Competition, Timing of Entry and Welfare in a Preemption Game

Rossella Argenziano[a] Philipp Schmidt-Dengler[b]

May 2013

Abstract

We show that in a preemption game of entry into a Cournot market, increasing the number of competitors beyond duopoly does not bring forward the time of first entry. We also show that all entries, except the first one, occur earlier than socially optimal.

KEYWORDS: Timing Games, Preemption, Dynamic Entry.

JEL CLASSIFICATION: C73, L13.

1 Introduction

In preemption games of entry, there is an early mover advantage which gives players an incentive to enter early, even if the entry cost declines over time. In particular, Fudenberg and Tirole (1985) show that in a duopoly, preemption motives bring first entry forward relative to monopoly, until rents are equalized. Intuition thus suggests that the presence

[a]University of Essex, Economics Department, Wivenhoe Park, Colchester CO4 3SQ, United Kingdom, Email: rargenz@essex.ac.uk

[b]Corresponding Author. University of Mannheim, Department of Economics, L7 3-5, 68131 Mannheim, Germany, Email: p.schmidt-dengler@uni-mannheim.de, Phone: +49-621 181 1832. Also affiliated with CEPR, CES-Ifo and ZEW.

of more competitors should bring forward first entry even more. We rely on an extension of Fudenberg and Tirole's (1985) rent equalization result to examine the effect of further competitors on the timing of entry. We show that the first entry time in duopoly represents a lower bound for the first entry time in any oligopoly. Against intuition, more competitors may actually *delay* first entry. Having examined the effect of competition on entry times, we turn to address welfare. We show that in preemption games, all entries except the first one occur too early from a welfare point of view. Whether first entry occurs too early as well, depends on the parameters of the game. The mechanism is similar to the one in Mankiw and Whinston (1986). While Mankiw and Whinston (1986) compare the number of entrants under free entry to the socially optimal number of firms in a static framework, we take the number of potential entrants as given, and compare the equilibrium entry times to the socially optimal entry times in a dynamic model.

2 The Model

We model entry in a new market as an infinite horizon dynamic game in continuous time. Our model and assumptions correspond to those in Fudenberg and Tirole (1985), restricting attention to the case of a new market. We allow for $N \geq 2$ potential entrants. Each firm has to decide whether and when to enter a new market. More precisely, at each instant in time each firm that has not yet entered the market observes the number and the identity of the firms already present in the market and chooses one of two actions: "Enter" or "Wait." Entry is irreversible. We restrict attention to pure strategies[1] and focus on subgame-perfect Nash equilibria. Before entry, a firm receives no profits. We assume that post-entry profits are generated by Cournot competition. Let P denote the market price and Q the aggregate output. Following Corchon (2008) we assume that the inverse demand function is given by the general form $P(Q) = A - bQ^{\alpha}$, with $b\alpha > 0$ and $\alpha > -1$. This specification nests

[1] As in Simon and Stinchcombe (1989) we interpret continuous time as "discrete time, but with a grid that is infinitely fine."

both linear and isoelastic demand. Marginal cost is constant at k and identical for all firms. Assuming $(A-k)\,b > 0$ and $-A\alpha < kN$ guarantees that output and market price are positive. When m firms are in the market, each of them receives instantaneous post-entry profits $\pi(m) = \alpha m^{1/\alpha-1}\,(A-k)^{1+1/\alpha}\,/b^{1/\alpha}\,(m+\alpha)^{1+1/\alpha}$. Hence, there is an early mover advantage: Earlier entrants receive higher profits for some time, because they have fewer competitors in the market.

As suggested by Fudenberg and Tirole (1985), we assume that the present value at time zero of the cost of entering the market at time t is given by $c(t) = c \cdot \exp\left(-(\rho + r)t\right)$, where c is the cost of entering at time zero, r denotes the discount rate, and ρ is an exponential decay parameter: The current value cost of entry declines over time at a decreasing rate. The exogenous delay in entry cost and the early mover advantage in profits make this game a preemption game. We assume that $c > \pi\,(1)\,/r$: At time zero, the entry cost exceeds discounted monopoly profits. Thus, no firm enters at the beginning of the game, even if it could thereby preempt all other firms and earn monopoly profits forever. Moreover, we assume that there exists a finite τ such that $c \cdot \exp\left(-\rho\tau\right) < \pi\,(N)\,/r$. Eventually, entry is profitable for all players.

Let the outcome of the game be such that the vector of entry times is $(T^1, T^2, ..., T^j, ..., T^N)$. If firm i is the j-th entrant, then firm i's payoff is

$$V_i^j(T^1, T^2, ..., T^j, ..., T^N) = \sum_{m=j}^{N} \pi(m) \int_{T^m}^{T^{m+1}} \exp\left(-rs\right) ds - c(T^j),$$

where $T^{N+1} \equiv +\infty$. In what follows, we will denote by $t_j\,(N)$ the j-th equilibrium entry time in a game with N entrants.

3 Results

To describe the equilibrium outcome of the game, we start by defining *stand-alone entry times* (see Katz and Shapiro (1987)). Consider the hypothetical problem of firm i, if it could act as a single decision maker and select the optimal time to enter the market paying cost $c(t)$, thereby receiving flow payoff of $\pi(m)$ forever, for $m \in \{1, ..., N\}$. This firm would choose t to maximize the following payoff: $f_m(t) \equiv \pi(m) \exp(-rt)/r - c(t)$. We denote the solution to this problem as T_m^*, and define it as the m-th *stand-alone entry time.* Observe that T_m^* is well-defined for every $m \in 1, ..., N$ as the solution to $f_m'(t) = 0 \Longleftrightarrow -\pi(m) \exp(-rt) - c'(t) = 0$. The condition is easily interpreted: when considering a marginal delay of entry, foregone discounted profits $\pi(m) \exp(-rt)$ are weighed against cost savings $c'(t)$.

The subgame-perfect Nash equilibrium outcome of the entry game is unique up to a permutation of firms. The construction of the equilibrium entry times relies on the fact that firms must receive the same payoff. Otherwise, there would be an incentive to preempt firms earning a higher payoff. The equilibrium entry times can be constructed recursively.[2] The last entrant solves a single-agent optimization problem maximizing her payoff $f_N(t)$. The last entry time is thus equal to the last stand-alone entry time: $t_N(N) = T_N^*$. The previous $N-1$ entry times are calculated recursively using a rent equalization condition. We distinguish two cases.

First, suppose the stand-alone entry time T_j^* is earlier than the next equilibrium entry time $t_{j+1}(N)$.[3] Then, the threat of preemption brings the j-th equilibrium investment time forward such that $t_j(N) < T_j^* < t_{j+1}(N)$. In particular, $t_j(N)$ solves the rent equalization condition that equates to zero the difference between the j-th and the $(j+1)$-th entrant's equilibrium payoff. It is the value of t that solves the following equation:

$$\pi(j) \int_t^{t_{j+1}(N)} \exp(-rs)\, ds - c(t) + c(t_{j+1}(N)) = 0. \tag{1}$$

Second, suppose instead that the stand-alone entry time of the j-th entrant is later than the next entry time, i.e. $T_j^* \geq t_{j+1}(N)$. This occurs if the preemption race to be the $(j+1)$-th entrant is sufficiently intense. Formally, this is the case if the difference between

[2]See Proposition 1 in Argenziano and Schmidt-Dengler (2013).

[3]This is always the case for $j = N-1$ because $T_{N-1}^* < T_N^* = t_N$ by construction.

the equilibrium payoff of the $(j+1)$-th entrant and of the next entrant whom she is not clustered with is positive when evaluated at T_j^*. Denoting by $t_{j+k}(N)$ the next entry time strictly larger than $t_{j+1}(N)$ the condition can be written as:

$$\pi(j+1)\int_{T_j^*}^{t_{j+k}(N)} \exp(-rs)\,ds - c(T_j^*) + c(t_{j+k}(N)) > 0 \tag{2}$$

In this second case, the j-th entrant has no incentive to enter earlier than the $(j+1)$-th. Consequently, the j-th and $(j+1)$-th entries occur jointly, i.e. they are "clustered." That is $t_j(N) = t_{j+1}(N) \leq T_j^*$.

It follows that in both cases any equilibrium entry time t_j, except for the last one, is earlier than the corresponding stand-alone entry time T_j^*: The preemption motive brings entry forward. Based on this characterization of the equilibrium outcome, we investigate two questions. First, we ask whether the presence of more than two potential entrants, i.e. more competition, brings forward the time of first entry in the market. The counterintuitive result that we found is that this is never the case. Formally:

Proposition 1 *The time of first entry in a duopoly game is earlier than the time of first entry in any game with more than two firms.*

To capture the intuition, compare the entry games with two and three firms. In the game with two firms, last entry takes place at the stand alone time T_2^*. First entry takes place at $t_1(2)$, the time that solves the rent equalization condition (**??**). Adding a third firm brings forward the second entry time, because the last two entrants compete in a preemption race for the role of second entrant. In turn, an early second entry makes first entry less attractive, because monopoly profits can be earned for a shorter period. Hence, the earlier the second entry is, the less intense the preemption race to be the first entrant becomes. This effect delays the first entry in triopoly, relative to duopoly. Panel (a) in Figure 1 illustrates this mechanism.

The only possible exception to this conclusion is when in the triopoly game the first and second entries are clustered. If the preemption motive to be second rather than third is sufficiently strong, not only are the first and second entry clustered, but they could also be pushed back even beyond the time of first entry of a duopoly game. For this to occur, the difference between duopoly profits and triopoly profits must be sufficiently large. In particular, it must be large relative to the difference between monopoly and duopoly profits, which is not the case in our symmetric Cournot setting. Panel (b) in Figure 1 illustrates how the first entry time varies with the number of potential entrants.

The second question we address is how the equilibrium entry times in the Cournot entry game compare to those that would be socially optimal. In order to answer it, we need to make a further parametric restriction.

Proposition 2 *Suppose that demand is linear (i.e. $\alpha = 1$). (i) The last $(N - 1)$ entries occur earlier than socially optimal. (ii) If the discount rate r is sufficiently large, relative to the cost decline parameter ρ, then the first entry occurs later than socially optimal.*

These results are very similar in spirit to Mankiw and Whinston (1986). They show that in a static setting with imperfect competition, the presence of a "business stealing effect"(i.e. additional entry causing incumbents to reduce sales), creates a bias towards excessive entry. Entry may be less than socially optimal, but at most by one firm. Similarly, our result says that all entries occur too early, with the only possible exception being first entry. Consider the stand-alone incentive to enter, as described by $f_m(t)$: a potential entrant takes into account his own profits from entry but ignores both the consumer surplus it generates and the business stealing effect which hurts earlier entrants. For entries after the first one, the latter effect dominates the former, hence the stand alone entry time is earlier than the socially optimal one. Since the equilibrium entry time is even earlier than the stand alone one, entry occurs too early. First entry instead, may occur too late. The reason is that there is no business stealing effect and the entrant does not take into account the consumer surplus it generates. Hence, the stand alone entry time is later than the socially optimal one. Although the preemption motive forces the first entrant to enter earlier than at the first stand-alone time, the second part of the proposition identifies a condition such that the former effect offsets the latter. When the discount rate is large relative to the cost decay parameter, the time of first entry is later than the socially optimal one. The proposition also shows that the numerical results in Mills (1991) are of more general nature. Mills (1991) studies a similar model, with growing demand rather than falling cost, and a fixed cost of entry. In numerical examples, he also finds that first entry is later than socially optimal, and later entries occur too early.

Our results suggests several lines of future research. On the theory side, it is desirable to examine the robustness of our findings to other forms of strategic interaction generating post-entry profits, for example considering a market with differentiated products or more generally, competition among asymmetric firms.[4] On the empirical side, the prediction of Proposition 1 that more potential entrants do not result in earlier first entry can be tested. A setting with an exogenous variable shifting the number of potential entrants is needed for such a test. Similarly, when data to infer demand and profits are available, the welfare effects of entry analyzed in Proposition 2 can be quantified.

Acknowledgments

Financial support from ESRC under grant RES-000-22-1906 is gratefully acknowledged. We thank Luigi Balletta for comments and suggestions.

Appendix

Proof of Proposition ??

Consider the game with N potential entrants. Either $t_1(N) < t_2(N)$, or $t_1(N) = t_2(N)$. First, we show that $t_1(2) \leq t_1(N)$ if $t_1(N) < t_2(N)$. Then, we show that the same result

[4] See for example Argenziano and Schmidt-Dengler (2012) for a preemption game of entry with asymmetric firms.

holds if $t_1(N) = t_2(N)$.

Suppose $t_1(N) < t_2(N)$. Since $t_1(N)$ is defined as the smallest solution to condition (**??**), $\partial t_1(N) / \partial t_2(N) = [-\pi(1)\exp(-rt_1(N)) - c'(t_1(N))] / [-\pi(1)\exp(-rt_2(N)) - c'(t_2(N))]$. This derivative is negative because $t_1(N) < T_1^* < t_2(N)$. Therefore, $t_2(N) \leq T_2^* = t_2(2)$ implies $t_1(2) \leq t_1(N)$.

Next, suppose instead that $t_1(N) = t_2(N)$. In particular, suppose that the first M entries occur simultaneously, with $M \in \{2, ..., N-1\}$: $t_1(N) = t_2(N) = ... = t_M(N) < T_M^* < t_{M+1}(N)$. Then, $t_1(N)$ is the solution to the rent equalization condition that equates the payoff of the first M entrants to the payoff of the $(M+1)$-th entrant: $\pi(M)\int_t^{t_{M+1}(N)} e^{-rs}ds - c(t) + c(t_{M+1}(N)) = 0$. Since $t_1(2)$ solves the rent equalization condition $\pi(1)\int_t^{T_2^*} e^{-rs}ds - c(t) + c(T_2^*) = 0$, we can conclude that $t_1(2) \leq t_1(N)$ iff $\pi(1)\int_t^{T_2^*} e^{-rs}ds - c(t) + c(T_2^*) \geq \pi(M)\int_t^{t_{M+1}(N)} e^{-rs}ds - c(t) + c(t_{M+1}(N))$ for every $t < T_1^*$. Since $[\pi(1) - \pi(M)]\int_t^{T_1^*} e^{-rs}ds > 0 > [c(t_{M+1}(N)) - c(T_2^*)]$, the above condition holds if

$$[\pi(1) - \pi(M)]\int_{T_1^*}^{T_2^*} e^{-rs}ds > \pi(M)\int_{T_2^*}^{t_{M+1}(N)} e^{-rs}ds.$$

Since $t_{M+1}(N) < T_{M+1}^*$, the latter condition holds if

$$[\pi(1) - \pi(M)]\int_{T_1^*}^{T_2^*} e^{-rs}ds > \pi(M)\int_{T_2^*}^{T_{M+1}^*} e^{-rs}ds.$$

Since e^{-rs} is decreasing in s, a sufficient condition for the above is that $[\pi(1) - \pi(M)][T_2^* - T_1^*] > [\pi(M)]\left[T_{M+1}^* - T_2^*\right]$ which can be rewritten as $\pi(1)[T_2^* - T_1^*] > \pi(M)\left[T_{M+1}^* - T_1^*\right]$. Since T_m^* solves $-\pi(m)e^{-rt} - c'(t) = 0$, hence it is equal to $\ln(c(\rho + r)/\pi(m))^{1/\rho}$, the above condition can be rewritten as $\ln(\pi(1)/\pi(2)) > \ln(\pi(1)/\pi(M+1)) / (\pi(1)/\pi(M))$.

We first show that the condition is satisfied for $M = 2$. Then, we show that the right-hand side of the expression is decreasing in M. Since the left-hand side is constant in M, this proves that the condition is satisfied for any $M \geq 2$.

Consider the case $M = 2$. Replacing $\pi(1)$, $\pi(2)$ and $\pi(M+1)$ with the corresponding expressions, the condition becomes

$$\ln\left(2^{(\alpha-1)/\alpha}\left((\alpha+2)\,/\,(\alpha+1)\right)^{(\alpha+1)/\alpha}\right) > \ln\frac{\left(3^{(\alpha-1)/\alpha}\left((\alpha+3)\,/\,(\alpha+1)\right)^{(\alpha+1)/\alpha}\right)}{2^{(\alpha-1)/\alpha}\left((\alpha+2)\,/\,(\alpha+1)\right)^{(\alpha+1)/\alpha}}$$

which is satisfied for any α in the domain. Now consider the term $\ln\left(\pi(1)/\pi(M+1)\right)/\left(\pi(1)/\pi(M)\right)$. Let $s(M)$ be defined as $\pi(1)/\pi(M)$. The derivative $\partial\left(\ln\left(s(M+1)\right)/s(M)\right)/\partial M$ is equal to $\left(s(M)\,s'(M+1)\,/s(M+1) - s'(M)\ln\left(s(M+1)\right)\right)/\left(s(M)\right)^2$. This derivative is negative because $\left[s'(M+1)\,/s(M+1)\right]/\left[s'(M)\,/s(M)\right] < 1 < \ln\left(s(M+1)\right)$. In particular, the first inequality holds because the derivative of $s'(M)\,/s(M)$ with respect to M is equal to $\left(2M+\alpha-2M\alpha-2M^2-\alpha^2\right)/\left[(M+\alpha)^2 M^2\right] < 0$. The second inequality holds for $M=2$, because $s(3) = 3^{1-1/\alpha}\left((\alpha+3)\,/\,(\alpha+1)\right)^{1+1/\alpha} > e$. Since the logarithmic function is increasing, this proves that it holds also for $M>2$. This concludes the proof. ■

Proof of Proposition ??.

Part (i). We prove that each stand-alone entry time is earlier than the corresponding welfare maximizing entry time. Since each of the last $(N-1)$ entry times is earlier than the corresponding stand-alone entry time, the result follows. In the case of Cournot competition with a linear demand function and constant marginal cost, profits are equal to $\pi(m) = (a-k)^2/b(m+1)^2$, consumer surplus is equal to $CS(m) = m^2(a-k)^2/2b(m+1)^2$ and total surplus is equal to $TS(m) = m(m+2)(a-k)^2/2b(m+1)^2$. The change in instantaneous welfare after the m-th entry is: $\Delta TS(m) = \left(TS(m) - TS(m-1)\right) = (2m+1)(a-k)^2/2bm^2(m+1)^2$. For any vector of entry times $(T^1, ..., T^N)$, total welfare is given by $\sum_{m=1}^{N}\int_{T^m}^{T^{m+1}} TS(m)\exp(-rs)\,ds - \sum_{m=1}^{N} c(T^m)$. Therefore, the socially optimal m-th entry time T_m^w solves the following first order condition: $\Delta TS(m)\exp\left(-rT_m^w\right) = -c'(T_m^w)$. Compare this condition to the first order condition that characterizes the stand alone entry time T_m^*: $\pi(m)e^{-rT_m^*} = -c'(T_m^*)$. If the left hand side of the former condition is larger than the left hand side of the latter condition, $T_m^* < T_m^w$. Substituting the expressions for $TS(m)$ and $\pi(m)$, this is the case if $(2m+1)(a-k)^2/2bm^2(m+1)^2 < (a-k)^2/b(m+1)^2$, which holds for all $m \geq 2$. Hence, since the last $(N-1)$ equilibrium entry times are earlier than the corresponding stand alone entry times, they are also earlier than the welfare maximizing entry times. ■

Part (ii). It follows immediately from the analysis above that for $\Delta TS(1) > \pi(1)$. Thus the first stand alone entry time is later than the socially optimal one. We now characterize the condition that guarantees that also the first equilibrium entry time is later than the socially optimal one. Proposition **??** implies that a condition that guarantees that $t_1(2) > T_1^W$ also guarantees that $t_1(N) > T_1^W$. We now characterize such a condition. The function $D_1(t) \equiv \pi(1) \int_t^{T_2^*} e^{-rs} ds - c(t) + c(T_2^*)$ describes the incentive to be the first, rather than the second entrant, evaluated at time t. It is a strictly quasiconcave function of t, maximized at T_1^*. $t_1(2)$ solves $D_1(t) = 0$, while $D_1\left(T_1^W\right)$ describes the preemption incentive to be first, evaluated at T_1^W. If the latter expression is negative, then $T_1^W < t_1(2)$. Since $T_1^W = \ln\left(8bc(\rho+r)/3(a-k)^2\right)^{1/\rho}$, $D_1\left(T_1^W\right)$ can be rewritten as:

$$\left(\frac{(a-k)^2}{b(\rho+r)}\right)^{(\rho+r)/\rho}\left(\frac{1}{c}\right)^{r/\rho}\left\{\frac{\rho+r}{4r}\left[\left(\frac{3}{8}\right)^{r/\rho}-\left(\frac{1}{9}\right)^{r/\rho}\right]-\left[\left(\frac{3}{8}\right)^{(\rho+r)/\rho}-\left(\frac{1}{9}\right)^{(\rho+r)/\rho}\right]\right\}.$$

Since $\left((a-k)^2/b(\rho+r)\right)^{(\rho+r)/\rho}(1/c)^{r/\rho} > 0$, the expression has the same sign as the term in curly brackets, which is negative for $r/\rho > \widetilde{(r/\rho)} \approx 1.275$. ■

References

[1] Argenziano, R., and P. Schmidt-Dengler (2012). Inefficient Entry Order in Preemption Games *Journal of Mathematical Economics*, 445- 460.

[2] Argenziano, R. and P. Schmidt-Dengler (2013), Clustering in N-Player Preemption Games, *Journal of the European Economic Association,* forthcoming.

[3] Corchón, L.C.(2008), Welfare Losses under Cournot Competition, *International Journal of Industrial Organization*, 26, 1120–1131.

[4] Fudenberg, D., and J. Tirole (1985), Preemption and Rent Equalization in the Adoption of a New Technology, *Review of Economic Studies*, 52, 383-401.

[5] Katz, M. and C. Shapiro (1987), R & D Rivalry with Licensing or Imitation, *American Economic Review*, 77, 402-420.

[6] Mankiw, N.G., and M.D. Whinston (1986), Free entry and social inefficiency, *The RAND Journal of Economics*, 48-58.

[7] Mills, D.E. (1991), Untimely Entry, *The Journal of Industrial Economics*, 659-670.

[8] Simon, L., and M. Stinchcombe (1989), Extensive Form Games in Continuous Time: Pure Strategies, *Econometrica*, 57, 1171-1214.

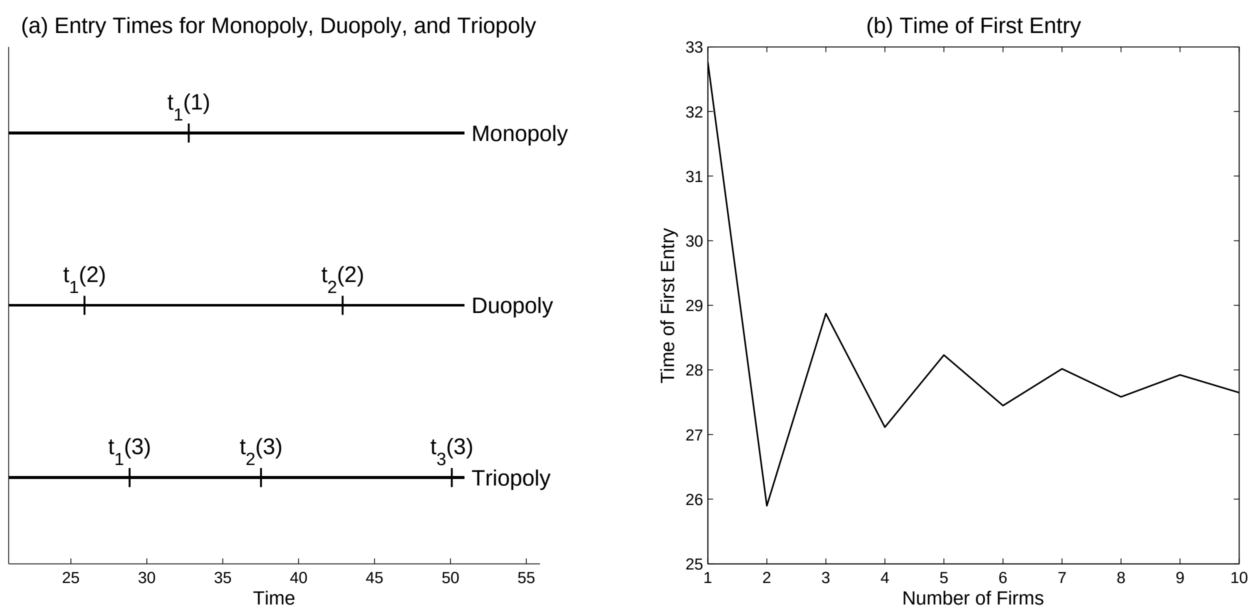

Figure 1: The number of potential entrants and entry times. Panel (a) illustrates the mechanism behind Proposition 1. Going from monopoly to duopoly brings forward the first entry time ($t_1(2) < t_1(1)$). Adding a third firm brings forward the second entry time ($t_2(3) < t_2(2)$) and delays the first entry time ($t_1(3) < t_1(2)$). Panel (b) illustrates the effect of varying the number of firms on the first entry time $t_1(N)$. Both figures are drawn for the linear demand model with parameters $A = 5$, $\alpha = 0$, $b = k = 1$, $\rho = .08$, $r = .03$, and $c = 500$.