Complexity-Sensitive Decision Procedures for Abstract Argumentation

KR 2012 (Rome)

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Introduction

- Formal Models of Argumentation are a vivid field within KR and AI.
- Underlying model: Dung's abstract argumentation frameworks.
- Simple, yet powerful formalism:
  - Some important reasoning problems are hard even for the second level of the polynomial hierarchy.
  - However, certain fragments show milder complexity.

Introduction (ctd.)

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  - If an instance is from a tractable fragment, the procedure should terminate in polynomial time.
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  - If an instance is from a tractable fragment, the procedure should terminate in polynomial time.
  - If an instance of a second-level problem is from an NP-fragment, the procedure should terminate after a single call (or small number of calls) to an NP-oracle (e.g. a SAT solver).
- Taking a SAT-solver as underlying engine
  - gives access to the sophisticated SAT solver technology
  - might even pay off for general instances (where an exponential number of calls is required).

Main Contributions

In this work we concentrate on complexity sensitive procedures for argumentation reasoning problems at the second level of the polynomial hierarchy (i.e. for preferred, semi-stable and stage semantics):

- We identify new NP/coNP fragments
- We augment NP fragments by certain parameterizations (syntactic distance and semantic parameters)
- We provide a novel schema for complexity sensitive procedures (in the talk: focus on preferred semantics).

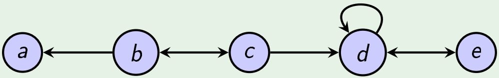
Dung's Abstract Argumentation Frameworks

Definition

An argumentation framework (AF) is a pair (A, R) where

- A is a set of arguments
- $R \subseteq A \times A$ is a relation representing the conflicts ("attacks")

Example

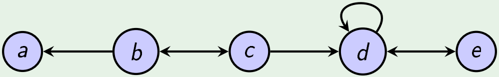


Basic Properties

Conflict-Free Sets

Given an AF $F = (A, R)$.
A set $S \subseteq A$ is conflict-free in F, if, for each $a, b \in S$, $(a, b) \notin R$.

Example



$cf(F) = \{\{a, c, e\}, \{b, e\}, \{a, c\}, \{a, e\}, \{c, e\}, \{a\}, \{b\}, \{c\}, \{e\}, \emptyset\}$

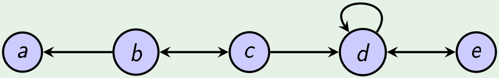
Basic Properties (ctd.)

Admissible Sets [Dung, 1995]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is admissible in F, if

- S is conflict-free in F
- each $a \in S$ is defended by S in F
  - $a \in A$ is defended by S in F, if for each $b \in A$ with $(b, a) \in R$, there exists a $c \in S$, such that $(c, b) \in R$.

Example



$adm(F) = \{\{a, c, e\}, \{b, e\}, \{a, c\},$ ~~$\{a, e\}$~~$, \{c, e\},$ ~~$\{a\}$~~$, \{b\}, \{c\}, \{e\}, \emptyset\}$

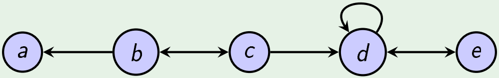
Semantics

Complete Extension [Dung, 1995]

Given an AF (A, R). A set $S \subseteq A$ is complete in F, if

- S is admissible in F
- each $a \in A$ defended by S in F is contained in S
  - $a \in A$ is defended by S in F, if for each $b \in A$ with $(b, a) \in R$, there exists a $c \in S$, such that $(c, b) \in R$.

Example



$com(F) = \{\{a, c, e\}, \{b, e\},$ ~~$\{a, c\}$~~, ~~$\{c, e\}$~~, $\{b\},$ ~~$\{c\}$~~, $\{e\}, \emptyset\}$

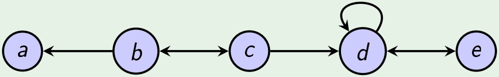
Semantics (ctd.)

Preferred Extensions [Dung, 1995]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a preferred extension of F, if

- S is admissible in F
- for each $T \subseteq A$ admissible in F, $S \not\subset T$

Example



$prf(F) = \{\{a, c, e\}, \{b, e\},$ ~~$\{a, c\}$~~, ~~$\{c, e\}$~~, ~~$\{b\}$~~, ~~$\{c\}$~~, ~~$\{e\}$~~, $\emptyset\}$

Semantics (ctd.)

Stable Extensions [Dung, 1995]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a stable extension of F, if

- S is conflict-free in F
- for each $a \in A \setminus S$, there exists a $b \in S$, such that $(b, a) \in R$

Semantics (ctd.)

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Semi-Stable/Stage Extensions [Verheij 1996, Caminada 2006]

Based on admissible/conflict-free sets but maximizing the "range".

Main Reasoning Problems

Credulous Acceptance

$Cred_\sigma$: Given AF $F = (A, R)$ and $a \in A$; is a contained in *at least one* σ-extension of F?

Skeptical Acceptance

$Skept_\sigma$: Given AF $F = (A, R)$ and $a \in A$; is a contained in *every* σ-extension of F?

Computational Complexity

σ	$Cred_\sigma$	$Skept_\sigma$
stb	NP-c	coNP-c
com	NP-c	P-c
prf	NP-c	Π_2^P-c
sem	Σ_2^P-c	Π_2^P-c
stg	Σ_2^P-c	Π_2^P-c

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Implications:

- for AFs with unique preferred extension, $Skept_{prf}$ becomes easier;
- for coherent AFs ($stb = prf$), $Skept_{prf}$ becomes easier;
- for stablecons AFs ($stb \neq \emptyset$), *sem* and *stg* become easier.

NP / coNP Fragments

Goal: identify classes of instances where reasoning tasks fall into first level of polynomial hierarchy.

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Table: Complexity when the AF belongs to a sub-class $\mathcal{G}$.

$\mathcal{G}$	$Skept_{prf}$	$Cred_{sem}$	$Skept_{sem}$	$Cred_{stg}$	$Skept_{stg}$
acyc	P-c	P-c	P-c	P-c	P-c
ocf	coNP-c	NP-c	coNP-c	NP-c	coNP-c
wcyc	coNP-c	Σ_2^P-c	Π_2^P-c	Σ_2^P-c	Π_2^P-c
uniqpref	in NP	in NP	in NP	Σ_2^P-c	Π_2^P-c
coherent	coNP-c	NP-c	coNP-c	NP-c	coNP-c
stablecons	Π_2^P-c	NP-c	coNP-c	NP-c	coNP-c

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Table: Complexity when parameterized by the distance to a sub-class $\mathcal{G}$

$\mathcal{G}$	$Skept_{prf}$	$Cred_{sem}$	$Skept_{sem}$	$Cred_{stg}$	$Skept_{stg}$
acyc	FPT	FPT	FPT	Σ_2^P-c	Π_2^P-c
ocf	Π_2^P-c	Σ_2^P-c	Π_2^P-c	Σ_2^P-c	Π_2^P-c
wcyc	Π_2^P-c	Σ_2^P-c	Π_2^P-c	Σ_2^P-c	Π_2^P-c
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coherent	Π_2^P-c	Σ_2^P-c	Π_2^P-c	Σ_2^P-c	Π_2^P-c

Semantical Parameterization

Idea: Take number of extensions into account, e.g.

- sol_σ^k as the class of AFs with at most k σ-extensions;
- coherent^k as the class of AFs where $|prf(F) \setminus stb(F)| \leq k$;
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Table: Complexity when the AFs belong to a sub-class $\mathcal{G}$ (for fixed k).

$\mathcal{G}$	$Skept_{prf}$	$Cred_{sem}$	$Skept_{sem}$	$Cred_{stg}$	$Skept_{stg}$
sol_σ^k	in P^{NP}	in P^{NP}	in P^{NP}	in P^{NP}	in P^{NP}
coherent^k	Π_2^P-c	in P^{NP}	in P^{NP}	in P^{NP}	in P^{NP}
$\text{stablecons}_\sigma^k$	–	in P^{NP}	in P^{NP}	in P^{NP}	in P^{NP}

CEGARTIX for Skeptical Acceptance in Preferred Semantics

Input: AF $\mathcal{F} = (A, R)$, argument $\alpha \in A$

1. $\varphi \leftarrow \varphi_{com}(\mathcal{F}) \wedge \neg x_\alpha$
2. **while** (φ is satisfiable)
   1. find model I of φ
   2. **while** (there is model I' of $\varphi_{com}(\mathcal{F}) \wedge \bigwedge_{x \in I \cap X} x \wedge (\bigvee_{x \in X \setminus I} x) \wedge \neg x_\alpha$)
      $I \leftarrow I'$
   3. **if** ($\varphi_{com}(\mathcal{F}) \wedge \bigwedge_{x \in I \cap X} x \wedge (\bigvee_{x \in X \setminus I} x)$ is unsatisfiable) **then** *reject*
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3. *accept*

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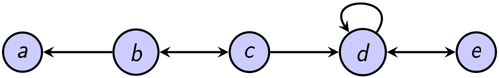

Skeptical acceptance of argument e:

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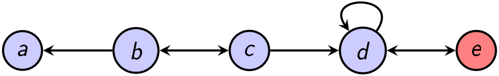

Skeptical acceptance of argument e:

Step 1: initialize φ: $\varphi \leftarrow \varphi_{com}(\mathcal{F}) \wedge \neg x_e$

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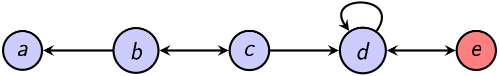

Skeptical acceptance of argument e:

Step 2: φ is satisfiable, e.g. $\emptyset$ is complete and $e \notin \emptyset$.

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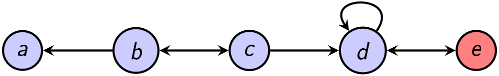

Skeptical acceptance of argument e:

Step 2.1: We keep the model representing $\emptyset$.

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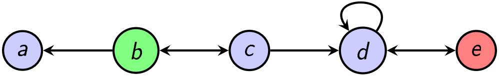

Skeptical acceptance of argument e:

Step 2.2: maximizing the complete set without adding e.
We can add b and thus we obtain the extension $\{b\}$.

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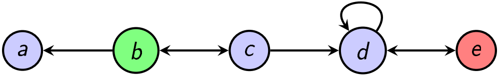

Skeptical acceptance of argument e:

Step 2.2: $\{b\}$ is maximal (given $\neg x_e$)

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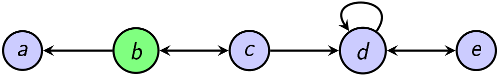

Skeptical acceptance of argument e:

Step 2.3: $\{b, e\}$ is a superset of $\{b\}$ and thus $\{b\}$ is not a counter-example for e being skeptically accepted.

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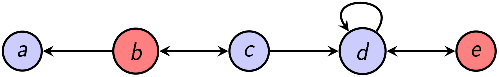

Skeptical acceptance of argument e:

Step 2.4: We exclude $\{b\}$ from further investigation by adding the clause $x_a \vee x_c \vee x_d \vee x_e$.

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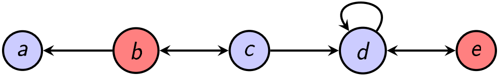

Skeptical acceptance of argument e:

Step 2: φ is unsatisfiable, as there is no complete extension neither containing b or e.

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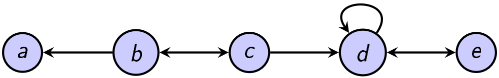

Skeptical acceptance of argument e:

Step 3: Skeptically accept e

Experimental Evaluation (ctd).

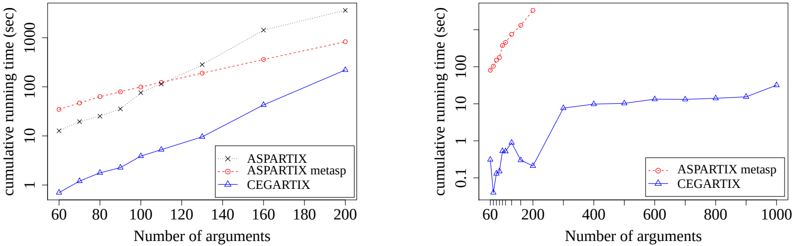

Figure: Comparison of CEGARTIX (using Minisat) and ASPARTIX (using claspD); cumulative running times over the random instances (left) and grid instances (right).

Conclusion

- Investigation of complexity of hard argumentation problems when frameworks are from certain fragments
  - focus on fragments which lower the complexity to NP/coNP;
  - only semantical parameterizations allow to extend these fragments.
- Novel algorithm which is complexity-sensitive for such a parameterization
  - in the paper: further shortcuts; also semi-stable and stage semantics.
- First experiments are promising, but further analyses required.
- Next Step: investigation of further fragments and parameters.
- System available under `www.dbai.tuwien.ac.at/research/project/argumentation/cegartix/`

Experimental Evaluation - Setting

Questions:

- Dedicated vs. SAT-based approach
- Does incremental SAT-solving help.

Machine: OpenSUSE with Intel Xeon processors (2.33 GHz) and 49 GB memory

Systems:

- ASPARTIX: gringo v3.0.3, claspD v1.1.1
- ASPARTIX (based on metasp): gringo v3.0.3, claspD v1.1.1
- CEGARTIX incremental: MINISAT v2.2.0
- CEGARTIX: MINISAT v2.2.0
- CEGARTIX: clasp v2.0.5

Benchmark instances:

- Fully Random Graphs varying the edge density
- Random graphs based on a grid structure

Experimental Evaluation - SAT-solvers

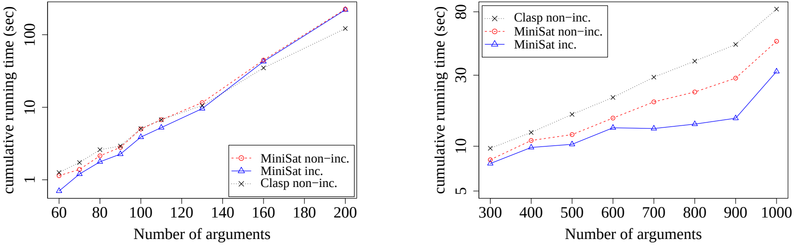

Figure: Comparison of different variants of CEGARTIX (non-incremental and incremental applications of Minisat, non-incremental application of Clasp): cumulative running times over the random instances (left) and grid instances (right).