Multivariate forecasting with VAR models

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Overview

**Univariate forecasting**
- Model-free: smoothing, filtering
- Models: ARIMA, GARCH

**Multivariate forecasting**
- Feedback? No, yes
  - Open loop system (single equation)
    - Multiple regression
    - Transfer function
  - Closed loop system (multiple equation)
    - Stationary: VAR, SVAR
    - Nonstationary: VEC, SVEC

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Definition of VAR(p)
Stationary vector autoregressive process

A VAR consists of a set of K-endogenous variables

\[ y_t = (y_{1t}, \ldots, y_{kt}, \ldots, y_{Kt}) \text{ for } k = 1, \ldots, K \]

A VAR(p) process is defined as

\[ y_t = \Phi_1 y_{t-1} + \ldots + \Phi_p y_{t-p} + u_t \]

where \( \Phi_i \) are \((K \times K)\) coefficient matrices for \( i = 1, \ldots, p \) and \( u_t \) is \( K \)-dimensional white noise.

The VAR(p) process is stable (stationary series), if

\[ \det(I_K - \Phi_1 z - \ldots - \Phi_p z^p) \neq 0 \text{ for } |z| \leq 1 \] (1)
Definition of bivariate VAR(1)

Stationary bivariate vector autoregressive process

Bivariate VAR(1) process \( y_t = \Phi_1 y_{t-1} + u_t \)

with \( u_t^T = (u_{1t}, u_{2t}) \) and \( \Phi_1 = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} \)

consists of two equations:

\[
\begin{align*}
    y_{1t} &= \phi_{11} y_{1,t-1} + \phi_{12} y_{2,t-1} + u_{1t} \\
    y_{2t} &= \phi_{21} y_{1,t-1} + \phi_{22} y_{2,t-1} + u_{2t}
\end{align*}
\]

\[\rightarrow\] Concept of Granger causality
Finding optimal time lag $p \rightarrow$ information criterion

**Model specification pitfall**

Number of parameters increases tremendously with more lags

Coefficients are estimated by OLS on each equation

Structural analyses

1. Granger causality
2. Impulse response analysis
3. Forecast error variance decomposition
naive forecast: Minimum mean square error (MMSE)

For a VAR(1) process

\[ y_t = \Phi_1 y_{t-1} + u_t \]

one-step-ahead forecast: \( \hat{y}_N(1) = \hat{\Phi}_1 y_N \)

two-step ahead forecast: \( \hat{y}_N(2) = \hat{\Phi}_1 \hat{y}_N(1) = \hat{\Phi}_1^2 y_N \)

Forecasts for horizons \( h \) are therefore obtained with

\[ \hat{y}_N(h) = \hat{\Phi}_1^h y_N \]
Extensions

- VARMA, VMA
- VARX, VARMAX (including exogenous variables)

imposing more restrictions: (model reduction)
- Structural VAR (SVAR)
- Bayesian VAR (BVAR)
VAR model and Cointegration

- before: stationary time series (→ stability condition)
- now: nonstationary data

one could difference the data, but not adequate in the presence of cointegration.

Cointegration

\( y_t \sim I(d) \) is cointegrated, if there exists a \( k \times 1 \) fixed vector \( \beta \neq 0 \), so that \( \beta y_t \) is integrated of order \(< d \).

→ Assume: \( y_{1t} \) and \( y_{2t} \) are \( I(1) \). They are cointegrated when a linear combination of \( y_{1t} \) and \( y_{2t} \) exists with \( (y_{1t} - \beta y_{2t}) \sim I(0) \)
Consider the bivariate VAR(2)

\[ y_t = \Phi_1 y_{t-1} + \Phi_2 y_{t-2} + u_t \]

with the matrix polynomial for \( z = 1 \) (\( \rightarrow \) stability condition (1))

\[ \Phi(1) = (I - \Phi_1 - \Phi_2) = \Pi \]

\( rank(\Pi) \) equals the cointegration rank of the system \( y_t \)

0 ... no cointegration (\( \rightarrow \) difference VAR)
1 ... one cointegrating vector (\( \rightarrow \) VECM)
2 ... process is stable (\( \rightarrow \) VAR)
Implementing cointegration in a VAR(2) using the VECM form:

\[ \Delta y_t = y_t - y_{t-1} = \Pi y_{t-1} + \Gamma_1 \Delta y_{t-1} + u_t \]

where \( \Gamma_1 = -\Phi_2 \) is the transition matrix and \( \Pi = \alpha \beta \) holds

- \( \alpha \) as the »loading matrix« (speed of adjustment)
- \( \beta \) consisting the independent cointegrating vector
- \( \beta Y_{t-1} \) as the lagged disequilibrium error
- \( \Pi y_{t-1} \) as the error correction term (long-run part)

(to catch the idea: consider bivariate VAR(1) equation:
\[ \Delta y_{1t} = \alpha_1 (y_{1,t-1} - \beta y_{2,t-1}) + u_{1t} \] with long-run equilibrium \( y_{1t} = \beta y_{2t} \))

- estimation by reduced rank regression and forecast as in VAR
Applications of VAR/VEC

VAR
- economics and finance
growth rates of macroeconomic time series and some asset returns

VEC
- economics
  - Money demand: money, income, prices and interest rates
  - Growth theory: income, consumption and investment
  - Fisher equation: nominal interest rates and inflation
- finance
cointegration between prices of the same asset trading on different markets due to arbitrage