## APPENDIX B

Tables I–IV present estimates of the critical values for the $\hat{Z}_\alpha$, $\hat{Z}_t$, $\hat{P}_u$, and $\hat{P}_z$ statistics. The tables allow for cointegrating regressions with up to five explanatory variables ($n \leqslant 5$). Critical values are provided for Models (10) and (16) and for cointegrating regressions with a constant term and trend.

The critical values were generated using the Monte Carlo method with 10000 iterations and 500 observations. All the computations were performed on an IBM/AT using the GAUSS programming language. The random innovations were drawn from the standard normal random number generator in GAUSS (i.e., "RNDNS"). Thus $\Omega = I$ and $\rho^2 = 0$ for the generated data, thereby simplifying the computation of the statistics.

Approximate 95% confidence intervals for the critical values were computed using the method described in Rohatgi (1984, pp. 496–500). In order to provide some indication of the degree of precision in the estimates, we present the approximate 95% confidence intervals for $n = 1$ (refer to the rows labelled $\Delta_1$). Confidence intervals for $n \geqslant 2$ are available from the authors on request.

### *Usage*

For Tables I and II ($\hat{Z}_\alpha$ and $\hat{Z}_t$): Reject the null hypothesis of no cointegration if the computed value of the statistic is *smaller* than the appropriate critical value. For example, for a regression with a constant term and one explanatory variable (i.e. $n = 1$), we reject at the 5% level if the computed value of $\hat{Z}_\alpha$ is less than $-20.4935$ or the computed value of $\hat{Z}_t$ is less than $-3.3654$.

For Tables III and IV ($\hat{P}_u$ and $\hat{P}_z$): Reject the null hypothesis of no cointegration if the computed value of the statistic is *greater* than the appropriate critical value. For example, for a regression with two explanatory variables (i.e., $n = 2$) but no constant term, we reject at the 5% level if the computed value of $\hat{P}_u$ is greater than 32.9392 or the computed value of $\hat{P}_z$ is greater than 71.2751.

TABLE Ia

CRITICAL VALUES FOR THE $\hat{Z}_\alpha$ STATISTIC (STANDARD)

| | Size | | | | | | |
|---|---|---|---|---|---|---|---|
| $n$ | 0.1500 | 0.1250 | 0.1000 | 0.0750 | 0.0500 | 0.0250 | 0.0100 |
| 1 | −10.7444 | −11.5653 | −12.5438 | −13.8123 | −15.6377 | −18.8833 | −22.8291 |
| 2 | −16.0164 | −17.0148 | −18.1785 | −19.6142 | −21.4833 | −25.2101 | −29.2688 |
| 3 | −21.5353 | −22.6211 | −23.9225 | −25.5236 | −27.8526 | −31.5432 | −36.1619 |
| 4 | −26.1698 | −27.3952 | −28.8540 | −30.9288 | −33.4784 | −37.4769 | −42.8724 |
| 5 | −30.9022 | −32.2654 | −33.7984 | −35.5142 | −38.0934 | −42.5473 | −48.5240 |
| $\Delta_1$ | (−0.2009) | (−0.1866) | (−0.2210) | (−0.2863) | (−0.5282) | (−0.5053) | (−0.8794) |
| | (+0.2283) | (+0.2338) | (+0.2941) | (+0.3163) | (+0.3899) | (+0.6036) | (+0.6801) |

TABLE Ib

CRITICAL VALUES FOR THE $\hat{Z}_\alpha$ STATISTIC (DEMEANED)

| | Size | | | | | | |
|---|---|---|---|---|---|---|---|
| $n$ | 0.1500 | 0.1250 | 0.1000 | 0.0750 | 0.0500 | 0.0250 | 0.0100 |
| 1 | −14.9135 | −15.9292 | −17.0390 | −18.4836 | −20.4935 | −23.8084 | −28.3218 |
| 2 | −19.9461 | −21.0371 | −22.1948 | −23.8739 | −26.0943 | −29.7354 | −34.1686 |
| 3 | −25.0537 | −26.2262 | −27.5846 | −29.5083 | −32.0615 | −35.7116 | −41.1348 |
| 4 | −29.8765 | −31.1512 | −32.7382 | −34.7110 | −37.1508 | −41.6431 | −47.5118 |
| 5 | −34.1972 | −35.4801 | −37.0074 | −39.1100 | −41.9388 | −46.5344 | −52.1723 |
| $\Delta_1$ | (−0.2646) | (−0.2664) | (−0.3035) | (−0.2660) | (−0.4174) | (−0.6163) | (−0.9824) |
| | (+0.1834) | (+0.3011) | (+0.3329) | (+0.3348) | (+0.4319) | (+0.4834) | (+1.1440) |

TABLE Ic

CRITICAL VALUES FOR THE $\hat{Z}_\alpha$ STATISTIC (DEMEANED AND DETRENDED)

| | Size | | | | | | |
|---|---|---|---|---|---|---|---|
| $n$ | 0.1500 | 0.1250 | 0.1000 | 0.0750 | 0.0500 | 0.0250 | 0.0100 |
| 1 | −20.7931 | −21.8068 | −23.1915 | −24.7530 | −27.0866 | −30.8451 | −35.4185 |
| 2 | −25.2884 | −26.4865 | −27.7803 | −29.7331 | −32.2231 | −36.1121 | −40.3427 |
| 3 | −30.2547 | −31.6712 | −33.1637 | −34.9951 | −37.7304 | −42.5998 | −47.3590 |
| 4 | −34.6336 | −36.0288 | −37.7368 | −39.7286 | −42.4593 | −47.1068 | −53.6142 |
| 5 | −38.9959 | −40.5939 | −42.3231 | −44.5074 | −47.3830 | −52.4874 | −58.1615 |
| $\Delta_1$ | (−0.2514) | (−0.3946) | (−0.3466) | (−0.3908) | (−0.5445) | (−0.6850) | (−0.9235) |
| | (+0.2771) | (+0.3020) | (+0.3044) | (+0.4081) | (+0.5049) | (+0.6158) | (+0.8219) |

TABLE IIa

CRITICAL VALUES FOR THE $\hat{Z}_t$ AND ADF STATISTICS (STANDARD)

| | Size | | | | | | |
|---|---|---|---|---|---|---|---|
| $n$ | 0.1500 | 0.1250 | 0.1000 | 0.0750 | 0.0500 | 0.0250 | 0.0100 |
| 1 | −2.2584 | −2.3533 | −2.4505 | −2.5822 | −2.7619 | −3.0547 | −3.3865 |
| 2 | −2.7936 | −2.8797 | −2.9873 | −3.1105 | −3.2667 | −3.5484 | −3.8395 |
| 3 | −3.2639 | −3.3529 | −3.4446 | −3.5716 | −3.7371 | −3.9895 | −4.3038 |
| 4 | −3.6108 | −3.7063 | −3.8068 | −3.9482 | −4.1261 | −4.3798 | −4.6720 |
| 5 | −3.9438 | −4.0352 | −4.1416 | −4.2521 | −4.3999 | −4.6676 | −4.9897 |
| $\Delta_1$ | (−0.0232) | (−0.0247) | (−0.0269) | (−0.0328) | (−0.0439) | (−0.0382) | (−0.0600) |
| | (+0.0211) | (+0.0228) | (+0.0218) | (+0.0347) | (+0.0318) | (+0.0601) | (+0.0755) |

TABLE IIb

CRITICAL VALUES FOR THE $\hat{Z}_t$ AND ADF STATISTICS (DEMEANED)

| | Size | | | | | | |
|---|---|---|---|---|---|---|---|
| $n$ | 0.1500 | 0.1250 | 0.1000 | 0.0750 | 0.0500 | 0.0250 | 0.0100 |
| 1 | −2.8639 | −2.9571 | −3.0657 | −3.1982 | −3.3654 | −3.6420 | −3.9618 |
| 2 | −3.2646 | −3.3513 | −3.4494 | −3.5846 | −3.7675 | −4.0217 | −4.3078 |
| 3 | −3.6464 | −3.7306 | −3.8329 | −3.9560 | −4.1121 | −4.3747 | −4.7325 |
| 4 | −3.9593 | −4.0528 | −4.1565 | −4.2883 | −4.4542 | −4.7075 | −5.0728 |
| 5 | −4.2355 | −4.3288 | −4.4309 | −4.5553 | −4.7101 | −4.9809 | −5.2812 |
| $\Delta_1$ | (−0.0290) | (−0.0261) | (−0.0232) | (−0.0296) | (−0.0424) | (−0.0389) | (−0.0582) |
| | (+0.0186) | (+0.0263) | (+0.0317) | (+0.0380) | (+0.0304) | (+0.0415) | (+0.0501) |

TABLE IIc

CRITICAL VALUES FOR THE $\hat{Z}_t$ AND ADF STATISTICS (DEMEANED AND DETRENDED)

| | Size | | | | | | |
|---|---|---|---|---|---|---|---|
| $n$ | 0.1500 | 0.1250 | 0.1000 | 0.0750 | 0.0500 | 0.0250 | 0.0100 |
| 1 | −3.3283 | −3.4207 | −3.5184 | −3.6467 | −3.8000 | −4.0722 | −4.3628 |
| 2 | −3.6613 | −3.7400 | −3.8429 | −3.9754 | −4.1567 | −4.3854 | −4.6451 |
| 3 | −3.9976 | −4.0808 | −4.1950 | −4.3198 | −4.4895 | −4.7699 | −5.0433 |
| 4 | −4.2751 | −4.3587 | −4.4625 | −4.5837 | −4.7423 | −5.0180 | −5.3576 |
| 5 | −4.5455 | −4.6248 | −4.7311 | −4.8695 | −5.0282 | −5.3056 | −5.5849 |
| $\Delta_1$ | (−0.0259) | (−0.0246) | (−0.0244) | (−0.0259) | (−0.0350) | (−0.0469) | (−0.0629) |
| | (+0.0246) | (+0.0281) | (+0.0205) | (+0.0301) | (+0.0288) | (+0.0507) | (+0.0722) |

TABLE IIIa

CRITICAL VALUES FOR THE $\hat{P}_u$ STATISTIC (STANDARD)

| | Size | | | | | | |
|---|---|---|---|---|---|---|---|
| $n$ | 0.1500 | 0.1250 | 0.1000 | 0.0750 | 0.0500 | 0.0250 | 0.0100 |
| 1 | 17.2146 | 18.6785 | 20.3933 | 22.7588 | 25.9711 | 31.8337 | 38.3413 |
| 2 | 22.9102 | 24.6299 | 26.7022 | 29.4114 | 32.9392 | 39.2236 | 46.4097 |
| 3 | 28.9811 | 31.0664 | 33.5359 | 36.5407 | 40.1220 | 46.3395 | 55.7341 |
| 4 | 34.5226 | 36.4575 | 39.2826 | 41.8969 | 46.2691 | 53.3683 | 63.2149 |
| 5 | 39.7187 | 41.7669 | 44.3725 | 47.6970 | 51.8614 | 59.6040 | 69.4939 |
| $\Delta_1$ | (−0.4356) | (−0.3845) | (−0.3833) | (−0.5797) | (−0.6274) | (−1.3218) | (−1.4320) |
| | (+0.3777) | (+0.3842) | (+0.4706) | (+0.5793) | (+0.6159) | (+0.8630) | (+1.4875) |

TABLE IIIb

CRITICAL VALUES FOR THE $\hat{P}_u$ STATISTIC (DEMEANED)

| | Size | | | | | | |
|---|---|---|---|---|---|---|---|
| $n$ | 0.1500 | 0.1250 | 0.1000 | 0.0750 | 0.0500 | 0.0250 | 0.0100 |
| 1 | 24.1833 | 25.8456 | 27.8536 | 30.3123 | 33.7130 | 39.9288 | 48.0021 |
| 2 | 29.3836 | 31.4238 | 33.6955 | 36.4757 | 40.5252 | 46.6707 | 53.8731 |
| 3 | 35.1077 | 37.4543 | 39.6949 | 42.8111 | 46.7281 | 53.9710 | 63.4128 |
| 4 | 40.5469 | 42.5683 | 45.3308 | 48.6675 | 53.2502 | 61.2555 | 71.5214 |
| 5 | 45.3177 | 47.6684 | 50.3537 | 53.5654 | 57.7855 | 65.8230 | 76.7705 |
| $\Delta_1$ | (−0.3913) | (−0.4662) | (−0.5310) | (−0.5323) | (−0.5064) | (−1.0507) | (−1.6209) |
| | (+0.4424) | (+0.5441) | (+0.4507) | (+0.7081) | (+0.8326) | (+1.3312) | (+2.1805) |

TABLE IIIc

CRITICAL VALUES FOR THE $\hat{P}_u$ STATISTIC (DEMEANED AND DETRENDED)

| | Size | | | | | | |
|---|---|---|---|---|---|---|---|
| $n$ | 0.1500 | 0.1250 | 0.1000 | 0.0750 | 0.0500 | 0.0250 | 0.0100 |
| 1 | 36.9055 | 38.8150 | 41.2488 | s44.2416 | 48.8439 | 56.0886 | 65.1714 |
| 2 | 41.2115 | 43.4320 | 46.1061 | 49.3671 | 53.8300 | 60.8745 | 69.2629 |
| 3 | 46.9643 | 49.2906 | 52.0015 | 55.4625 | 60.2384 | 68.4051 | 78.3470 |
| 4 | 51.9689 | 54.3205 | 57.3667 | 60.8175 | 65.8706 | 74.4712 | 84.5480 |
| 5 | 56.0522 | 58.6310 | 61.6155 | 65.3514 | 70.7416 | 79.0043 | 91.0392 |
| $\Delta_1$ | (−0.5294) | (−0.5724) | (−0.6764) | (−0.7143) | (−1.0116) | (−1.2024) | (−2.1849) |
| | (+0.5171) | (+0.5187) | (+0.6762) | (+0.7989) | (+0.8773) | (+1.2936) | (+2.2679) |

TABLE IVa

CRITICAL VALUES FOR THE $\hat{P}_z$ STATISTIC (STANDARD)

| | Size | | | | | | |
|---|---|---|---|---|---|---|---|
| $n$ | 0.1500 | 0.1250 | 0.1000 | 0.0750 | 0.0500 | 0.0250 | 0.0100 |
| 1 | 30.0137 | 31.7517 | 33.9267 | 36.6646 | 40.8217 | 47.2452 | 55.1911 |
| 2 | 56.7679 | 59.1613 | 62.1436 | 65.6162 | 71.2751 | 79.5177 | 89.6679 |
| 3 | 92.7621 | 95.7974 | 99.2664 | 103.8454 | 109.7426 | 119.3793 | 131.5716 |
| 4 | 135.2724 | 138.9636 | 143.0775 | 148.4109 | 155.8019 | 166.3516 | 180.4845 |
| 5 | 186.4277 | 190.6337 | 195.6202 | 201.9621 | 210.2910 | 224.0976 | 237.7723 |
| $\Delta_1$ | (−0.4804) | (−0.4493) | (−0.5646) | (−0.7120) | (−0.8406) | (−1.1662) | (−1.2202) |
| | (+0.4633) | (+0.5042) | (+0.6770) | (+0.9202) | (+0.7319) | (+1.5961) | (+1.7356) |

TABLE IVb

CRITICAL VALUES FOR THE $\hat{P}_z$ STATISTIC (DEMEANED)

| | Size | | | | | | |
|---|---|---|---|---|---|---|---|
| $n$ | 0.1500 | 0.1250 | 0.1000 | 0.0750 | 0.0500 | 0.0250 | 0.0100 |
| 1 | 42.5452 | 44.8266 | 47.5877 | 50.7511 | 55.2202 | 61.4556 | 71.9273 |
| 2 | 74.1493 | 76.8850 | 80.2034 | 84.4027 | 89.7619 | 97.8734 | 109.4525 |
| 3 | 113.5617 | 116.6933 | 120.3035 | 125.4579 | 132.2207 | 142.5992 | 153.4504 |
| 4 | 160.8156 | 164.7394 | 168.8572 | 174.2575 | 182.0749 | 194.7555 | 209.8054 |
| 5 | 215.2089 | 219.5757 | 225.2303 | 232.4652 | 241.3316 | 255.5091 | 270.5018 |
| $\Delta_1$ | (−0.4629) | (−0.7383) | (−0.6744) | (−0.6903) | (−1.0214) | (−0.7998) | (−1.8177) |
| | (+0.4873) | (+0.7355) | (+0.5972) | (+0.6662) | (+0.7440) | (+2.0530) | (+2.4081) |

TABLE IVc

CRITICAL VALUES FOR THE $\hat{P}_z$ STATISTIC (DEMEANED AND DETRENDED)

| | Size | | | | | | |
|---|---|---|---|---|---|---|---|
| $n$ | 0.1500 | 0.1250 | 0.1000 | 0.0750 | 0.0500 | 0.0250 | 0.0100 |
| 1 | 66.2417 | 68.8271 | 71.9586 | 75.7349 | 81.3812 | 90.2944 | 102.0167 |
| 2 | 106.6198 | 109.9751 | 113.4929 | 118.3710 | 124.3933 | 133.6963 | 145.8644 |
| 3 | 154.8402 | 158.6619 | 163.1050 | 168.7736 | 175.9902 | 188.1265 | 201.0905 |
| 4 | 210.3150 | 214.3858 | 219.5098 | 225.6645 | 234.2865 | 247.3640 | 264.4988 |
| 5 | 273.3064 | 277.9294 | 284.0100 | 291.2705 | 301.0949 | 315.7299 | 335.9054 |
| $\Delta_1$ | (−0.5433) | (−0.7346) | (−0.8305) | (−0.6905) | (−0.8651) | (−1.6500) | (−2.3915) |
| | (+0.6819) | (+0.5862) | (+0.7373) | (+1.1280) | (+1.4149) | (+1.8572) | (+2.1024) |

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