Answers and comments to 2010 test in “Econometrics of Seasonality”

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1. This problem contained a typo, as the three trimester levels for the dummy constants are called ‘quarterly’ instead of ‘trimester’, which may have led to some confusion. The intention was clear, however, as three constants were given. Solution requires a simple application of the formula

$$A = R^{-1}\Gamma,$$

which yields the vector $A = (1, 1, -\frac{\sqrt{3}}{3})$. Clearly, $\frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$. The matrix R contains the values $\cos\frac{2\pi t}{3}$ for $t = 1, 2, 3$ in its second column and the corresponding sine terms in its third column. The rows of R^{-1} have comparable shapes, they just differ by a scaling factor.

2. For $S = 2$, the general regression equation for the HEGY test assumes the form

$$\Delta_2 y_t = \pi_1 y_{t-1}^{(1)} + \pi_2 y_{t-1}^{(2)} + \sum_{j=1}^{p-2} \psi_j \Delta_2 y_{t-j} + u_t.$$

   (a) Clearly, t–tests test for the nulls $\pi_1 = 0$ and $\pi_2 = 0$, this is standard econometrics. Of more interest is the meaning of these hypotheses. Note that $y_t^{(1)} = (1+B)y_t$ and $y_t^{(2)} = (1-B)y_t$, while $\Delta_2 = (1-B)(1+B)$. Thus, if $\pi_1 = 0$, the factor $(1-B)$ is present in the unique autoregressive representation of y_t, and ‘there is a unit root at $+1$’. Similarly, if $\pi_2 = 0$, the factor $(1+B)$ is present, and ‘there is a unit root at -1’.

   (b) The test statistic for the long-run root always has a Dickey-Fuller distribution (asymptotic, under the null), independent of S. The test statistic for the Nyqvist (shortest) frequency always has the same properties, independent of S, with a potential sign correction depending on the definition of transforms. For $S = 2$, no new distributions can arise, the good old Diceky-Fuller tables can be used.

   (c) Augmenting terms always have the same shape as the dependent variable. They must be stationary under the null *and* under the alternative, which is true for $\Delta_2 y_{t-j}$ for any j, where both potential unit roots have been differenced away. If a third-order autoregression fits the levels, only one term $\Delta_2 y_{t-1}$ appears. The one-one transformation from (ϕ_1, ϕ_2, ϕ_3) to (π_1, π_2, ψ_1) is a simple mathematical exercise.

3. A seasonal random walk (y_t) is essentially defined by the equation

$$y_t = y_{t-S} + \varepsilon_t,$$

while the assumed properties of ε_t may differ across sources. White noise is usually required. The process should be defined as 'started' in $t = S$, for example, with starting values $y_0, \ldots, y_{S-1}$, as it 'has unit roots' and does not allow a useful definition for $t \in \mathbf{Z}$. If a constant is added to the right-hand side, the process (y_t) is called a *drifting* seasonal random walk.