Analysis of the Relationship between Commodity Futures and Spot Prices
An Applied Time Series Analysis
(Stationarity, Cointegration, Granger-Causality)

Georg Valentin Lehecka

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Introduction

Aims of this empirical analysis

- Examination of *stationarity* of daily commodity futures and spot prices
- Examination of *cointegration relationships* of daily commodity futures and spot prices
- Examination of *Granger-cause relationships* of daily commodity futures and spot prices
Introduction: Spot and Futures Prices

Spot Prices

The spot price of a commodity is the price that is quoted for immediate (spot) settlement (payment and delivery).

Futures Prices

The futures price of a commodity is the price that is quoted for a certain delivery time and a certain amount in the future.
Introduction: Spot Prices

Daily National Grain Market Summary

Grains closed slightly mixed after Tuesday’s active trade. Corn closed mostly unchanged as it rallied late to close off its losses, as dry conditions in Argentina has curbed selling interest. Soybeans had slight gains as soybeans are also tied to the dry conditions in South America. Wheat had losses in Chicago and Minneapolis, with slight gains in K.C. Wheat closed slightly mixed. Corn traded steady to 3 cents higher. Soybeans closed 2-12 cents higher.

<table>
<thead>
<tr>
<th>TRUCK BIDS:</th>
<th>DATE</th>
<th>CHANGE</th>
<th>YEAR AGO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wheat:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kansas City (HRW ORD)</td>
<td>6.89</td>
<td>up 1</td>
<td>7.98</td>
</tr>
<tr>
<td>Minneapolis (DNS)</td>
<td>8.94</td>
<td>dn 5 1/4</td>
<td>9.29</td>
</tr>
<tr>
<td>Portland (SWW)</td>
<td>6.05-6.25</td>
<td>dn 5</td>
<td>7.60-7.70</td>
</tr>
<tr>
<td>St. Louis (SRW)</td>
<td>6.63</td>
<td>dn 3</td>
<td>8.02</td>
</tr>
<tr>
<td>Corn, US No 2 Yellow:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kansas City</td>
<td>6.77</td>
<td>up 3</td>
<td>6.01-6.04</td>
</tr>
<tr>
<td>Minneapolis</td>
<td>No Bid</td>
<td>N/A</td>
<td>No Bid</td>
</tr>
<tr>
<td>So. Iowa</td>
<td>6.37 1/2-6.43 1/2</td>
<td>unch</td>
<td>5.93 1/4-5.97 1/4</td>
</tr>
<tr>
<td>Omaha</td>
<td>6.47-6.50</td>
<td>unch</td>
<td>5.91-5.95</td>
</tr>
<tr>
<td>Soybeans, US No 1 Yellow:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kansas City</td>
<td>12.20</td>
<td>up 12</td>
<td>13.55-13.59</td>
</tr>
<tr>
<td>Minneapolis</td>
<td>No Bid</td>
<td>N/A</td>
<td>No Bid</td>
</tr>
<tr>
<td>So. Iowa</td>
<td>11.99-12.02</td>
<td>up 2 1/2</td>
<td>13.57-13.71 1/2</td>
</tr>
<tr>
<td>Cent. Il Processor</td>
<td>12.20-12.25</td>
<td>up 12 1/2-2 1/2</td>
<td>13.71 1/2-13.86 1/2</td>
</tr>
</tbody>
</table>
Introduction: Futures Prices

Georg Valentin Lehecka

Relationship between Commodity Futures and Spot Prices
### Description of Data

<table>
<thead>
<tr>
<th>Commodity</th>
<th>Exchange</th>
<th>Time Period</th>
<th>Data Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lean hog</td>
<td>CBOT</td>
<td>1995-11-02 to 2011-12-30</td>
<td>4065</td>
</tr>
<tr>
<td>Feeder cattle</td>
<td>CBOT</td>
<td>2001-01-02 to 2011-12-30</td>
<td>2763</td>
</tr>
<tr>
<td>Kansas wheat</td>
<td>KCBOT</td>
<td>2002-01-02 to 2011-12-30</td>
<td>2468</td>
</tr>
<tr>
<td>Minneapolis wheat</td>
<td>MGE</td>
<td>2002-01-02 to 2011-12-30</td>
<td>2465</td>
</tr>
<tr>
<td>Crude oil</td>
<td>NYME</td>
<td>1986-01-02 to 2011-12-30</td>
<td>6514</td>
</tr>
<tr>
<td>Heating oil</td>
<td>NYME</td>
<td>1986-06-02 to 2011-12-30</td>
<td>6406</td>
</tr>
</tbody>
</table>

- Futures prices are obtained from the particular exchange (data of the next-to-delivery contract month, respectively).
- Spot prices are obtained from USDA (grains), CME (meats), and U.S. EIA (energy) reports, respectively.
Description of Data

Lean hog prices

Feeder cattle prices

Kansas wheat prices

Minneapolis wheat prices

Crude oil prices

Heating oil prices

Relationship between Commodity Futures and Spot Prices
ADF Test

$$\Delta X_t = a + bt + \phi X_{t-1} + \sum_{j=1}^{p-1} \alpha_j \Delta X_{t-j} + u_t$$

$H_0 : \phi = 0 \iff$ unit root in the characteristic polynomial $\implies$ nonstationarity

$H_1 : \phi < 0 \iff$ no unit root in the characteristic polynomial $\implies$ (trend) stationarity.

The lag order $p$ is determined by minimizing AIC for autoregressions with a constant and a linear trend. $X_t$ are logarithms of the nominal price series.
### Tests on Stationarity

**ADF test results**

<table>
<thead>
<tr>
<th>Commodity</th>
<th>Series</th>
<th>ADF</th>
<th>ADF-I(1)</th>
<th>Lags</th>
<th>Implication</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lean hog</td>
<td>Spot</td>
<td>-3.87**</td>
<td>-23.87***</td>
<td>3</td>
<td>I(0)</td>
</tr>
<tr>
<td></td>
<td>Futures</td>
<td>-3.69**</td>
<td>-61.19***</td>
<td>1</td>
<td>I(0)</td>
</tr>
<tr>
<td>Feeder cattle</td>
<td>Spot</td>
<td>-1.90</td>
<td>-17.38***</td>
<td>10</td>
<td>I(1)</td>
</tr>
<tr>
<td></td>
<td>Futures</td>
<td>-2.23</td>
<td>-47.43***</td>
<td>1</td>
<td>I(1)</td>
</tr>
<tr>
<td>Kansas wheat</td>
<td>Spot</td>
<td>-2.33</td>
<td>-50.68***</td>
<td>0</td>
<td>I(1)</td>
</tr>
<tr>
<td></td>
<td>Futures</td>
<td>-2.21</td>
<td>-48.56***</td>
<td>0</td>
<td>I(1)</td>
</tr>
<tr>
<td>Minneapolis wheat</td>
<td>Spot</td>
<td>-1.94</td>
<td>-48.87***</td>
<td>0</td>
<td>I(1)</td>
</tr>
<tr>
<td></td>
<td>Futures</td>
<td>-2.01</td>
<td>-48.13***</td>
<td>0</td>
<td>I(1)</td>
</tr>
<tr>
<td>Crude oil</td>
<td>Spot</td>
<td>-3.29*</td>
<td>-39.20***</td>
<td>5</td>
<td>I(1)</td>
</tr>
<tr>
<td></td>
<td>Futures</td>
<td>-3.42**</td>
<td>-81.41***</td>
<td>0</td>
<td>I(0)</td>
</tr>
<tr>
<td>Heating oil</td>
<td>Spot</td>
<td>-2.76</td>
<td>-43.18***</td>
<td>4</td>
<td>I(1)</td>
</tr>
<tr>
<td></td>
<td>Futures</td>
<td>-2.70</td>
<td>-80.18***</td>
<td>0</td>
<td>I(1)</td>
</tr>
</tbody>
</table>

Note: Single (*), double (**), and triple asterisks (***) denote significance at the 10%, 5%, and 1% levels, respectively.
Tests on Cointegration

Johansen Procedure (Multivariate DF Model)

If the re-written vector representation

$\Delta X_t = \mu + \Pi X_{t-1} + \psi_1 \Delta X_{t-1} + \psi_2 \Delta X_{t-2} + \ldots + \psi_{p-1} \Delta X_{t-p+1} + u_t$

with $X = (X_1, \ldots, X_n)'$ is cointegrated, then $\Pi$ is singular and can be represented as

$\Pi = \alpha \beta'$,

with $(n \times r)$-matrices $\alpha$, $\beta$ of rank $r$. The $r$ columns of $\beta$ are cointegrating vectors that describe equilibrium relations. The coefficients in $\alpha$ are called loading coefficients and describe how the variables react to deviations from equilibrium.
Tests on Cointegration

The Johansen procedure step by step:

1. All variables $X_j, j = 1, ..., n$ should be either I(1) or I(0).
2. Determine the VAR lag order $p$ by multivariate information criteria.
3. Determine the cointegrating rank $r$ by sequences of hypothesis tests: estimate $\beta$.
4. Estimate the full EC-VAR model given $p$ and $r$ to estimate $\alpha$ and $\Psi_j$. 
Cointegration test results

<table>
<thead>
<tr>
<th>Commodity</th>
<th>r ≤ 1</th>
<th>r = 0</th>
<th>Lags</th>
<th>Implication</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lean hog</td>
<td>11.79***</td>
<td>109.56***</td>
<td>12</td>
<td>system is stationary</td>
</tr>
<tr>
<td>Feeder cattle</td>
<td>2.00</td>
<td>27.31***</td>
<td>23</td>
<td>cointegration r = 1</td>
</tr>
<tr>
<td>Kansas wheat</td>
<td>2.60</td>
<td>12.36</td>
<td>2</td>
<td>no cointegration</td>
</tr>
<tr>
<td>Minneapolis wheat</td>
<td>2.65</td>
<td>19.11**</td>
<td>12</td>
<td>cointegration r = 1</td>
</tr>
<tr>
<td>Crude oil</td>
<td>0.88</td>
<td>105.90***</td>
<td>22</td>
<td>cointegration r = 1</td>
</tr>
<tr>
<td>Heating oil</td>
<td>0.98</td>
<td>82.54***</td>
<td>22</td>
<td>cointegration r = 1</td>
</tr>
</tbody>
</table>

Note: Single (*), double (**), and triple asterisks (***)) denote significance at the 10%, 5%, and 1% levels, respectively. The standard model with a constant is used. The VAR lag order is determined by minimizing multivariate AIC. X_t are logarithms of nominal price series.
Cointegration Relationships

For example, **feeder cattle** spot and futures prices cointegrate (the rank \( r = 1 \)) and the system can be written as

\[
\begin{bmatrix}
\Delta \text{Spot}_t \\
\Delta \text{Futures}_t
\end{bmatrix} =
\begin{bmatrix}
0.0103 \\
-0.0018
\end{bmatrix} +
\begin{bmatrix}
-0.0906 \\
0.0174
\end{bmatrix} \underbrace{\begin{bmatrix} 1 \\ -0.98 \end{bmatrix}}_{\alpha} \begin{bmatrix}
\text{Spot}_{t-1} \\
\text{Futures}_{t-1}
\end{bmatrix}

+ \begin{bmatrix}
\psi_{11} & \psi_{12} \\
\psi_{12} & \psi_{22}
\end{bmatrix} \begin{bmatrix}
\Delta \text{Spot}_{t-1} \\
\Delta \text{Futures}_{t-1}
\end{bmatrix} + \ldots + \begin{bmatrix}
\delta_{1,t} \\
\delta_{2,t}
\end{bmatrix}.
\]
In the case of two time series, $X_1$ and $X_2$, $X_1$ Granger-cause $X_2$ if $X_2$ can be better predicted using the histories of both $X_1$ and $X_2$ than it can by using the histories of $X_2$ alone.

**Granger-Causality Test**

$X_{2,t}$ is not Granger-causal for $X_{1,t}$ iif the bivariate VAR($p$) process of the form

$$
\begin{bmatrix}
X_{1,t} \\
X_{2,t}
\end{bmatrix} = \sum_{i=1}^{p} \begin{bmatrix}
\gamma_{11,i} & \gamma_{12,i} \\
\gamma_{12,i} & \gamma_{22,i}
\end{bmatrix} \begin{bmatrix}
X_{1,t-i} \\
X_{2,t-i}
\end{bmatrix} + \begin{bmatrix}
u_{1,t} \\
u_{2,t}
\end{bmatrix},
$$

has $\gamma_{12,i} = 0$, for all $i = 1, 2, \ldots, p$. 
Notes on Granger-causality tests:

■ It requires checking whether specific coefficients are zero, therefore standard tests for zero restrictions are applied ($\chi^2$- or $F$-test based on the Wald principle).

■ They have nonstandard asymptotic properties if the VAR contains $I(m)$ variables with $m > 0$ (or possible cointegration).

■ This can be overcome in fitting VAR processes whose order exceeds the true order.

■ A lag augmented model with $m$ additional lags can be used in the test.

■ The hypothesis of zero coefficients has to be tested on only the first $p$ coefficients.

Tests on Granger-Causality

Granger causality test results

<table>
<thead>
<tr>
<th>Commodity</th>
<th>Spot $\rightarrow$ Futures</th>
<th>Futures $\rightarrow$ Spot</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lean hog</td>
<td>0.01***</td>
<td>0.00***</td>
</tr>
<tr>
<td>Feeder cattle</td>
<td>0.00***</td>
<td>0.00***</td>
</tr>
<tr>
<td>Kansas wheat</td>
<td>0.73</td>
<td>0.00***</td>
</tr>
<tr>
<td>Minneapolis wheat</td>
<td>0.00***</td>
<td>0.00***</td>
</tr>
<tr>
<td>Crude oil</td>
<td>0.00***</td>
<td>0.00***</td>
</tr>
<tr>
<td>Heating oil</td>
<td>0.00***</td>
<td>0.00***</td>
</tr>
</tbody>
</table>

Note: Single (*), double (**), and triple asterisks (***) denote significance at the 10%, 5%, and 1% levels, respectively. A VAR model with a constant is used. The same lag orders as in cointegration tests are used. $m$ (the maximum order of integration of both time series) additional lags are included. Logarithms of nominal price series are used.
Implications of Granger-causality test results:

- Granger-causality between commodity spot and futures prices appears generally to be bi-directional. Futures prices cause spot prices and vice versa.

- This suggests that no profitable arbitrage exists, new information appears to be reflected by spot and futures prices simultaneously.
Summary and Conclusion

Test Results

- **Stationarity:** Unit roots tests suggest that daily commodity spot and futures prices appear generally to be **non-stationary**.

- **Cointegration:** Tests on cointegration suggest that daily commodity spot and futures prices appear generally to be **cointegrated**.

- **Granger-causality:** Tests on Granger-causality suggest generally **bi-directional causality** between daily commodity spot and futures prices.