Test 'Nonlinear time series'
Tentative answers

December 1, 2009

1. The model as stated corresponds to the Wold representation, as the only root of the MA polynomial $\theta(z) = 1 + 0.5z$ is 2 and thus larger than one. The invertible MA representation, however, corresponds to the Wold representation. This answers (a), while we cannot answer whether the process is linear, as ε_t is only given as 'white noise'. For a linear process, we need a representation in $iid(0, \sigma^2)$ errors, according to the definition, which answers (b).

2. A threshold autoregression is given by the equations $X_t = 0.5X_{t-1} + \varepsilon_t$ for $X_{t-2} > 0$ and $X_t = 0.4X_{t-1} + \varepsilon_t$ for $X_{t-2} \leq 0$, with (ε_t) specified as Gaussian white noise.

   (a) (X_t) is stationary, as all submodels are stable and the variance is identical across regimes (slide # 37).

   (b) The delay is 2 (slide # 36).

   (c) I separate the observations into those for the first regime and those for the second one. Within each of the regimes, I estimate the coefficients by OLS.

   (d) The original sheet on the web page had a typo. It should be $X_{t-1} > 0$ and $X_{t-1} \leq 0$. Then, any given (positive or negative) value X_t for a single time point t implies that the whole trajectory always remains above zero (positive) or below zero (negative). The coefficients cannot be estimated for the 'other' regime, as no observations are available. This is a good example for a non-mixing process.

3. Checking ARCH models for stationarity conditions: (a) is an ill-defined model, as the constant is zero (slide # 52), it degenerates and is not stationary; (b) is a properly defined stationary (in both definitions) ARCH model (slide # 53); (c) is an IGARCH model (slide # 65), strictly stationary but with infinite variance and thus not covariance-stationary.