A Structural Model of Aggregate US Job Flows: Another Look

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Summary

A recent article (Collard *et al.*, 2002) published in this journal presented a structural model of aggregate job flows. Unrestricted estimation of the model yields parameter estimates that would imply an umemployment rate of 99 percent. Instead of solving this problem by fixing one of the parameters, as originally attempted by the authors, I add moments regarding the employment rate. The new results call for a reevaluation of the model.

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1 Introduction

This paper presents the results of re-estimating a model of aggregate job flows in the United States by Collard *et al.* (2002). The paper is motivated by the fact that without further restricting the model the estimated parameters imply an unemployment rate of 99 percent. Instead of fixing one of the parameters I will add more moments regarding the employment rate in the economy in order to estimate the model. The structure of the paper is as follows. Section 2 briefly sketches the model and the estimation procedure. Section 3 presents estimation results and compares them to those obtained by Collard *et al.* Section 4 concludes.

2 A Model of Aggregate US Job Flows

Collard *et al.* (2002) develop a simple matching model that allows for endogenous separation and tractable heterogeneity. The economy consists of two employment pools, indexed by $j = 1, 2$. Each firm i in pool j faces a linear production function

$$Y_{i,j,t} = \eta_{j,t} N_{i,j,t}$$

$N_{i,j,t}$ is the employment level, and $\eta_{j,t}$ describes the technology, which follows an $AR(1)$ process:

$$log(\eta_{j,t}) = \rho_\eta log(\eta_{j,t-1}) + \sigma_\eta \nu_{j,t}, \qquad \nu_{j,t} \sim nid(0,1)$$

The Gaussian white noise $\nu_{i,t}$ satisfies $\nu_{1,t} = -\nu_{2,t}$. The matching technology is described by

$$H_{j,t} = \bar{H} V_{j,t}^{\alpha} U_t^{1-\alpha} \qquad \text{for } j = 1, 2$$

where $\alpha \in (0,1)$ and $\bar{H} \geq 0$. The cost of posting vacancies $V_{i,j,t}$ is given by

$$\psi(V_{i,j,t}, V_{j,t}) = \frac{\omega}{2} \frac{V_{i,j,t}^2}{V_{j,t}} \qquad \text{for } j = 1, 2$$

The cost of firing $F_{i,j,t}$ is given by

$$\phi(F_{i,j,t}, N_{j,t}) = \varphi \frac{F_{i,j,t}^2}{(1-s)N_{j,t} - F_{i,j,t}} \qquad \text{for } j = 1, 2$$

where s denotes an exogenous separation rate. Firms now maximize expected discounted future profits:

$$\max_{\{V_{i,j,t+\tau}, F_{i,j,t+\tau}\}_{\tau=0}^{\infty}} E_t \{ \sum_{\tau=0}^{\infty} (1+r)^{-\tau} [\Pi_{i,j,t+\tau} - \psi(V_{i,j,t+\tau}, V_{i,t+\tau)}) - \phi(F_{i,j,t+\tau}, N_{j,t+\tau})] \}$$

subject to

$$\begin{aligned}
N_{i,j,t+1} &= (1-s)N_{i,j,t} + q_{j,t}Vi,j,t - F_{i,j,t} \qquad (X_{i,j,t}) \\
V_{i,j,t} &\geq 0 \qquad\qquad\qquad\qquad\qquad (\lambda_{i,j,t}) \\
F_{i,j,t} &\geq 0 \qquad\qquad\qquad\qquad\qquad (\mu_{i,j,t})
\end{aligned}$$

where $r \in (0,1)$ denotes the firm's discount factor and $q_{j,t} = H_{j,t}/V_{j,t}$, the rate at which vacancies are filled. $X_{i,j,t}$, $\lambda_{i,j,t}$, and $\mu_{i,j,t}$ are the Lagrange multipliers. The profit flows, $\Pi_{i,j,t}$, are given by

$$\Pi_{i,j,t} = \eta_{i,j,t}N_{i,j,t} - w_t N_{i,j,t}$$

where w is the real wage, which follows an $AR(1)$ process:

$$\log(w_t) = \rho_\omega log(w_{t-1}) + \sigma_w \epsilon_t \qquad \epsilon_t \sim nid(0,1)$$

Standard optimization yields the following decision rules on hiring and firing:

$$\begin{aligned}
H_{j,t} &= I_{[X_{j,t}\geq 0]}\bar{H}^{\frac{1}{1-\alpha}}\omega^{\frac{\alpha}{\alpha-1}}X_{j,t}^{\frac{\alpha}{1-\alpha}}(1-N_{1,t}-N_{2,t}) \\
F_{j,t} &= I_{[X_{j,t}\leq 0]}(1-s)N_{j,t}(1-(\frac{\varphi - X_{j,t}}{\varphi})^{-\frac{1}{2}})
\end{aligned}$$

Thus we have a state space model in eleven dimensions describing the evolution of the variables $\{H_{1,t}, H_{2,t}, F_{1,t}, F_{2,t}, N_{1,t}, N_{2,t}, X_{1,t}, X_{2,t}, log(\eta_{1,t}), log(\eta_{2,t}), log(w_t)\}$. There is a vector θ of nine structural parameters $\{\alpha, r, s, \bar{H}, \omega, \varphi, \rho_\eta, \sigma_\eta, \rho_\omega, \sigma_\omega\}$. Collard *et al.* fix the discount rate r, as well as the parameters of the exogenous wage process, ρ_ω and σ_ω.[1] They further summarize the term $\bar{H}^{\frac{1}{1-\alpha}}\omega^{\frac{\alpha}{\alpha-1}}$ by one parameter ξ. Thus we are left with six structural parameters to be estimated. The model is estimated by simulated methods of moments, i.e. I simulate the model 100 times over a 1087 time period. Then I discard the initial 1000 observations, and compute time series on job creation, c_t, and job destruction, d_t:[2]

[1] $\rho_\omega = 0.8532$, $\sigma_\omega = 0.0073$, $r = 0.01$, where the parameters for the wage process are estimated from real wage data.

[2] The data are assembled and described in detail in Davis et al. (1996). They were obtained from `http://www.bsos.umd.edu/econ/haltiwanger/` and seasonally adjusted using the Census X-11 procedure in EViews.

Table 1: Moments used in SMM Estimation

Creation	Destruction	Cross-correlation
$E(c_t)$	$E(d_t)$	$E(c_t d_{t+i})$
$E(c_t^2)$	$E(d_t^2)$	$E(c_t^2 d_{t+i})$
$E(c_t^3)$	$E(d_t^3)$	$E(c_{t+i} d_t^2)$
$E(c_t^4)$	$E(d_t^4)$	$E(c_t^2 d_{t+i}^2)$
$E(c_t c_{t-1})$	$E(d_t d_{t-1})$	
$E(c_t^2 c_{t-1}^2)$	$E(d_t^2 d_{t-1}^2)$	

Note: $i = -1, 0, 1$

$$c_t = \frac{H_{1,t} + H_{2,t}}{.5(N_{1,t} + N_{2,t} + N_{1,t-1} + N_{2,t-1})}$$
$$d_t = \frac{F_{1,t} + sN_{1,t} + sN_{2,t} + F_{2,t}}{.5(N_{1,t} + N_{2,t} + N_{1,t-1} + N_{2,t-1})}$$

I first compute empirical moments, of the observed job creation and destruction data $\hat{\psi}_T$. Then I compute moments of the simulated series, $\tilde{\psi}_T^i$ for $i = 1, ..., N$, for a given starting value of θ, where $N = 100$ is the number of simulations, and construct their average $\tilde{\psi}_T^N = \frac{1}{N}\sum_{i=1}^N \psi_T^i(\theta)$. This procedure is repeated, searching over parameters until the criterion function, $J(\theta) = g'_{T,N} W_T g_{T,N}$, is minimized. Consequently,

$$\tilde{\theta}_T^N = \underset{\theta}{\text{argmin}}\, J(\theta),$$

where $g_{T,N} = (\hat{\psi}_T - \tilde{\psi}_T^N(\theta))$. W_T is a symmetric weighting matrix. As noted by Ingram and Lee (1991), $\sqrt{T}(\tilde{\theta}_T^N - \theta_0)$ is asymptotically normally distributed with covariance matrix $(1 + \frac{1}{N})(D'_\theta W_T D_\theta)^{-1})$, where $D_\theta = \frac{\partial g_{T,N}}{\partial \theta}$. Further, a global specification test can be performed, as $J - stat = TNJ(\theta)/(1+N)$ is asymptotically distributed as chi-square, with degrees of freedom equal to the number of overidentifying restrictions.[3]

3 Estimation

Collard *et al.* select 24 moments to be matched as described in Table 1. However, unrestricted estimation of the parameter vector yields unreasonable results; the estimated parameters imply that

[3]For a more detailed exposition of the model and the estimation technique, please see Collard *et al.* (2002).

the average share of the population not employed in the labor market is more than 99%. Collard *et al.* solve this problem by fixing the parameter ξ to .2 in order to match average participation rate in the US economy over the sample and estimate the five remaining parameters. They argue that

> "[...] since (*i*) transition to and from non-participation account for half of the flows into and out of employment and (*ii*) the number of individuals out of the labor force wanting a job is roughly equal to the number of unemployed workers (see e.g. [Blanchard and Diamond, 1989]), it is more reasonable to calibrate the model on the basis of the participation rate rather than the unemployment rate." (p.210)

Although this is obviously a valid argument for *not* using the unemployment rate, it is not a convincing argument for trying to match the participation rate. A further problem with the participation rate is that visual inspection of the series reveals that it simply exhibits a linear trend over the relevant time period, going from 60% to 70%.

In order to estimate the model including the parameter ξ I thus include four moments of the *employment rate*[4], $E(e_t), E(e_t^2), E(c_t e_t), E(d_t e_t)$, i.e. moments of the series $e_t = N_{1,t} + N_{2,t}$. Thus I also capture the effect of transition from non-participation to employment, but exclude those that just enter the labor force without being employed. The weighting matrix W_T is computed from the observed data, using the Newey-West (1987) estimator with 2 lags as suggested by Ingram and Lee (1991)[5]. To look for a minimum I use the function *fminsearch* in *Matlab*. In order to check for convergence I use a small perturbation of the parameters. Table 2 presents the parameter estimates obtained by the estimation including the employment rate. The last two columns contain the estimates and corresponding standard errors obtained by Collard *et al*.

The last two rows report the global specification test. We see that the model is now rejected by the data, when I try to match employment moments. Further it is interesting to see the new parameter estimates. The new α is even smaller than the one obtained by Collard *et al.* Further, the exogenous productivity shock exhibits a stronger persistence and a higher variance. The exogenous quit rate is one half percent smaller, and the firing cost parameter is about twice as large. These results are

[4]This is the series LHEPRR, "employment-population ratio: total, 16 Yrs+, seasonally adjusted" from `Citibase`.

[5]Collard *et al.* use the VARHAC estimator proposed by Den Hann and Levin (1997), which had to be modified to avoid inversion problems. I am grateful to Fabrice Collard for having provided their estimate, but since I was unable to replicate it and in order to avoid these numerical problems, I chose the Newey-West estimator. It should however be noted that parameter estimates and the minimized value of the criterion function are highly sensitive to the choice of covariance matrix estimator and lag length.

Table 2: Parameter Estimates

θ	New			Collard		
	$\hat{\theta}$	$\hat{\sigma}$	$t-stat$	$\hat{\theta}$	$\hat{\sigma}$	$t-stat$
α	0.2753	0.0411	6.7004	0.4085	0.1238	3.3001
ρ_η	0.6878	0.1175	5.8537	0.399	0.1236	3.2355
σ_η	0.3147	0.0846	3.7179	0.1601	0.0164	9.7499
s	0.0474	0.0002	222.5002	0.0518	0.0006	80.2689
φ	1.2861	0.8353	1.5396	0.5995	0.6788	0.8831
ξ	0.0335	0.0097	3.4529	0.2000	fixed	
$J-stat$		175.5629			27.6919	
$P-value$		< 0.0001			0.0896	

also reflected in the simulated values of the moments and associated diagnostic tests[6] as shown in Table 3. We see that compared to Collard *et al.* the new parameters are even worse at matching particularly the higher order moments. This is emphasized by the high values of t-statistics[7]. The low α makes the hiring process less sensitive to exogenous shocks, the high φ makes firing rather expensive. Toghether with a more persistent technology shock this leads to less variation in the two processes.

4 Concluding Remarks

In this paper I re-estimated a structural model of job flows in the United States in order to see whether the model was also capable of producing the dynamics of employment in the US, along with job creation and destruction. This seems a relevant question even if the model was primarily designed to capture the joint process of creation and destruction. The model is now globally rejected by the data. Again, the model cannot explain the high volatility, especially in the destruction process, which allows several conclusions. One possibility would be to simply dismiss the model in general as not capable of explaining the dynamics of job flows. Increasing the number of employment pools, as Collard *et al.* suggest, appears difficult, as the number of parameters freely estimated already had to be restricted due to 'identification problems'. Thus, improvement of the model requires

[6] Collard *et al.* show that each element of the vector of t-statistics

$$T_{T,N} = \{diag[\Omega_T - D_\theta(D'_\theta W_T D_\theta)^{-1}D'_\theta]\}^{-1/2}\sqrt{T}g_{T,N}$$

is distributed as $nid(0,1)$, where Ω and D_θ are being replaced by consistent estimates.

[7] These values also are very sensitive to the choice of covariance matrix estimator.

Table 3: Moments for Simulated and Observed Data

Moment	Observed			Simulated-New			Diagnostic	Simulated-Collard
	$\hat{\psi}$	$\hat{\sigma_T}$	$t-stat$	$\tilde{\psi_T^N}$	$\hat{\sigma_T^N}$	$t-stat$	Test	
$E(c_t)$	5.1937	0.1315	39.4996	4.8313	0.0831	58.1296	-4.5457	5.2382
$E(d_t)$	5.6486	0.2092	26.9970	4.8222	0.1240	38.8918	-6.7871	5.2384
$E(c_t^2)$	27.6664	1.4389	19.2270	23.4614	0.8555	27.4248	-5.1049	28.0018
$E(d_t^2)$	33.5496	2.7844	12.0489	23.5842	1.9677	11.9854	-5.1032	27.6658
$E(c_t^3)$	151.2422	12.0324	12.5696	114.5162	6.6980	17.0970	-5.6731	152.5911
$E(d_t^3)$	211.3374	29.2550	7.2240	118.7140	25.1809	4.7144	-3.6901	146.8952
$E(c_t^4)$	848.3012	91.1295	9.3087	561.9310	47.2276	11.8984	-6.2519	846.9521
$E(d_t^4)$	1420.5033	285.5702	4.9743	632.963	304.5642	2.0783	-2.5896	794.1571
$E(c_t c_{t-1})$	27.4633	1.4117	19.4545	23.4098	0.8343	28.0578	-5.0580	27.8311
$E(d_t d_{t-1})$	33.0508	2.6286	12.5733	23.3077	1.0742	21.6983	-9.3057	27.46535
$E(c_t^2 c_{t-1}^2)$	822.5197	88.0288	9.3438	557.5723	45.7870	12.1775	-5.9803	828.4348
$E(d_t^2 d_{t-1}^2)$	1335.7353	256.2747	5.2121	567.1511	78.9997	7.1792	-9.9257	766.4126
$E(c_t d_{t-1})$	29.3444	1.4065	20.8640	23.3179	0.8498	27.4401	-7.3600	27.3836
$E(c_t d_t)$	29.0993	1.1820	24.6195	23.2262	0.6019	38.5856	-10.5059	27.4178
$E(c_t d_{t+1})$	29.0024	1.1038	26.2748	23.2534	0.6237	37.2815	-9.8656	27.3916
$E(c_t^2 d_{t-1})$	156.687	10.6998	14.6439	113.3473	6.1959	18.2938	-7.2654	146.1199
$E(c_t^2 d_t)$	153.6605	8.7898	17.4818	112.5138	4.4457	25.3085	-9.9449	146.4704
$E(c_t^2 d_{t+1})$	152.9564	8.1807	18.6973	112.8071	4.5555	24.7628	-9.4318	146.2000
$E(c_{t-1} d_t^2)$	170.7388	13.1219	13.0117	113.5047	8.8440	12.8340	-6.5895	143.9331
$E(c_t d_t^2)$	171.1174	13.8265	12.3761	112.9717	8.0341	14.0615	-7.4002	143.9853
$E(c_{t+1} d_t^2)$	175.1020	16.1620	10.8342	114.1911	11.3763	10.0376	-5.4187	144.3294
$E(c_t^2 d_{t-1}^2)$	936.5830	100.9229	9.2802	556.3058	67.8842	8.1949	-5.6819	766.3041
$E(c_t^2 d_t^2)$	893.7780	76.6273	11.6640	544.8315	37.2764	14.6160	-9.7691	770.3897
$E(c_t^2 d_{t+1}^2)$	886.6385	70.1858	12.6327	548.4076	43.9795	12.4696	-7.9221	776.8512
$E(e_t)$	60.0157	0.3588	167.2772	61.9328	0.3830	161.6837	5.2485	
$E(e_t^2)$	3605.6541	43.1053	83.6476	3837.4311	45.8279	83.7357	5.3194	
$E(c_t e_t)$	310.6792	6.5542	47.4012	298.9330	3.6807	81.2170	-3.3476	
$E(d_t e_t)$	338.0197	11.4593	29.4974	298.5753	7.1455	41.7852	-5.5878	

adding elements like aggregate shocks without enlarging the parameter space. There are also several problems with respect to the use of the employment rate to generate additional moment conditions. Although it should be stationary, there is some trending behavior in it, which is determined outside the model. This includes increasing labor market participation of women, which increases the rate, and an aging society, which lowers the rate. Further, the rate is computed for the overall economy, including services, whereas the Davis-Haltiwanger data set on job creation and destruction only reflects the manufacturing sector.

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