

# Second test in Macro-econometrics

Robert M. Kunst

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1. Integration and cointegration in theory and practice.[11 points]

- (a) The concept of a first-order integrated (I(1)) variable is more general than the concept of the random walk. Provide an example for an I(1) variable that is not a random walk.
- (b) If two I(1) variables  $X, Y$  are cointegrated, what is the cointegrating vector?
- (c) If we apply the cointegrating vector to the two cointegrated variables  $X$  and  $Y$ , will the result  $aX + bY$  usually be white noise?
- (d) Presume you wish to test for cointegration using the Engle-Granger method, by regressing  $Y$  on  $X$ , with both variables I(1). Least-squares regression and a Dickey-Fuller test are available to you. You run this Dickey-Fuller test on the OLS residuals. Can you use the  $p$ -value indicated by your software? What could be the problem?

2. Two error-correction systems. [12 points]

- (a) The dynamic behavior of two macroeconomic aggregates  $X$  and  $Y$  follows a VAR(1) model

$$\begin{bmatrix} X_t \\ Y_t \end{bmatrix} = \begin{bmatrix} 1 & 0.2 \\ 0 & 0.2 \end{bmatrix} \begin{bmatrix} X_{t-1} \\ Y_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}.$$

Write the model in its error-correction form  $\Delta x_t = \mathbf{\Pi}x_{t-1} + \varepsilon_t$  for  $x = (X, Y)'$ , and particularly determine the matrix  $\mathbf{\Pi}$ . This impact matrix should have a rank of 1. Can you confirm this?

- (b) Try and find a factorization of  $\mathbf{\Pi} = \alpha\beta'$ . Provide the cointegrating vector  $\beta$ . [Hint: we assume that explosive roots and unit roots other than one are not present in this system.]
- (c) The dynamic behavior of two other macroeconomic aggregates  $X$  and  $Y$  follows a VAR(1) model

$$\begin{bmatrix} X_t \\ Y_t \end{bmatrix} = \begin{bmatrix} 0.8 & 0.2 \\ -0.1 & 1.1 \end{bmatrix} \begin{bmatrix} X_{t-1} \\ Y_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}.$$

Again, re-write it in its error-correction form. Again, the impact matrix should have a rank of one. Can you confirm this?

- (d) Try and find a factorization of  $\mathbf{\Pi} = \alpha\beta'$ . Provide the cointegrating vector  $\beta$ . [Hint: we assume that explosive roots and unit roots other than one are not present in this system.] What is so special about the system in (a)–(b)?

3. Panels [12 points] Someone wishes to estimate the dependence of tourism on the availability of sea beaches in a panel of 27 EU countries for annual data 2005–2016. At first, two variables are available, the annual overnight stays  $N$  and the total length of the coastline in kilometers  $B$ . A first idea is the static model (with  $\varepsilon$  containing potential country effects)

$$N_{it} = \beta_0 + \beta B_{it} + \varepsilon_{it}$$

- (a) What is the main problem with this model, and why will you not be able to obtain a reasonable fixed-effects estimate for  $\beta$ ?
- (b) You modify the model after obtaining data on another variable  $T$ , with  $T_{it}$  representing the average temperature over the year  $t$  at the capital of country  $i$ , and now you estimate

$$N_{it} = \beta_0 + \beta_1 B_{it} + \beta_2 T_{it} + \varepsilon_{it}$$

A Hausman test rejects. What does this imply for your preference between fixed-effects and random-effects estimation?

- (c) You try the dynamic model

$$N_{it} = \beta_0 + \beta_1 B_{it} + \beta_2 T_{it} + \beta_3 N_{i,t-1} + \varepsilon_{it},$$

as tourists may come back to the same country in the next year if they have enjoyed their vacation. Can you continue with applying fixed effects here, or would it be preferable to try a better method?

- (d) What is the interpretation of individual effects in the most advanced dynamic model in (c)? Which other variables would you want to use as further covariates?