First test in Macro-econometrics

Robert M. Kunst

April 27, 2018

1. Stationarity and concepts [9 points]

   (a) Explain why a random walk and a white noise are not the same. What are the differences?

   (b) Every day a statistician chooses a different random distribution from a long list of distributions. All distributions have mean zero and variance 1. Then, the statistician generates a random number from the selected distribution. Does this define a white noise of daily data?

   (c) Explain the difference between a stationary process and a stable process. You may refer to the simple AR(1) model.

2. Characteristic polynomials [9 points]

   For each of the following ARMA processes, provide the characteristic AR polynomial $\phi(z)$ and the characteristic MA polynomial $\theta(z)$, determine the polynomial zeros (roots) and state whether they fulfill the conditions for the ARMA process to be uniquely defined and stable (asymptotically stationary). [Hint: the famous Pythagorean formula for the roots of second-order polynomials $az^2 + bz + c = 0$ is $\zeta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$]:

   (a) $X_t = 0.25X_{t-2} + \varepsilon_t$;

   (b) $X_t = 1.5X_{t-1} - 0.5X_{t-2} + \varepsilon_t$;

   (c) $X_t = 0.5X_{t-1} + \varepsilon_t + 0.5\varepsilon_{t-1}$.

3. Unit-root testing [9 points]
You wish to test for unit roots in non-trending data X_t using the Dickey-Fuller test. Your software package does not have a corresponding command, so you would like to run a regression. You remember that the Dickey-Fuller statistic can be obtained from a least-squares regression printout.

   (a) In a preliminary lag order search via AIC, you choose an AR(2) model as having the best fit to your data. Specify the response variable and all regressors for the regression that you would need to run. You know that the 'Dickey-Fuller statistic' is a t–statistic in your regression printout. For which regressor?

   (b) You know that the p–value indicated in your standard regression printout is incorrect. So, you look up the critical values in a table and you find that the DF–test rejects. What is your conclusion concerning the generating process for your data?

   (c) Presume you have found the following coefficient estimates in your regression: $* = -0.5* + 0.2* + \text{residuals}$, suppressing the intercept for simplicity and asterisks denoting the variables that you have specified in (a). Presume you interpret this model as an estimate of the data-generating AR(2) process. How would this AR(2) read in the usual notation $X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \varepsilon_t$; in other words, write down the implied $\phi_j, j = 1, 2$.

4. Conditional heteroskedasticity [8 points]
   Which of the following GARCH and ARCH models, if any, fulfills the condition for stability and potential covariance stationarity (if the stability condition holds, the process will be covariance stationary if started in $t = -\infty$)?

   (a) $h_t = \mathrm{E}(\varepsilon_t^2|\mathcal{I}_{t-1}) = 0.2 + 0.4\varepsilon_{t-1}^2 + 0.4h_{t-1}$;
   (b) $h_t = \mathrm{E}(\varepsilon_t^2|\mathcal{I}_{t-1}) = 0.2 - 0.4\varepsilon_{t-1}^2 + 0.4h_{t-1}$;
   (c) $h_t = \mathrm{E}(\varepsilon_t^2|\mathcal{I}_{t-1}) = 0.2 + 0.5\varepsilon_{t-1}^2 + 0.5\varepsilon_{t-2}^2$;
   (d) $h_t = \mathrm{E}(\varepsilon_t^2|\mathcal{I}_{t-1}) = 0.4\varepsilon_{t-1}^2 + 0.4\varepsilon_{t-2}^2$.