for could be formulated as follows. The site must

- have *moderate* slope
- have *favorable* aspect
- have *moderate* elevation
- be *close to* a lake
- be *not near* a major road
- not be located in a restricted area

All the conditions mentioned above (except the one for the restricted area) are vague, but correspond to the way we express these conditions in our languages and thinking. Using the conventional approach the above mentioned conditions would be translated into crisp classes, such as

- slope less than 10 degrees
- aspect between 135 degrees and 225 degrees, or the terrain is flat
- elevation between 1,500 meters and 2,000 meters
- within 1 kilometer from a lake
- not within 300 meters from a major road

If a location falls within the given criteria we would accept it, otherwise (even if it would be very close to the set threshold) it would be excluded from our analysis. If, however, we allow degrees of membership to our classes, we can accommodate also those locations that just miss a criterion by a few meters. They will just get a low degree of membership, but will be included in the analysis. Usually, we assign a degree of membership to a class as a value between zero and one, where zero indicates no membership and one represents full membership. Any value in between can be a possible degree of membership.

### 1.8.2 Fuzziness versus Probability

Degrees of membership as values ranging between zero and one look very similar to probabilities, which are also given as a value between zero and one. We might be tempted to assume that fuzziness and probability are basically the same. There is, however, a subtle, yet important, difference.

*Probability* gives us an indication with which likelihood an event will occur. Whether it is going to happen, is not sure depending on the probability. *Fuzziness* is an indication to what degree something belongs to a class (or phenomenon). We

know that the phenomenon exists. What we do not know, however, is its extent, i.e., to which degree members of a given universe belong to the class. In the following sections we will establish the mathematical basis to deal with vague and fuzzy concepts.

## 1.2 Crisp Sets and Fuzzy Sets

In general set theory an element is either a member of a set or not. We can express this fact with the characteristic function for the elements of a given universe to belong to a certain subset of this universe. We call such a set a *crisp set*.

**Definition 1 (Characteristic function).** Let  $A$  be a subset of a universe  $X$ . The *characteristic function*  $\chi_A$  of  $A$  is defined as  $\chi_A : X \rightarrow \{0,1\}$  with

$$\chi_A(x) = \begin{cases} 1 & \text{iff } x \in A \\ 0 & \text{iff } x \notin A \end{cases}$$

In this way we always can clearly indicate whether an element belongs to a set or not. If, however, we allow some degree of uncertainty as to whether an element belongs to a set, we can express the membership of an element to a set by its membership function.

**Definition 2 (Fuzzy set).** A *fuzzy set*  $A$  of a universe  $X$  is defined by a *membership function*  $\mu_A$  such that  $\mu_A : X \rightarrow [0,1]$  where  $\mu_A(x)$  is the *membership value* of  $x$  in  $A$ . The universe  $X$  is always a crisp set.

If the universe is a finite set  $X = \{x_1, x_2, \dots, x_n\}$ , then a fuzzy set  $A$  on  $X$  is expressed as  $A = \mu_A(x_1)/x_1 + \mu_A(x_2)/x_2 + \dots + \mu_A(x_n)/x_n = \sum_{i=1}^n \mu_A(x_i)/x_i$ .

The term  $\mu_A(x_i)/x_i$  indicates the membership value to fuzzy set  $A$  for  $x_i$ . The symbol “/” is called *separator*,  $\Sigma$  and “+” function as *aggregation* and *connection* of terms.

If the universe is an infinite set  $X = \{x_1, x_2, \dots\}$ , then a fuzzy set  $A$  on  $X$  is expressed as  $A = \int_X \mu_A(x)/x$ . The symbols  $\int$  and “/” function as aggregation and separator.<sup>1</sup>

The *empty fuzzy set*  $\emptyset$  is defined as  $\forall x \in X, \mu_{\emptyset}(x) = 0$ .

For every element of the universe  $X$  we trivially have  $\forall x \in X, \mu_X(x) = 1$ , i.e., the universe is always crisp.

A membership function assigns to every element of the universe a degree of membership (or membership value) to a fuzzy set. This membership value must be between zero (no membership) and one (definite membership). All other values

<sup>1</sup> Note that the symbols  $\Sigma$ ,  $+$ , and  $\int$  are not to be interpreted in their usual meaning as sum, addition, and integral.

between zero and one indicate to which degree an element belongs to the fuzzy set. It is important to note that the membership degree of 1 does not need to be obtained for members of a fuzzy set.

**Example 1.** Let us take three persons A, B, and C and their respective heights as 185cm (A), 165cm (B) and 186cm (C). We want to assign the different persons to classes for short, average, and tall people, respectively.

If we take a crisp classification and set the class boundaries to  $(-, 165]$  for short,  $(165, 185]$  for average, and  $(185, -)$  for tall, we see that A falls into the average class, B into the short class, and C into the tall class. We also see that A is nearly as tall as C, and yet they fall into different classes. The characteristic functions of the three classes are displayed in Table 1.

Table 1. Characteristic function for height classes

	Short	Average	Tall
A	0	1	0
B	1	0	0
C	0	0	1

When we choose a fuzzy set approach, we need to define three membership functions for the three classes, respectively (Figure 1).

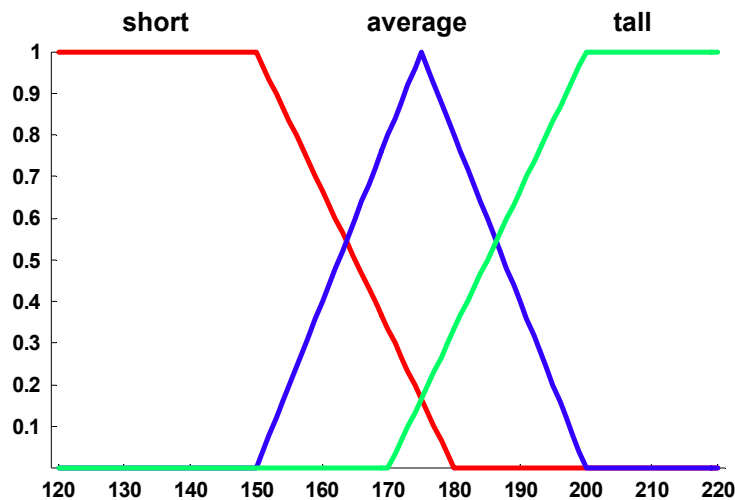


Figure 1. Membership functions for “short”, “average”, and “tall”

For *short* we select a linear membership function that produces a membership value of one for persons shorter than 150cm and decreases until it reaches zero at 180cm.

The membership function for the *average* class produces values equal zero for persons shorter than 150cm, it then increases until it reaches one at 175cm. From there it decreases until it reaches zero at 200cm.

The membership function for the *tall* class is zero up to 170cm. From there it increases until it reaches one at 200cm. The membership values for the three persons to the three classes are given in Table 2.

Table 2. Membership values for the height classes

	Short	Average	Tall
A	0.00	0.60	0.50
B	0.50	0.60	0.00
C	0.00	0.56	0.53

Using the fuzzy set approach we can much better express the fact that A and C are nearly the same height and that both have a higher degree of membership to the average class than to short or tall, respectively.

### 1.3 Membership Functions

The selection of a suitable membership function for a fuzzy set is one of the most important activities in fuzzy logic. It is the responsibility of the user to select a function that is a best representation for the fuzzy concept to be modeled. The following criteria are valid for all membership functions:

- The membership function must be a real valued function whose values are between 0 and 1.
- The membership values should be 1 at the center of the set, i.e., for those members that definitely belong to the set.
- The membership function should fall off in an appropriate way from the center through the boundary.
- The points with membership value 0.5 (crossover point) should be at the boundary of the crisp set, i.e., if we would apply a crisp classification, the class boundary should be represented by the crossover points.

We know two types of membership functions: (i) linear membership functions and (ii) sinusoidal membership functions. Figure 2 shows the linear membership function. This function has four parameters that determine the shape of the function. By choosing proper values for  $a$ ,  $b$ ,  $c$ , and  $d$ , we can create S-shaped, trapezoidal, triangular, and L-shaped membership functions.

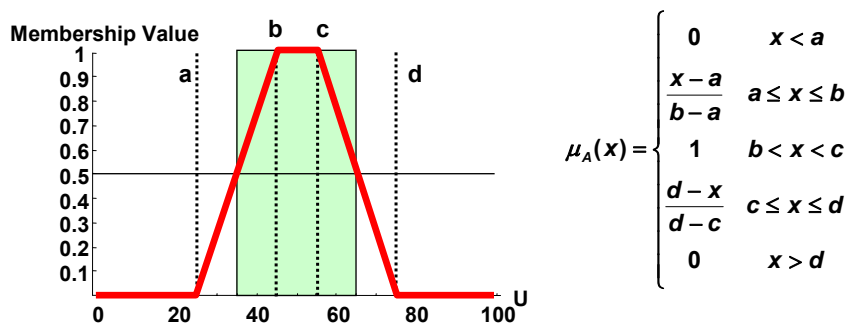


Figure 2. Linear membership function

If a rounded shape of the membership function is more appropriate for our purpose we should choose a sinusoidal membership function (Figure 3). As with linear membership functions we can achieve S-shaped, bell-shaped, and L-shaped membership functions by proper selection of the four parameters.

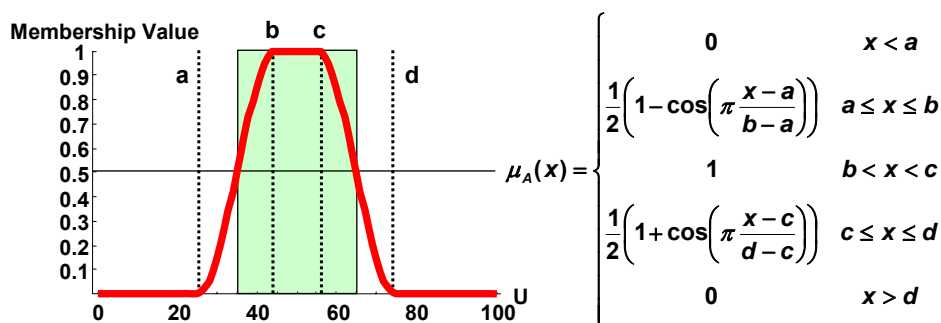


Figure 3. Sinusoidal membership function

A special case of the bell-shaped membership functions is the Gaussian function derived from the probability density function of the normal distribution with two parameters  $c$  (mean) and  $\sigma$  (standard deviation). Although this membership

function is derived from probability theory, it is used here as a membership function for a fuzzy set.

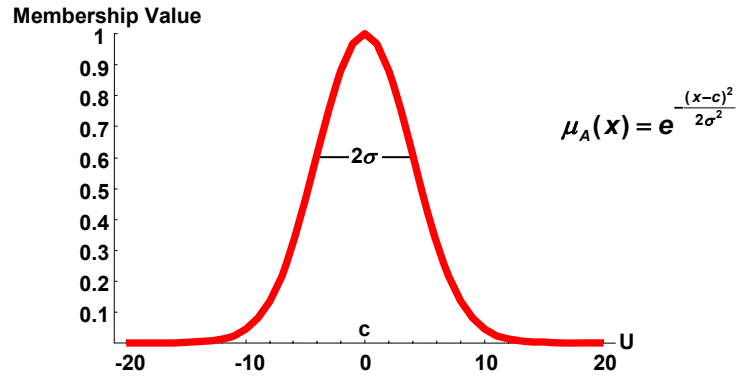


Figure 4. Gaussian membership function

**Example 2.** The membership functions in Example 1 are linear functions with the following parameters:

$$\mu_{\text{Short}}(x) = \begin{cases} 1 & x \leq 150 \\ \frac{180-x}{30} & 150 < x \leq 180 \\ 0 & x > 180 \end{cases}$$

$$\mu_{\text{Average}}(x) = \begin{cases} 0 & x \leq 150 \\ \frac{x-150}{25} & 150 < x \leq 175 \\ \frac{200-x}{25} & 175 \leq x \leq 200 \\ 0 & x > 200 \end{cases}$$

$$\mu_{\text{Tall}}(x) = \begin{cases} 0 & x \leq 170 \\ \frac{x-170}{30} & 170 < x \leq 200 \\ 0 & x > 200 \end{cases}$$

## 1.4 Operations on Fuzzy Sets

Operations on fuzzy sets are defined in a similar way as for crisp sets. However, not all rules for crisp set operations are also valid for fuzzy sets. Like for crisp sets we have subset, union, intersection, and complement. In addition, there are alternate operations for union and intersection of fuzzy sets.

**Definition 3 (Support).** All elements of the universe  $X$  that have a membership value greater than zero for a fuzzy set  $A$  are called the *support* of  $A$ , or  $\text{supp}(A) = \{x \in X \mid \mu_A(x) > 0\}$ .

**Example 3.** The support of the fuzzy set for short people (Example 1) is those persons who are shorter than 150cm.

**Definition 4 (Height).** The *height* of a fuzzy set  $A$  is the largest membership value in  $A$ , written as  $\text{hgt}(A)$ . If  $\text{hgt}(A) = 1$  then the set is called *normal*.

**Example 4.** The height of the fuzzy sets Short, Average, and Tall is 1. They are all normal fuzzy sets.

We can always normalize a fuzzy set by dividing all its membership values by the height of the set.

**Definition 5 (Equality).** Two fuzzy sets  $A$  and  $B$  are *equal* (written as  $A = B$ ) if for all members of the universe  $X$  their membership values are equal, i.e.,  $\forall x \in X, \mu_A(x) = \mu_B(x)$ .

Subsets in fuzzy sets are defined by fuzzy set inclusion.

**Definition 6 (Inclusion).** A fuzzy set  $A$  is *included* in a fuzzy set  $B$  (written as  $A \subseteq B$ ) if for every element of the universe the membership values for  $A$  are less than or equal to those of  $B$ , i.e.,  $\forall x \in X, \mu_A(x) \leq \mu_B(x)$ .

When we look at the graph of the membership functions a fuzzy set  $A$  will be included in fuzzy set  $B$  when the graph of  $A$  is completely covered by the graph of  $B$  (Figure 5).

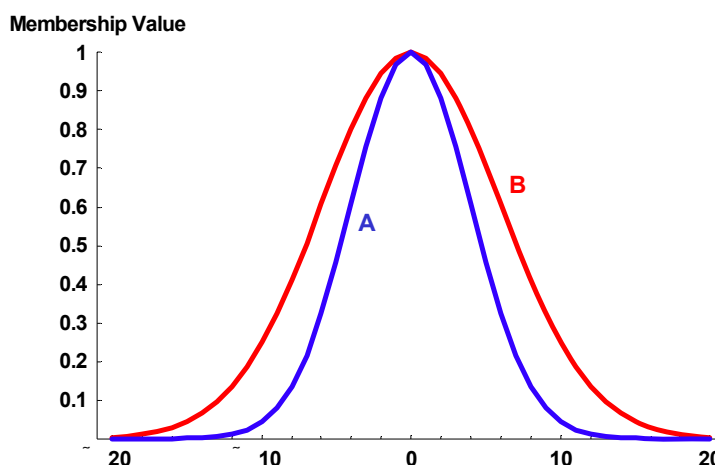


Figure 5. Set inclusion

For the union of two fuzzy sets we have more than one operator. The most common ones are presented here.

**Definition 7 (Union).** The *union* of two fuzzy sets  $A$  and  $B$  can be computed for all elements of the universe  $X$  by one of the three operators:

1.  $\mu_{A \cup B}(x) = \max(\mu_A(x), \mu_B(x))$
2.  $\mu_{A \cup B}(x) = \mu_A(x) + \mu_B(x) - \mu_A(x) \cdot \mu_B(x)$
3.  $\mu_{A \cup B}(x) = \min(1, \mu_A(x) + \mu_B(x))$

The max-operator is a *non-interactive* operator in the sense that the membership values of both sets do not interact with each other. In fact, one set could be completely ignored in a union operation when it is included in the other. The two other operators are called *interactive*, because the membership value of the union is determined by the membership values of both sets.

**Example 5.** Figure 6 illustrates the union operators for the fuzzy sets Short and Average from Example 1.

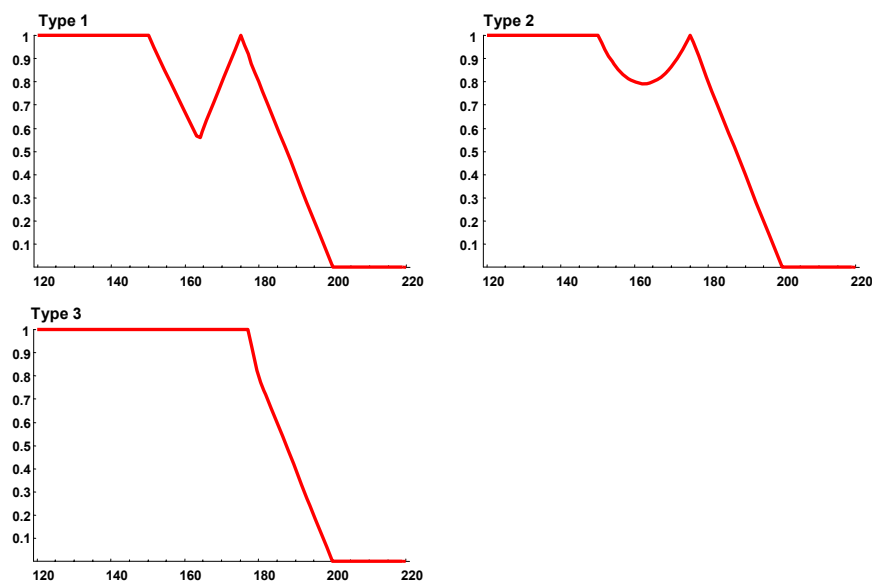


Figure 6. Fuzzy set union operators

**Definition 8 (Intersection).** The *intersection* of two fuzzy sets  $A$  and  $B$  can be computed for all elements of the universe  $X$  by one of the three operators:

1.  $\mu_{A \cap B}(x) = \min(\mu_A(x), \mu_B(x))$
2.  $\mu_{A \cap B}(x) = \mu_A(x) \cdot \mu_B(x)$
3.  $\mu_{A \cap B}(x) = \max(0, \mu_A(x) + \mu_B(x) - 1)$

The min-operator is non-interactive, the two others are interactive operators as explained above.

**Example 6.** Figure 7 illustrates the intersection of the fuzzy sets Short and Average from Example 1.

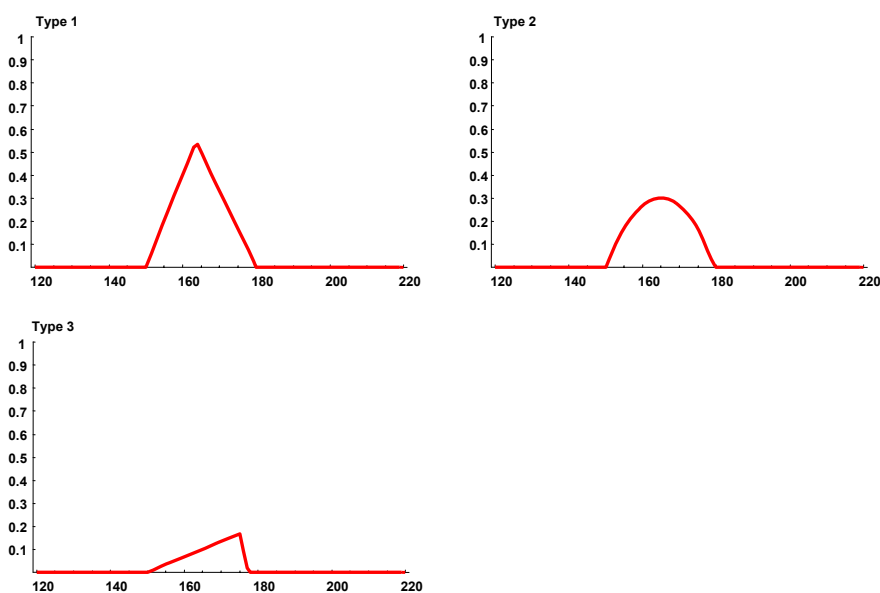


Figure 7. Fuzzy set intersection



**Definition 9 (Complement).** The *complement* of a fuzzy set  $A$  in the universe  $X$  is defined as  $\forall x \in X, \mu_{\bar{A}}(x) = 1 - \mu_A(x)$ .

**Example 7.** Figure 8 shows the fuzzy set Average from Example 1 and its complement.

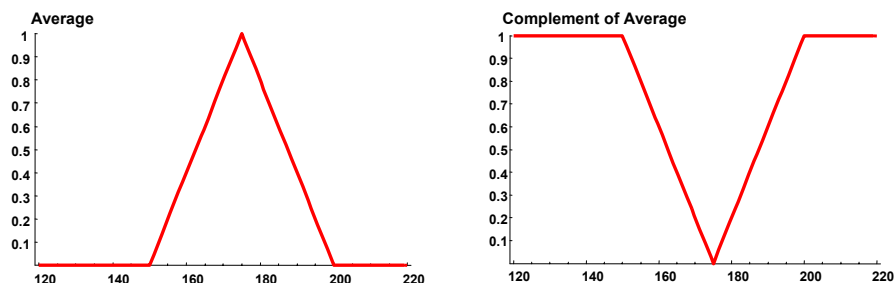


Figure 8. Fuzzy set and its complement

Many rules for set operations are valid for both crisp and fuzzy sets. Table 3 shows the rules that are valid for both.

Table 3. Rules for set operations valid for crisp and fuzzy sets

1.	$A \cup A = A$	idempotent law
2.	$A \cap A = A$	
3.	$(A \cup B) \cup C = A \cup (B \cup C)$	associativity
4.	$(A \cap B) \cap C = A \cap (B \cap C)$	
5.	$A \cup B = B \cup A$	commutativity
6.	$A \cap B = B \cap A$	
7.	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	distributivity
8.	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	
9.	$\overline{A \cup B} = \bar{A} \cap \bar{B}$	DE MORGAN's law
10.	$\overline{A \cap B} = \bar{A} \cup \bar{B}$	
11.	$\overline{\bar{A}} = A$	double complement

Table 4 shows those rules that in general are valid for crisp sets but not for fuzzy sets.

Table 4. Rules valid only for crisp sets

1.	$A \cup \bar{A} = X$	law of the excluded middle
2.	$A \cap \bar{A} = \emptyset$	law of contradiction

Figure 9 illustrates that the law of the excluded middle and the law of contradiction does not generally hold for fuzzy sets.

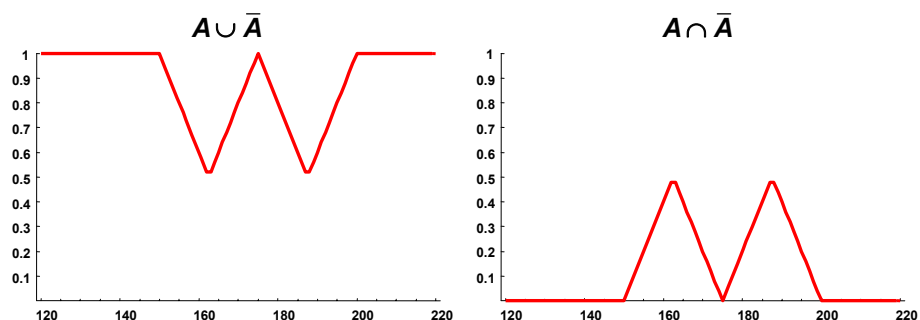


Figure 9. Law of the excluded middle and law of contradiction for fuzzy set Average.

## 1.5 Alpha-Cuts

If we wish to know all those elements of the universe that belong to a fuzzy set and have at least a certain degree of membership, we can use  $\alpha$ -level sets.

**Definition 10 ( $\alpha$ -Cut).** A (weak)  $\alpha$ -cut (or  $\alpha$ -level set)  $A_\alpha$  with  $0 < \alpha \leq 1$  is the set of all elements of the universe such that  $A_\alpha = \{x \in X \mid \mu_A(x) \geq \alpha\}$ . A strong  $\alpha$ -cut  $A_{\bar{\alpha}}$  is defined as  $A_{\bar{\alpha}} = \{x \in X \mid \mu_A(x) > \alpha\}$ .

**Example 8.** The 0.8-cut of the fuzzy set Tall contains all those persons who are 194cm or taller.

With  $\alpha$ -level sets we can identify those members of the universe that typically belong to a fuzzy set.

## 1.6 Linguistic Variables and Hedges

In mathematics variables usually assume numbers as values. A *linguistic variable* is a variable that assumes linguistic values which are words (*linguistic terms*). If, for example, we have the linguistic variable “height”, the linguistic values for height could be “short”, “average”, and “tall”. These linguistic values possess a certain degree of uncertainty or vagueness that can be expressed by a membership function to a fuzzy set. Often, we modify a linguistic term by adding words like “very”, “somewhat”, “slightly”, or “more or less” and arrive at expressions such as “very tall”, “not short”, or “somewhat average”.

Such modifiers are called *hedges*. They can be expressed with operators applied to the fuzzy sets representing linguistic terms (see Table 5).

Table 5. Operators for hedges

Operator	Expression
Normalization	$\mu_{\text{norm}(A)}(x) = \frac{\mu_A(x)}{\text{hgt}(\mu_A)}$
Concentration	$\mu_{\text{con}(A)}(x) = \mu_A^2(x)$
Dilation	$\mu_{\text{dil}(A)}(x) = \sqrt{\mu_A(x)}$
Negation	$\mu_{\text{not}(A)}(x) = \mu_A^-(x) = 1 - \mu_A(x)$
Contrast intensification	$\mu_{\text{int}(A)}(x) = \begin{cases} 2\mu_A^2(x) & \text{if } \mu_A(x) \in [0, 0.5] \\ 1 - 2(1 - \mu_A(x))^2 & \text{otherwise} \end{cases}$

The following Table 6 shows the models being used to represent hedges for linguistic terms.

Table 6. Hedges and their models

Hedge	Operator
very A	con(A)
more or less A (fairly A)	dil(A)
plus A	$A^{1.25}$
not A	not(A)
slightly A	int(norm(plus A $\cap$ not(very A)))

**Example 9.** Figure 10 shows the membership functions for Tall, Very Tall, and Very Very Tall.

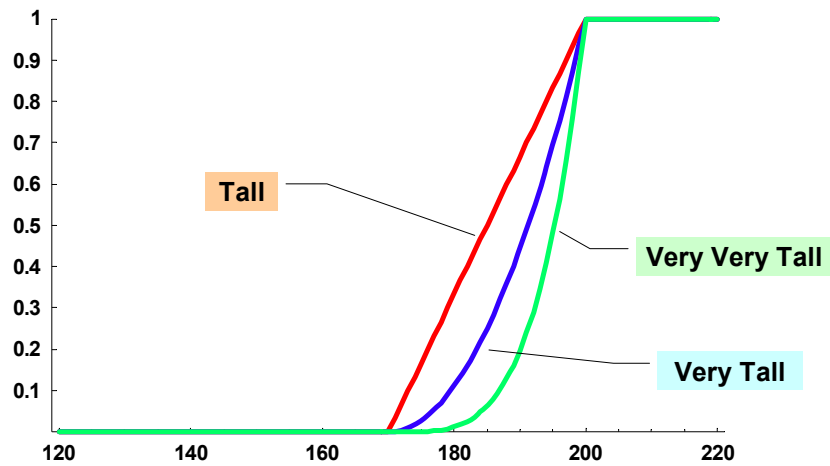


Figure 10. Membership functions for Tall, Very Tall, and Very Very Tall

**Example 10.** Figure 11 shows the membership functions for Tall and Not Very Tall.

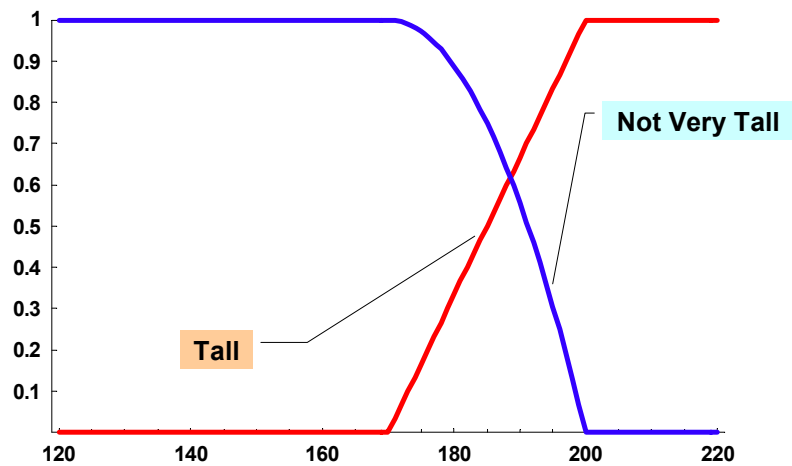


Figure 11. Membership function for Tall and Not Very Tall

**Example 11.** Figure 12 shows the membership functions for Tall and Slightly Tall.

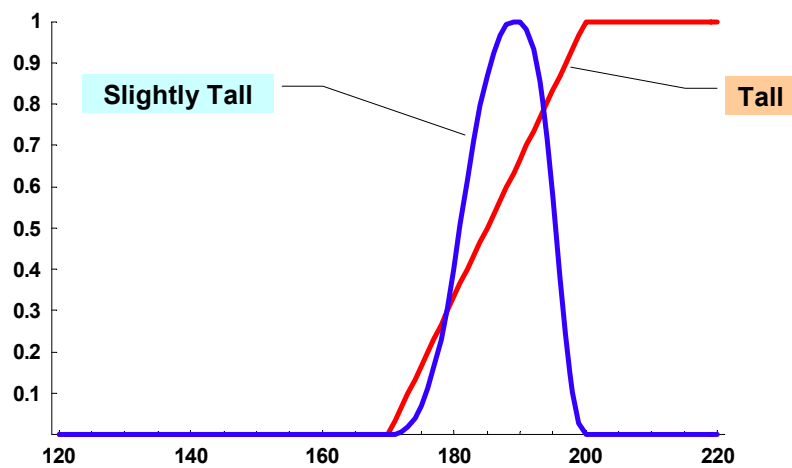


Figure 12. Membership function for Tall and Slightly Tall



1. Apply the input values to the premise variables for every rule and compute the minimum of  $\mu_{A_i}(x_0)$  and  $\mu_{B_i}(y_0)$ :

$$\text{Rule}_1: m_1 = \min(\mu_{A_1}(x_0), \mu_{B_1}(y_0))$$

$$\text{Rule}_2: m_2 = \min(\mu_{A_2}(x_0), \mu_{B_2}(y_0))$$

$$\vdots$$

$$\text{Rule}_n: m_n = \min(\mu_{A_n}(x_0), \mu_{B_n}(y_0))$$

2. Cut the membership function of the consequence  $\mu_{C_i}(z)$  at  $m_i$ :

$$\text{Conclusion of rule}_1: \mu_{C_1}(z) = \min(m_1, \mu_{C_1}(z)) \quad \forall z \in C_1$$

$$\text{Conclusion of rule}_2: \mu_{C_2}(z) = \min(m_2, \mu_{C_2}(z)) \quad \forall z \in C_2$$

$$\vdots$$

$$\text{Conclusion of rule}_n: \mu_{C_n}(z) = \min(m_n, \mu_{C_n}(z)) \quad \forall z \in C_n$$

3. Compute the final conclusion by determining the union of all individual conclusions from step 2:

$$\mu_C(z) = \max(\mu_{C_1}(z), \mu_{C_2}(z), \dots, \mu_{C_n}(z))$$

The result of the final conclusion is a fuzzy set. For practical reasons we need a definite value for the consequence variable. The process to determine this value is called *defuzzification*. There are several methods to defuzzify a given fuzzy set. One of the most common is the *center of gravity* (or center of area).

For a discrete fuzzy set the center of area is computed as

$$z_0 = \frac{\sum \mu_C(z) \cdot z}{\sum \mu_C(z)}$$

For a continuous fuzzy set this becomes

$$z_0 = \frac{\int \mu_C(z) \cdot z dz}{\int \mu_C(z) dz}$$

**Example 13.** Given the speed of a car and the distance to a car in front of it, we would like to determine whether we should break, maintain the speed, or accelerate. Assume the following set of rules for the given situation:

- Rule 1 If the distance between the cars is short and the speed is low then maintain speed
- Rule 2 If the distance between the cars is short and the speed is high then reduce speed
- Rule 3 If the distance between the cars is long and the speed is low then increase speed
- Rule 4 If the distance between the cars is long and the speed is high then maintain speed

Distance, speed, and acceleration are linguistic variables with the values “short”, “long”, “high”, “low”, and “reduce”, “maintain”, and “increase”, respectively. They can be modeled as fuzzy sets (Figure 14).

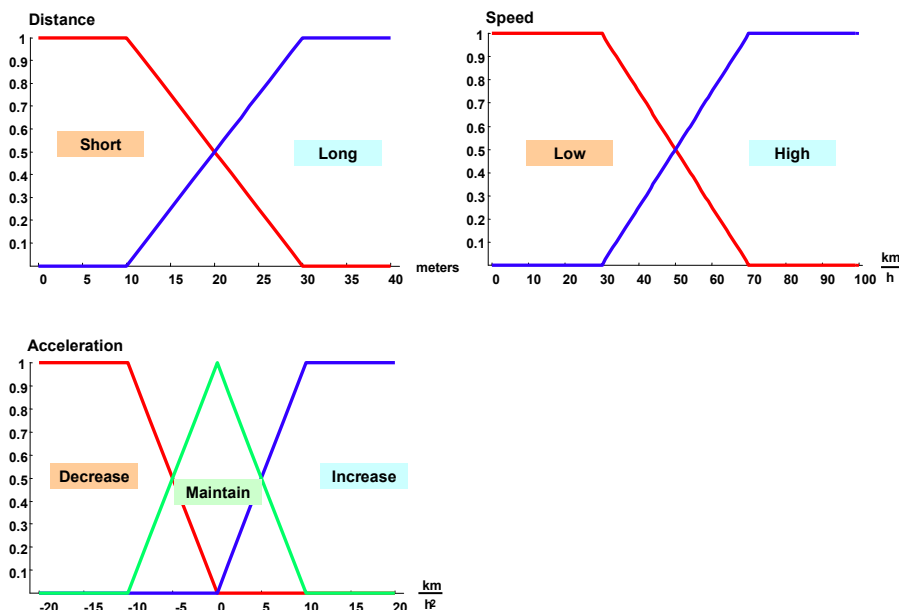


Figure 14. Fuzzy sets of the rules

With a given distance  $x_0 = 15$  meters and a speed of  $y_0 = 60$  km/h we perform step 1. The results are shown in Table 7.

Table 7. Fuzzy inference step 1

Rule	Short	Long	Low	High	Min
1	0.75		0.25		0.25
2	0.75			0.75	0.75
3		0.25	0.25		0.25
4		0.25		0.75	0.25

Now we must cut the membership function for the conclusion variable at the minimum values from step 1. The result is illustrated in Figure 15.

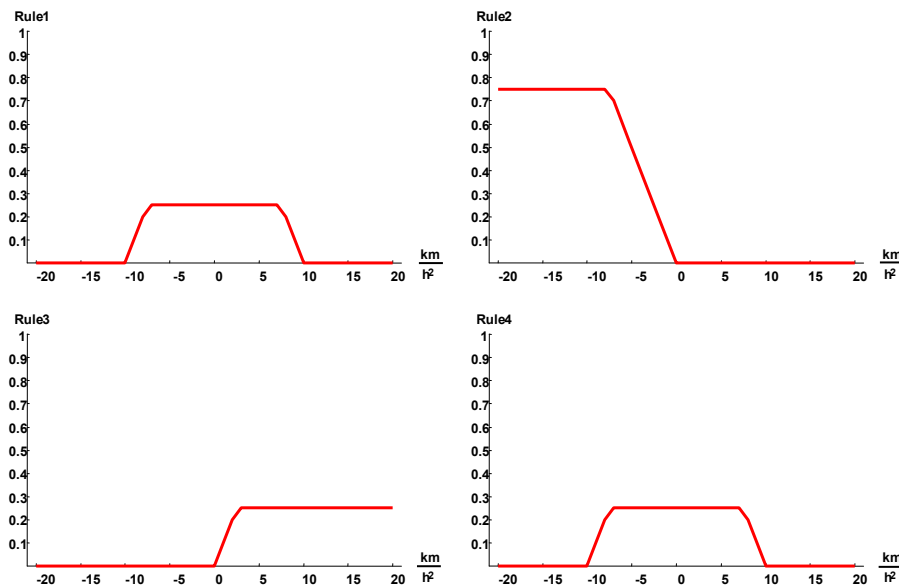


Figure 15. Fuzzy inference step 2

Finally, we must combine the individual membership functions from step 2 to the final result and defuzzify it. The union of the four membership functions is displayed in Figure 16. The final value after defuzzification is  $-5.46$  and is indicated by the blue dot. The conclusion of this fuzzy inference is that when the distance between the cars is 15 meters and the speed is 60 km/h, then we have to break gently to reduce the speed.

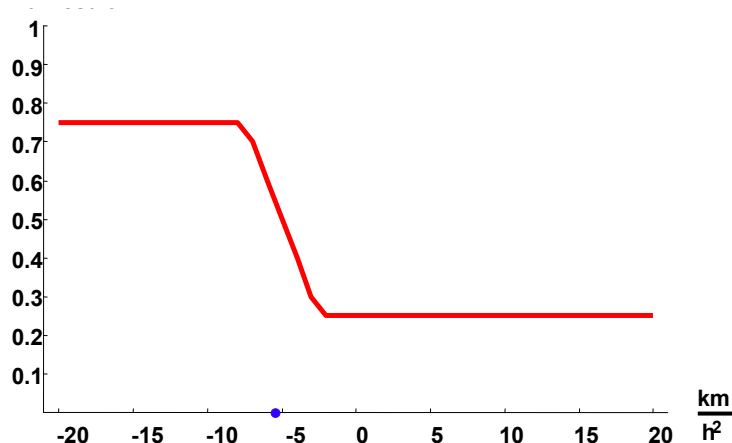


Figure 16. Fuzzy inference final result

### 1.8.2 Simplified Method

Often, the defuzzification process is too time-consuming and complicated. An alternative approach is the simplified method where the conclusion is a real value  $c$  instead of a fuzzy set. It is based on a generalized *modus ponens* of the form:

$$\begin{array}{l}
 p \Rightarrow q : \left\{ \begin{array}{l} \text{If } x \text{ is } A_1 \text{ and } y \text{ is } B_1 \text{ then } z \text{ is } c_1 \\ \text{If } x \text{ is } A_2 \text{ and } y \text{ is } B_2 \text{ then } z \text{ is } c_2 \\ \vdots \\ \text{If } x \text{ is } A_n \text{ and } y \text{ is } B_n \text{ then } z \text{ is } c_n \end{array} \right. \\
 \hline
 p_1 : \quad \quad \quad x \text{ is } A', y \text{ is } B' \\
 q_1 : \quad \quad \quad z \text{ is } c'
 \end{array}$$

Premise<sub>1</sub> becomes a set of rules as illustrated in Figure 17. The premise variables are fuzzy sets; the conclusion is a real number (fuzzy singleton).

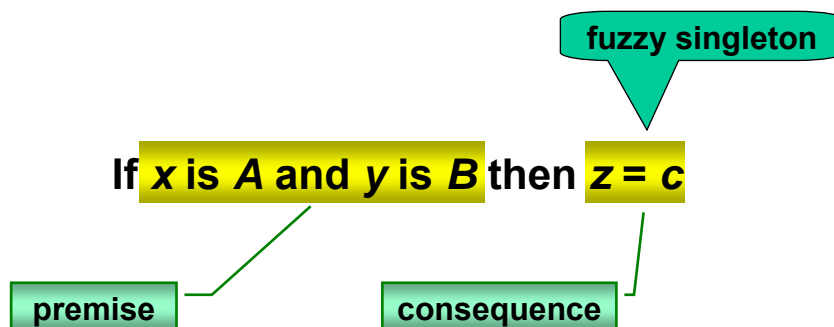


Figure 17. Simplified Method

The reasoning process is then straightforward in analogy to the previous method with the difference that the result is not a fuzzy set that needs to be defuzzified but we can compute the final result directly after step 2 in the algorithm.

The algorithm works as outlined in the following procedure. Let  $x_0$  and  $y_0$  be the input for the premise variables.

1. Apply the input values to the premise variables for every rule and compute the minimum of  $\mu_{A_i}(x_0)$  and  $\mu_{B_i}(y_0)$  :

$$\text{Rule}_1: m_1 = \min(\mu_{A_1}(x_0), \mu_{B_1}(y_0))$$

$$\text{Rule}_2: m_2 = \min(\mu_{A_2}(x_0), \mu_{B_2}(y_0))$$

$$\vdots$$

$$\text{Rule}_n: m_n = \min(\mu_{A_n}(x_0), \mu_{B_n}(y_0))$$

2. Compute the conclusion value per rule as:

$$\text{Conclusion of rule}_1: c'_1 = m_1 \cdot c_1$$

$$\text{Conclusion of rule}_2: c'_2 = m_2 \cdot c_2$$

$$\vdots$$

$$\text{Conclusion of rule}_n: c'_n = m_n \cdot c_n$$

3. Compute the final conclusion as:

$$c' = \frac{\sum_{i=1}^n c'_i}{\sum_{i=1}^n m_i}$$

**Example 14.** Given the slope and the aspect maps of a region and the following set of rules, we can conduct a risk analysis based on degrees of risk ranging from 1 (low risk) to 4 (very high risk). The fuzzy sets for flat and steep slope are displayed in Figure 18 and Figure 19-

- Rule 1 If slope is flat and aspect is favorable then risk is 1
- Rule 2 If slope is steep and aspect is favorable then risk is 2
- Rule 3 If slope is flat and aspect is unfavorable then risk is 1
- Rule 4 If slope is steep and aspect is unfavorable then risk is 4

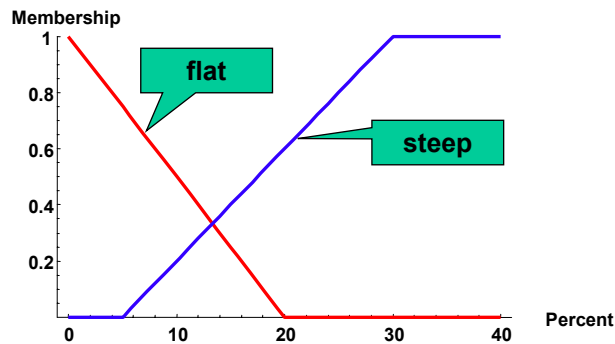


Figure 18. Membership functions for flat and steep slope

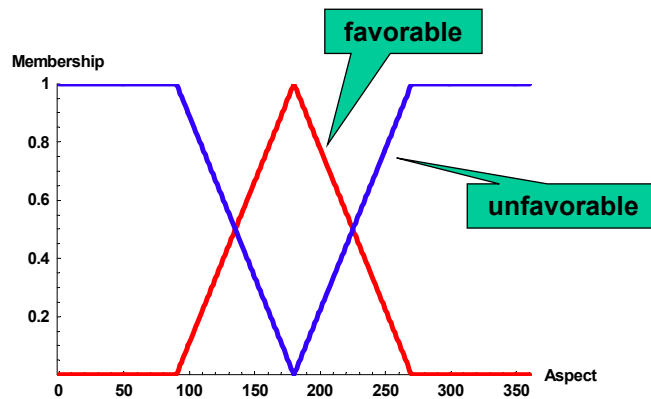


Figure 19. Membership functions for favorable and unfavorable aspect.



For a slope of 10 percent and an aspect of 180 degrees we have the following results:

	Slope (s)	Aspect (a)	Min(s,a)	Conclusions
Rule1	0.5	1	0.5	0.5
Rule2	0.2	1	0.2	0.4
Rule3	0.5	0	0	0
Rule4	0.2	0	0	0

For the final result we get  $c' = \frac{0.5+0.4+0+0}{0.5+0.2+0+0} = 1.29$ , which means a low risk.

## 1.8 Applications in GIS

Many spatial phenomena are inherently fuzzy or vague or possess indeterminate boundaries. Fuzzy logic has been applied for many areas in GIS such as fuzzy spatial analysis, fuzzy reasoning, and the representation of fuzzy boundaries. The following example illustrates how a fuzzy set can be computed from a given grid data set.

### 1.8.1 Objective

The objective of this analysis is to determine high elevation in the area covered by the 1 : 24,000 topographic map sheet of Boulder, Colorado

### 1.8.2 Fuzzy Concepts

Elevation is considered *high* when it is above 1,700 meters. We represent the features meeting the criterion as a fuzzy set with a sinusoidal membership function (Figure 20) defined as

$$\mu_{\text{high elevation}}(x) = \begin{cases} 0 & x \leq 1700 \\ \frac{1}{2} \left( 1 - \cos \left( \pi \frac{x-1700}{300} \right) \right) & 1700 < x \leq 2000 \\ 1 & x > 2000 \end{cases}$$

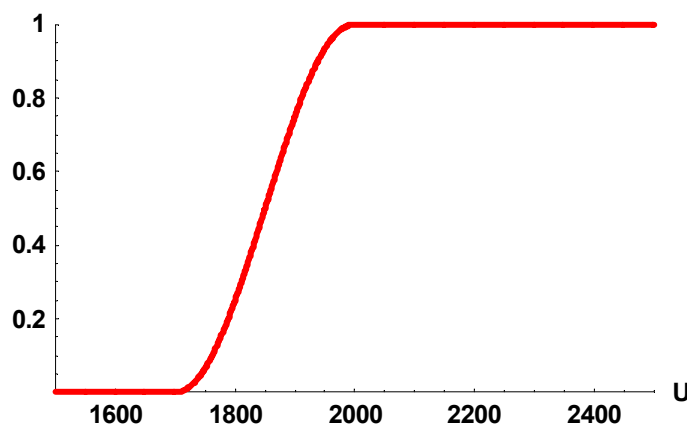


Figure 20. Membership function for “high elevation”

### 1.8.3 Software Approach

The 1 : 24K DEM was downloaded from the USGS and imported into ArcGIS as a grid ELEVATION. In principle, there are several ways to solve the problem: we can use ArcInfo GRID, ArcMap Spatial Analyst, or ArcView 3.x Spatial Analyst.

We can even create our own fuzzy logic tool using the scripting environment of the geoprocessor in ArcGIS 9. In the following, all approaches are illustrated. The grid involved is ELEVATION. The fuzzy set will be a grid FELEVATION whose values are between zero and one.

### 1.8.3.1 ArcInfo GRID

To compute the fuzzy set we use an AML script that is run from ArcInfo GRID and the DOCELL block:

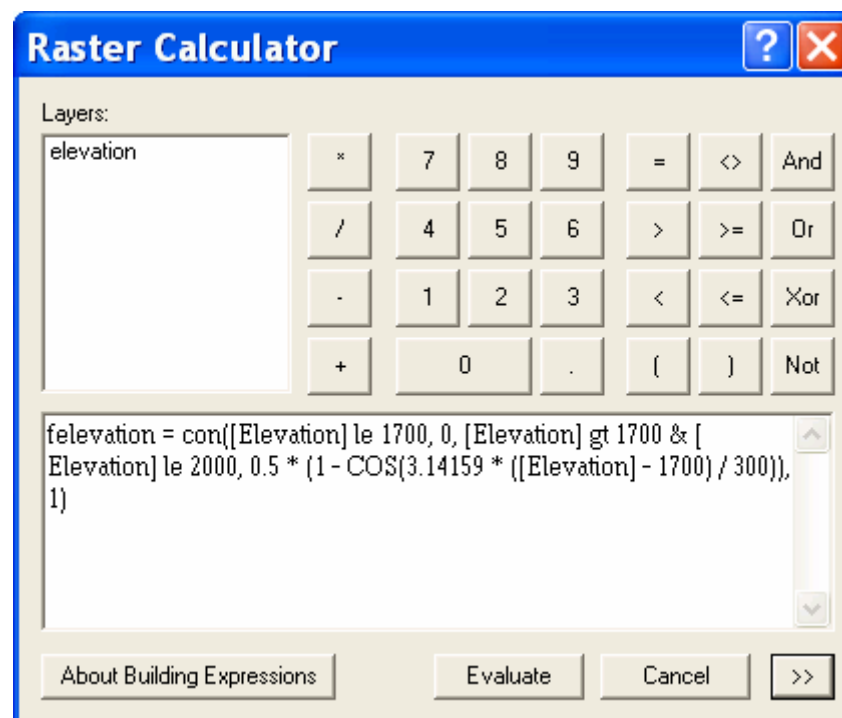
```
/*
/* high elevation
/* =====
/*
dozell
  if (elevation le 1700) felevation = 0
  if (elevation gt 1700 & elevation le 2000) ~
    felevation = 0.5 * (1 - COS(3.14159 * (elevation - 1700) / 300))
  if (elevation gt 2000) felevation = 1
end
```

We can also use the GRID CON command:

```
/* high elevation
/* =====
/*
felevation = con(elevation le 1700, 0, elevation gt 1700 & elevation ~
le 2000, 0.5 * (1 - COS(3.14159 * (elevation - 1700) / 300)), 1)
```

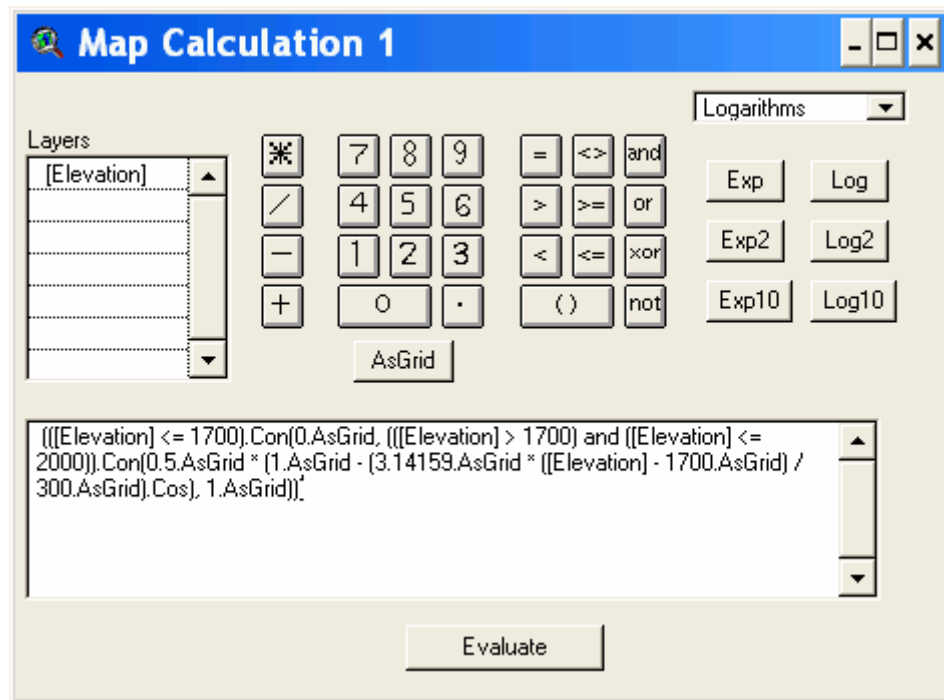
### 1.8.3.2 ArcMap Spatial Analyst

To solve the problem we use the raster calculator of the Spatial Analyst. The following screen dump shows the command to produce the required fuzzy set.



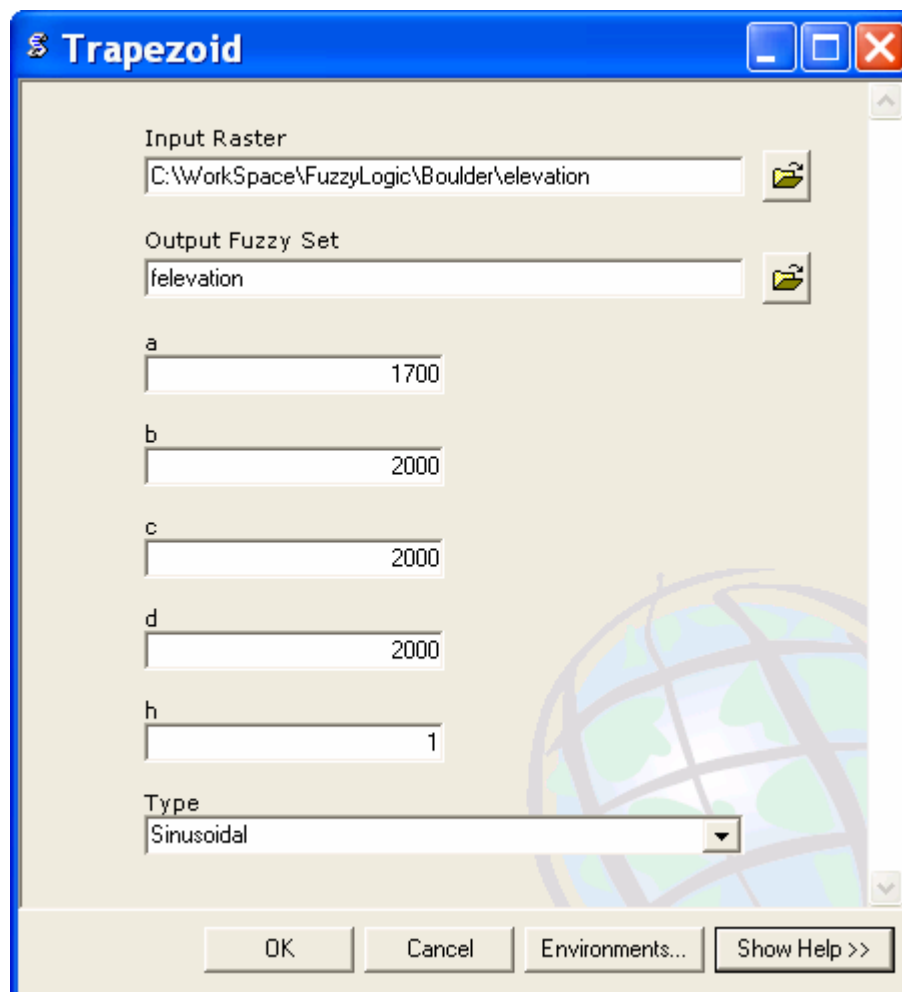
### 1.8.3.3 ArcView 3.x

If you do not have ArcGIS available, the same results can be achieved by using requests in the ArcView GIS Spatial Analyst map calculator. The following screen dump shows how to use the Avenue **con** request for computing the fuzzy set for high elevation.



#### 1.8.3.4 ArcGIS 9 Script

We have written a Python script that generates a fuzzy raster data set from a given input raster data set. This script is used here as a tool in the ArcToolbox.



### 1.8.4 Result

Figure 21 shows the result of the analysis with a fuzzy logic approach (left map) and a crisp approach (right map). The grid size has been set to 10 meters according to the grid cell size of the elevation model.

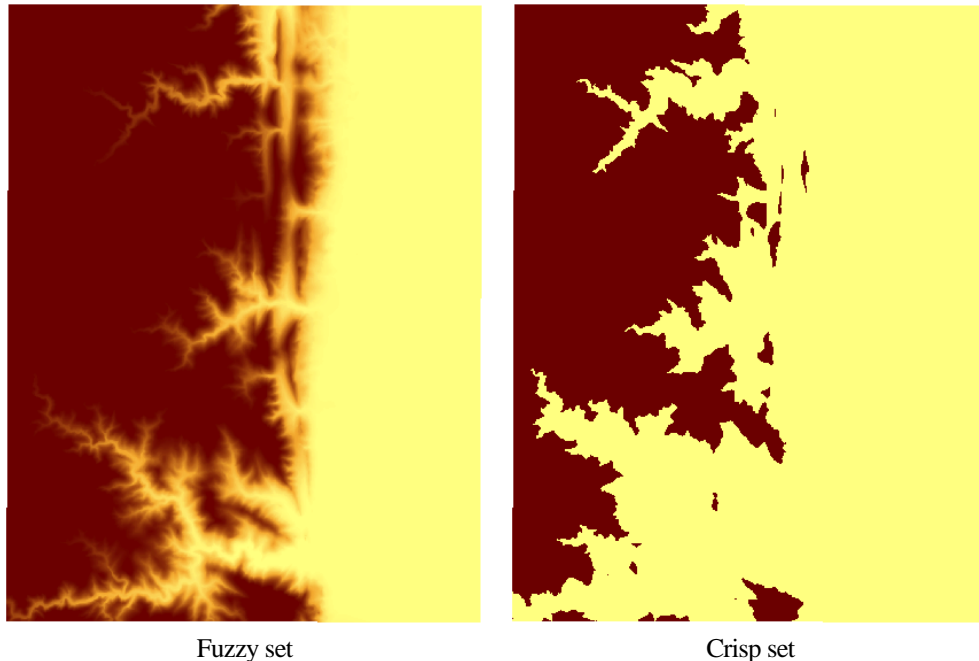


Figure 21. Analysis with a fuzzy logic approach (left) and a crisp approach (right)

## 1.9 Exercises

*Exercise 1* Determine a linear membership function for “moderate elevation” when the ideal elevation is between 400 and 600 meters.

*Exercise 2* Determine a Gaussian membership function for the aspect “south.”

*Exercise 3* Use the 1:24,000 digital topographic data set of Boulder, Colorado and determine a suitable site with the following characteristics:

- (i) moderate slope
- (ii) favorable aspect
- (iii) moderate elevation
- (iv) near a lake or reservoir
- (v) not very close to a major road, and
- (vi) not in a park or military reservation.

Choose suitable membership functions for the fuzzy terms.

*Exercise 4* Design a simple fuzzy reasoning system for avalanche risk in the Rocky Mountains (Boulder, CO area). The variables involved are slope, aspect, snow cover change. For simplicity we do not consider surface cover. The snow cover change must be simulated. The rules are given as:

Rule 1: If the slope is very steep and the aspect is unfavorable and the snow cover change is big then the risk is very high.

- Rule 2: If the slope is moderate and the aspect is unfavorable and the snow cover change is big then the risk is moderate.
- Rule 3: If the slope is steep and the aspect is unfavorable and the snow cover change is small then the risk is low.
- Rule 4: If the slope is not steep and the aspect is unfavorable and the snow cover change is big then the risk is moderate.

## 1.10 Literature

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