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"Particle Oscillations, Entanglement and Decoherence"

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Contents

1 Introduction

The phenomena of flavor mixing and particle oscillations have been a basis for new discoveries for the last 50 years, e.g. the discovery of CP violation in the studies of neutral kaons or the fact that, in contrast to the Standard Model of particle physics, neutrinos must have mass, to name but two. It is an interesting topic to combine these phenomena with two other fundamental principles of quantum mechanics: entanglement and decoherence. Precise measurements of correlations of flavor entangled particles can be used for testing different models of decoherence, that can be applied to implement the loss of coherence in a system due to interactions with the environment.

In Section 2, a short introduction to particle oscillations is given using the example of neutrino oscillations. In that section, two different approaches with plane waves and wave packets and the problems with them are discussed. (A more appropriate but more complex treatment in the language of quantum field theory is omitted in this thesis.) In Sections 3 and 4, a different ansatz for unstable particles with density matrices is shown, with the focus not so much on the oscillation itself, but more on the special case of a system of two entangled oscillating mesons and a model to describe decoherence in this system.

2 Neutrino Oscillations

2.1 Flavor and Mass Eigenstates

To understand particle oscillations it is necessary to understand the concepts of *flavor eigenstates* and *mass eigenstates*. Flavor oscillations can occur if these do not coincide, which is known as flavor mixing. In the formalism of quantum mechanics this can be described as follows [1]: the Hamiltonian H can be split into $H = H_{\text{prop}} + H_{\text{int}}$, where H_{prop} describes the propagation of the particles and H_{int} the interactions producing them. If H_{int} conserves flavor, whereas H_{prop} does not, there are two different bases that can be used, each diagonalizing one of the Hamiltonians: the *flavor basis* is made up of eigenstates of both H_{int} and the flavor operator, the *mass basis* is made up of eigenstates of H_{prop}. So each flavor eigenstate can be written as a linear combination of mass eigenstates (see (2.1)) et vice versa. A particle is always produced and detected in a flavor eigenstate (since the production is described by H_{int}), which is a superposition of two or more mass eigenstates that propagate in space. The interference of these propagating mass eigenstates eventually gives rise to the phenomenon of flavor oscillations.

2.2 Plane Wave Treatment

The illustration of particle oscillations with plane waves is, even though suffering from some serious problems (see Section 2.2.1), the "standard" treatment for neutrino oscillations and can be found in any textbook or review article on this topic (see for example [1] or [2] (chapter 12)). It provides a good overview on what is going on and leads to the correct oscillation length in the ultra-relativistic limit.

Let states with indices with Greek letters denote flavor eigenstates $|\nu_\alpha\rangle$, $\alpha = e, \mu, \tau$, and states with indices with Latin letters mass eigenstates $|\nu_i\rangle$, $i = 1, 2, 3$. The relation between flavor and mass eigenstates is then given by:

$$|\nu_\alpha(t=0)\rangle = \sum_i U_{\alpha i} |\nu_i(0)\rangle \tag{2.1}$$

The mixing matrix U (corresponding to the CKM-matrix in the case of quarks) is unitary if we restrict ourselves to stable particles such as neutrinos. The pure flavor states are orthogonal

$$\langle \nu_\alpha(0) | \nu_\beta(0) \rangle = \delta_{\alpha\beta} \tag{2.2}$$

whereas the the same relation for the mass eigenstates can be found from (2.1) to be

$$\langle \nu_i(0) | \nu_j(0) \rangle = \sum_\alpha U^{-1}_{j\alpha} U^{-1\dagger}_{\alpha i} = \left(U^\dagger U\right)^{-1}_{ji} = \delta_{ji} \tag{2.3}$$

where the last step is only true for $U^\dagger = U^{-1}$, i.e. for stable particles. If we start with a neutrino in a flavor eigenstate ν_α at time $t = 0$ and position $\mathbf{x} = 0$, the transition amplitude for finding a state ν_β at t and $\mathbf{x}$ is [1]

$$\mathcal{M}_{\alpha\to\beta} = \langle \nu_\beta(0) | \mathrm{e}^{-i\hat{H}_{\mathrm{prop}} t + i\hat{\mathbf{P}}\cdot\mathbf{x}} | \nu_\alpha(0) \rangle \tag{2.4}$$

To solve this, we have to write $|\nu_\alpha\rangle$ in the basis of mass eigenstates, because those are the eigenstates of the energy and momentum operator, i.e. $\hat{H}_{\mathrm{prop}} |\nu_i\rangle = E_i |\nu_i\rangle$ and $\hat{\mathbf{P}} |\nu_i\rangle = \mathbf{p}_i |\nu_i\rangle$.

$$\mathcal{M}_{\alpha\to\beta} = \langle \nu_\beta(0) | \sum_i U_{\alpha i}\, \mathrm{e}^{-iE_i t + i\mathbf{p}_i\cdot\mathbf{x}} | \nu_i(0) \rangle \tag{2.5}$$

Transforming back to flavor states with the inverse of (2.1)

$$|\nu_i(0)\rangle = \sum_\gamma U^{-1}_{i\gamma} |\nu_\gamma(0)\rangle \tag{2.6}$$

and using the orthogonality relation (2.2) yields:

$$\mathcal{M}_{\alpha\to\beta} = \sum_i U_{\alpha i} U^{-1}_{i\beta}\, \mathrm{e}^{-i\phi_i} \tag{2.7}$$

[1] $\hbar = c = 1$

with $\phi_i = E_i t - \mathbf{p}_i \cdot \mathbf{x}$. The final probability of finding a ν_β is

$$P_{\alpha\to\beta}(t,\mathbf{x}) = |\mathcal{M}_{\alpha\to\beta}|^2 = \sum_{i,j} U_{\alpha i} U^{-1}_{i\beta} U^*_{\alpha j} \left(U^*_{j\beta}\right)^{-1} \mathrm{e}^{-i(\phi_i-\phi_j)} \tag{2.8}$$

The phase $\phi_i = E_i t - \mathbf{p}_i \cdot \mathbf{x}$ can be simplified by using relativistic kinematics and some approximations. The mass eigenstates represent on-shell particles, so $E_i = \sqrt{p_i^2 + m_i^2}$. If we assume that all the differences in the masses and in the momenta are very small, we can expand this term around average mass squared m^2 and momentum $\mathbf{p}$, so that $m_i^2 = m^2 + \delta m_i^2$ and $\mathbf{p}_i = \mathbf{p} + \delta \mathbf{p}_i$. The phase ϕ_i then reads

$$\phi_i \approx Et - \mathbf{p}\cdot\mathbf{x} + \underbrace{(\mathbf{v}t - \mathbf{x})}_{=0} \cdot \delta\mathbf{p}_i + \frac{\delta m_i^2}{2E} t \tag{2.9}$$

with $E = \sqrt{p^2 + m^2}$ and $\mathbf{v} = \frac{\mathbf{p}}{E}$. So the phase difference is just $(\phi_i - \phi_j) = \frac{\Delta m_{ij}^2}{2E} t$ with $\Delta m_{ij}^2 = m_i^2 - m_j^2$. Because in oscillation experiments one can only measure the distance $|\mathbf{x}|$ and never the time t, we should also take that into account and use $t = \frac{|\mathbf{x}|}{v} = \frac{|\mathbf{x}|E}{p}$. Because it is known that neutrino masses have to be extremely small, they are always ultra-relativistic and we can therefore approximate $E \approx p$ (which is the same as setting $v = 1$ and $t = |\mathbf{x}|$). So finally we can write

$$\phi_i - \phi_j \approx \frac{\Delta m_{ij}^2}{2E} |\mathbf{x}| = 2\pi \frac{|\mathbf{x}|}{L_{ij}^{\mathrm{osc}}} \tag{2.10}$$

with *oscillation length* L_{ij}^{osc}

$$L_{ij}^{\mathrm{osc}} = \frac{4\pi E}{\Delta m_{ij}^2} \tag{2.11}$$

That this really results in some “oscillation” effects is most easily seen if we restrict ourselves to only two kinds of neutrinos, let's say $\alpha, \beta = e, \mu$ and $i, j = 1, 2$. Then the 2×2 unitary matrix U can be made real by using the invariance of (2.8) under the transformation $U_{\alpha i} \to \mathrm{e}^{i\psi_\alpha} U_{\alpha i}\, \mathrm{e}^{i\psi_i}$ with arbitrary ψ_α, ψ_i [3], which just leaves the normal two dimensional rotation matrix with a mixing angle θ.

$$U_{2\times 2} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \tag{2.12}$$

Using (2.8), the probability of a conversion of a ν_e at $(t = 0, \mathbf{x} = 0)$ to a ν_μ at $(t, \mathbf{x})$ is

$$P_{\nu_e\to\nu_\mu}(\mathbf{x}) = \frac{1}{2}\sin^2(2\theta)\left(1 - \cos\left(2\pi \frac{|\mathbf{x}|}{L_{12}^{\mathrm{osc}}}\right)\right) \tag{2.13}$$

with $L_{12}^{\text{osc}} = \frac{4\pi E}{m_1^2 - m_2^2}$. The probability of finding a ν_μ, even though we know a ν_e was initially produced, oscillates in space between 0 and $\sin^2(2\theta)$. So the probability can only reach 1 in the case of maximal mixing $\theta = \frac{\pi}{4}$.

2.2.1 Problems with the plane wave treatment

Although the plane wave ansatz yields a formula for flavor oscillations and the oscillation length, it is quite obvious that there are some problems and inconsistencies (see Refs. [1][4]). It is not clear how it can be consistent with energy and momentum conservation if a flavor state is written as a superposition of three mass eigenstates with three different, but definite, momenta and energies. For example, if we assume that a ν_μ is produced in the decay of a pion, $\pi^+ \to \mu^+ \nu_\mu$, that would imply that the pion and the myon have three different possible configurations for their energies and momenta as well. Apart from that, we should be able (at least in principle) to find out which of the mass eigenstates is propagating, just by measuring energy and momentum of the neutrino accurate enough (which can be done either at the detector or by measuring the energies and momenta of the pion and the myon). But then we would have a single mass eigenstate and no longer a superposition of two or more of them, and a mass eigenstate cannot oscillate, since it is an eigenstate of H_{prop}, only the superposition can lead to flavor oscillations. But this is not taken into account in (2.13). Furthermore, a plane wave is infinitely spread in space and therefore it seems quite meaningless to speak of an oscillation "length", because the distance that the particle has travelled is not determined at all.

2.3 Wave Packet Approach

To solve the issues mentioned in Section 2.2.1, it is necessary to give up the plane wave treatment and take a new approach with wave packets instead, see for example [5][6][7]. The idea is to write the i-th mass eigenstate as a Gaussian wave packet centered around the momentum p_i (for convenience this will be done only in one dimension), so that the flavor eigenstate at (t, x) is given by

$$|\nu_\alpha(t,x)\rangle = \sum_i U_{\alpha i} \int \frac{\mathrm{d}p}{\sqrt{2\pi}} \left(\frac{1}{\sqrt{2\pi}\sigma_p} \right)^{1/2} \mathrm{e}^{\frac{-(p-p_i)^2}{4\sigma_p^2}} \, \mathrm{e}^{-iE_i(p)t+ipx} \, |\nu_i\rangle \tag{2.14}$$

where σ_p denotes the momentum uncertainty and $E(p) = \sqrt{p^2 + m_i^2}$. By the same arguments that were used in Section 2.2, this term can be simplified to $E_i(p) \approx E_i + v_i(p - p_i)$, with $E_i := E_i(p_i)$. Then the integral in (2.14) becomes

$$\left(\frac{\sqrt{2\pi}}{\sigma_p} \right)^{1/2} \mathrm{e}^{-iE_i t + ip_i x} \int \mathrm{d}p \exp\left[\frac{-(p-p_i)^2}{4\sigma_p^2} - i(v_i t - x)(p - p_i) \right] \tag{2.15}$$

Completing the square yields a Gaussian integral and eventually the whole integral is

$$\left(\frac{1}{\sqrt{2\pi}\sigma_x}\right)^{1/2} \exp\left[-iE_i t + ip_i x - \frac{(v_i t - x)^2}{4\sigma_x^2}\right] \tag{2.16}$$

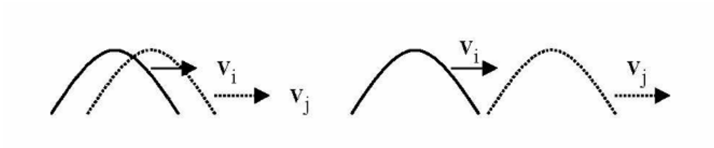
where $\sigma_x = \frac{1}{2\sigma_p}$ is the width of the wave packet in position space. The probability of finding a ν_β at (t,x), if a ν_α was produced at $(0,0)$, is

$$\begin{aligned} P_{\alpha\to\beta}(t,x) = &\sum_{i,j} U_{\alpha i} U_{i\beta}^{-1} U_{\alpha j}^* \left(U_{j\beta}^*\right)^{-1} \frac{1}{\sqrt{2\pi}\sigma_x} \\ &\times \exp\left[-i(E_i - E_j)t + i(p_i - p_j)x - \frac{1}{4\sigma_x^2}\left((v_i t - x)^2 + (v_j t - x)^2\right)\right] \end{aligned} \tag{2.17}$$

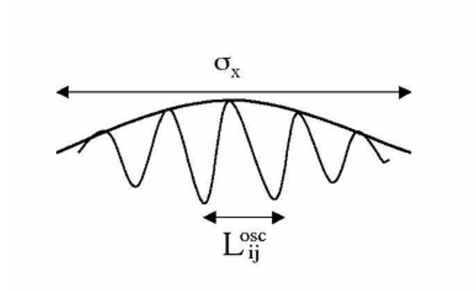
The first line is only a normalization factor and the mixing matrices, but the new features of the wave packet approach are all in the exponential factors in the second line, so we will only concentrate on them and write the matrices just as $A_{ij}^{\alpha\beta} := U_{\alpha i} U_{i\beta}^{-1} U_{\alpha j}^* \left(U_{j\beta}^*\right)^{-1}$ from now on. As mentioned above, in real oscillation experiments one can only measure the length x but never the time t, so we should get rid of the time to get the probability $P_{\alpha\to\beta}(x)$, depending only on the point x where the particle is detected. One way of doing this is shown in [5]: If we assume that the time is completely unknown in the experiment, we can integrate over it, which is again done by completing the squares in the exponent:

$$\begin{aligned} P_{\alpha\to\beta}(x) = \int \mathrm{d}t\, P_{\alpha\to\beta}(t,x) = &\sum_{ij} A_{ij}^{\alpha\beta} \sqrt{\frac{2}{v_i^2 + v_j^2}} \\ &\times \exp\left[i\left(\frac{v_i + v_j}{v_i^2 + v_j^2}(E_i - E_j) - (p_i - p_j)\right)x\right] \quad (a) \\ &\times \exp\left[-\frac{x^2 (v_i - v_j)^2}{4\sigma_x^2 (v_i^2 + v_j^2)}\right] \quad (b) \\ &\times \exp\left[-\frac{(E_i - E_j)^2}{4\sigma_p^2 (v_i^2 + v_j^2)}\right] \quad (c) \end{aligned} \tag{2.18}$$

The normalization factor $\sqrt{\frac{2}{v_i^2+v_j^2}}$ is almost exactly 1 for extremely relativistic particles and will therefore be neglected from now on. The first exponential factor (a) is just an oscillating phase and will lead to the same term as the plane wave treatment in the ultra-relativistic limit [5]: $(a) \to \exp\left[-i2\pi \frac{x}{L_{ij}^{\text{osc}}}\right]$ with $L_{ij}^{\text{osc}} = \frac{4\pi E}{\Delta m_{ij}^2}$. The factors (b) and (c) in (2.18) are new and only due to the wave packet ansatz for the mass eigenstates and will give rise to two different sources of decoherence in neutrino oscillations.



(a) Decoherence due to the non-overlapping of the different mass eigenstates wave packets.



(b) Decoherence due to a wave packet width σ_x larger than the oscillation length.

Figure 1: Possible reasons for decoherence in neutrino oscillations [1]

How can they be interpreted? First we take a look at the factor (b): $\exp\left[-\frac{x^2(v_i-v_j)^2}{4\sigma_x^2(v_i^2+v_j^2)}\right]$. If we define the *coherence length* L_{ij}^{coh} as

$$L_{ij}^{\text{coh}} = \frac{2\sigma_x\sqrt{v_i^2+v_j^2}}{|v_i-v_j|} \tag{2.19}$$

this factor can be written as $\exp\left[-\left(\frac{x}{L_{ij}^{\text{coh}}}\right)^2\right]$. So the oscillation is exponentially damped if $x \gtrsim L^{\text{coh}}$. This is due to the fact that we described the mass eigenstates as wave packets localized in space, but because these different wave packets can have different group velocities, they will not overlap anymore if they can propagate too far, see Fig. 1(a). If they do not overlap, coherence is lost and no oscillation can be observed. Of course, L^{coh} then should be smaller if the difference in velocity is greater or the wave packets are narrower - and indeed $L_{ij}^{\text{coh}} \propto \frac{\sigma_x}{|v_i-v_j|}$.

The factor (c): $\exp\left[-\frac{(E_i-E_j)^2}{4\sigma_p^2(v_i^2+v_j^2)}\right]$ is directly related to the problems mentioned in Section 2.2.1 and brings Heisenberg's uncertainty principle into play. It can be written as $\exp\left[-c\left(\frac{\Delta p_{ij}}{\sigma_p}\right)^2\right]$, with $\Delta p_{ij} = p_i - p_j$ under the assumption that $E_i \approx p_i$ (the factor $c = \frac{1}{4(v_i^2+v_j^2)}$ does not really change the physical interpretation of this term). So oscillations will be destroyed if $\Delta p_{ij} \gtrsim \sigma_p$, that is if momentum uncertainty is of order or smaller than the difference in the momenta, which means we could resolve which mass eigenstate really propagates (because Δp_{ij} is of the same order as Δm_{ij}^2). In this case it is clear that no oscillation can take place, as was discussed in Section 2.2.1, but one can also look at this from a different angle: the term can also be rewritten as $\exp\left[-\tilde{c}\left(\frac{\sigma_x}{L_{ij}^{\rm osc}}\right)^2\right]$, so oscillations are suppressed if $\sigma_x \gtrsim L_{ij}^{\rm osc}$, which is again quite clear because if the uncertainty on the position of the neutrino is of the same order as the oscillation length, any oscillation will be completely washed out, see Fig. 1(b). In principle, nothing forbids to make energy and momentum measurements as accurate as necessary to determine which single mass eigenstate propagates in space, but this can only be done at the prize of a greater uncertainty on the position, which will eventually destroy any observable oscillation effects. So the knowledge of which mass eigenstate contributes and the observation of neutrino oscillations exclude each other. This is analogous to other well known quantum mechanical interference phenomena, e.g. the famous double slit experiment: as long as two possible ways are accessible for the particle, an interference pattern can be observed at a screen behind the slits, but if it is determined which way a particle really goes, coherence is lost and no interference pattern will be seen. Interference effects in quantum mechanics can only occur if two or more amplitudes coherently contribute to the total probability of finding the system in a given final state, i.e. it is indistinguishable which "way" (literally in the case of the double slit experiment, but also in a more abstract sense) is chosen. The same is true for particle oscillations (which are in fact nothing but interference effects): the total amplitude for a (flavor eigenstate) neutrino to propagate from one point in space to another, is the sum of the amplitudes of the different mass eigenstates, that are kinematically allowed in this process. If one reduces this to only one single mass eigenstate (for example by measuring energy and momentum accurate enough), there are no two amplitudes that could add up and so no oscillation can take place.

In the case of only two neutrinos, considering only ν_e, ν_μ and ν_1, ν_2, with the real 2×2 mixing matrix $U_{2\times 2}$, the probability of flavor changing from ν_e to ν_μ with the wave packet ansatz (compare with (2.13) for plane waves) is

$$P_{\nu_e \to \nu_\mu}(x) = \frac{1}{2}\sin^2(2\theta)\left(1 - \mathrm{e}^{-\left(\frac{x}{L_{12}^{\rm coh}}\right)^2}\mathrm{e}^{-\left(\frac{\sigma_x}{L_{12}^{\rm osc}}\right)^2}\cos\left(2\pi\frac{x}{L_{12}^{\rm osc}}\right)\right) \tag{2.20}$$

So if either the distance x is of order or greater than the coherence length $L_{12}^{\rm coh}$ or the wave packet width σ_x is of order or greater than the oscillation length $L_{12}^{\rm osc}$, the oscillating term $\cos\left(2\pi\frac{x}{L_{12}^{\rm osc}}\right)$ is damped and (2.20) reduces to a constant term

$$\tilde{P}_{\nu_e \to \nu_\mu} = \frac{1}{2}\sin^2(2\theta) \tag{2.21}$$

From (2.21) one can directly see that the suppression of oscillation effects does *not* mean that flavor changing cannot occur. The probability of finding a ν_μ if a ν_e was produced, is then simply the probability that the inital ν_e propagates as a ν_1 ($P_{\nu_e \to \nu_1}$) times the probability that a ν_1 is measured as a ν_μ ($P_{\nu_1 \to \nu_\mu}$), plus the probability of the same with a ν_2 as the intermediate state instead. Because coherence is lost, these probabilities can be simply summed up in the classic way: $\tilde{P}_{\nu_e\to\nu_\mu} = P_{\nu_e\to\nu_1} \times P_{\nu_1\to\nu_\mu} + P_{\nu_e\to\nu_2} \times P_{\nu_2\to\nu_\mu}$. So (2.21) can also be obtained that way:

$$\tilde{P}_{\nu_e\to\nu_\mu} = |U_{e1}|^2|U_{1\mu}^{-1}|^2 + |U_{e2}|^2|U_{2\mu}^{-1}|^2 = 2\cos^2\theta\,\sin^2\theta = \frac{1}{2}\sin^2(2\theta) \tag{2.22}$$

So the decohrence terms in (2.20) lead to a more "classical" behavior of the particles, in the sense that the probabilities (and not the amplitudes) of ending up in the same final state via two different intermediate states are simply added and no interference takes place.

2.4 Experimental Bounds on the Parameters

For three different mass and flavor eigenstates, the 3×3 mixing matrix can be parameterized in the following way [8]:

$$\begin{aligned} U &= U^{23}U^{13}U^{12} \\ &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix} \begin{pmatrix} \cos\theta_{13} & 0 & \sin\theta_{13} \\ 0 & 1 & 0 \\ -\sin\theta_{13} & 0 & \cos\theta_{13} \end{pmatrix} \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{aligned} \tag{2.23}$$

where a possible CP violating phase is neglected. So there are 5 independent parameters altogether, that can be determined from neutrino oscillations: 3 mixing angles θ_{ij} and 2 mass squared differences Δm_{ij}^2. There are different types of neutrino oscillation experiments: at reactors, at accelerators or with atmospheric or solar neutrinos, all varying very much in their typical energy E and distance from the source to the detector L. This is very import, since the sensitivity for Δm_{ij}^2 is depending on the ratio $\frac{E}{L}$ [8]. Because the oscillating term in (2.20) has the form $\cos\left(\frac{\Delta m_{ij}^2 L}{2E}\right)$, it is not possible to determine Δm_{ij}^2 if either

$$\frac{\Delta m_{ij}^2 L}{2E} \gg 1 \tag{2.24}$$

because the oscillations will be too fast and only an average is measurable, or

$$\frac{\Delta m_{ij}^2 L}{2E} \ll 1 \tag{2.25}$$

because the oscillations will be too slow and no flavor changing can be observed. So different experiments are sensitive to different values of Δm_{ij}^2, such that

$$\Delta m_{ij}^2 \sim \frac{E}{L} \tag{2.26}$$

The sensitivity for Δm_{ij}^2 can reach from $\sim 10^2\,\text{eV}^2$ in some short-baseline accelerator experiments to $\sim 10^{-12}\,\text{eV}^2$ for solar neutrinos [8].

The best fits for the parameters up to now are [9]:

$$\sin^2(2\theta_{12}) = 0.857^{+0.023}_{-0.025}$$
$$\sin^2(2\theta_{23}) > 0.95$$
$$\sin^2(2\theta_{13}) = 0.098^{+0.013}_{-0.013}$$

$$\Delta m_{21}^2 = 7.50^{+0.19}_{-0.20}\ 10^{-5}\,\text{eV}^2$$
$$\Delta m_{32}^2 = 2.32^{+0.12}_{-0.08}\ 10^{-3}\,\text{eV}^2$$

3 Neutral Kaons

Apart from neutrinos, some mesons (namely the neutral K- and B-mesons) are very often used to study oscillation effects. In Section 3.1 a short introduction of oscillations of K-mesons will be given, in Section 3.2 two additional phenomena occurring in particle oscillations will be discussed: entanglement and decoherence in entangled systems.

3.1 Kaon Oscillations

K-mesons (or kaons) are mesons consisting of one s-quark and one light u- or d-quark (and the corresponding anti-quarks). Oscillations can only be observed with neutral kaons, which are K^0 with a quark content $d\bar{s}$ and its anti-particle $\bar{K}^0$ with $\bar{d}s$. These are the flavor eigenstates corresponding to $(\nu_e, \nu_\mu, \nu_\tau)$ in the case of neutrinos. A transition between K^0 and $\bar{K}^0$ can happen via two exchanges of a W-boson (see Fig. 2).

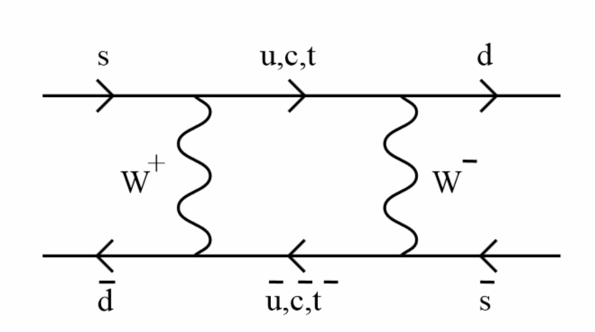


Figure 2: K^0-$\bar{K}^0$ oscillation

The mass eigenstates (corresponding to (ν_1, ν_2, ν_3)) are constructed as eigenstates of CP transformations, a charge conjugation C (switch between particle and anti-particle) plus a parity transformation P. Because K^0 and $\bar{K}^0$ are particle and anti-particle, we can write:

$$
\begin{aligned}
C\,|K^0\rangle &= |\bar{K}^0\rangle \\
C\,|\bar{K}^0\rangle &= |K^0\rangle
\end{aligned}
\tag{3.1}
$$

Kaons belong to the pseudoscalar meson-octet, so they have odd parity:

$$
\begin{aligned}
P\,|K^0\rangle &= -\,|K^0\rangle \\
P\,|\bar{K}^0\rangle &= -\,|\bar{K}^0\rangle
\end{aligned}
\tag{3.2}
$$

From (3.1) and (3.2) one can see that neither $|K^0\rangle$ nor $|\bar{K}^0\rangle$ are CP eigenstates, but they can be constructed from linear combinations of the flavor eigenstates [2].

$$
\begin{aligned}
|K_S\rangle &= \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle) \\
|K_L\rangle &= \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle)
\end{aligned}
\tag{3.3}
$$

$$
\begin{aligned}
CP\,|K_S\rangle &= +\,|K_S\rangle \\
CP\,|K_L\rangle &= -\,|K_L\rangle
\end{aligned}
\tag{3.4}
$$

The subscripts "S" and "L" are used because they are called "K-short" and "K-long", indicating that they differ very much in their mean lifetimes ($\tau_{K_S} = 8.59 \times 10^{-11}s$ and $\tau_{K_L} = 5.11 \times 10^{-8}s$ [9]). Because the kaons are not stable, they cannot be treated in the same way as the neutrinos, but we have to introduce an "effective mass" Hamiltonian to describe the decay properly. (A detailed derivation of this so called *Wigner-Weisskopf-Approximation* is given in Ref. [10], appendix I.) $|K_S\rangle$ and $|K_L\rangle$ are then eigenstates of this Hamiltonian:

$$
H\,|K_{S,L}\rangle = \left(m_{S,L} - \frac{i}{2}\Gamma_{S,L}\right)|K_{S,L}\rangle \tag{3.5}
$$

[2] Due to CP violation, the mass eigenstates $|K_S\rangle$ and $|K_L\rangle$ are in fact *not* the CP eigenstates, but

$$
\begin{aligned}
|K_S\rangle &= \frac{1}{\sqrt{2}\sqrt{1+|\epsilon|^2}}\left((1+\epsilon)\,|K^0\rangle - (1-\epsilon)\,|\bar{K}^0\rangle\right) \\
|K_L\rangle &= \frac{1}{\sqrt{2}\sqrt{1+|\epsilon|^2}}\left((1+\epsilon)\,|K^0\rangle + (1-\epsilon)\,|\bar{K}^0\rangle\right)
\end{aligned}
$$

But all effects of CP violation are negligible small in this context, so we can assume perfect CP symmetry and set $\epsilon = 0$ from now on.

with $m_{S,L}$ and $\Gamma_{S,L}$ being the masses and decay widths of the states. Because of the term $\frac{i}{2}\Gamma_{S,L}$, the probability of finding the system in the state $|K_{S,L}\rangle$ decreases exponentially in time, just as it should.

$$\langle K_{S,L}(t)|K_{S,L}(t)\rangle = \mathrm{e}^{-\Gamma_{S,L}t} \tag{3.6}$$

(if the state was normalized at time $t = 0$)

Because this effective mass Hamiltonian is not hermetian, the energy in this system would not be conserved. To resolve this problem one would have to look at the whole system, i.e. the initial state and the final states, (an open-quantum-system formulation is given in Ref. [11]). The Hilbert space of this system can be written as $\mathbf{H_{tot}} = \mathbf{H_s} \oplus \mathbf{H_f}$, where $\mathbf{H_s}$ is the space of the system we are looking at and $\mathbf{H_f}$ the space of the decay products. The Hamiltonian of the total Hilbert space $\mathbf{H_{tot}}$ would be hermitian, but because we are not interested in the decay products in our considerations about particle oscillations, we can stay in the space $\mathbf{H_s}$ of the kaons and work with the non-hermitian Hamiltonian as in (3.5). Because of this, one cannot use the *von Neumann equation*

$$\frac{\partial}{\partial t}\rho = i[\rho, H] \tag{3.7}$$

in its usual form with the commutator $[\rho, H]$ for the time evolution of a density matrix ρ, but of course

$$\frac{\partial}{\partial t}\rho = i(\rho H^\dagger - H\rho) \tag{3.8}$$

instead. With the definitions in (3.3),(3.5) and (3.8), we can now tackle the $K^0\bar{K}^0$-strangeness-oscillations ([12][13]).

$|K^0\rangle$ and $|\bar{K}^0\rangle$ are states with well-defined strangeness, so they are eigenstates of the strangeness operator S:

$$\begin{aligned} S\,|K^0\rangle &= +\,|K^0\rangle \\ S\,|\bar{K}^0\rangle &= -\,|\bar{K}^0\rangle \end{aligned} \tag{3.9}$$

In the basis $\{K^0, \bar{K}^0\}$, S can be written as

$$S = |K^0\rangle\langle K^0| - |\bar{K}^0\rangle\langle\bar{K}^0| \tag{3.10}$$

But it will be more useful to write S in the basis $\{K_S, K_L\}$, so with (3.3) one finds:

$$S = |K_S\rangle\langle K_L| + |K_L\rangle\langle K_S| \tag{3.11}$$

Let's say we start with a state with strangeness +1, i.e. $|K^0\rangle$, at $t = 0$ and we want to know the strangeness expectation value $\langle S\rangle$ at a given time t. First, one has to transform this in the basis of $\{K_S, K_L\}$, because only for them we know the time evolution. Doing so we get the density matrix at $t = 0$.

$$\rho(0) = |K^0\rangle\langle K^0| = \frac{1}{2}(|K_S\rangle\langle K_S| + |K_L\rangle\langle K_L| + |K_S\rangle\langle K_L| + |K_L\rangle\langle K_S|) \tag{3.12}$$

Applying (3.8) we get

$$\rho(t) = \frac{1}{2}\left(|K_S\rangle\langle K_S|\,\mathrm{e}^{-\Gamma_S t} + |K_L\rangle\langle K_L|\,\mathrm{e}^{-\Gamma_L t} + |K_S\rangle\langle K_L|\,\mathrm{e}^{i\Delta m t-\Gamma t} + |K_L\rangle\langle K_S|\,\mathrm{e}^{-i\Delta m t-\Gamma t}\right) \tag{3.13}$$

$$\Delta m = m_L - m_S \qquad\qquad \Gamma = \frac{\Gamma_S + \Gamma_L}{2} \tag{3.14}$$

We were interested in the strangeness $\langle S\rangle = \mathrm{Tr}(S\rho)$, so letting S act on ρ yields:

$$S\rho(t) = \frac{1}{2}\left(|K_L\rangle\langle K_S|\,\mathrm{e}^{-\Gamma_S t} + |K_S\rangle\langle K_L|\,\mathrm{e}^{-\Gamma_L t} + |K_L\rangle\langle K_L|\,\mathrm{e}^{i\Delta m t-\Gamma t} + |K_S\rangle\langle K_S|\,\mathrm{e}^{-i\Delta m t-\Gamma t}\right) \tag{3.15}$$

Finally, after taking the trace, the desired result is (note: $\langle K_S|K_L\rangle = 0$ is only true because of the assumption of CP symmetry):

$$\begin{aligned}\langle S\rangle &= \frac{1}{2}\left(\mathrm{e}^{i\Delta m t-\Gamma t} + \mathrm{e}^{-i\Delta m t-\Gamma t}\right)\\ &= \cos(\Delta m\, t)\mathrm{e}^{-\Gamma t}\end{aligned} \tag{3.16}$$

The strangeness expectation value oscillates with angular frequency Δm, which means that the probability of finding either K^0 or $\bar{K}^0$ oscillates just as was found for the neutrino flavor states. The damping factor $\mathrm{e}^{-\Gamma t}$ accounts for the fact that the kaons are not stable. From these oscillation effects it is possible to determine the mass difference Δm with great accuracy: $\Delta m = (3.483 \pm 0.006) \times 10^{-12}\,\mathrm{MeV}$ [9]

3.2 Entanglement and Decoherence

After studying the "normal" kaon oscillation, we can now take a look at the more interesting case of two kaons entangled in their strangeness. The entanglement in strangeness can be treated analogous to two spin-$\frac{1}{2}$ particles entangled in their spin. The states $|K^0\rangle$ and $|\bar{K}^0\rangle$ correspond to $|\uparrow\rangle$ and $|\downarrow\rangle$ respectively, the strangeness operator S is just the Pauli matrix σ_z. One can define a singlet state $|\psi^-\rangle$ with zero total strangeness (notation: $|\cdot\rangle\,|\cdot\rangle := |\cdot\rangle \otimes |\cdot\rangle$)

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle\,|\bar{K}^0\rangle - |\bar{K}^0\rangle\,|K^0\rangle) \tag{3.17}$$

A transformation to the basis $\{K_S, K_L\}$ yields

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}(|K_S\rangle\,|K_L\rangle - |K_L\rangle\,|K_S\rangle) \tag{3.18}$$

With a short notation for the 2-particle states $|e_1\rangle = |K_S\rangle\,|K_L\rangle$ and $|e_2\rangle = |K_L\rangle\,|K_S\rangle$, $|\psi^-\rangle$ has the simple form:

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}(|e_1\rangle - |e_2\rangle) \tag{3.19}$$

In order to destroy the coherence in the system and study the effects of that, (3.8) can be modified by adding a so called *dissipator* $D[\rho]$

$$\frac{\partial}{\partial t}\rho = i\rho H^\dagger - iH\rho - D[\rho] \tag{3.20}$$

The dissipator encodes the effects of interactions of the system with the environment and will eventually lead to decoherence. One choice of $D[\rho]$ to describe decoherence in a system of two entangled particles is given in Ref. [14]:

$$D[\rho] = \lambda(P_1\rho P_2 + P_2\rho P_1) \tag{3.21}$$

where $P_i = |e_i\rangle\,\langle e_i|$ is the projector on the 2-particle state $|e_i\rangle$ and λ is the *decoherence parameter* ($\lambda \geq 0$). With this choice, (3.20) is a special case of the *Lindblad equation* with

$$D_{\text{Lindblad}}[\rho] = \frac{1}{2}\sum_i \left(A_i^\dagger A_i\rho + \rho A_i^\dagger A_i - 2A_i\rho A_i^\dagger\right) \tag{3.22}$$

(set $A_i = \sqrt{\lambda}P_i$), which is the most general ansatz for a master equation of the type (3.20) to describe the time evolutoin of a density matrix ρ.

We can now investigate how the new term $D[\rho]$ changes the time evolution of the density matrix of entangled kaons [13]. If we start with the singlet state $|\psi^-\rangle$ at $t = 0$, the density matrix is:

$$\begin{aligned}\rho(0) &= \frac{1}{2}\left(|e_1\rangle\,\langle e_1| + |e_2\rangle\,\langle e_2| - |e_1\rangle\,\langle e_2| - |e_2\rangle\,\langle e_1|\right) \\ &= \sum_{i,j=1,2} \rho_{ij}(0)\,|e_i\rangle\,\langle e_j|\end{aligned} \tag{3.23}$$

From (3.21) we get the dissipator ($\langle e_i | e_j \rangle = \delta_{ij}$):

$$D[\rho] = -\lambda \left(|e_1\rangle \langle e_2| + |e_2\rangle \langle e_1| \right) \tag{3.24}$$

For the 2-particle case (3.5) now reads

$$H \hat{=} H \otimes \mathbb{1} + \mathbb{1} \otimes H \Rightarrow H |e_i\rangle = \Big(m_S + m_L - \frac{i}{2}(\Gamma_S + \Gamma_L) \Big) |e_i\rangle \tag{3.25}$$

The four components of the density matrix decouple and from (3.20) we get:

$$\begin{aligned}
\frac{\partial}{\partial t}\rho_{11}(t) &= (-2\Gamma)\rho_{11}(0) \\
\frac{\partial}{\partial t}\rho_{22}(t) &= (-2\Gamma)\rho_{22}(0) \\
\frac{\partial}{\partial t}\rho_{12}(t) &= (-2\Gamma - \lambda)\rho_{12}(0) \\
\frac{\partial}{\partial t}\rho_{21}(t) &= (-2\Gamma - \lambda)\rho_{21}(0)
\end{aligned} \tag{3.26}$$

with $\Gamma = \frac{\Gamma_S + \Gamma_L}{2}$. Solving this yields the exponential factors for the density matrix:

$$\rho(t) = \frac{1}{2} \mathrm{e}^{-2\Gamma t} \Big(|e_1\rangle \langle e_1| + |e_2\rangle \langle e_2| - \mathrm{e}^{-\lambda t} \big(|e_1\rangle \langle e_2| + |e_2\rangle \langle e_1| \big) \Big) \tag{3.27}$$

So the dissipator suppresses the off-diagonal elements of ρ, whereas the diagonal elements are not affected at all, while the whole state is decaying with decay width 2Γ.

For two particles in a spin singlet state we expect, that if one is measured with spin up, the other will be measured with spin down et vice versa. In the above case of the two neutral kaons in a strangeness singlet state and the von Neumann equation modified with a dissipator given in (3.21), there are two new issues to be considered:

- each of the two kaons will individually oscillate, as was discussed in Section 3.1
- due to the suppression of the off-diagonal elements of the density matrix, the system will not be in a pure state for $t > 0$ if $\lambda \neq 0$ (see Ref. [13])

The next few pages will again follow Ref. [13], with some calculations shown more explicitly.

To investigate the effects of the two issues mentioned above, imagine the following situation: the two kaons are produced in the state $|\psi^-\rangle$ at $t = 0$ and freely propagate to the left and the right, where their strangeness can be measured, i.e. we can test the kaons for either being a K^0 or a $\bar{K}^0$. If the right moving particle is tested for being a K^0 at time t_r, i.e. having strangeness $+1$, we can write the operator for doing so as

$S_r^+ = \mathbb{1} \otimes |K^0\rangle\langle K^0|$ and then take the partial trace over the space of the right moving particle, and thereby get the single particle density matrix now only for the left moving particle.

$$\rho_l(t = t_r; t_r) := \mathrm{Tr}_r \left(S_r^+ \rho(t_r) \right) \tag{3.28}$$

This remaining one-particle state $\rho_l(t > t_r; t_r)$ then freely propagates, but because there is no decoherence assumed for a single particle (we want to test the entanglement of the two particle state), this time the time evolution in (3.8) without the dissipator is used. At $t = t_l > t_r$ this particle is tested for being a $\bar{K}^0$, so we use the operator $S^- = |\bar{K}^0\rangle\langle\bar{K}^0|$. So the expectation value for finding a K^0 at the right at $t = t_r$ and a $\bar{K}^0$ at the left at $t = t_l$ is:

$$P_\lambda(\bar{K}^0, t_l; K^0, t_r) := \mathrm{Tr}(S^- \rho_l(t_l; t_r)) \tag{3.29}$$

Now we can do these calculations explicitly. From (3.27) we know the density matrix at $t = t_r$. First we write this in the basis $\{K_S, K_L\}$

$$\begin{aligned}\rho(t_r) = \frac{1}{2}\mathrm{e}^{-2\Gamma t_r}\Big(&|K_S\rangle\langle K_S| \otimes |K_L\rangle\langle K_L| + |K_L\rangle\langle K_L| \otimes |K_S\rangle\langle K_S| \\ &- \mathrm{e}^{-\lambda t_r}\big(|K_S\rangle\langle K_L| \otimes |K_L\rangle\langle K_S| + |K_L\rangle\langle K_S| \otimes |K_S\rangle\langle K_L| \big)\Big)\end{aligned} \tag{3.30}$$

We do the same with the operator S_r^+ and then follow the steps described above to get $\rho_l(t = t_r; t_r)$:

$$S_r^+ = \mathbb{1} \otimes \frac{1}{2}\Big(|K_S\rangle\langle K_S| + |K_L\rangle\langle K_L| + |K_S\rangle\langle K_L| + |K_L\rangle\langle K_S| \Big) \tag{3.31}$$

$$\begin{aligned}S_r^+ \rho(t_r) = \frac{1}{4}\mathrm{e}^{-2\Gamma t_r}\Big(&|K_S\rangle\langle K_S| \otimes (|K_L\rangle\langle K_L| + |K_S\rangle\langle K_L|) \\ &+ |K_L\rangle\langle K_L| \otimes (|K_S\rangle\langle K_S| + |K_L\rangle\langle K_L|) \\ &- \mathrm{e}^{-\lambda t_r}\Big(|K_S\rangle\langle K_L| \otimes (|K_L\rangle\langle K_S| + |K_S\rangle\langle K_S|) \\ &+ |K_L\rangle\langle K_S| \otimes (|K_S\rangle\langle K_L| + |K_L\rangle\langle K_L|)\Big)\Big)\end{aligned} \tag{3.32}$$

$$\begin{aligned}\mathrm{Tr}_r(S_r^+ \rho(t_r)) &= \frac{1}{4}\mathrm{e}^{-2\Gamma t_r}\Big(|K_S\rangle\langle K_S| + |K_L\rangle\langle K_L| - \mathrm{e}^{-\lambda t_r}\big(|K_S\rangle\langle K_L| + |K_L\rangle\langle K_S| \big)\Big) \\ &=: \rho_l(t = t_r; t_r)\end{aligned} \tag{3.33}$$

We have already calculated the time evolution for a single particle density matrix $\rho_l(t; t_r)$ in (3.13) and so we get

$$\rho_l(t_l; t_r) = \frac{1}{4}\mathrm{e}^{-2\Gamma t_r}\Big(|K_S\rangle\langle K_S|\,\mathrm{e}^{-\Gamma_S\Delta t} + |K_L\rangle\langle K_L|\,\mathrm{e}^{-\Gamma_L\Delta t} - \mathrm{e}^{-\lambda t_r - \Gamma\Delta t}\big(|K_S\rangle\langle K_L|\,\mathrm{e}^{i\Delta m\Delta t} + |K_L\rangle\langle K_S|\,\mathrm{e}^{-i\Delta m\Delta t}\big)\Big) \tag{3.34}$$

Doing the analogous calculations to (3.32) and (3.33) but this time with

$$S^- = \frac{1}{2}\Big(|K_S\rangle\langle K_S| + |K_L\rangle\langle K_L| - |K_S\rangle\langle K_L| - |K_L\rangle\langle K_S| \Big) \tag{3.35}$$

eventually yields:

$$\begin{aligned}\mathrm{Tr}(S^-\rho_l(t_l; t_r)) &= \frac{1}{8}\mathrm{e}^{-2\Gamma t_r}\Big(\mathrm{e}^{-\Gamma_S\Delta t} + \mathrm{e}^{-\Gamma_L\Delta t} + 2\mathrm{e}^{-\Gamma\Delta t}\cos(\Delta m\Delta t)\mathrm{e}^{-\lambda t_r}\Big)\\ &=: P_\lambda(\bar{K}^0, t_l; K^0, t_r)\end{aligned} \tag{3.36}$$

If we do analogous calculations for arbitrary combinations of K^0 and $\bar{K}^0$ on the left and the right side, we find the expectation values for finding like or unlike strangeness on both sides. If we do not restrict ourselves to do the measurement on the right first, we must replace t_r in the exponent in (3.36) by $\min(t_r, t_l)$, because decoherence only occurs in the two particle system, so the dissipator only acts until the first measurement is made.

$$\begin{aligned}P_{\lambda,unlike}(t_l, t_r) &= \frac{1}{4}\mathrm{e}^{-2\Gamma t_r}\Big(\mathrm{e}^{-\Gamma_S\Delta t} + \mathrm{e}^{-\Gamma_L\Delta t} + 2\mathrm{e}^{-\Gamma\Delta t}\cos(\Delta m\Delta t)\mathrm{e}^{-\lambda\min(t_r,t_l)}\Big)\\ P_{\lambda,like}(t_l, t_r) &= \frac{1}{4}\mathrm{e}^{-2\Gamma t_r}\Big(\mathrm{e}^{-\Gamma_S\Delta t} + \mathrm{e}^{-\Gamma_L\Delta t} - 2\mathrm{e}^{-\Gamma\Delta t}\cos(\Delta m\Delta t)\mathrm{e}^{-\lambda\min(t_r,t_l)}\Big)\end{aligned} \tag{3.37}$$

The *asymmetry* is defined as

$$A_\lambda(t_l, t_r) = \frac{P_{\lambda,unlike}(t_l, t_r) - P_{\lambda,like}(t_l, t_r)}{P_{\lambda,unlike}(t_l, t_r) + P_{\lambda,like}(t_l, t_r)} \tag{3.38}$$

Using the results from (3.37) yields

$$A_\lambda(t_l, t_r) = \frac{\cos(\Delta m\Delta t)}{\cosh(\frac{1}{2}\Delta\Gamma\Delta t)}\,\mathrm{e}^{-\lambda\min(t_r,t_l)} \tag{3.39}$$

How can this result be interpreted? For non-oscillating, stable particles without any decoherence, one would expect $A_{\lambda=0}(t_l, t_r) = 1$ for all times t_l and t_r if the two particles were initially in a singlet state, because only unlike strangeness will be measured for the two particles. This result is indeed found in (3.39) and (3.37) if we set $\lambda = 0$ (no decoherence), $\Delta m = 0$ (no oscillations) and $\Gamma_{S,L} = 0 \Rightarrow \Delta\Gamma = 0$ (stable particles). If we allow oscillations ($\Delta m \neq 0$), the probabilities of like and unlike strangeness and therefore the asymmetry oscillate with $\cos(\Delta m \Delta t)$. Both kaons oscillate individually, but as long as both are measured at the *same* time, one will still find only unlike strangeness. But because of the oscillations this will not be true anymore if there is a time difference between the two measurements. If Δt is exactly half of the oscillation period $T = \frac{2\pi}{\Delta m}$, the asymmetry will be -1 (which means finding only like strangeness), after a whole oscillation period it will be $+1$ again. Because of the different lifetimes of the mass-eigenstates, an additional factor occurs in (3.39) that damps the asymmetry for greater time differences Δt. The probability of finding like and unlike strangeness for two entangled kaons (without decoherence) is shown in Fig. 3. If we now allow $\lambda > 0$, there is the additional factor of $e^{-\lambda \min(t_r,t_l)}$. So the asymmetry is exponentially damped, the stronger the later the first measurement is made, which means that the two measurements are getting independent of each other - the correlation due to entanglement is lost, decoherence has taken place.

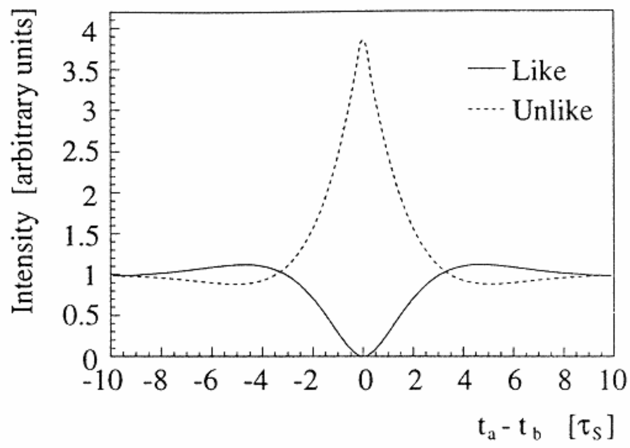


Figure 3: like and unlike strangeness without decoherence [15]

3.3 Experimental Data

The predicted asymmetry calculated above can be directly tested in experiments at accelerators, for example at the CPLEAR detector at CERN [15] or the KLOE experiment at DAΦNE [16]. At DAΦNE, e^+e^- are collided with a center of mass energy of $\sqrt{s} = 1019$ MeV to produce Φ mesons. The Φ is a neutral meson with strangeness $S = 0$ and $J^{PC} = 1^{--}$, and decays via the strong interaction into $K^0\bar{K}^0$ with a branching ratio of 34.2%. Because both P and C are symmetries in strong interactions, the two kaons have to be in a 1^{--} state as well, which is just the singlet state $|\psi^-\rangle$ defined in Section 3.2. At CPLEAR the kaons were produced in proton-antiproton collisions. To determine the strangeness of a kaon at a given position, an absorber can be placed at that position and the strangeness of the outgoing particles, produced via the strong interaction of the kaons with the nuclei of the material, can be measured. With this method it is possible to test a neutral kaon for being a K^0 or $\bar{K}^0$ by looking for either K^+, or K^- and Λ particles [15]:

$$K^0 + \text{nucleus} \to K^+ + X \tag{3.40}$$

$$\begin{aligned} \bar{K}^0 + \text{nucleus} &\to K^- + X \\ \bar{K}^0 + \text{nucleus} &\to \Lambda + X \end{aligned} \tag{3.41}$$

At the experiment at the CPLEAR detector at CERN it was possible to measure the asymmetry for two different experimental configurations (see Fig. 4). The detector

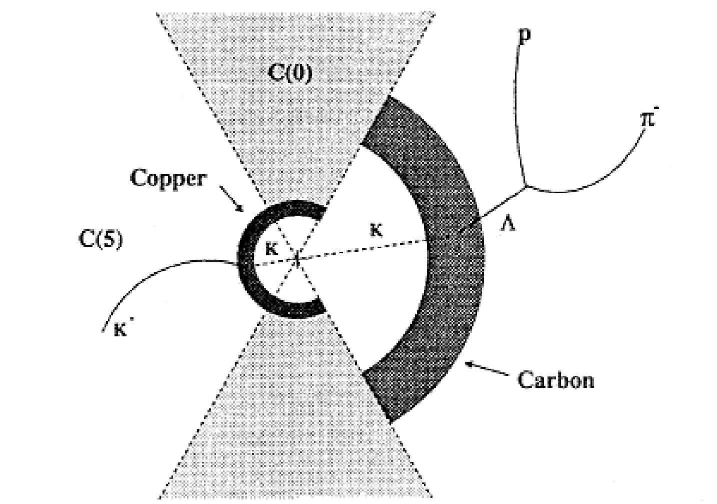


Figure 4: CPLEAR detector [15]

consists of two cylindrical absorbers, one with radius 2cm (copper) and one with radius 7cm (carbon). Depending on the direction of the kaons, they either both hit the absorber made of copper, so the difference of their propagation length is $\Delta l = 0$, or one hits the copper and one the outer absorber made of carbon, giving a difference of $\Delta l = 5$cm. Knowing the energy of the kaons, it is easy to get Δt from Δl and so (3.39) can be tested directly in this experiment. Fig. 5 shows the result of the measured asymmetry at CPLEAR: The solid line is the expected asymmetry without any decoherence, the crosses show the actually measured asymmetry for $\Delta l = 0$cm and $\Delta l = 5$cm. In Ref. [13], a fit was made with the data from CPLEAR, to obtain the experimental bounds for the decoherence parameter λ:

$$\lambda = \left(1.84^{+2.50}_{-2.17}\right) \times 10^{-12}\ \text{MeV} \tag{3.42}$$

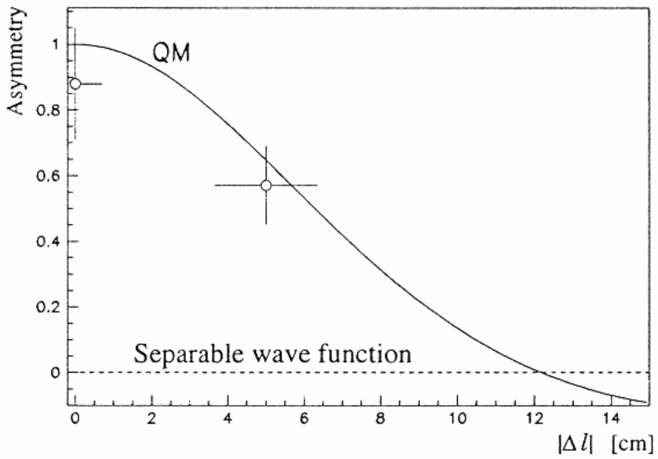


Figure 5: asymmetry for the two experimental configurations [15]

This is clearly compatible with $\lambda = 0$, but an upper bound $\lambda < 4.34 \times 10^{-12}$ MeV is found.

More recent data and more accurate bounds on the decoherence parameter λ and parameters of other possible models of decoherence were determined at the KLOE and KLOE-2 experiments at DAΦNE, given in Refs. [16][17]. In any case, up to now no clear deviation from the predictions of quantum mechanics without any decoherence could be found in these experiments. In [17] the KLOE-2 collaboration announces, that they expect to decrease the experimental uncertainties by one order of magnitude within the next years.

4 B-Mesons

4.1 Entangled B-Mesons and Semileptonic Decays

B-mesons are very similar to kaons, but with a bottom quark instead of a strange quark, so B^0 and $\bar{B}^0$ have quark content $d\bar{b}$ and $\bar{d}b$ respectively (there also exists a strange B-meson with the down quark replaced by a strange quark, i.e. B_s^0 has quark content $s\bar{b}$). The formalism to describe oscillations and entanglement of B-mesons is the same as that used for the kaons in Section 3. So again, assuming CP symmetry, the mass eigenstates B_H (corresponding to K_L) and B_L (corresponding to K_S), “H” and “L” for “heavy” and “light”, can be constructed as CP eigensates .

$$
\begin{aligned}
|B_L\rangle &= \frac{1}{\sqrt{2}}(|B^0\rangle - |\bar{B}^0\rangle) \\
|B_H\rangle &= \frac{1}{\sqrt{2}}(|B^0\rangle + |\bar{B}^0\rangle)
\end{aligned}
\tag{4.1}
$$

They are eigenstates of the effective mass Hamiltonian analogous to (3.5) with masses $m_{H,L}$ and decay widths $\Gamma_{H,L}$, and $\langle B_H|B_L\rangle = 0$ (again only true for CP symmetry). Furthermore, the description of oscillating entangled two-particle states and the model for decoherence with the dissipator (3.21) can be applied just in the same way.

But there is a different way of measuring the flavor in experiments with B-mesons, on which we will concentrate in this section: The flavor of the meson can be determined from semileptonic decays, i.e. the decay of the meson by the weak interaction in a hadron and a charged lepton (plus the corresponding neutrino, see Fig. 6). So B^0 and $\bar{B}^0$ can

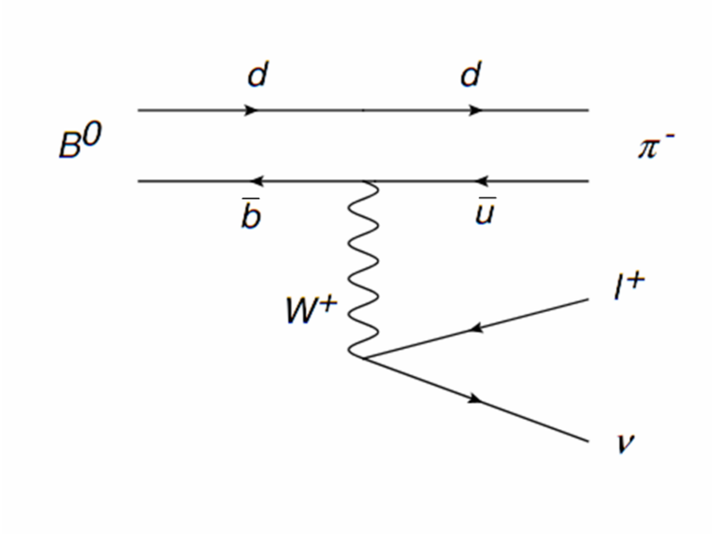


Figure 6: decay of a B^0 into a π^-, a positive charged lepton and a neutrino

be identified by

$$\begin{aligned} B^0 &\to X\,\ell^+\,\nu \\ \bar{B}^0 &\to X\,\ell^-\,\bar{\nu} \end{aligned} \tag{4.2}$$

where X denotes a hadron and $\ell = e, \mu$. If $P_{\lambda_B}(\bar{B}^0, t_l; B^0, t_r)$ denotes the expression corresponding to (3.36) for the case of B-mesons, i.e. the probability of finding a $\bar{B}^0$ at time t_l on the left side and a B^0 at time t_r at the right side, the probability of measuring a dilepton event with a negative charged lepton (from a semileptonic decay of a $\bar{B}^0$) on the left side and a positve charged lepton (from a semileptonic decay of a B^0) at the right side, can be obtained by integrating over the time [14]:

$$P_{-+} = \Gamma_\ell^2 \int_0^\infty \mathrm{d}t_l \int_0^\infty \mathrm{d}t_r\, P_{\lambda_B}(\bar{B}^0, t_l; B^0, t_r) \tag{4.3}$$

where Γ_ℓ is the inclusive semileptonic decay rate. The other three possibilities are then of course given by

$$
\begin{aligned}
P_{++} &= \Gamma_\ell^2 \int_0^\infty \mathrm{d}t_l \int_0^\infty \mathrm{d}t_r \, P_{\lambda_B}(B^0, t_l; B^0, t_r) \\
P_{--} &= \Gamma_\ell^2 \int_0^\infty \mathrm{d}t_l \int_0^\infty \mathrm{d}t_r \, P_{\lambda_B}(\bar{B}^0, t_l; \bar{B}^0, t_r) \\
P_{+-} &= \Gamma_\ell^2 \int_0^\infty \mathrm{d}t_l \int_0^\infty \mathrm{d}t_r \, P_{\lambda_B}(B^0, t_l; \bar{B}^0, t_r)
\end{aligned}
\tag{4.4}
$$

The integrals can be solved and yield [14]

$$
\begin{aligned}
P_{++} = P_{--} &= \frac{\Gamma_\ell^2}{4\Gamma^2}\left(\frac{1}{1-y^2} - \frac{1}{1+x^2}(1-\zeta)\right) \\
P_{+-} = P_{-+} &= \frac{\Gamma_\ell^2}{4\Gamma^2}\left(\frac{1}{1-y^2} + \frac{1}{1+x^2}(1-\zeta)\right)
\end{aligned}
\tag{4.5}
$$

with

$$
x = \frac{\Delta m}{\Gamma} \qquad y = \frac{\Delta\Gamma}{2\Gamma} \qquad \zeta = \frac{\lambda_B}{2\Gamma + \lambda_B} \tag{4.6}
$$

So the method of flavor tagging with semileptonic decays provides us with a measurable quantity, which we can use to try to determine the decoherence parameter λ_B from experiment, as will be shown in the next section.

4.2 Experimental Data

The correlated $B^0\bar{B}^0$-pairs can be produced in a similar way to the kaons, but with a heavier Υ-meson instead of the Φ-meson. Usually the B-mesons are produced in e^+e^- collisions at the energy of the $\Upsilon(4S)$ resonance ($m = 10.58$ GeV), which decays into $B^0\bar{B}^0$ with a branching ratio of 49% [9].

The parameter ζ is linked to the ratio of like-sign dilepton events to opposite-sign dilepton events R, that can be measured directly in experiments

$$
R = \frac{P_{++} + P_{--}}{P_{+-} + P_{-+}} \tag{4.7}
$$

With the results in (4.5) and the approximation $y = 0$, solving (4.7) for ζ yields

$$
\zeta = 1 - \frac{1-R}{1+R}(x^2 + 1) \tag{4.8}
$$

In Ref. [14], results from ARGUS and CLEO were combined to obtain the value $R_{\text{exp}} = 0.189 \pm 0.044$, data from LEP experiments were used fo find $x_{\text{exp}} = 0.740 \pm 0.031$. With these experimental values the numerical estimate $\zeta = -0.06 \pm 0.10$ was obtained. More recent data were combined in Ref. [18], yielding $x_{\text{exp}} = 0.771 \pm 0.007$ and $R_{\text{exp}} = 0.222 \pm 0.022$ (R is related to the mixing parameter χ_d, $R = \frac{\chi_d}{1-\chi_d}$). With these results we find $\zeta = -0.015 \pm 0.047$, and with $\Gamma = (4.3360 \pm 0.0020) \times 10^{-10}$ MeV [18] the decoherence parameter λ_B is

$$\lambda_B = (-1.3 \pm 4.0) \times 10^{-11} \text{ MeV} \tag{4.9}$$

So again no clear deviation from $\lambda_B = 0$ can be found.

5 Conclusion

The aim of this thesis was to give a brief introduction to flavor oscillations and then combine this with the concept of decoherence. First we have demonstrated how particle oscillations can occur if flavor eigenstates do not coincide with mass eigenstates and how the oscillation length can be derived when we assume that the states can be written as plane waves. To resolve the problems that come along with this assumption, another approach with wave packets was illustrated. Just from this simple ansatz we found two new decoherence terms that damp the oscillations if either the wave packets do not overlap or the momentum resolution becomes of order of the mass differences. Finally a third ansatz for dealing with particle oscillations with density matrices was shown. This formalism easily allows for describing oscillations of unstable particles and entangled two-particle systems, and can be modified to include a model of decoherence by adding a dissipator term in the von Neumann equation. We have seen that flavor oscillations of K- and B-mesons can be used for testing this model of decoherence by comparing the flavor asymmetry in an entangled state with the predictions of quantum mechanics. Upper bounds on the decoherence parameters λ and λ_B can be found - up to now all experimental results are compatible with $\lambda_{(B)} = 0$.

Both, flavor oscillations and a theory of decoherence, are very active fields of research both theoretically and experimentally. We have seen that particle oscillation, besides being a highly interesting topic on its own, provides us with an accurate method for determining mass differences of elementary particles. In the case of neutrinos, this very phenomenon even led to the discovery that they have mass at all, while it is still not possible today to determine the absolute values of their masses from other experiments. How to properly include neutrino masses in the Standard Model is still not entirely clear. Also CP violation and Bell inequalities can be studied using the effects of flavor mixing.

Decoherence is a key point for understanding the transition from quantum mechanics to our macroscopic "classical" world. We have seen that even a system of quite heavy particles (B-mesons are more than five times havier than protons) can form an entangled state over a clearly macroscopic distance (7cm for kaons in the CPLEAR experiment discussed in Section 3.3), in perfect agreement with the predictions of quantum mechanics. Maybe new experiments and more precise measurements can give more accurate bounds on the effects of inevitable interactions with the environment in such systems. A better understanding of quantum decoherence is essential for a better understanding of quantum mechanics as a whole, as well as for possible future technical applications such as quantum computing.

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