# First test in Macro-econometrics

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1. Stationarity [9 points]

   (a) A process $(X_t)$ has time-constant expectation $\mathrm{E}X_t$ and time-constant variance $\mathrm{var}X_t$. Why is this not sufficient for covariance stationarity? Which condition is missing?

   (b) How is a white noise $(\varepsilon_t)$ defined? Is a white noise stationary? Why? Are $\varepsilon_t$ and $\varepsilon_s$ for $s \neq t$ independent?

   (c) How is a random walk defined? Why is the random walk not stationary?

2. Characteristic polynomials [10 points]

   (a) For the following AR processes, provide the characteristic AR polynomial $\phi(z)$, determine the polynomial zeros (roots) and state whether they fulfill the conditions for the AR(2) process to be stable (asymptotically stationary). [Hint: the famous Pythagorean formula for the roots of second-order polynomials $az^2 + bz + c = 0$ is $\zeta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$]:

      i. $X_t = 0.25X_{t-2} + \varepsilon_t$;

      ii. $X_t = X_{t-1} - 0.5X_{t-2} + \varepsilon_t$.

   (b) Consider the ARMA(1,1) process

$$X_t = 2X_{t-1} + \varepsilon_t - 2\varepsilon_{t-1} \quad .$$

   Provide its characteristic AR and MA polynomials $\phi(z)$ and $\theta(z)$? Is it stationary? Is it white noise? Which condition is violated?

   (c) Why is it not correct to say that the processes in part (a) are stationary?

3. Unit-root testing [8 points]
You wish to test for unit roots in non-trending data $X_t$ using the Dickey-Fuller test. Your software package indeed has a corresponding command, but you would like to check the results by running a regression. You remember that the Dickey-Fuller statistic can be obtained from a least-squares regression printout.

   (a) In a preliminary lag order search via AIC, you choose an AR(2) model as having the best fit to your data. Specify the response variable and all regressors for the regression that you would need to run.

   (b) A basic regression printout just contains coefficient estimates, standard errors of all coefficients, and the $R^2$. How do you obtain the Dickey-Fuller statistic from such a printout?

   (c) Presume the regression printout also contains p–values for all coefficients, which are typically based on a standard normal or $t$ distribution for the $t$–statistics. Can you use these p–values to decide on the unit-root hypothesis?

   (d) Given that the DF–test fails to reject, what is your conclusion concerning the generating process for your data?

4. Conditional heteroskedasticity [8 points]
   Which of the following GARCH models, if any, fulfills the condition for stability and potential covariance stationarity (in this case, $(X_t)$ is covariance stationary if started at $t = -\infty$)?

   (a) $h_t = \mathrm{E}(\varepsilon_t^2|\mathcal{I}_{t-1}) = 0.2 + 0.4\varepsilon_{t-1}^2 + 0.4h_{t-1}$;

   (b) $h_t = \mathrm{E}(\varepsilon_t^2|\mathcal{I}_{t-1}) = 0.4\varepsilon_{t-1}^2 + 0.4h_{t-1}$;

   (c) $h_t = \mathrm{E}(\varepsilon_t^2|\mathcal{I}_{t-1}) = 1 + 0.5\varepsilon_{t-1}^2 + 0.5h_{t-1}$;

   (d) $h_t = \mathrm{E}(\varepsilon_t^2|\mathcal{I}_{t-1}) = 1 - 0.2\varepsilon_{t-1}^2 + 0.2h_{t-1}$.