ELSEVIER

8 October 2001

PHYSICS LETTERS A

Physics Letters A 289 (2001) 21–26

www.elsevier.com/locate/pla

Bell inequality and *CP* violation in the neutral kaon system

R.A. Bertlmann *, W. Grimus, B.C. Hiesmayr

Institute for Theoretical Physics, University of Vienna, Boltzmanngasse 5, A-1090 Vienna, Austria

Received 6 July 2001; accepted 23 August 2001
Communicated by P.R. Holland

Abstract

For the entangled neutral kaon system we formulate a Bell inequality sensitive to *CP* violation in mixing. Via this Bell inequality we obtain a bound on the leptonic *CP* asymmetry which is violated by experimental data. Furthermore, we connect the Bell inequality with a decoherence approach and find a lower bound on the decoherence parameter which practically corresponds to Furry's hypothesis. © 2001 Elsevier Science B.V. All rights reserved.

PACS: 03.65.Ta; 03.65.Ud; 13.25.Es; 11.30.Er
Keywords: Bell inequality; Nonlocality; Entangled states; Decoherence; Neutral kaons; *CP* violation

Recently, there has been great interest in investigating entangled massive particle systems, e.g., neutral kaons [1–5]. In analogy to spin-$1/2$ particles or to polarized photons [6], neutral kaons also can be described by a "quasi-spin", a view which is especially useful in this connection (see, e.g., Ref. [4]). They are ideal systems to test the EPR–Bell correlations for massive systems. A general test of quantum mechanics (QM) versus local realistic theories (LRT) is performed via Bell inequalities [7]. In the kaon case we have the freedom of choosing different detection times and different quasi-spins. They play the role of the different angle choices in the spin-$1/2$ or photon case. Experimentally such systems are produced at the Φ resonance, for instance, in the e^+e^- collider DAΦNE at Frascati or in $p\bar{p}$ collisions in the CPLEAR experiment at CERN.

An interesting feature of the kaon systems is that the kaons reveal *CP* violation and, amazingly, it turns out that Bell inequalities for such systems imply bounds on the physical *CP* violation parameters [8,9], which can be checked experimentally, indeed, not necessarily in experiments with entangled kaons.

It was Uchiyama [8] who first found that a Bell inequality with different quasi-spin eigenstates leads to an inequality for the *CP* violation parameter ε. The derivation relied on a specific phase convention for the kaon states. Such a specific choice, although customary in kaon physics, is a certain drawback for the physical interpretation of Bell inequalities since their formulation should be as general and loophole free as possible (see, e.g., Ref. [10]).

It is the purpose of the present Letter to optimize the Bell inequality (BI) for such entangled kaons by exploiting the phase freedom in the definition of the kaon states. In this way we will clarify the relation between Uchiyama's BI and *CP* violation in mixing.

Quantum mechanically we are considering entangled states of $K^0\bar{K}^0$ pairs, in analogy to the entangled

* Corresponding author.
E-mail addresses: bertlman@ap.univie.ac.at (R.A. Bertlmann), grimus@doppler.thp.univie.ac.at (W. Grimus), hies@thp.univie.ac.at (B.C. Hiesmayr).

PII: S0375-9601(01)00577-1

spin up and down pairs, or photon pairs. They are created through the reaction $e^+e^- \to \Phi \to K^0\bar{K}^0$ in a $J^{PC} = 1^{--}$ quantum state, and are thus antisymmetric under C and P, and are described at the time $t = 0$ by the entangled state

$$\big|\psi(t=0)\big\rangle = \frac{1}{\sqrt{2}}\big\{\big|K^0\big\rangle_l \otimes \big|\bar{K}^0\big\rangle_r - \big|\bar{K}^0\big\rangle_l \otimes \big|K^0\big\rangle_r\big\}, \tag{1}$$

which can be rewritten in the $K_S K_L$ basis as

$$\big|\psi(t=0)\big\rangle = \frac{N_{SL}}{\sqrt{2}}\big\{|K_S\rangle_l \otimes |K_L\rangle_r - |K_L\rangle_l \otimes |K_S\rangle_r\big\} \tag{2}$$

with $N_{SL} = N^2/(2pq)$. Then the neutral kaons fly apart and will be detected on the left (l) and right (r) side of the source. Of course, during their propagation the K^0 and $\bar{K}^0$ oscillate and K_S, K_L decays will occur.

What is Uchiyama's inequality? Imagine the following gedanken experiment. Two neutral kaons are produced at the Φ resonance, each one in a definite quasi-spin state. The probability of measuring the short lived state K_S on the left side and the antikaon $\bar{K}^0$ on the right side, at the time $t = 0$, is denoted by $P(K_S, \bar{K}^0)$, and analogously the probabilities $P(K_S, K_1^0)$ and $P(K_1^0, \bar{K}^0)$. Then under the usual hypothesis of Bell's locality the following Bell inequality can be derived [8]:

$$P\big(K_S, \bar{K}^0\big) \leqslant P\big(K_S, K_1^0\big) + P\big(K_1^0, \bar{K}^0\big). \tag{3}$$

Generalizations can be found in Ref. [4]. Although this BI is rather formal because it involves the unphysical CP-even state $|K_1^0\rangle$, it implies an inequality on the physical CP violation parameter ε, which is experimentally testable. The procedure to derive this inequality is as follows.

In QM we describe the neutral kaons by three sets of quasi-spin eigenstates. Let us begin with the strangeness eigenstates. They distinguish the K^0 from its antiparticle $\bar{K}^0$ by

$$S\big|K^0\big\rangle = +\big|K^0\big\rangle,\qquad S\big|\bar{K}^0\big\rangle = -\big|\bar{K}^0\big\rangle. \tag{4}$$

As the K mesons are pseudoscalars, their parity P is negative and charge conjugation C transforms K^0 and $\bar{K}^0$ into each other so that we conventionally have for the combined transformation CP:

$$CP\big|K^0\big\rangle = -\big|\bar{K}^0\big\rangle,\qquad CP\big|\bar{K}^0\big\rangle = -\big|K^0\big\rangle. \tag{5}$$

From this follows that the orthogonal linear combinations

$$\big|K_1^0\big\rangle = \frac{1}{\sqrt{2}}\big\{\big|K^0\big\rangle - \big|\bar{K}^0\big\rangle\big\},\qquad \big|K_2^0\big\rangle = \frac{1}{\sqrt{2}}\big\{\big|K^0\big\rangle + \big|\bar{K}^0\big\rangle\big\} \tag{6}$$

are eigenstates of CP,

$$CP\big|K_1^0\big\rangle = +\big|K_1^0\big\rangle,\qquad CP\big|K_2^0\big\rangle = -\big|K_2^0\big\rangle, \tag{7}$$

a quantum number conserved in strong interactions.

Due to weak interactions, which are CP-violating, the kaons decay and the "physical" states are the short and long lived states

$$|K_S\rangle = \frac{1}{N}\big\{p\big|K^0\big\rangle - q\big|\bar{K}^0\big\rangle\big\},\qquad |K_L\rangle = \frac{1}{N}\big\{p\big|K^0\big\rangle + q\big|\bar{K}^0\big\rangle\big\}. \tag{8}$$

They are eigenstates of the non-Hermitian "effective mass" Hamiltonian. In a particular phase convention, the weights are expressed by [11]

$$p = 1 + \varepsilon, \qquad q = 1 - \varepsilon, \quad \text{and} \qquad N^2 = |p|^2 + |q|^2, \tag{9}$$

where ε is the complex CP-violating parameter, associated with the neutral kaon decay into the isospin-0 two-pion state (CPT invariance is assumed; thus the short and long lived states contain the same CP-violating parameter $\varepsilon_S = \varepsilon_L = \varepsilon$).

Note that the two states $|K^0\rangle$ and $|\bar{K}^0\rangle$ can be regarded as the quasi-spin states up $|\Uparrow\rangle$ and down $|\Downarrow\rangle$, and the operators acting in this quasi-spin space are expressible by Pauli matrices; the strangeness operator S can be identified with the Pauli matrix σ_3, the CP operator with $(-\sigma_1)$ and CP violation in the effective Hamiltonian is proportional to σ_2 [4].

Calculating now the probabilities of Eq. (3) within quantum mechanics the Bell inequality (3) turns into an inequality for the CP-violating parameter ε:

$$\mathrm{Re}\,\{\varepsilon\} \leqslant |\varepsilon|^2. \tag{10}$$

Inequality (10) is obviously violated by the experimental value of ε, having an absolute value of order 10^{-3} and a phase of about 45° [12]. In this way *CP* violation in $K^0\bar{K}^0$ mixing is related to the violation of a Bell inequality.

Alternatively, we could choose K_L instead of K_S and K_2^0 instead of K_1^0 in the BI (3) and arrive at the same inequality (10).

However, as already mentioned above, the derivation of inequality (10) relies on a specific choice of the phases of the kaon states. In particular, the choice of the weights in Eq. (9), where the *CP* violation parameter ε enters, is a convention such that the relative phase of the decay amplitudes $K^0 \to \pi\pi$ and $\bar{K}^0 \to \pi\pi$, both $\pi\pi$ states with isospin $I=0$, is 180° (see, for instance, Ref. [11]). However, the BI (3) involves only the two-dimensional space generated by the basis elements $|K^0\rangle$ and $|\bar{K}^0\rangle$ and has nothing to do with decays. This suggests to dispense with the phase convention (9) and rather use the phase freedom to define the unphysical state $|K_1^0\rangle$.

This we can achieve by having a phase in the *CP* transformation:

$$\begin{aligned} CP|K^0\rangle &= e^{i\alpha}|\bar{K}^0\rangle, \\ CP|\bar{K}^0\rangle &= e^{-i\alpha}|K^0\rangle, \end{aligned} \tag{11}$$

where we have chosen $(CP)^2 = \mathbf{1}$. In Eq. (5) the phase α has been fixed for convenience to $\alpha = 0$, but in general it is arbitrary and without any physical significance. So the *CP* eigenstates are the following linear combinations:

$$\begin{aligned} |K_1^0\rangle &= \frac{1}{\sqrt{2}}\{|K^0\rangle - e^{i\alpha}|\bar{K}^0\rangle\}, \\ |K_2^0\rangle &= \frac{1}{\sqrt{2}}\{|K^0\rangle + e^{i\alpha}|\bar{K}^0\rangle\}, \end{aligned} \tag{12}$$

and with this definition the quantum mechanical probabilities are

$$\begin{aligned} P_{\mathrm{QM}}(K_1^0, \bar{K}^0) &= \frac{1}{4}, \\ P_{\mathrm{QM}}(K_S, \bar{K}^0) &= \frac{1}{2N^2}|p|^2, \\ P_{\mathrm{QM}}(K_S, K_1^0) &= \frac{1}{4N^2}|pe^{i\alpha} - q|^2. \end{aligned} \tag{13}$$

Note that besides α there is also the relative phase of p and q, which is still not fixed.

We insert probabilities (13) into the Bell inequality (3) and obtain

$$\mathrm{Re}\{e^{i\alpha}pq^*\} \leqslant |q|^2. \tag{14}$$

Now we choose α such that it compensates the relative phase χ of the complex weights p and q:

$$\mathrm{Re}\{e^{i\alpha}pq^*\} = \mathrm{Re}\{e^{i(\alpha+\chi)}|p||q|\} = |p||q|. \tag{15}$$

Clearly, inequality (14) is optimal for $\alpha + \chi = 0$ and we finally find an inequality independent of any phase conventions,

$$|p| \leqslant |q|. \tag{16}$$

Inequality (16) is experimentally testable! Let us consider the semileptonic decays of the K mesons, in particular the leptonic asymmetry

$$\delta_l = \frac{\Gamma(K_L \to \pi^- l^+ \nu_l) - \Gamma(K_L \to \pi^+ l^- \bar{\nu}_l)}{\Gamma(K_L \to \pi^- l^+ \nu_l) + \Gamma(K_L \to \pi^+ l^- \bar{\nu}_l)}, \tag{17}$$

where l represents either an electron or a muon. If *CP* were conserved, we would have $\delta_l = 0$. Experimentally, however, the asymmetry is nonvanishing,[1] namely

$$\delta_l = (3.27 \pm 0.12) \times 10^{-3}, \tag{18}$$

and is thus a clear sign of *CP* violation. On the other hand, we recall the $\Delta S = \Delta Q$ rule for the decays of the strange particles. It implies that—due to their quark contents—the kaon $K^0(\bar{s}d)$ and the anti-kaon $\bar{K}^0(s\bar{d})$ have definite decays

$$\begin{aligned} K^0 &\xrightarrow{\bar{s} \to \bar{u} l^+ \nu_l} \pi^- + l^+ + \nu_l, \\ \bar{K}^0 &\xrightarrow{s \to u l^- \bar{\nu}_l} \pi^+ + l^- + \bar{\nu}_l. \end{aligned} \tag{19}$$

Thus, l^+ and l^- tag K^0 and $\bar{K}^0$, respectively, in the K_L state, and the leptonic asymmetry (17) is expressed by the probabilities of finding a K^0 and a $\bar{K}^0$ in the K_L state:

$$\delta_l = \frac{|p|^2 - |q|^2}{|p|^2 + |q|^2} \equiv \delta. \tag{20}$$

[1] It is the weighted average over electron and muon events, see Ref. [12].

Then inequality (16) turns into the bound

$$\delta \leqslant 0 \tag{21}$$

for the leptonic asymmetry which measures *CP* violation. It is in contradiction to the experimental value (18) which is definitely positive. In this sense *CP* violation is related to the violation of a Bell inequality.

On the other hand, we can replace $\bar{K}^0$ by K^0 in the BI (3) and along the same lines as discussed before we obtain the inequality

$$|q| \leqslant |p|, \tag{22}$$

independent of any phase conventions. Inequalities (16) and (22), however, imply the strict equality

$$|p| = |q|, \tag{23}$$

which is in contradiction to experiment. Thus the premises of LRT are only compatible with strict *CP* conservation in $K^0\bar{K}^0$ mixing. Conversely, *CP* violation in $K^0\bar{K}^0$ mixing, no matter which sign the experimental asymmetry (17) actually has, always leads to a violation of a BI, either of inequality (16) or of (22).

Another interesting feature is the connection of the Bell inequality with the decoherence approach, see Ref. [13]. With a simple modification of the quantum-mechanical probabilities, namely by multiplying the interference term of the amplitudes by $(1-\zeta)$, where ζ is the decoherence parameter, we can achieve a continuous factorization of the wavefunction (see, e.g., Refs. [14,15]). When does this approach represent a local realistic theory, thus satisfying a Bell inequality? For $\zeta = 0$ we have pure QM, the violation of a BI and thus a nonlocal situation. On the other hand, for $\zeta = 1$, called Furry's hypothesis [16], there is total decoherence or spontaneous factorization of the wavefunction. Then the BI is satisfied and a LRT may describe the physical phenomena.

However, what can we say for ζ values between 0 and 1? Let us consider again the Bell inequality (3) and recalculate it with the modified probabilities, in order to find a bound on ζ. We choose for the entangled state the $K_S K_L$ basis representation (2) and modify the probabilities as described above:

$$P(f_1, f_2) = \frac{N^4}{8|p|^2|q|^2}\Big|\langle f_1|_l \otimes \langle f_2|_r \times \big\{|K_S\rangle_l \otimes |K_L\rangle_r - |K_L\rangle_l \otimes |K_S\rangle_r\big\}\Big|^2$$

$$\begin{aligned}\to P_\zeta(f_1, f_2) = \frac{N^4}{8|p|^2|q|^2}\Big\{&\big|\langle f_1|K_S\rangle_l\big|^2\big|\langle f_2|K_L\rangle_r\big|^2 \\ &+ \big|\langle f_1|K_L\rangle_l\big|^2\big|\langle f_2|K_S\rangle_r\big|^2 \\ &- 2(1-\zeta)\,\mathrm{Re}\big\{\langle f_1|K_S\rangle_l^*\langle f_2|K_L\rangle_r^* \\ &\times \langle f_1|K_L\rangle_l\langle f_2|K_S\rangle_r\big\}\Big\}.\end{aligned} \tag{24}$$

Then we find the following probabilities modified by ζ:

$$\begin{aligned}P_\zeta\big(K_1^0, \bar{K}^0\big) &= P_{\mathrm{QM}}\big(K_1^0, \bar{K}^0\big) - \zeta\frac{1}{8}\big(1-\eta^2\big), \\ P_\zeta\big(K_S, \bar{K}^0\big) &= P_{\mathrm{QM}}\big(K_S, \bar{K}^0\big) - \zeta\frac{1}{4}\big(1-\eta^2\big), \\ P_\zeta\big(K_S, K_1^0\big) &= P_{\mathrm{QM}}\big(K_S, K_1^0\big) + \zeta\frac{1}{8\eta^2}\big(1-\eta^2\big)^2,\end{aligned} \tag{25}$$

where

$$\eta = \frac{|q|}{|p|} \tag{26}$$

is a measure for *CP* violation in $K^0\bar{K}^0$ mixing. Note that all ζ terms are independent of the phase α; it enters only in the quantum mechanical probability $P_{\mathrm{QM}}(K_S, K_1^0)$ (see Eq. (13)).

Inserting now probabilities (25) into the Bell inequality (3), choosing α—like before in Eq. (15)—such that it compensates the relative phase χ of the weights p and q and expressing η by

$$\eta^2 = \frac{1-\delta}{1+\delta}, \tag{27}$$

we find the bound

$$\frac{(1-\delta)}{\delta}\big(\sqrt{1-\delta^2} - 1 + \delta\big) \leqslant \zeta. \tag{28}$$

The expansion to order δ gives

$$1 - \frac{3}{2}\delta \lesssim \zeta. \tag{29}$$

Numerically, from the experimental value (18) we get the bound

$$0.9951 \pm 0.0002 \lesssim \zeta, \tag{30}$$

which is, due to our optimal choice of the phases in inequality (14), a slight improvement as compared to the numerical bound of 0.987 of Ref. [13]. Thus, the decoherence parameter ζ has to be very close to one; hence, Furry's hypothesis or spontaneous factorization has to take place totally. Intuitively, we would have expected that there exist local realistic theories which allow at least partially for an interference term; see, for instance, Refs. [17,18].

On the other hand, we can compare this result with the experimentally determined $\zeta^{K_S K_L} = 0.13^{+0.16}_{-0.15}$ (see Ref. [15]), where $\zeta = 1$ is excluded by many standard deviations. This means that for experimental reasons a LRT equivalent to a modification of QM in the $K_S K_L$ basis choice is definitely excluded!

However, the situation changes when modifying the quantum-mechanical probabilities in the $K^0\bar{K}^0$ basis [14,15]. Then we obtain

$$\begin{aligned} P_\zeta\big(K_1^0, \bar{K}^0\big) &= P_{\mathrm{QM}}\big(K_1^0, \bar{K}^0\big), \\ P_\zeta\big(K_S, \bar{K}^0\big) &= P_{\mathrm{QM}}\big(K_S, \bar{K}^0\big), \\ P_\zeta\big(K_S, K_1^0\big) &= P_{\mathrm{QM}}\big(K_S, K_1^0\big) \\ &\quad + \zeta \frac{1}{2N^2} \mathrm{Re}\big\{e^{i\alpha} pq^*\big\}, \end{aligned} \tag{31}$$

which implies with inequality (3) the lower bound

$$1 - \sqrt{\frac{1-\delta}{1+\delta}} \leqslant \zeta. \tag{32}$$

To order δ we have

$$\delta \lesssim \zeta$$

and, numerically,

$$0.0033 \pm 0.0001 \lesssim \zeta. \tag{33}$$

Comparing this bound with the experimentally determined $\zeta^{K^0\bar{K}^0} \sim 0.4 \pm 0.7$ [15], we see that we cannot discriminate between QM and LRT in this case.

Summarizing, we have related Uchiyama's Bell inequality (3)—valid for the entangled $K^0\bar{K}^0$ state with negative C parity—with CP violation in $K^0\bar{K}^0$ mixing. Avoiding to involve any phase convention referring to K^0 and $\bar{K}^0$ decays, we have shown that Uchiyama's inequality necessarily requires the CP-violating leptonic asymmetry δ to be zero, in contradiction to experiment. In this way, $\delta \neq 0$ is a manifestation of the entanglement of the considered state. [2] Amazingly, the nonzero result of δ, obtained from measurements at one-particle states, gives us information about the entanglement of the two-particle state produced at the Φ resonance. Moreover, connecting the BI with the decoherence parameter ζ, then the premises of locality and reality are only compatible with a practically totally factorized wavefunction, i.e., with $\zeta = 1$ (Furry's hypothesis), and not with a partially contributing interference term.

[2] We want to stress that in the case of Uchiyama's Bell inequality (3), since it is considered at $t = 0$, it is rather contextuality [19] than nonlocality which is tested.

Acknowledgement

This research was supported by the FWF Project No. P14143-PHY of the Austrian Science Foundation and by the Project No. 2001-11 of the Austrian–Czech Republic Scientific Collaboration.

References

[1] A. Bramon, M. Nowakowski, Phys. Rev. Lett. 83 (1999) 1; B. Ancochea, A. Bramon, M. Nowakowski, Phys. Rev. D 60 (1999) 094008.
[2] G.C. Ghirardi, R. Grassi, T. Weber, in: G. Pancheri (Ed.), Proc. of Workshop on Physics and Detectors for DAΦNE, the Frascati Φ Factory, April 9–12, INFN–Laboratori Nazionali di Frascati, Frascati, 1991.
[3] B.C. Hiesmayr, Diploma thesis, University of Vienna (1999).
[4] R.A. Bertlmann, B.C. Hiesmayr, Phys. Rev. A 63 (2001) 062112.
[5] R.A. Bertlmann, W. Grimus, B.C. Hiesmayr, in: R.A. Bertlmann, A. Zeilinger (Eds.), Quantum [Un]speakables, Springer, Heidelberg, 2001.
[6] G. Weihs, T. Jennewein, C. Simon, H. Weinfurter, A. Zeilinger, Phys. Rev. Lett. 81 (1998) 5039; N. Gisin, A. Go, Am. J. Phys. 69 (2001) 264.
[7] J.S. Bell, Speakable and Unspeakable in Quantum Mechanics, Cambridge University Press, Cambridge, 1987.
[8] F. Uchiyama, Phys. Lett. A 231 (1997) 295.
[9] F. Benatti, R. Floreanini, Phys. Rev. D 57 (1998) 1332; F. Benatti, R. Floreanini, Eur. Phys. J. C 13 (2000) 267.
[10] M. Genovese, C. Novero, E. Predazzi, Phys. Lett. B 513 (2001) 401.
[11] G.C. Branco, L. Lavoura, J.P. Silva, CP Violation, Oxford University Press, Oxford, 1999.
[12] D.E. Groom et al., Eur. Phys. J. C 3 (1998) 1.
[13] B.C. Hiesmayr, Found. Phys. Lett. 14 (2001) 231.
[14] R.A. Bertlmann, W. Grimus, Phys. Lett. B 392 (1997) 426; G.V. Dass, K.V.L. Sarma, Eur. Phys. J. C 5 (1998) 283; R.A. Bertlmann, W. Grimus, Phys. Rev. D 58 (1998) 034014.
[15] R.A. Bertlmann, W. Grimus, B.C. Hiesmayr, Phys. Rev. D 60 (1999) 114032.
[16] W.H. Furry, Phys. Rev. 49 (1936) 393, 476.

[17] J. Six, Phys. Lett. A 150 (1990) 243.
[18] F. Selleri, Lett. Nuovo Cimento 36 (1983) 521;
P. Privitera, F. Selleri, Phys. Lett. B 296 (1992) 261;
R. Foadi, F. Selleri, Phys. Lett. B 461 (1999) 123.
[19] J.S. Bell, Rev. Mod. Phys. 38 (1966) 447;
S. Kochen, E.P. Specker, J. Math. Mech. 17 (1967) 59;
N.D. Mermin, Rev. Mod. Phys. 65 (1993) 803.