An analytical model for the amplitude of lee waves forming on the boundary layer inversion

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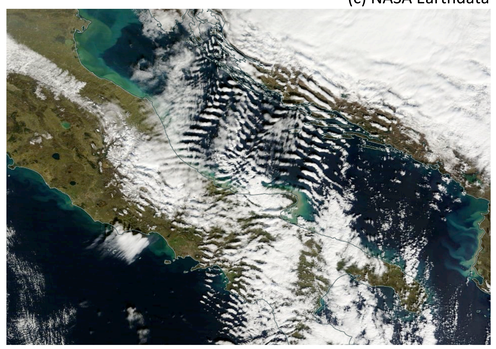
Introduction

Lee waves are horizontally propagating internal gravity waves that are trapped vertically in a layer of finite depth. In this layer they can extend horizontally over hundreds of kilometers. When condensation occurs in their crests, waves can be frequently observed in satellite images in the lee of mountainous areas around the globe (see image below).

(c) NASA Earthdata



Lee wave train emerging from the Croatian coast. (MODIS image on 10 Feb. 2015)

Motivation

The factors governing the **wavelength** of lee waves are **well understood** from **linear theory**, but the **amplitude is still relatively unexplored**.
Large amplitude lee waves can cause low level turbulence in atmospheric rotors and pose a hazard for aviation.

Aim of this study:

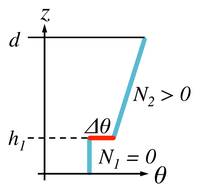
Formulate a simple analytical model for the amplitude of waves on the boundary layer inversion

$$A = f(N_2, U, \Delta\theta, h_1, \ldots)$$

Analytical amplitude model

The idea:

Describe the energy source and the wave propagation with two separate models. A similar approach has been used almost a century ago to determine the amplitude of waves on the water surface



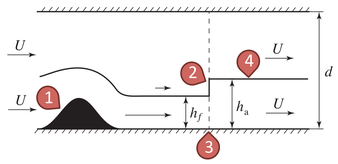
Large amplitude lee waves can be explained with shallow water theory:

1. **High obstacle**: the flow is partially blocked and a **hydraulic jump** forms
2. **Dissipation** in the jump: the **flow loses energy** in the form of turbulence
3. Rayleigh (1912) found that the **rate of energy loss** depends on the **jump size (h_a - h_f)**
4. Favre (1935) found that in **weak jumps** energy does not dissipate but radiate through **laminar waves**
5. Lemoine (1948) coupled Rayleigh's energy loss model with a linear wave model

In this work, we extend Lemoine's approach to determine the amplitude of atmospheric lee waves on the boundary layer inversion.

Details

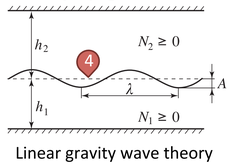
Energy source model (Klemp et al. 1997):



Two-layer hydraulic theory

$$\Delta EF = -\frac{\hat{\rho}U^3 d(d-h_a)(d-2h_f)(h_f-h_a)^3}{2h_f^2(d-h_f)^2(2d-h_f-h_a)}$$

Wave propagation model:



Linear gravity wave theory

$$A = \sqrt{\frac{4EF_x}{\hat{\rho}C^3}}\left\{n_1\left[\frac{e^{2n_1h_1} - e^{-2n_1h_1} + 4n_1h_1}{(e^{n_1h_1} - e^{-n_1h_1})^2}\right] + n_2\left[\frac{e^{2n_2h_2} - e^{-2n_2h_2} + 4n_2h_2}{(e^{-n_2h_2} - e^{n_2h_2})^2}\right]\right\}^{-\frac{1}{2}}$$

Assuming (5) $EF_x = \Delta EF$ we obtain

$$A = f(N_2, U, \Delta\theta, h_1 = h_a, h_f, \lambda)$$

Water tank experiments

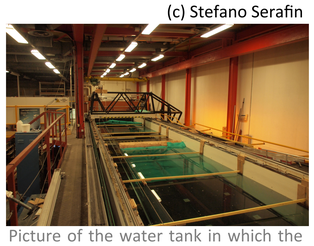
Laboratory experiments → useful to study waves because the flow can be controlled
→ environmental influences are minimized

Can the analytical model reproduce the measured amplitudes in the water tank experiments?

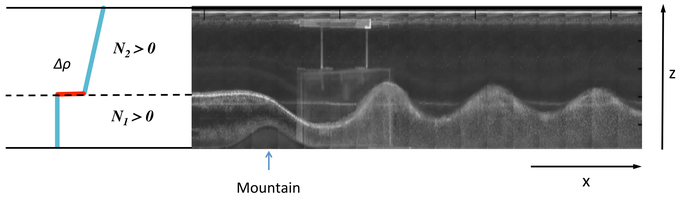
Experiment description:

- Data from the HyIV-CNRS-SECORO experiment
- Measurements of laboratory flows in a 20 m long water tank at CNRM (Météo France)
- Two-layer flow setup according to a predefined density profile (see figure below)
- A mountain was towed with various speeds along the bottom boundary
- Different flow responses occur depending on Froude number $Fr = U/(g'h_1)^{1/2}$ and nondim. mountain height H/h_1 (see Figure ★ on the right side)

(c) Stefano Serafin



Picture of the water tank in which the experiments were carried out.



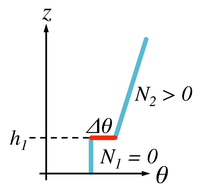
Numerical simulations

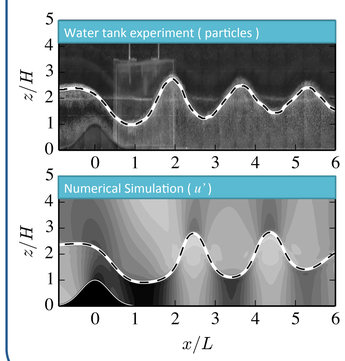
Numerical models → Most advanced tools to simulate flows
→ Give the best estimate of the flow field in the water tank experiments

Are numerical simulations more accurate than the analytical model in reproducing the amplitudes in the water tank?

Details:

Model	CM1 R16 (Bryan Cloud Model)
Domain size	2880 x 145 gridpoints in x and z direction
Resolution	Δx = 50 m and Δz = 50 m below z = 2 km (stretching to Δz = 200 m above)
Mountain	H = 1000 m and L = 5000 m
U	10 m/s





Scaling of interfacial flows

Q: How to determine the simulation setup from the laboratory experiments if the scale is different?
(laboratory ~ 1 m, simulations ~ 1000 m)

A: Using the nondimensional numbers:

- NH/U
- H/L

Scale the mountain wave response

- Fr
- H/h_1

Scale the flow transition over the obstacle

Q: Will the lee wavelength and amplitude (λ, $A \sim u'$) be in scale as well?

A: Yes. It can be shown that:

- $L/\lambda \sim H/h_1$ and Fr
- $|u'/U| \sim H/h_1$, Fr and NH/U

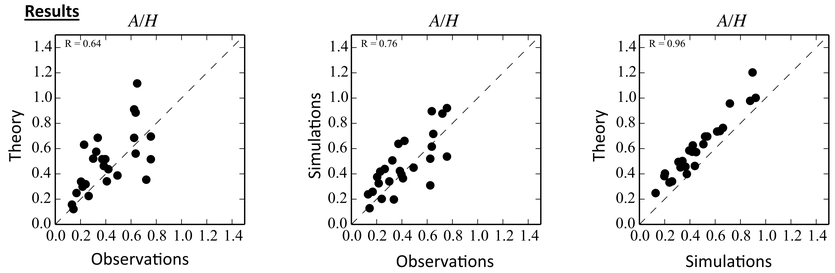
Verification of the analytical model

Parameters for analytical model

1) Upstream flow profile: $(N_2,\ U,\ \Delta\theta,\ h_1 \text{ and } d)$ a priori known

2) Jump parameters: $(h_a,\ h_f \text{ and } \lambda)$ determined from the interface displacement

Results



Lee waves and hydraulic jumps

It can be shown that **hydraulic jumps** correspond to the limiting case of **breaking lee waves**.

In general, gravity waves break when $|u'/U| \gtrsim 1$ (Baines 1995)

Internal gravity waves (mountain waves)

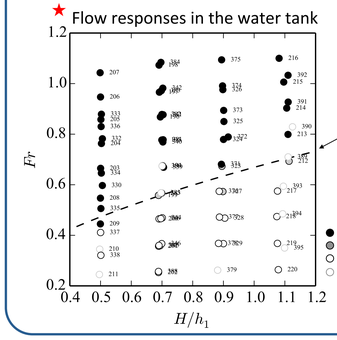
$$|u'/U| \sim NH/U$$

(e.g. Baines 1995)

Interfacial waves (lee waves)

$$|u'/U| \sim \left|\left[\left(\frac{1}{2}\frac{H}{h_1}Fr^{-2}\right)^2 - (NH/U)^2\right]^{1/2}\right|$$

★ Flow responses in the water tank

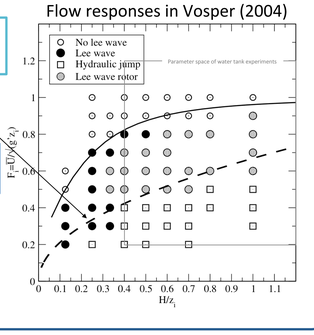


Regime transition
$|u'/U| = 1$

dashed lines

The dashed lines reproduce the regime transition between lee waves and hydraulic jumps

Flow responses in Vosper (2004)



Take-home message:

Interfacial lee wave dynamics:

- Similar dynamics as surface water waves
- Hydraulic jumps -> limiting case of breaking lee waves

Analytical model:

- The lee wave amplitude can be determined analytically
- The key element is to describe the energy source with hydraulic theory
- The accuracy of the analytical model is comparable to that of simulations

Outlook:

Test the analytical model with atmospheric observations

Test the accuracy of other hydraulic models

Extend the model to resonant lee waves

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