A NONPARAMETRIC TEST FOR SEASONAL UNIT ROOTS

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Abstract

We consider a nonparametric test for the null of seasonal unit roots in quarterly and monthly time series that builds on the RUR (range unit root) test by Aparicio, Escribano, and Sipols, so we tentatively use the name RURS (for RUR–seasonal). We present some evidence on the quarterly version uses eight test statistics at the seasonal frequencies with high-contrast unexplored limiting distributions. We present some evidence on the size and power of our procedure and provide illustrations by empirical applications.

The RUR test

The RUR test is a nonparametric test for unit roots that was suggested by Aparicio, Escribano, and Sipols (2006. Journal of Time Series Analysis, AES). Essentially, parametric unit-root tests use test statistics that measure the correlation of levels and differences, which are zero for random walks. Nonparametric unit-root tests use test statistics that exploit other characteristic properties of random walks, such as the rate of expansion of trajectories or the number of level crossings. The RUR statistic counts the frequency of new extrema within the trajectory.

For a given realization \( x_j, j = 1, \ldots, n \), define

\[ x_{j+1} = \max_{1 \leq i \leq j} x_i, \quad x_{j-1} = \min_{1 \leq i \leq j} x_i. \]

Then, \( x_{j+1} - x_{j-1} \) defines the sequence of ranges of the series. Any time it increases over \( j = 1, \ldots, n \), this is called a record. The number of records until \( n \) is denoted as \( R(n) \).

AER show that \( R(n) = O(n^{1/2}) \) for a random walk with independent increments. One can also show that \( R(n) = O(\log n) \) for many stationary processes. This motivates that the RUR (range unit root) statistic

\[ R(n) = \sum_{i=1}^{n} x_{i+1} - x_{i-1} \]

can be the basis for a consistent test, if the null is a random walk and the alternative is stationarity.

The quarterly RURS statistics

AERS do not address the issue of the null non-similarity of the RUR test. Assume \((x_t)\) is I(0) but not a random walk. We suggest to eliminate serial correlation under the null by regressing \( x_t \) on p IIR selected lags

\[ \Delta x_t = \beta_0 + \sum_{i=1}^{p} \beta_i \Delta x_{t-i} + \epsilon_t \quad (2 \geq p \geq 2). \]

Estimation residuals \( \epsilon_t \) are accumulated according to

\[ \Delta x_t = \sum_{i=0}^{n} \Delta x_{t+i}, \]

such that \( \Delta x_t \) is ideally a pure random walk without drift. The same can be done for the constructed variables \( x_t^{(p)}, x_t^{(2p)}, x_t^{(4p)} \).

Deterministic terms. In the RURS construction, drifts and deterministic seasonal patterns are eliminated. The RURS test is conducted as a one-sided test. The right tail of the AES limit distribution can only materialize in the presence of upward-linear trends.

Handling RUR non-similarity

AERS do not address the issue of the null non-similarity of the RUR test. Assume \((x_t)\) is I(1) but not a random walk. We suggest to eliminate serial correlation under the null by regressing \( x_t \) on p IIR selected lags

\[ \Delta x_t = \beta_0 + \sum_{i=1}^{p} \beta_i \Delta x_{t-i} + \epsilon_t \quad (2 \geq p \geq 2). \]

Estimation residuals \( \epsilon_t \) are accumulated according to

\[ \Delta x_t = \sum_{i=0}^{n} \Delta x_{t+i}, \]

such that \( \Delta x_t \) is ideally a pure random walk without drift. The same can be done for the constructed variables \( x_t^{(p)}, x_t^{(2p)}, x_t^{(4p)} \).

Deterministic terms. In the RURS construction, drifts and deterministic seasonal patterns are eliminated. The RURS test is conducted as a one-sided test. The right tail of the AES limit distribution can only materialize in the presence of upward-linear trends.

Simulated quantiles

<table>
<thead>
<tr>
<th>model</th>
<th>1%</th>
<th>5%</th>
<th>10%</th>
<th>90%</th>
<th>95%</th>
<th>99%</th>
</tr>
</thead>
<tbody>
<tr>
<td>SWR ( n = 100 )</td>
<td>±1.99</td>
<td>1.14</td>
<td>1.49</td>
<td>2.42</td>
<td>2.86</td>
<td>3.25</td>
</tr>
<tr>
<td>SWR ( n = 500 )</td>
<td>±1.69</td>
<td>1.10</td>
<td>1.52</td>
<td>2.38</td>
<td>2.78</td>
<td>3.19</td>
</tr>
<tr>
<td>SWR ( n = 1000 )</td>
<td>±1.58</td>
<td>1.04</td>
<td>1.48</td>
<td>2.29</td>
<td>2.69</td>
<td>3.08</td>
</tr>
</tbody>
</table>

Notes: \( \pm \) correspond to the \( J_1 \) and \( J_2 \) statistics; \( \pm \) to \( J_1 \) and \( J_2 \). Left tails to be used for testing. Values are close to AES values but they are not identical.

An empirical application

The transformation to random walks works only for \( \phi_t \) and \( \phi_t \). All other frequencies, the asymptotic distribution of the RURS statistic is unknown. Limit trajectories are not Brownian motion. The expansion rate of any RURS statistic will again be \( O(n^{1/2}) \).

Example: testing at frequency \( \pi/3 \). Assume \((x_t)\) is a monthly SWR \( x_t = x_{t-12} + \epsilon_t \). Then, the dynamic transformation

\[ y_t = (1 + \sqrt{3}i + \sqrt{3}i + \sqrt{3}i - B^3 - \sqrt{3}i - \sqrt{3}i) \]

will be a pure unit-root process of the form \( 1 - \sqrt{3}i - \sqrt{3}i - \epsilon_t \) at the angular frequency \( \pi/3 \). For these processes, a parallel procedure can be used to purge them from serial correlation under the null. Consider the auxiliary regression

\[ (1 - \sqrt{3}i - \sqrt{3}i) \epsilon_t = \mu + \sum_{j=1}^{p} \sqrt{3}i \epsilon_{t-j} + \epsilon_t \quad (t \geq 3). \]

Purged trajectories evolve from accumulating estimation residuals \( \hat{\epsilon}_t = \epsilon_t - \mu - \sum_{j=1}^{p} \sqrt{3}i \epsilon_{t-j} \). The same can be done for frequencies \( \pi/3, 2\pi/3, 5\pi/6 \). The monthly RURS test uses 8 test statistics, two at \( \pi/3 \) and one at each of the other six seasonal frequencies.