Questions for the General Relativity II exam, SS 2019

The candidate will draw randomly three questions from the list below, and will be asked to write an exhaustive report on the topics drawn.

This list of questions gives a faithful representation of the contents of the lectures and tutorials in SS2019.

i. Lie bracket, Jacobi identity, Levi-Civita connection, Riemann curvature tensor and its properties,

ii. Local inertial coordinates, Geodesic deviation (Jacobi equation), tidal forces

iii. Einstein equations and matter: examples of energy-momentum tensors, dust in general relativity, the continuity equation

iv. The Schwarzschild metric: Eddington-Finkelstein extension; Time functions; the black hole; what happens at $r = 0$;

v. The Schwarzschild metric: Stationary observers, the interpretation of m, the Flamm paraboloid

vi. The Kruskal-Szekeres extension of the Schwarzschild metric

vii. The Schwarzschild metric: Conformal Carter-Penrose diagram

viii. The Schwarzschild metric: Geodesics, the interpretation of E, circular timelike geodesics,

ix. The Schwarzschild metric: Circular null geodesics, gravitational redshift, weak field light bending

x. The Schwarzschild metric: Perihelion/periastron precession

xi. (not relevant for students in SS 2017 and 2018) Perfect fluids, general-relativistic Euler equations and their Newtonian limit, Newtonian thermodynamics interpretation

xii. (not relevant for students in SS 2019) Linearized Einstein equations, TT-gauge, linearized waves

xiii. (not relevant for students in SS 2019) Slowly varying weak gravitational fields, quadrupole formula, chirp mass

xiv. Spherically symmetric static stars: (not relevant for students in SS 2017 and 2018) $g = -e^{\nu}dt^2 + e^{\lambda}dr^2 + r^2 d\Omega^2$; the following equations are admitted:

$$G^0{}_0 = e^{-\lambda}\left(\frac{1}{r^2} - \frac{\lambda'}{r}\right) - \frac{1}{r^2} = -8\pi\rho \ , \ G^1{}_1 = e^{-\lambda}\left(\frac{1}{r^2} + \frac{\nu'}{r}\right) - \frac{1}{r^2} = 8\pi p \ ,$$

derivation of the TOV equation

xv. (not relevant for students in SS 2017 and 2018) Spherically symmetric static stars; the following equations are admitted:
$g = -e^{\nu}dt^2 + e^{\lambda}dr^2 + r^2 d\Omega^2$,
$m' = 4\pi\rho r^2$, $e^{-\lambda(r)} = 1 - \dfrac{2m(r)}{r}$, $p' = -\dfrac{(\rho + p)(4\pi p r^3 + m(r))}{r(r - 2m(r))}$
Newtonian limit, Buchdahl inequality, Chandrasekhar mass

xvi. (not relevant for students in SS 2019) The Lie derivative, an axiomatic approach, relation to isometries

xvii. (not relevant for students in SS 2017 and 2018) Transporting tensor fields, flows of vector fields

xviii. (not relevant for students in SS 2017 and 2018) The Lie derivative: definition through flows

xix. Isometries, Killing vectors, maximally symmetric space-times

xx. FRWL metrics: Hubble law, cosmological red-shift formula, the red shift-factor z and distance, the deceleration parameter

xxi. Einstein equations for a FRWL metric: the following equations are admitted:
$\dot{R}^2 = \dfrac{\kappa K}{3R^2} + \dfrac{\kappa E}{3R} + \dfrac{1}{3}\Lambda R^2 - k$,
$\partial_t(\rho R^3) + p\partial_t(R^3) = 0$,
solutions with $p = 0$, behavior for R small and large.