# Bound sets for the biobjective team orienteering problem with time windows

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When solving orienteering problems, a fleet of vehicles is used to visit a set of *control points*. To each control point $i$ is associated a profit $p_i$. Due to operational constraints it is impossible to visit all control points; the objective is to maximize the total profit. In the team orienteering problem with time windows (TOPTW), these constraints are the size of the fleet and hard time windows at control points. We consider a biobjective extension of the TOPTW, which we call the BITOPTW, where the second objective is the minimization of total travel costs. We use the *Pareto* approach, meaning that each objective is equally important and that the decision maker's preference between these two objectives is unknown. Therefore the output of an optimization algorithm for the BITOPTW is a set of non-dominated solutions.

Although the two objectives are maximization of total profit and minimization of total cost, in the following we consider, without loss of generality, a biobjective minimization problem. Transforming the BITOPTW to a biobjective minimization problem can be achieved by multiplying all $p_i$ values by $-1$. Similarly to Ehrgott and Gandibleux (2007), we consider *bound sets*. Lower bound sets are obtained by solving exactly a relaxation of the BITOPTW, while upper bound sets are produced by heuristic means.

In order to produce a valid lower bound set, we formulate the BITOPTW as a set packing problem, where each element (control point) can be covered at most once by the selected subsets (routes). The linear relaxation of this model is solved using column generation. The subproblem is an elementary shortest path problem with resource constraints, which we solve using dynamic programming with an algorithm similar to the one described by Feillet et al. (2004). To produce a valid lower bound set, we solve a succession of single-objective versions of the BITOPTW, using weighted-sum aggregations of the two objectives. We prove that the set of aggregation functions that we consider is sufficient to generate the convex hull of the Pareto-optimal set in the objective space for the relaxed problem. Therefore the obtained set is a valid lower bound set for the BITOPTW.

The upper bound set is produced using multi-directional search (MDLS), a metaheuristic for multiobjective optimization introduced by Tricoire (2012). MDLS requires single-objective local search for each objective; for that purpose we use large neighborhood search (see Shaw (1998)), and implement a small set of *destroy* and *repair* operators.

We provide experimental data regarding the gap between lower and upper bound sets. Since we are comparing continuous sets with discrete sets, specific performance indicators are developed and used.

## References

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