

Chemical Crystallography and Structural Chemistry

(VO 270287)

Lecture 8

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Previous Lecture

1. Model building
2. Refinement
3. Constraints & Restraints

Today's Lecture

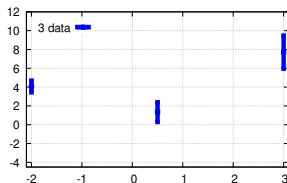
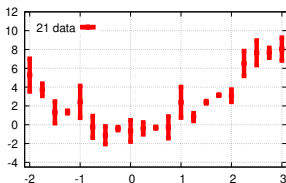
1. Example of constraints and data:parameter ratio
2. Validation

Example: Stabilisation through restraints

Two hypothetic measurements:

Experiment 1: high resolution, 21 pairs of measurements $(x_1, y_1), \dots, (x_{21}, y_{21})$
and errors $\sigma_1, \dots, \sigma_{21}$

Experiment 2: low resolution, 3 pairs of measurements $(x_1, y_1), \dots, (x_3, y_3)$
and errors $\sigma_1, \dots, \sigma_3$



Example: Stabilisation through restraints

Testing two models:

$$\text{Model 1: } g(x) = g_2x^2 + g_1x + g_0$$

$$\text{Model 2: } h(x) = h_3x^3 + h_1x + h_0$$

Either model has three parameters, g_0, g_1, g_2 and h_0, h_1, h_3 respectively. These parameters correspond to e.g. the model coordinates (x_i, y_i, z_i) , or the ADPs U_i .

We will fit both models to the data to find out which model better represents the data.

Example: Stabilisation through restraints

Least-squares-minimisation:

$$\text{minimise: } \sum_{i=1}^N \frac{1}{\sigma_i^2} (y_i - g(x_i))^2 \quad \text{model 1}$$

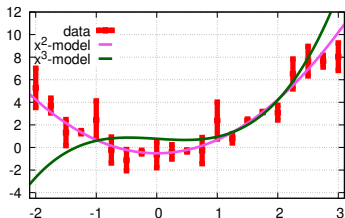
$$\text{minimise: } \sum_{i=1}^N \frac{1}{\sigma_i^2} (y_i - h(x_i))^2 \quad \text{model 2}$$

- **Experiment 1:** $N = 21$ data points
- **Experiment 2:** $N = 3$ data points

We will start with the high resolution **experiment 1**

Example: Stabilisation through restraints

experiment 1: high resolution; high data to parameter ratio = 21:3=7



$$\text{Model 1: } 1.2x^2 + 0.0x - 0.5$$

rmsd: 1.07

$$\text{Model 2: } 0.5x^3 - 0.3x - 0.8$$

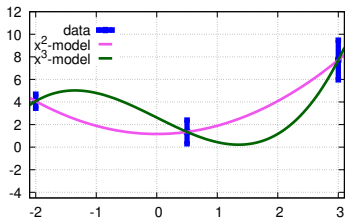
rmsd: 4.74

The *root mean square deviation* rmsd between model and data corresponds to the crystallographic $R1$ value.

The lower rmsd 1.07 clearly favours model 1. The pink curve also visually fits the data better than the green curve.

Example: Stabilisation through restraints

experiment 2: low resolution, low data to parameter ration = 3:3 = 1



model 1: $0.7x^2 + 0.0x + 1.2$

rmsd: 0

model 2: $0.5x^3 - 2.7x - 2.6$

rmsd: 0

When there are as many parameters as data points, any model can be fitted perfectly to the data. We cannot distinguish between the two models

Example: Stabilisation through restraints

experiment 2: low resolution with constraint

For some reason we know that the data must pass through the point (0,0).
For the two models this means

$$\begin{aligned}0 &= g(0) \\ &= g_2 * 0^2 + g_1 * 0 + g_0 \\ \Rightarrow g_0 &= 0\end{aligned}$$

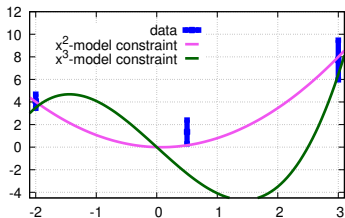
and analogously

$$h_0 = 0$$

The constraint reduced the number of parameters, only two parameters per model

Example: Stabilisation through restraints

experiment 2: low resolution with constraint



model 1: $0.9x^2 - 0.1x$

rmsd: 1.13

model 2: $0.8x^3 - 4.9x$

rmsd: 3.7

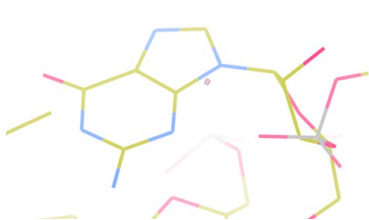
Due to the constraint, data to parameter ratio = 3:2 = 1.5. Now there is an *rmsd*, and it favours (again) the first model.

Summary & model building

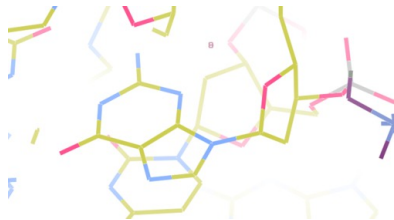
- (*cf.* phase problem)
- phases are calculate from the model
- model phases and observed data yield the electron density map, and electron difference map
- model building improves the model in large steps
- refinement optimises the model against the data
- medium resolution data or poor quality data require restraints and constraints in order to create a chemically sensible model

Model quality and data quality: structure validation

Atom coordinates \neq model accuracy



Guanine model in ribosome, data resolution 3.1 Å



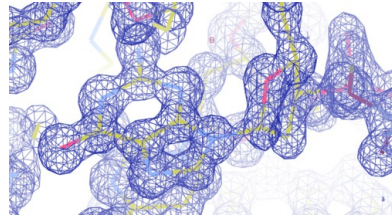
Guanine model in Z-DNA, at resolution 1.0 Å

The coordinates of the model do not reveal the data quality, nor the model quality.

model coordinates = interpretation of data



Guanine model **with map** in ribosome, data resolution 3.1Å



Guanine model **with map** in Z-DNA, at resolution 1.0 Å

Only in combination with the data can we judge the model quality

Once more: data to parameter ratio

Example Ciprofloxacin ($a = 9.5\text{\AA}$, $b = 9.9\text{\AA}$, $c = 11.0\text{\AA}$, $\alpha = 94.2^\circ$, $\beta = 100.2^\circ$, $\gamma = 91.3^\circ$)

- $FC_{17}N_3O_9H_{30}$: $60 \times 9 = 540$ Parameter

data resolution 0.43 Å: 26'308 reflections $\hat{=}$ 48.7 data points per parameter: very high, reliable refinement

data resolution 0.8 Å: 2'926 reflections $\hat{=}$ 5.4 data points per parameter: medium, refinement needs checking

Once more: data to parameter ratio

Example Ribosome ($a = 401.4\text{\AA}$, $b = 401.4\text{\AA}$, $c = 175.9\text{\AA}$, $\alpha = \beta = \gamma = 90^\circ$, $P4_12_12$)

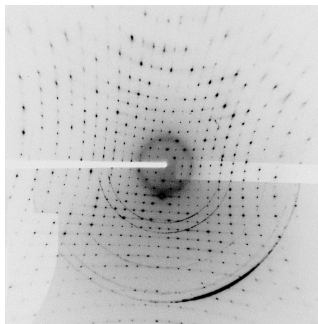
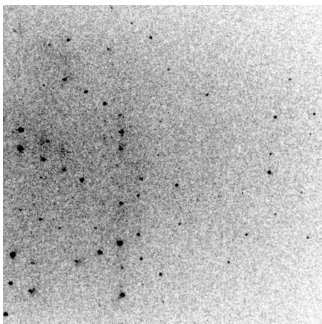
- PDB ID 1J5E: 51'atoms atoms = 207'768 parameters
- data resolution 3.05 Å 238'205 reflections

$$\frac{238'205}{207'768} = 1.15$$

Even at such low data to parameter ratio can a reasonable model be built and refined. It is important to be aware of differences in the interpretation of the data

Quality indicators

Example data quality



Important quality indicators

R_{meas} relative difference between:

1. measured data
2. calculated data

data completeness : fraction of measured data w.r.t. theoretically possible data

multiplicity (*alias: redundancy*): how often every unique reflection was measured (on average)

signal strength $I(hkl)/\sigma_{I(hkl)} < 1$: noise

CC_{1/2}

1. split data set into two random halves
2. calculated correlation coefficient between symmetry equivalent reflections

R-values for data

The classic data quality indicator is R_{int} , alias R_{merge} or R_{sym} :

$$R_{\text{int}} = \sum_h \sum_j \frac{|I_{hj} - \langle I_h \rangle|}{\langle I_h \rangle}$$

R_{int} mathematically increases with multiplicity, although data quality improves with multiplicity

R_{int} is typically shown in publications. It is, however, obsolete and should not be published. R_{meas} *alias* $R_{\text{r.i.m.}}$ should be published instead:

$$R_{\text{meas}} = \sum_h \frac{n_h}{n_h - 1} \sum_j \frac{|I_{hj} - \langle I_h \rangle|}{\langle I_h \rangle}$$

Example data statistics (XPREP)

Resolution	#Data	#Theory	%Complete	Redundancy	Mean I	Mean I/s	Rmerge
Inf - 2.46	196	197	99.5	39.27	215.01	110.27	0.0300
2.46 - 1.13	1762	1825	96.5	14.86	75.32	42.01	0.0453
1.13 - 0.89	1972	2123	92.9	8.71	25.52	19.00	0.0895
0.89 - 0.77	2007	2258	88.9	6.81	10.84	10.39	0.1425
0.77 - 0.69	1864	2499	74.6	3.37	5.66	5.76	0.1885
0.69 - 0.62	2108	3360	62.7	2.24	2.88	3.29	0.2890
0.62 - 0.57	1929	3542	54.5	1.44	1.51	1.79	0.4191
0.57 - 0.54	1123	2367	47.4	1.10	0.90	1.14	0.5593

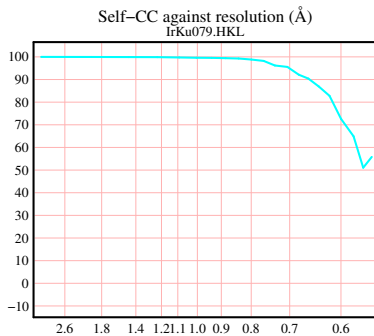
0.64 - 0.54	3720	7014	53.0	1.43	1.47	1.76	0.4170
Inf - 0.54	12961	18171	71.3	5.08	20.64	13.61	0.0514

Merged [A], lowest resolution = 11.49 Angstroms

CC1/2, and resolution cut-off

- CC1/2 should be close to 100% throughout resolution range
- where CC1/2 drops below 70%, noise becomes significant, and data at higher resolution can be excluded from refinement
- $I/\sigma(I)$ should be about 2, where CC1/2 about 70%
- $I/\sigma(I)$ should be about 1, where CC1/2 about 40% (in cases very resolution cut-off is critical)

Example $CC_{1/2}$, and resolution cut-off



$CC_{1/2}$ vs. data resolution

R-values for the model

$$R = R1 = \sum_h \frac{||F_h(data)| - |F_h(model)||}{|F_h(data)|}$$

weighted R-value:

$$wR = \sum_h \frac{|w_h|F_h(data)| - |F_h(model)||}{w_h|F_h(data)|}$$

weighted intensity based R-value:

$$wR2 = R_B = \sqrt{\sum_h \frac{|w_h(I_h(data) - I_h(model))|^2}{w|I_h(data)|^2}}$$

Small molecules: $R1$ of the refined model 2-5 %.

GooF

Goodness of Fit

$$GooF = \sqrt{\frac{\sum_h w_h (F_h^2(data) - F_h^2(model))^2}{n - p}}$$

- Takes number of parameters (p) and number of data (n) into account
- Ideally ≈ 1 , increases with worse model

model: residual density

SHELXL calculates the “highest peak” and “deepest hole” in the electron density map. Units are electrons, e.g. at the beginning of model building:

Electron density synthesis with coefficients Fo-Fc

Highest peak 4.95 at 0.5434 0.9981 0.3231 [0.04 A from RU01

Deepest hole -3.34 at 0.0057 0.4976 0.3299 [0.99 A from RU02

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Mean =        0.00,        Rms deviation from mean =        0.34 e/A<sup>3</sup>

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model: residual density

SHELXL calculates the “highest peak” and “deepest hole” in the electron density map. Units are electrons, e.g. for the refined model:

Electron density synthesis with coefficients Fo-Fc

Highest peak 0.50 at 0.6610 0.1969 0.4278 [0.69 A from C006

Deepest hole -1.22 at 0.2635 0.6156 0.2132 [0.04 A from P003

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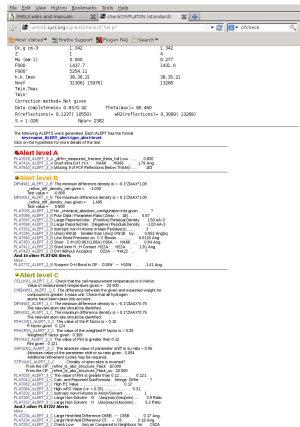
Mean = 0.00, Rms deviation from mean = 0.06 e/Å<sup>3</sup>

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checkCIF (PLATON web-based)



The screenshot shows the checkCIF web interface. At the top, it says "checkCIF is sponsored by Crystallography Journals Online". Below that, it explains that checkCIF reports on the consistency and quality of crystal structure determinations. There is a section for "Please upload your CIF using the form below" with a "Browse" button. Below this are several checkboxes for validation options: "Full validation of CIF and structure factors", "Validation of CIF only (no structure factors)", "Check CIF for warnings", "Check CIF for errors", "Full validation of CIF and structure factors", and "Validation of CIF only (no structure factors)". There is also a "Send CIF for checking" button. At the bottom, there is a "CheckCIF" section with a "Prepublication check for submissions to RSC journals" and a "Download CIF editor (checkCIF) from the RSC" link.



The screenshot shows the checkCIF results page. At the top, it displays the CIF file name and the checkCIF version. Below this, there is a table of CIF data:

Item	Value	Warning	Alert
Cell dim. (Å)	1.342	1.342	
Z	2		
No. Obs (h)	2,800	2,217	
R _{int}	0.027	0.027	
R _{sigma}	0.041	0.041	
h, k, l max	36, 35, 21		
Total	13089		
Test			
Correction method: Not given			
Data completeness: 0.995 (0)		0.995	
Reflections: 6, 1221 (1000)		6 (0.5%)	
S = 1.029	None = 2982		

Below the table, there are sections for "Alert level A", "Alert level B", and "Alert level C". Each section lists specific CIF errors and their severity. For example, under "Alert level A", there is a warning about the maximum of observed reflections being less than 2000. Under "Alert level B", there are warnings about the maximum of observed reflections being less than 1000 and the maximum of observed reflections being less than 500. Under "Alert level C", there are warnings about the maximum of observed reflections being less than 200, the maximum of observed reflections being less than 100, and the maximum of observed reflections being less than 50.

Every published structure *should* have a checkCIF report. There are different alert levels of decreasing severity. Reviewers typically require that a structure should **not** contain A- or B-alerts.

Summary

- A model without data does not reflect data quality
- Data quality: data resolution, multiplicity, R-values, I/σ_I , $CC_{1/2}$
- Model quality: R1-values, GooF, residual density
- available for everyone: checkCIF <http://checkcif.iucr.org> (with or without data)
- *ALERT levels* A, B, ...
- (Analogously for macromolecular structures: <http://molprobity.biochem.duke.edu/>)

End of lecture