Inefficient Entry Order in Preemption Games *

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July 2012

Abstract

In a preemption game, players decide when to take an irreversible action. Delaying the action exogenously increases payoffs, but there is an early mover advantage. Riordan (1992) shows that in a preemption game with two asymmetric players, players act in decreasing order of efficiency. This provides a microfoundation to the assumption that entry in a market occurs in the order of profitability, commonly used in the empirical analysis of market entry. We provide a counterexample showing that with more than two players this intuitive result can be reversed. We present a preemption game of entry into a new market. The potential entrants are three asymmetric firms: one "efficient" firm with high post-entry profits, and two "inefficient firms". We show that the set of parameters such that the equilibrium entry order does not reflect the efficiency ranking is nonempty, and analyze which changes in post-entry profits preserve this entry order.

Keywords: Timing Games, Preemption, Dynamic Entry.
Jel Classification: C73, L13.

*A previous version of this paper was circulated under the title "Preemption and the Efficiency of Entry". We thank Riccardo Martina, Pierre Regibeau, Nicolas Schutz, Francesco Squintani, and seminar audiences at the University of Manchester, the University of Naples "Federico II" and at the seventh IIOC in Boston for helpful comments. Financial support from ESRC under grant RES-000-22-1906 is gratefully acknowledged.
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1 Introduction

A preemption game is a game of timing: Players have to decide when to take an action. The cost of taking the action declines over time. A player earns a positive profit flow upon acting. This profit flow is decreasing in the number of players who have already acted. Delaying the action exogenously increases payoffs, but the early mover advantage gives players an incentive to act early. A fundamental implication of the analysis by Riordan (1992) is that if the game is played by two players, and one of them is more efficient, in the sense that he receives higher profit flows upon acting, the unique subgame-perfect equilibrium outcome of the game is that the efficient player is the first to act. This paper shows that when such a game is played by more than two players this intuitive result is sometimes reversed: Adding a third player qualitatively changes the equilibrium outcome.

An important application of preemption games is the analysis of firms entry into a new market. In the recent empirical literature on entry games with asymmetric potential entrants, a modelling problem that needs to be addressed is the inherent multiplicity of equilibria. Following Berry (1992), this problem is often addressed assuming that entry occurs in the order of profitability. We show that if the underlying game is a preemption game, the assumption is problematic: At any given point in time, the firms observed in the market are not necessarily the most efficient among the potential entrants.

To prove that with more than two players the order of action does not necessarily reflect their relative efficiency, we provide a simple example involving three players. We consider one efficient firm ("type A" firm) and two inefficient firms ("type B" firms) playing a preemption game of entry in a new market. The cost of entry declines exogenously over time. Post-entry profits are declining in the number of rivals already in the market, and are higher for firm A than for the type B firms, conditional on the number and type of competitors.

We show that with a general payoff structure, the unique subgame-perfect equilibrium outcome may be such that the order of entry is $B-A-B$. That is, one of the inefficient firms enters first, the efficient firm follows strictly later, to be followed by the remaining inefficient firm. We prove that the set of parameters such that the order of entry is $B-A-B$ is nonempty, and analyze which changes in flow profit parameters preserve this entry order.

In Appendix A, we numerically obtain the range of parameters for which the equilibrium entry order is $B-A-B$ in an example where profits are derived from Cournot competition.

Moreover, we illustrate the possible effects of an inefficient entry order on social welfare.[1]

The intuition behind our result is as follows. When a firm considers entering first in the market, it takes into account for how long it will earn monopoly profits. Thus the incentive to enter first, to preempt its rivals, depends on the timing of second entry, which in turn depends on the intensity of the preemption race in the ensuing two-player subgame. If firm A enters first, the resulting subgame among the two type B firms can involve a very intense preemption race, and thus second entry will occur relatively soon. If however a type B firm enters first, the resulting subgame among firm A and the remaining type B firm involves relatively weak preemption incentives: The second entrant is firm A, and it can afford to wait long and enter at a relatively low cost, because B is a weak competitor. As we show, this can result in a first entrant of type A earning monopoly profits for a shorter period than a first entrant of type B. This shorter monopoly period can outweigh the higher monopoly flow profit that the more efficient firm A would earn. As a result, in equilibrium one of the inefficient type B firms will enter first. The exact timing of first entry will be determined by a preemption race among the two inefficient firms. Rents among the two inefficient type B firms are equalized.

The above intuition also explains how the primitives of the model determine whether the first entrant is one of the less efficient firms. A decrease in the monopoly profits for A decreases its incentive to be first. An increase in the monopoly profits for the type B firms increases their incentive to be first. Next, consider the changes in the duopoly and triopoly profits that make the preemption race between two type B firms to be second rather than third more intense. Any such change brings forward the time of second entry, conditional on A being the first entrant. Therefore, it decreases A's incentive to be first, because it shortens the period for which A would earn monopoly profits. Similarly, consider the changes in the duopoly and triopoly profits that make the preemption race to be second rather than third between A and a type B firm less intense. Any such change delays the time of second entry, conditional on one of the B firms being the first entrant. It increases the incentive for a type B firm to be first, because it prolongs the period for which it would earn monopoly profits.

[1] Throughout the paper, we say that entry is "efficient" whenever the entry order reflects the firms' profitability ranking, and "inefficient" otherwise. We discuss the impact of the entry order on welfare in Section 4 and present numerical results in Appendix A.

1.1 Related literature

Our model builds on the classic literature on two-player preemption games.[2] We rely on Fudenberg and Tirole (1985) to derive the outcome of the two-player symmetric subgame played by two inefficient firms. From Riordan's (1992) analysis, we obtain the outcome of the two-player asymmetric subgame. Riordan (1992) shows that in a two-player asymmetric game, the more efficient firm always enters first. We contribute to the literature by showing that with more than two players the order of entry may not reflect the efficiency ranking.

While our paper focuses on a triopoly model with exogenous asymmetry in post-entry profits, a number of papers explore different forms of asymmetry in the context of real options duopoly models. Mason and Weeds (2010) consider a model in which asymmetry of the post-entry profits can be endogenously generated by a first-mover advantage: Firms are ex-ante identical but the first mover can gain an advantage that persists even after the late mover acts. Hence, the entry order always reflects the (ex-post) efficiency order. Pawlina and Kort (2006) investigate the consequences of an exogenous asymmetry in the entry cost. They identify three classes of equilibria. In all of them, the firm with the lowest cost always invests weakly earlier than the opponent. Femminis and Martini (2011) consider a duopoly model in which firms are ex-ante identical but the entry cost of the late mover is endogenously lower, due to a spillover effect from the action of the early mover. In our model, entry costs are exogenous and symmetric.

Another possible explanation of the fact that the order of entry in a market does not always reflect relative efficiency is ongoing technological progress: Later entrants can be endogenously more efficient because the choice to wait allows them to develop a better production process, or a higher-quality product. This explanation is analyzed by Dutta, Lach and Rustichini (1995) for the case of costless technological progress, and by Hoppe and Lehmann-Grube (2001) for the more general case of potentially costly R&D. In this paper, we provide an alternative explanation that is more suitable for markets in which the asymmetry in productive efficiency is due to factors that are given at the time the new market opens, rather than to ongoing technological process. For example, the asymmetry may be due to differences in managerial skills, geographical location, tax treatment, or access to distribution channels and input markets.

[2] In her survey on techonology adoption, Hoppe (2002) provides an excellent overview of both the theoretical and empirical literature on preemption games.

Finally, our result relates to the work by Quint and Einav (2005). They show that the assumption that the order of entry in a market reflects relative efficiency can be rationalized by assuming that entry is the outcome of a war of attrition where entry costs are sunk gradually. Our analysis shows that if the underlying game is a preemption game this result may be reversed: At any given point in time, the firms observed in the market are not necessarily the most efficient ones.

2 Model

We model entry in a new market as an infinite horizon dynamic game in continuous time. Our assumptions correspond to those made by Fudenberg and Tirole (1985) and Riordan (1992), when specialized to the case of a new market, and with a third firm added to the model. In particular, we consider a model with one efficient firm, firm A, and two identical "type B" firms, less efficient than A. Each firm has to decide whether and when to enter a new market. More precisely, at each instant in time each firm that has not yet entered the market observes the number and the identity of the firms already present in the market and chooses one of two actions: "Enter" or "Wait." Entry is irreversible. As in Simon and Stinchcombe (1989), we restrict play to pure strategies and interpret continuous time as "discrete time, but with a grid that is infinitely fine." The solution concept we use is subgame perfect Nash equilibrium. In section 2.2. below, we discuss in more detail how we model strategies in games in continuous time and address the issue of possible non-existence of subgame-perfect equilibria.

Before entry, a firm receives no profits.[3] Upon entry, firm i (for $i = A, B$) earns flow profits $\pi_i(m, -i)$, where m is the total number of firms that have entered, hence $m \in \{1, 2, 3\}$, and $-i$ stands for the identity of rival firms that have entered. For example, $\pi_B(2, A)$ stands for the profits of a type B firm in duopoly if its rival is firm A. The

[3] The assumption that pre-entry profits are independent of the number of firms already in the market is essential for obtaining a unique equilibrium outcome. This assumption has been previously adopted by Bouis, Huisman and Kort (2009), who study dynamic investment in oligopoly in a real options framework, and by Argenziano and Schmidt-Dengler (2011), who study the existence of clusters of simultaneous investments in N-player preemption games.

following profits are relevant in our model:

	Firm A	Type B firms	
Monopoly	$\pi_A(1)$	$\pi_B(1)$	
Duopoly	$\pi_A(2)$	$\pi_B(2, A)$	$\pi_B(2, B)$
Triopoly	$\pi_A(3)$	$\pi_B(3)$	

In monopoly there are no rival firms and in triopoly the identity of rivals is uniquely identified by the identity of firm i. Similarly, in a duopoly firm A will always oppose a type B firm. Hence, to economize on notation, we leave out the $-i$ term in the monopoly and triopoly cases as well as for firm A's duopoly profits. We denote by $\boldsymbol{\pi} = (\pi_A(1), ..., \pi_B(3))$ a flow profit structure, i.e. the set of all flow profits relevant in the model.

All the above profits are positive. For a given firm, profits decline in the number of competitors. Moreover, firm A's higher efficiency is reflected in payoffs. Firm A always earns higher profits than a type B firm, for a given number of competitors. Also, a type B firm earns lower duopoly profits if its opponent is firm A than if its opponent is the other type B firm, i.e. profits decline in the efficiency of rival firms.[4] Formally:

Assumption 1

(i) $\pi_i(m, -i) > 0 \qquad \forall(m, -i)$

(ii)

$$\begin{aligned}
&\pi_A(1) > \pi_B(1) \\
&\pi_A(2) > \pi_B(2, B) > \pi_B(2, A) \\
&\pi_A(3) > \pi_B(3) \\
&\pi_A(1) > \pi_A(2) > \pi_A(3) \\
&\pi_B(1) > \pi_B(2, B) > \pi_B(2, A) > \pi_B(3)
\end{aligned}$$

We denote the present value at time zero of the cost of entering the market at time t by $c(t)$. Following the literature,[5] we assume the following:

Assumption 2

The current value cost function $c(t)e^{rt}$ is (i) strictly decreasing and (ii) strictly convex.

[4] We follow Fudenberg and Tirole (1985) and Riordan (1992) in assuming that post-entry flow profits are parameters rather than deriving them from equilibrium behavior. We also assume that profits are independent of the time of entry, which implies that there is no ongoing technological progress. Dutta, Lach and Rustichini (1995) and Hoppe and Lehman-Grube (2001) analyze the case where efficiency depends on the time of entry.

[5] See Fudenberg and Tirole (1985) and Riordan (1992).

The cost of investing declines over time. This may capture upstream process innovations or economies of learning and scale. Moreover, cost declines at a decreasing rate.

The payoff function for firm i, conditional upon a given entry order in which i is the j-th entrant, as a function of its own entry time t_j and the competitors' entry times t_{-j} is:

$$f_i\left(t_j, t_{-j}\right) \equiv \sum_{m=1}^{3} I\left[j \leq m\right] \cdot \int_{t_m}^{t_{m+1}} \pi_i(m, -i) e^{-rs} ds - c\left(t_j\right)$$

where $I\left[\cdot\right]$ is the indicator function and $t_4 \equiv +\infty$. Before t_j, firm i receives zero profits. Then, it receives flow profits $\pi_i(m, -i)$ depending on the number and identity of the competitors present in the market. Finally, $c\left(t_j\right)$ denotes entry cost.

The next assumption guarantees that entry at time 0 is not profitable, and that all firms enter the market eventually.

Assumption 3 *(i) At time zero, for all firms, investment cost exceeds discounted monopoly profits:* $\frac{\pi_i(1)}{r} - c(0) < 0$. *(ii) Eventually, investment is profitable for all firms:* $\exists \tau$ *such that* $c\left(\tau\right) e^{r\tau} < \frac{\pi_i(3)}{r}$.

Assumption 3(i) guarantees that investing at time zero is too costly. No firm would invest immediately, even if it could thereby preempt all other firms and earn monopoly profits $\pi_i\left(1\right)$ forever. Assumption 3(ii) ensures that the value of investing becomes positive in finite time: The cost of investing eventually reaches a level sufficiently low, that it becomes profitable to invest, even for a type B firm facing maximum competition. This guarantees that the last investment occurs in finite time.

Next, to highlight the trade-offs faced by each firm, we extend the terminology in Katz and Shapiro (1987) and define the *stand-alone entry time* for the profit flow $\pi_i(m, -i)$. Consider the hypothetical problem of firm i, if it could act as a single decision maker and select the optimal time to make an investment which costs $c\left(t\right)$ and guarantees flow payoff of $\pi_i(m, -i)$ forever. Let $c\left(t\right)$ and $\pi_i(m, -i)$ satisfy assumptions 1, 2 and 3. This firm would choose t to maximize the following function:

$$g_{i,m,-i}\left(t\right) \equiv \frac{\pi_i(m, -i)}{r} e^{-rt} - c\left(t\right). \tag{1}$$

We denote the solution to this problem as $T_i^*(m, -i)$ and will refer to it as the *stand-alone entry time* for the profit flow $\pi_i(m, -i)$. It follows immediately from the assumptions that

$g_{i,m,-i}(t)$ is strictly quasi-concave and that $T_i^*(m,-i)$ is well defined for every $m \in \{1,2,3\}$ and $i = A, B$ as the solution to:[6]

$$g'_{i,m,-i}(t) = 0 \Longleftrightarrow -\pi_i(m,-i)e^{-rt} - c'(t) = 0.$$

The condition is easily interpreted: a marginal delay of entry implies foregone profits $\pi_i(m,-i)e^{-rt}$ and cost savings $c'(t)$. Given the quasiconcavity of $g_{i,m,-i}(t)$ in t, it follows from the implicit function theorem that $T_i^*(m,-i)$ is decreasing in $\pi_i(m,-i)$. Hence, the following inequalities follow from assumption 1(ii):

$$\begin{aligned}
&T_A^*(1) < T_B^*(1) \\
&T_A^*(2) < T_B^*(2,B) < T_B^*(2,A) \\
&T_A^*(3) < T_B^*(3) \\
&T_A^*(1) < T_A^*(2) < T_A^*(3) \\
&T_B^*(1) < T_B^*(2,B) < T_B^*(2,A) < T_B^*(3).
\end{aligned}$$

It is clear that firm A's stand-alone entry time, *for a given rank in the entry order,* is always earlier than that of a less efficient firm: By delaying entry, A would forego a higher profit than a type B firm would.

2.1 Modelling continuous-time preemption games

To model strategies in a preemption game of complete information with observable actions in continuous time, we follow Hoppe and Lehmann-Grube (2005) in adopting the framework introduced by Simon and Stinchcombe (1989). A key question in this class of models is how to associate an outcome to a strategy profile. Simon and Stinchcombe (1989) identify "....a class of continuous-time strategies with the following property: when restricted to an arbitrary, increasingly fine sequence of discrete-time grids, any profile of strategies drawn from this class generates a convergent sequence of outcomes, whose limit is independent of the sequence of grids."[7] They then define this limit as the outcome of the continuous-time strategy profile. Hoppe and Lehmann-Grube (2005) were the first to adopt this approach to model innovation timing games, and to clarify how to do so.

Another issue to be addressed is the issue of nonexistence of a subgame-perfect equilib-

[6] See Claim 1 in Appendix B.
[7] Simon and Stinchcombe (1989), abstract.

rium in pure strategies in preemption games. This is due to the possibility of coordination failures.[8] Since we adopt the Simon and Stinchcombe (1989) framework, we need to explicitly rule out the possibility of coordination failures,[9] and we do so using a randomization device as in Katz and Shapiro (1987), Dutta, Lach and Rustichini (1995), and Hoppe and Lehmann-Grube (2005):

Assumption 4
If n firms plan to enter at the same instant t (with $n \in \{2,3\}$), then only one firm, each with probability $\frac{1}{n}$, succeeds.

Assumption 4 rules out the possibility of coordination failures and thus ensures existence of an equilibrium in pure strategies.

In what follows, we will denote by t_j the j-th equilibrium investment time. All proofs are relegated to Appendix B.

2.2 Example

We conclude this section presenting a simple example that satisfies our assumptions and that forms the basis of the numerical analysis in Appendix A.

Example *Post-entry flow profits arise from Cournot competition. The inverse demand function is given by $P(Q) = Q^{-\eta}$, where Q is total output in the industry and $\eta \in (0,1)$ is the elasticity. Firms' cost functions are given by $K_i(q_i) = k_i q_i$ for $i = A, B$. Marginal costs are: $k_A = 1$ and $k_B \in \left(1, \frac{1}{1-\eta}\right)$.[10] The resulting profit structure satisfies Assumption 1. The current value cost of entry declines exponentially at rate $\alpha > 0$: $c(t)e^{rt} = \overline{c}e^{-\alpha t}$ where $\overline{c}$ is a positive constant.[11] This cost function satisfies Assumption 2. Assumption 3 is satisfied for any $\overline{c} > \frac{\eta}{1-\eta}\frac{(1-\eta)^{\frac{1}{\eta}}}{r}$.[12]*

[8] See the example in Simon and Stinchcombe (1989, p. 1178-1179).

[9] This observation is due to Hoppe and Lehmann-Grube (2005).

[10] The upper bound on k_B guarantees that all firms produce strictly positive quantities in all possible market structures (see Corchon (2007)).

[11] This choice of cost function is motivated by the example given in Fudenberg and Tirole (1985).

[12] In equilibrium, $\frac{\pi_A(1)}{r} = \frac{\eta}{1-\eta}\frac{(1-\eta)^{\frac{1}{\eta}}}{r}$ and $\frac{\pi_B(3)}{r} = \frac{((\eta-1)k_B+1)^2}{(3-\eta)\eta(1+2k_B)}\left(\frac{1+2k_B}{3-\eta}\right)^{-\frac{1}{\eta}}\frac{1}{r} > 0$. Thus Assumption 3(i) is satisfied if $\overline{c} > \frac{\eta}{1-\eta}\frac{(1-\eta)^{\frac{1}{\eta}}}{r} = \frac{\pi_A(1)}{r}$ and Assumption 3(ii) is satisfied for any finite $\overline{c}$ since $c(t)e^{rt} = \overline{c}e^{-\alpha t} \to 0 < \frac{\pi_B(3)}{r}$.

3 Inefficient Entry Order

In this section, we show that the efficient entry order result in the two-firm game analyzed by Riordan (1992) can be reversed when there are more than two firms. More precisely, we provide a counterexample in which there is one efficient firm and two inefficient firms. We show that for a nonempty set of parameter values the unique equilibrium outcome involves entry order $B-A-B$. We analyze which changes in post-entry profit parameters preserve this entry order.

The first step in our construction is to establish that last entry in the game must occur no later than $T_B^*(3)$.

Lemma 1 *In any SPNE, all firms enter the market in finite time, and last entry occurs no later than* $T_B^*(3)$.

This result follows from Assumption 3: Eventually, the entry cost becomes sufficiently small, that even entry with the lowest possible profits dominates staying out of the market. Last entry must occur by $T_B^*(3)$ because afterwards the foregone profit flow from delaying entry always exceeds the reduction in the entry cost, for either type of firm.

As it is standard in the preemption games literature, we continue solving the game using backwards induction. In order to construct the incentive of each type of firm to be the first entrant in the market, we have to characterize the outcome of two-firm subgames which start after first entry has occurred. For expositional purposes, we present the outcome of these subgames for the case where first entry has occurred sufficiently early that second entry occurs strictly later.[13] Section 3.1 considers the symmetric two-firm subgame between two inefficient type B firms following entry by firm A. Section 3.2 considers the asymmetric two-firm subgame between firm A and one type B firm following entry by the other type B firm. Having characterized the outcomes following first entry by either type of firm, we study the incentives to invest first in this game. Section 3.3 characterizes conditions under which the unique equilibrium outcome involves entry order $B-A-B$. Section 3.4 shows that the set of model primitives resulting in this entry order is nonempty and analyzes which changes in flow profit parameters preserve it.

[13] Appendix B provides the complete characterization, allowing for first entry to occur at any time $t \leq T_B^*(3)$.

3.1 The symmetric two-firm subgame

Suppose firm A is the first to enter the market. Here we consider the ensuing symmetric preemption game played by the two inefficient type B firms. It is analogous to the game analyzed by Fudenberg and Tirole (1985). The following characterization follows directly from their analysis. Figure 1 illustrates.

By Lemma 1, last entry in the subgame occurs no later than $T_B^*(3)$. Hence, either both firms enter exactly at $T_B^*(3)$, or one of them enters at some time t earlier than $T_B^*(3)$, taking the role of leader of the subgame. In the latter case, the remaining firm takes the role of follower and enters exactly at $T_B^*(3)$ because it chooses its entry time solving the single-agent optimization problem $\max_t g_{B,3}(t)$.

Consequently, the payoff of the leader and follower are:

$$\begin{aligned} L_B^{BB}(t) &= \pi_B(2,A)\int_t^{T_B^*(3)} e^{-rs}ds + \pi_B(3)\int_{T_B^*(3)}^{+\infty} e^{-rs}ds - c(t) \\ F_B^{BB}(t) &= \pi_B(3)\int_{T_B^*(3)}^{+\infty} e^{-rs}ds - c\left(T_B^*(3)\right) \end{aligned}$$

respectively.[14] $F_B^{BB}(t)$ is constant with respect to t, and positive, by assumption 3. $L_B^{BB}(t)$ is strictly quasi-concave and maximized at $T_B^*(2,A)$.[15] Now consider the incentive for each firm to preempt the competitor and be the leader of the subgame, rather than the follower. At time t, a firm prefers to be the leader rather than the follower if the following expression is positive:

$$D_B^{BB}(t) = L_B^{BB}(t) - F_B^{BB}(t) = \pi_B(2,A)\int_t^{T_B^*(3)} e^{-rs}ds - \left[c(t) - c(T_B^*(3))\right].$$

The function $D_B^{BB}(t)$ is strictly quasiconcave. At time zero it is negative: The leader's payoff curve is negative by assumption 3, hence it is below the follower's curve. At $T_B^*(3)$, leader and follower curves intersect, so $D_B^{BB}(T_B^*(3))$ is zero. The reason is that if the first entry in the subgame occurs at $T_B^*(3)$, the rival follows immediately, and both firms obtain the same payoff. Since the leader's curve is maximized before $T_B^*(3)$, the two curves must also intersect in a point to the left of $T_B^*(3)$, that we denote by T_B^{BB}: The earliest point where $D_B^{BB}(t)$ is zero. The following result follows from Fudenberg and Tirole (1985).

[14] In our analysis of two-firm subgames, subscripts of the $L(\cdot)$ and $F(\cdot)$ functions denote the type of the firm. Superscripts denote the firms active in the subgame.

[15] Notice that $L_B^{BB}(t)$ is equal to $g_{B,2,A}(t)$ minus a constant.

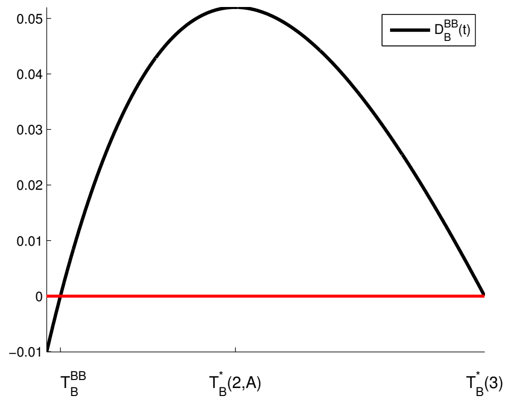

Figure 1: Preemption in the symmetric two-firm (BB) subgame. The figure is drawn using our example with entry cost parameters $\bar{c} = 20$ and $\alpha = 0.08$, interest rate $r = 0.03$, demand elasticity $\eta = 0.5$, type B cost $k_B = 1.04$. The $D_B^{BB}(t)$ curve describes the relative benefit of being the leader rather than the follower. Whenever it is positive, the incentive to preempt is positive. Hence, the first investment must occur at the earliest point where $D_B^{BB}(t)$ is zero: T_B^{BB}.

Lemma 2 *If the first entrant in the three-firm game is A, and first entry occurs strictly earlier than T_B^{BB}, then second entry occurs at T_B^{BB} and third entry occurs at $T_B^*(3)$.*

The mechanism at work is well-known: after A has entered the market, the most profitable outcome for each of the B firms is to be leader of the subgame, entering at the time when the leader's curve is maximized, $T_B^*(2, A)$. The opponent would then follow at $T_B^*(3)$. The leader would receive a higher payoff than the follower. This cannot occur in equilibrium, because the firm who takes the role of follower could profitably deviate and preempt the opponent by investing at $T_B^*(2, A) - \varepsilon$. This highlights that leader investment cannot take place at any time when earlier preemption is profitable. As a consequence, first investment must occur weakly before the first intersection of the leader and follower payoff functions, when the leader's payoff is not larger than the follower's. In equilibrium, it occurs exactly at the first intersection of the two curves, i.e. when $D_B^{BB}(t)$ first equals zero.

3.2 The asymmetric two-firm subgame

Now suppose that the first firm to enter the market is a type B firm. The ensuing subgame is an asymmetric preemption game played by one efficient firm of type A and one inefficient firm of type B. It is analogous to the game analyzed by Riordan (1992). The equilibrium characterization follows directly from his analysis. Figure 2 illustrates.

By Lemma 1, last entry in the subgame occurs no later than $T_B^*(3)$. If the first firm to enter in the subgame is A, at time t, then B follows exactly at the stand-alone entry time $T_B^*(3)$ because it solves the problem $\max_t g_{B,3}(t)$. Hence, firm A receives the leader payoff

$$L_A^{AB}(t) = \pi_A(2)\int_t^{T_B^*(3)} e^{-rs}ds + \pi_A(3)\int_{T_B^*(3)}^{\infty} e^{-rs}ds - c(t)$$

and firm B receives the follower payoff

$$F_B^{AB}(t) = \pi_B(3)\int_{T_B^*(3)}^{\infty} e^{-rs}ds - c(T_B^*(3)).$$

Suppose instead that firm B takes the role of leader and enters at time t. Provided that the subgame starts earlier than firm A's stand-alone entry time $T_A^*(3)$, firm A follows exactly at $T_A^*(3)$ because it solves the problem $\max_t g_{A,3}(t)$. The leader's and follower's payoff are

$$\begin{aligned} L_B^{AB}(t) &= \pi_B(2,B)\int_t^{T_A^*(3)} e^{-rs}ds + \pi_B(3)\int_{T_A^*(3)}^{\infty} e^{-rs}ds - c(t) \\ F_A^{AB}(t) &= \pi_A(3)\int_{T_A^*(3)}^{\infty} e^{-rs}ds - c(T_A^*(3)). \end{aligned}$$

respectively. Now consider the incentive for each firm to preempt the competitor and be the leader of the subgame, rather than the follower. At time t, firm A prefers to be the leader rather than the follower if the following expression is positive:

$$\begin{aligned} D_A^{AB}(t) &= L_A^{AB}(t) - F_A^{AB}(t) \\ &= \pi_A(2)\int_t^{T_B^*(3)} e^{-rs}ds - \pi_A(3)\int_{T_A^*(3)}^{T_B^*(3)} e^{-rs}ds - \left[c(t) - c(T_A^*(3))\right]. \end{aligned}$$

Similarly, at time t, firm B prefers to be the leader rather than the follower if the following expression is positive:

$$\begin{aligned} D_B^{AB}(t) &= L_B^{AB}(t) - F_B^{AB}(t) \\ &= \pi_B(2,B)\int_t^{T_A^*(3)} e^{-rs}ds + \pi_B(3)\int_{T_A^*(3)}^{T_B^*(3)} e^{-rs}ds - \left[c(t) - c(T_B^*(3))\right]. \end{aligned}$$

Both $D_A^{AB}(t)$ and $D_B^{AB}(t)$ are strictly quasiconcave.[16] Define T_B^{AB} as the earliest point such

[16]Notice that $D_A^{AB}(t)$ is equal to $g_{A,2}(t)$ minus a constant, and $D_B^{AB}(t)$ is equal to $g_{B,2,B}(t)$ minus a constant. Also, notice that both $F_B^{AB}(t)$ and $F_A^{AB}(t)$ are constant in t, hence it is sufficient to consider $D_A^{AB}(t)$ and $D_B^{AB}(t)$ to identify the equilibrium outcome (see Hoppe and Lehmann-Grube (2005)).

that B prefers to be the leader of the subgame.[17] Riordan (1992) showed that in a game with two asymmetric firms the more efficient firm enters first: Either at the earliest time when the less efficient firm prefers to be the leader (T_B^{AB} in our game) or at the efficient firm's stand-alone investment time ($T_A^*(2)$ in our game), whichever comes first. The following Lemma summarizes this result:

Lemma 3 *If the first entrant in the three-firm game is a type B firm, and first entry occurs strictly earlier than* $\min\left\{T_A^*(2), T_B^{AB}\right\}$, *then firm A enters second at* $\min\left\{T_A^*(2), T_B^{AB}\right\}$ *and the remaining B firm enters last at* $T_B^*(3)$.

To gain intuition for why the leader in the subgame must be A, observe that the preemption incentive for A is stronger than the preemption incentive for B. The two functions $D_A^{AB}(t)$ and $D_B^{AB}(t)$ describe the preemption incentive for A and B respectively, and the former is larger than the latter. Compare the first term in both functions. If firm A enters first in the subgame, it earns duopoly profits longer than a B would, because $T_A^*(3) < T_B^*(3)$. Moreover, the duopoly profits are higher for A than for $B : \pi_A(2) > \pi_B(2, B)$. Hence, the first term in $D_A^{AB}(t)$ is larger than the first term in $D_B^{AB}(t)$. Next, consider the third term in $D_A^{AB}(t)$ and $D_B^{AB}(t)$. Bringing entry forward to t is cheaper from $T_A^*(3)$ than from $T_B^*(3)$. Thus the last term $D_A^{AB}(t)$ is larger than the first term in $D_B^{AB}(t)$. To complete the argument, we must consider the second terms in $D_A^{AB}(t)$ and $D_B^{AB}(t)$. In particular we have to show that triopoly profits for B earned from $T_A^*(3)$ to $T_B^*(3)$ and the triopoly profits *not earned* by A over the same period do not offset the previous two effects. The intuition is as follows. By preempting B, firm A delays the date from which it earns triopoly profits from $T_A^*(3)$ to $T_B^*(3)$. In the interval $[T_A^*(3), T_B^*(3))$ triopoly profits are replaced by duopoly profits, so the total effect for A is still positive. Now consider firm B. By preempting A it brings forward the time from which it earns triopoly profits from $T_B^*(3)$ to $T_A^*(3)$. By definition of $T_B^*(3)$, bringing entry as a triopolist forward to the left of this point diminishes B's payoff: extra triopoly profits are more than offset by the increase in entry cost.

Given that $D_A^{AB}(t) > D_B^{AB}(t)$, we can conclude that in equilibrium B cannot enter first in this subgame. If it did, at some time t, it would have to be the case that at t firm B weakly prefers the leader's payoff to the follower's payoff: $D_B^{AB}(t) \geq 0$. But then $D_A^{AB}(t)$, which is larger than $D_B^{AB}(t)$, would be strictly positive, hence A could profitable deviate, preempting B and entering at $(t - \varepsilon)$.

[17] If $D_B^{AB}(t) < 0$ for every $t \leq T_A^*(3)$, let $T_B^{AB} = \infty$.

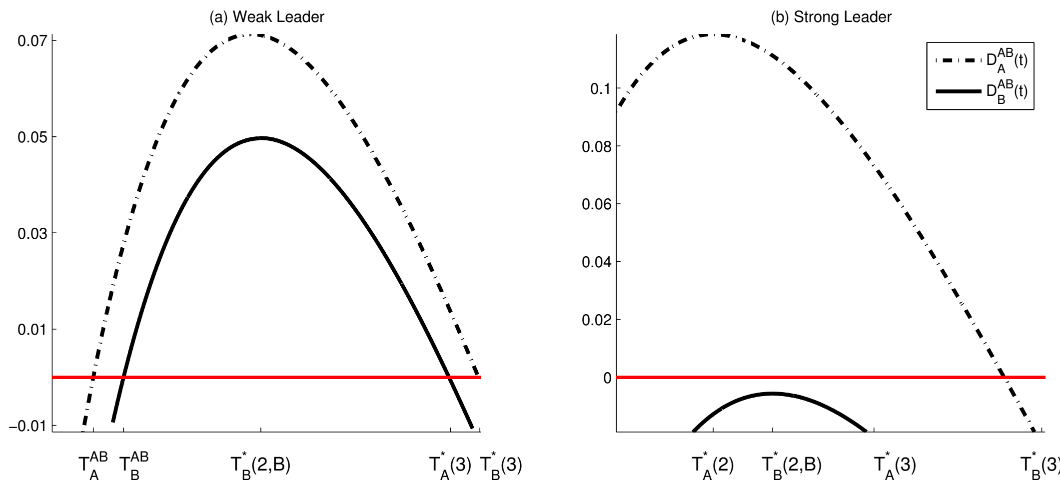

Figure 2: Weak and strong leader in the asymmetric two-player (AB) subgame. The figure is drawn using our example. All the parameter values are as in Figure 1, except for $B's$ cost. We set k_B equal to 1.01 in (2a) and 1.06 in (2b). Both panels show that the preemption incentive is larger for A than for B: $D_A^{AB}(t) > D_B^{AB}(t)$ everywhere. In (2a), the relative inefficiency of B with respect to A is small and hence A is a weak leader. A is bound by B's preemption incentive and enters at T_B^{AB}, the earliest point where B is indifferent between being the leader and the follower. In (2b), the inefficiency is larger. With $D_B^{AB}(t)$ always negative, B never finds it profitable to preempt. Hence, A is a strong leader and invests at the optimal stand-alone time $T_A^*(2)$.

Having identified the order of entry, we can turn to identifying timing of entry. Again we look at the properties of the preemption incentives $D_A^{AB}(t)$ and $D_B^{AB}(t)$. First, they are both negative at $t = 0$ by Assumption 3: preemption is too costly at time zero. Moreover, they are both strictly quasi-concave and have a maximum in $T_A^*(2)$ and $T_B^*(2, B)$ respectively. Finally, in $t = T_A^*(2)$, the function $D_A^{AB}(t)$ is strictly positive.

Following the argument in Riordan (1992), the equilibrium in the subgame must have the following features. First entry in the subgame cannot occur at t very close to zero, because $D_A^{AB}(t)$ and $D_B^{AB}(t)$ are both negative. From some $T_A^{AB} < T_A^*(2)$ onwards, $D_A^{AB}(t)$ becomes positive: firm A would rather be leader than follower in the subgame, and ideally it would like to enter first in the subgame at $T_A^*(2)$. If $D_B^{AB}(t)$ is negative for all entry times earlier than $T_A^*(2)$, firm B has no incentive to enter before $T_A^*(2)$. Firm A can therefore not only be the first to enter in the subgame, but also enter at its preferred time, i.e. $T_A^*(2)$. If instead $D_B^{AB}(t)$ is positive from some time $T_B^{AB} \in \left(T_A^{AB}, T_A^*(2)\right)$ onwards,[18] then the threat of preemption forces A to bring entry forward to $t = T_B^{AB}$. Following the terminology in Riordan (1992), we refer to firm A as a "weak leader" if $T_B^{AB} < T_A^*(2)$ and as a "strong

[18] The fact that $D_A^{AB}(t) > D_B^{AB}(t)$ guarantees that $T_A^{AB} < T_B^{AB}$.

leader" otherwise.

3.3 Inefficient entry in equilibrium

The analysis of the asymmetric two-firm subgame highlights the well-known result that in such a game the more efficient firm must be the leader because it has a stronger preemption incentive. A key force driving this result is that by preempting the opponent, A earns *higher* profits for a *longer period.* We now show that this mechanism is not necessarily at work in the three firm game. By investing first in the three-firm game, the efficient firm would earn a higher monopoly payoff than an inefficient firm would. However, it may do so for a *shorter period*. This may diminish the efficient firm's preemption incentive sufficiently so that one of the less efficient type B firms enters first in equilibrium. Figure 3 illustrates the analysis.

Suppose that the type A firm preempts its rivals and enters first, and that it does so earlier than T_B^{BB}. From Lemma 2, the B firms will follow at T_B^{BB} and $T_B^*(3)$ respectively, hence firm A will earn a leader payoff:

$$L_A(t) = \pi_A(1) \int_t^{T_B^{BB}} e^{-rs} ds + \pi_A(2) \int_{T_B^{BB}}^{T_B^*(3)} e^{-rs} ds + \pi_A(3) \int_{T_B^*(3)}^{\infty} e^{-rs} ds - c(t)$$

and each of the B firms will earn a follower payoff:

$$\begin{aligned} F_B(t) &= \pi_B(2, A) \int_{T_B^{BB}}^{T_B^*(3)} e^{-rs} ds + \pi_B(3) \int_{T_B^*(3)}^{\infty} e^{-rs} ds - c\left(T_B^{BB}\right) \\ &= \pi_B(3) \int_{T_B^*(3)}^{\infty} e^{-rs} ds - c\left(T_B^*(3)\right). \end{aligned}$$

where the second equality follows from the definition of T_B^{BB}.

Next, suppose that instead a type B firm enters first in the game, and it does so at a time t no later than $\min\left\{T_A^*(2), T_B^{AB}\right\}$. From Lemma 3, A enters at $\min\left\{T_A^*(2), T_B^{AB}\right\}$ and the remaining B firm at time $T_B^*(3)$. The early B firm obtains the leader payoff:

$$L_B(t) = \pi_B(1) \int_t^{\min\left\{T_A^*(2), T_B^{AB}\right\}} e^{-rs} ds + \pi_B(2, A) \int_{\min\left\{T_A^*(2), T_B^{AB}\right\}}^{T_B^*(3)} e^{-rs} ds + \pi_B(3) \int_{T_B^*(3)}^{\infty} e^{-rs} ds - c(t).$$

Firm A is preempted and thus earns the follower payoff:

$$F_A(t) = \pi_A(2) \int_{\min\left\{T_A^*(2), T_B^{AB}\right\}}^{T_B^*(3)} e^{-rs} ds + \pi_A(3) \int_{T_B^*(3)}^{\infty} e^{-rs} ds - c(\min\left\{T_A^*(2), T_B^{AB}\right\})$$

and the late B firm obtains follower's payoff $F_B(t)$.[19]

We can now write the incentive to be the first entrant in the game for an efficient and an inefficient firm, respectively.

Firm A would like to preempt its rivals and be the leader rather than the follower if

$$\begin{aligned} D_A(t) &= L_A(t) - F_A(t) \\ &= \pi_A(1) \int_t^{T_B^{BB}} e^{-rs} ds + \pi_A(2) \int_{T_B^{BB}}^{\min\left\{T_A^*(2), T_B^{AB}\right\}} e^{-rs} ds - \left[c(t) - c\left(\min\left\{T_A^*(2), T_B^{AB}\right\}\right)\right] \end{aligned}$$

is positive.[20] By preempting the rivals, firm A gains monopoly profits from time t until T_B^{BB}, achieves duopoly profits starting from T_B^{BB} rather than $\min\left\{T_A^*(2), T_B^{AB}\right\}$, and finally sustains a higher entry cost because it enters earlier.

Similarly, a type B firm prefers to be the leader rather than the follower if

$$\begin{aligned} D_B(t) &= L_B(t) - F_B(t) \\ &= \pi_B(1) \int_t^{\min\left\{T_A^*(2), T_B^{AB}\right\}} e^{-rs} ds + \pi_B(2, A) \int_{\min\left\{T_A^*(2), T_B^{AB}\right\}}^{T_B^{BB}} e^{-rs} ds - \left[c(t) - c\left(T_B^{BB}\right)\right] \end{aligned}$$

is positive. By preempting the rivals, a B firm gains monopoly profits from t until $\min\left\{T_A^*(2), T_B^{AB}\right\}$, achieves duopoly profits starting from $\min\left\{T_A^*(2), T_B^{AB}\right\}$ rather than T_B^{BB}, and finally sustains a higher entry cost because it enters earlier.

Consider the $D_i(t)$ functions, for $i = A, B$. In $t = 0$, they are both negative, because by Assumption 3 preemption is too costly at time zero. Moreover, they are both strictly quasi-concave[21] and have a maximum in $T_i^*(1)$ for $i = A, B$ respectively. Let their earliest

[19] Observe that a B firm's follower payoff is independent of whether the subgame following first entry is between two type B firms or one type A and one type B firm.

[20] Notice that both $F_B(t)$ and $F_A(t)$ are constant in t, hence it is sufficient to consider $D_A(t)$ and $D_B(t)$ to identify the equilibrium outcome (see Hoppe and Lehmann-Grube (2005)).

[21] Notice that $D_A(t)$ is equal to $g_{A,1}(t)$ minus a constant and $D_B(t)$ is equal to $g_{B,1}(t)$ minus a constant.

intersections with zero be defined as follows:

$$T_B^1 \equiv \begin{cases} \min\{\tau \text{ such that } D_B(\tau)=0\} \text{ if } D_B(t) \text{ admits at least one zero} \\ +\infty \text{ otherwise} \end{cases}$$

$$T_A^1 \equiv \begin{cases} \min\{\tau \text{ such that } D_A(\tau)=0\} \text{ if } D_A(t) \text{ admits at least one zero} \\ +\infty \text{ otherwise} \end{cases}$$

The following result holds:

Proposition 1 *If $T_B^1 < T_A^1$, the game admits a unique SPNE outcome, in which the entry order is $B-A-B$, and entry times are $t_1 = T_B^1$, $t_2 = \min\{T_A^*(2), T_B^{AB}\}$, $t_3 = T_B^*(3)$. In equilibrium, both inefficient firms achieve the same equilibrium payoff, and the efficient firm achieves a higher payoff.*

Suppose that $T_B^1 < T_A^1$. This implies that either the preemption incentive for A is always negative (if T_B^1 is finite and $T_A^1 = +\infty$) or that the preemption incentive for B is strictly positive whenever it is positive for A. Therefore, the efficient firm cannot be the leader of the three-firm game. The first entrant must be a type B firm. Consider the timing of entry. First entry takes place exactly at T_B^1. Clearly, no firm has an incentive to enter earlier, because for $t < T_B^1$, all firms prefer to be the follower. Moreover, first entry cannot take place later than T_B^1 because in a right-neighbourhood of T_B^1, $D_B(t)$ is positive. If first entry took place at $t > T_B^1$, one of the two B firms would enter third at $T_B^*(3)$ and receive $F_B(t)$. That firm would rather deviate, preempt the rivals, and be a leader at $T_B^1 + \varepsilon$. Following a logic that is analogous to that in Fudenberg-Tirole (1985), the preemption race between the two B firms guarantees that first entry takes place exactly at T_B^1 so that there is rent equalization for the two B firms and neither has an incentive to preempt further. Finally, $T_B^1 < T_A^1$ guarantees that $D_A(T_B^1)<0$: at T_B^1 firm A strictly prefers to be follower rather than leader, hence has no incentive to deviate and preempt.

Proposition 1 establishes that if $T_B^1 < T_A^1$ the entry order in equilibrium is $B-A-B$. The next remark provides the main intuition for this result:[22]

Remark 1 *A necessary condition for $T_B^1 < T_A^1$ is that $T_B^{BB} < \min\{T_A^*(2), T_B^{AB}\}$: first entry in a BB subgame must occur earlier than first entry in an AB subgame.*

[22] For a formal proof, see Claim 4 in Appendix B.

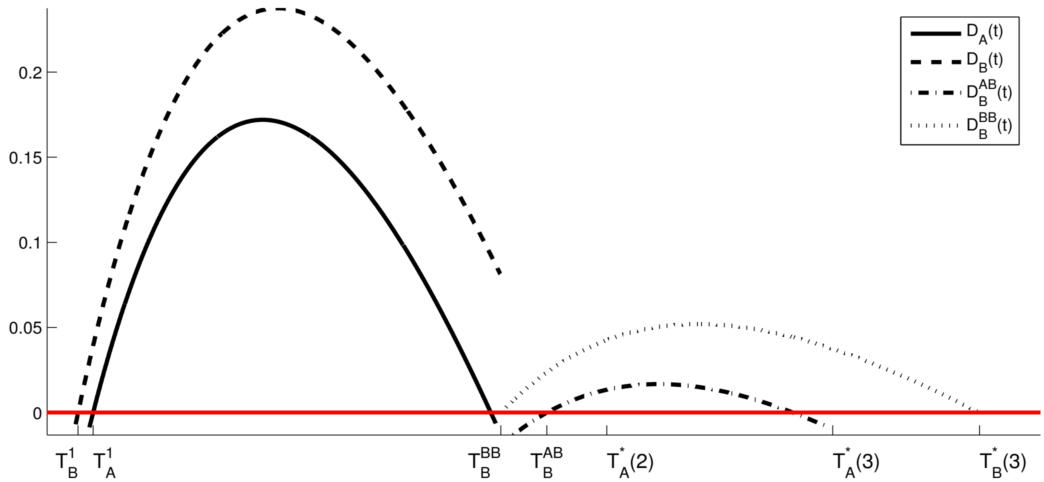

Figure 3: Inefficient entry order in the three player game. The figure is drawn using our example. All the parameter values are as in Figure 1. Second entry occurs earlier if the first entrant is A, than if it is B: $T_B^{BB} < T_B^{AB}$. Hence the necessary condition of Remark 1 is satisfied. The longer monopoly period for B outweighs the lower monopoly profits: $D_B^1(t) > D_A^1(t)$ everywhere, and the first entrant is a type-B firm, at T_B^1.

If the model primitives are such that this condition holds, then by preempting the opponents and investing first in the three-firm game, the efficient firm would earn a higher monopoly payoff than an inefficient firm, but it would earn it for a *shorter period*. Hence, it does not necessarily have a stronger incentive than the opponents to be first in the game. To see why the condition is necessary, suppose it were violated. In this case, the preemption incentive to be the first rather than the second entrant would always be stronger for A than for a type B firm, for the following reasons: The first entrant achieves monopoly profits for some time. For firm A, these profits are higher than for firm B, and they would be achieved for a longer time (until T_B^{BB}, rather than until $\min\left\{T_A^*(2), T_B^{AB}\right\}$). Moreover, by entering at t and being a leader rather than a follower, each firm sustains a higher entry cost. The cost would increase less for firm A, who would otherwise enter at $\min\left\{T_A^*(2), T_B^{AB}\right\}$, than for firm B, who would otherwise enter at T_B^{BB}. Finally, by being leader rather than follower, a firm changes the time from which it starts earning duopoly profits. For an A-type leader, this date would be delayed from $\min\left\{T_A^*(2), T_B^{AB}\right\}$ to T_B^{BB}. In that interval, duopoly profits would be replaced by monopoly profits, so the total effect would be positive. For a type B leader instead, this date would be anticipated from T_B^{BB} to $\min\left\{T_A^*(2), T_B^{AB}\right\}$. Nonetheless, the extra duopoly profits earned in this period would be more than offset by the increase in entry cost, so the total effect would be negative. In sum, if min $\left\{T_A^*(2), T_B^{AB}\right\} \leq T_B^{BB}$, then the total preemption incentive to be first rather than second is stronger for firm A than for a type B firm and $T_B^1 < T_A^1$ cannot hold.

3.4 Conditions for inefficient entry

In this subsection, we present our main result: We show that the set of parameter values that induce equilibrium entry order $B - A - B$ is nonempty and analyze which changes in one or more flow profit parameters preserve this entry order.

Proposition 2 ***(a)*** *For any cost function satisfying the model assumptions, the set of profit structures satisfying the model assumptions and inducing equilibrium entry order* $B-A-B$ *is nonempty.*
(b) *Let* $\widehat{\boldsymbol{\pi}}$ *be a profit structure such that the equilibrium entry order is* $B - A - B$*. Any profit structure* $\boldsymbol{\pi}$ *that satisfies the model assumptions and the following inequalities also induces equilibrium entry order* $B - A - B$*:*
(i) $\pi_A(1) \leq \widehat{\pi}_A(1)$*, (ii)* $\pi_B(2, A) \geq \widehat{\pi}_B(2, A)$*, (iii)* $\pi_B(3) \leq \widehat{\pi}_B(3)$*, (iv)* $\pi_B(1) \geq \widehat{\pi}_B(1)$*, (v)* $\pi_A(2) \leq \widehat{\pi}_A(2)$*, (vi)* $\pi_B(2, B) \leq \widehat{\pi}_B(2, B)$*.*

In Appendix A, we show that for the parametric example introduced in Section 2 the set of profit structures inducing equilibrium entry order $B - A - B$ is nonempty.
Part (a) of Proposition 2 shows that this existence result is more general: For *any* cost function, there exist profit structures inducing the inefficient entry order.

The intuition for the result in part (b) is the following: Starting from a profit structure that induces equilibrium entry order $B-A-B$, the entry order is preserved by any change in the parameter values that weakly decreases the incentive for firm A to be first and/or increases the incentive for the type B firms to be first.

Consider the incentive for firm A to preempt its rivals and be the first entrant. By doing so, firm A earns monopoly profits for some time. Any change in the parameter values that decreases either the level of A's monopoly profits or the interval of time for which they are earned, decreases A's incentive to be first. In particular, condition *(i)* describes a weak decrease in A's monopoly profits, while conditions *(ii)* and *(iii)* are related to the time of second entry. The time of second entry, after first entry by A, depends on the intensity of the preemption race in the ensuing BB subgame. An increase in $\pi_B(2, A)$, and/or a decrease in $\pi_B(3)$, increases the incentive for each of the type B firms to be second, rather than third. Hence, it brings forward the time of second entry. This shortens the interval of time for which firm A, if first entrant, earns monopoly profits, thus decreasing A's incentive to be first.

Next, consider the incentive for a type B firm to preempt its rivals and be the first entrant. Any change in the parameter values that increases either the level of the type B firms' monopoly profits or the interval of time for which they are earned, increases the type B firms' incentive to be first. In particular, condition *(iv)* describes a weak increase in the type B firms's monopoly profits, while conditions *(iii)*, *(v)* and *(vi)* are related to the time of second entry, hence to the features of the ensuing AB subgame. If A is a "strong leader" in this subgame, $\min\left\{T_A^*(2), T_B^{AB}\right\}$ is equal to $T_A^*(2)$. Therefore, the smaller $\pi_A(2)$, the later is second entry after a type B firm has entered first, and the stronger the incentive for a type B firm to be first. Similarly, if A is a "weak leader", the second entry time after first entry by a type B firm is equal to T_B^{AB}, which in turn is determined by the incentive B has to preempt A in the subgame. This incentive is increasing in both $\pi_B(2, B)$ and $\pi_B(3)$: by being second rather than third, B would receive duopoly profits for some time, and triopoly profits for a longer time (from $T_A^*(3)$ rather than from $T_B^*(3)$). Therefore, the smaller $\pi_B(2, B)$ or $\pi_B(3)$, the later is second entry after a type B firm has entered first, and the stronger the incentive for a type B firm to be first.

4 Concluding remarks

We analyzed a preemption game of entry into a new market with ex-ante asymmetric firms. It is well known from the literature that in a two-firm game the equilibrium entry order reflects the efficiency ranking. We show that this result can be reversed if the game is played by more than two firms. We present an example with one efficient firm and two inefficient firms. The set of parameters such that the unique equilibrium outcome involves first entry by one of the inefficient firms is shown to be nonempty. Moreover we investigate which changes in post-entry profit parameters preserve this entry order. Our result shows that the assumption that market entry occurs in the order of profitability, often made in the empirical entry literature, may be problematic.

We conclude with two remarks. First, an entry order that does not reflect the efficiency ranking has interesting welfare implications.[23] When the equilibrium entry order is $B-A-B$ rather than $A-B-B$, not only do consumers face a less efficient monopolist, they also face it for a longer period, as second entry is delayed. At the same time, there are savings in entry costs due to the delay of second entry. We do not obtain general analytical results on the total effect of the entry order on welfare. In Appendix A, we will however present numerical results in the context of the example introduced earlier, showing that the negative effect on the consumers' surplus of the inefficiency in the entry order B-A-B tends to dominate the savings in entry costs.[24]

Finally, we draw attention to the implications of a slightly more general model where *all three* firms differ in efficiency. That is, there are three firms, A, B and C, where C is strictly less efficient than B. While characterizing the equilibrium of such a game is beyond the scope of this paper, we point out here that an equilibrium with the least efficient firm C entering first is not feasible. The reason is that in such a candidate equilibrium, B would have a strict incentive to preempt C and enter first. This is because if firm B were to enter first, not only would it earn higher monopoly profits than firm C, but also for a longer period. Regardless of which of the inefficient firms enters first, the second entrant would be firm A, and it would invest later in an AC subgame than in an AB subgame. Hence the incentive to enter first is stronger for firm B than for firm C, ruling out an equilibrium entry order $C-A-B$.

Appendix A: Numerical Results

In this Appendix, we use the example introduced in Section 2 to numerically identify the range of parameter values that induce entry order $B-A-B$ and to illustrate the welfare implications of inefficient entry.

As in Figures 1 to 3, we fix $r = 0.03$ and $\overline{c} = 20$. For a range of values of α, η and k_B for which Assumptions 1-3 are satisfied, we compute the equilibrium entry order. Table 1 shows that for some pairs (α, η), there exists a range of values of k_B for which the entry order in equilibrium is $B-A-B$.

An immediate observation is that this range is bounded both above and below. This reflects the fact that an increase in k_B affects the incentives of the two types of firms to be

[23] We thank a referee to alerting us to these implications.

[24] See Figure 5 in Appendix A.

Table 1: Ranges of k_B for which the equilibrium order of entry is B-A-B

α	η			
	0.2	0.4	0.6	0.8
0.02	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
0.03	$\emptyset$	$\emptyset$	$\emptyset$	(1.1031, 1.1165)
0.04	$\emptyset$	$\emptyset$	(1.0580, 1.0718)	(1.0636, 1.1336)
0.05	(1.0150, 1.0165)	(1.0298, 1.0405)	(1.0387, 1.0781)	(1.0264, 1.1451)
0.06	(1.0118, 1.0174)	(1.0204, 1.0428)	(1.0193, 1.0825)	(1.0000, 1.1534)
0.07	(1.0082, 1.0181)	(1.0107, 1.0445)	(1.0000, 1.0859)	(1.0000, 1.1596)
0.08	(1.0044, 1.0186)	(1.0000, 1.0458)	(1.0000, 1.0885)	(1.0000, 1.1646)

first in a nonmonotonic way. In particular, consider the effect of a change in k_B on profit flows, illustrated in Figure 4.

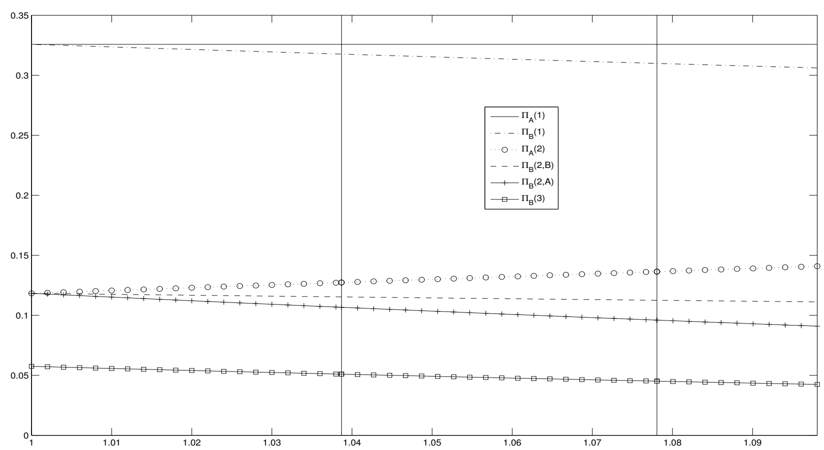

Figure 4: The effect of k_B on firms' profits for $\eta = 0.6$, $\alpha = 0.05$.

Consider firm A's incentive to be first. An increase in k_B, ceteris paribus, leaves $\pi_A(1)$ unchanged and reduces both $\pi_B(2, A)$ and $\pi_B(3)$. Therefore, while the level of monopoly profits for A is unaffected, the period of time for which A would earn them is affected in a nontrivial way. In the ensuing BB subgame, both the duopoly and the triopoly post-entry profits are decreased. Therefore, it is a priori unclear whether the time of second entry in the game is brought forward, or delayed.

Next, consider the type B firms. An increase in k_B reduces monopoly profits $\pi_B(1)$, thus reducing a type B firm's incentive to be first. It also affects the period of time for which they would be earned in a nontrivial way because it increases $\pi_A(2)$, and reduces

$\pi_B(2, B)$ and $\pi_B(3)$. This illustrates why the range of values of k_B for which the entry order in equilibrium is $B - A - B$ is bounded both below and above.

We now examine the effect of inefficient entry on welfare in Figure 5. We discuss what happens at the first vertical line, when the equilibrium order switches from $A - B - B$ to $B - A - B$. All the effects are analogous, but with the opposite sign, at the second vertical line, when the entry order switches back to $A - B - B$.

Panel (a) illustrates the effect of an increase in k_B on entry times. Both the first and third entry time (represented by the bottom and the top line respectively) are continuous in k_B. The second entry time (the middle line) is discontinuous: When the equilibrium order switches from $A - B - B$ to $B - A - B$, there is a discrete jump and second entry occurs later, because it is determined by the preemption race in an AB subgame, rather than in a BB subgame.

Panel (b) plots producer surplus and entry costs. The decrease in total entry costs at the first switch reflects the delay of second entry. The discontinuity in producer surplus has exactly the same sign and the same magnitude.[25] The intuition for this is the following: At the switch point, both $A - B - B$ and $B - A - B$ are equilibrium entry orders, and both A and the B firms are indifferent between being leader or follower. Hence, the discounted sum of the revenue flows of all players minus the sum of their entry costs is the same under both equilibria.

Observe that in the region of inefficient entry, entry times in panel (a) as well as surplus and cost in panel (b) are nonmonotone. This is due to the fact that as k_B increases sufficiently, A switches from being a weak leader of the AB subgame, to being a strong leader.

Panel (c) reports consumer surplus. The discontinuity at the switch point from $A-B-B$ to $B - A - B$ is caused by the fact that a low-cost monopolist is replaced by a high-cost monopolist, and moreover the monopoly period lasts longer.

Finally, panel (d) illustrates the resulting effect on overall welfare, where welfare is calculated as the sum of consumer and producer surplus, minus total investment costs. The discontinuity at the switch point reflects the discontinuity in consumer surplus because producer surplus net of entry costs is continuous in this example.

[25] We thank a referee for alerting us to this point.

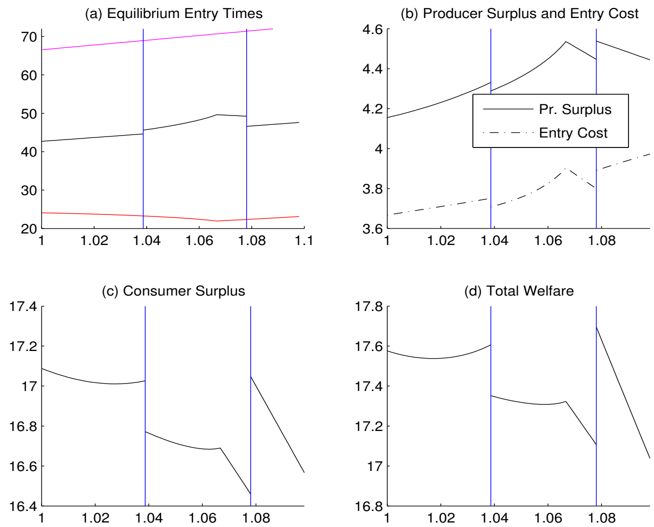

Figure 5: The effect of k_B on entry times, welfare, and its components for $\eta = 0.6$ and $\alpha = 0.05$. All welfare components are discounted at time zero at the common discount rate $r = 0.03$.

Appendix B: Proofs

In this appendix we present a result (Claim 1) regarding the properties of the function $g_{i,m,-i}(t)$. We then prove the results stated in the paper. Observe that, as in most preemption games, equilibrium is constructed starting from the end of the game and moving backwards. This requires the analysis of a large set of off-equilibrium path subgames which makes a lengthy proof with several intermediate steps necessary.

Claim 1 *(a) The function $g_{i,m,-i}(t)$ is strictly quasi-concave in t. (b) It admits a unique global maximum in $T_i^*(m,-i)$, defined as the solution to:*

$$g'_{i,m,-i}(t) = 0 \Longleftrightarrow -\pi_i(m,-i)e^{-rt} - c'(t) = 0.$$

(c) $g_{i,m,-i}(T_i^*(m,-i)) > 0.$

Proof of Claim 1. Part (a) We prove that the function is strictly quasiconcave, by showing that in every critical point of the function the second derivative is strictly negative. The first derivative $g'_{i,m,-i}(t)$ is equal to $\left[-\pi_i(m,-i)e^{-rt} - c'(t)\right]$ and the second derivative $g''_{i,m,-i}(t)$ is equal to $\left[r\pi_i(m,-i)e^{-rt} - c''(t)\right]$. Using $g'_{i,m,-i}(t) = 0$ we can rewrite $g''_{i,m,-i}(t)$

evaluated at any critical point as

$$g''_{i,m,-i}(t) = -c'(t)\, r - c''(t)\,. \tag{2}$$

By Assumption 2: $e^{rt}\left[c'(t) + rc(t)\right] < 0$ and $e^{rt}\left[2c'(t)\, r + c(t)\, r^2 + c''(t)\right] > 0$. Together, these two inequalities imply that expression (2) is negative.

Part (b) To prove that the stand-alone times are well-defined, i.e. that the function $g_{i,m,-i}(t)$ admits a critical point, we prove that $g'_{i,m,-i}(t)$ is positive at zero and negative for sufficiently large values of t. Assumptions 1 and 3 guarantee that $g_{i,m,-i}(t)$ is negative at zero and positive at a later time. Quasiconcavity then implies that $g'_{i,m,-i}(0) > 0$. Moreover, since $g_{i,m,-i}(t)$ is continuous and either always increasing or single peaked, it admits a limit as t goes to infinity. This limit must be greater than or equal to zero by assumption 3(ii). It must also be smaller than or equal to zero because $\lim_{t\to+\infty} g_{i,m,-i}(t) = -\lim_{t\to+\infty} c(t)$. Hence, the only possible candidate limit is zero. But if that is the case, since the function is positive from some τ onwards, it must approach zero from above. Hence it must be decreasing for t sufficiently large. We therefore conclude that the function $g_{i,m,-i}(t)$ admits a critical point.

Part (c) Assumptions 1, 2 and 3 imply that $g_{i,m,-i}(t)$ is strictly positive for any $t \geq T_i^*(m,-i)$, hence $g_{i,m,-i}(T_i^*(m,-i)) > 0$. ■

Proof of Lemma 1 We show that at any decision node with calendar time $\tau \geq T_B^*(3)$, with any number of active firms, all firms enter immediately. First, suppose only one type B firm is active at t. For $t \geq \tau$, the function $g_{B,3}(t)$ represents the firm's payoff from entering last at time t. By assumptions 1, 2 and 3, it is strictly positive for every t larger than some finite t'. Hence its maximum value, attained at $T_B^*(3)$, is strictly positive. Therefore, the firm will enter immediately. Similarly, suppose that only firm A is active at t. The function $g_{A,3}(t)$ represents the firm's payoff from entering last at time t. It is maximized at $T_A^*(3)$ and by assumptions 1, 2 and 3 it is strictly positive for every t larger than $T_A^*(3)$. It follows that at time τ the firm will enter immediately because $\tau \geq T_B^*(3) > T_A^*(3)$.

Next, suppose that at τ there are two active firms. If they both enter immediately, each receives payoff $g_{i,3}(\tau)$. If one of the two deviates to playing WAIT at τ, the other firm enters at τ, the game enters a subgame with one active firm, and it follows from the above that the deviating firm enters immediately as well. Hence the deviation does not affect the payoff and is not profitable. Similarly, if only one firm plays ENTER at τ and the other plays WAIT, the outcome is that both firms enter, sequentially, at τ and get payoff $g_{i,3}(\tau)$. The firm who initially plays WAIT has no incentive to deviate because it would not affect its payoff. The firm who initially plays ENTER has no incentive to deviate because it would then get either zero, or $g_{i,3}(t)$ for some $t \geq \tau$ and $g_{i,3}(t)$ is strictly decreasing in the interval considered. By a similar argument, in equilibrium it cannot be the case that both active firms play WAIT at τ because each of them would be better off by deviating. The argument for three active firms at time τ is analogous. ■

Proof of Lemma 2 We present a more general result that characterizes the equilibrium outcome of the BB subgames starting at any time $\tau < T_B^*(3)$ and implies Lemma **2**. It follows immediately from our assumptions and the analysis in Fudenberg and Tirole (1985) that, given the functions $L_B^{BB}(t)$ and $F_B^{BB}(t)$, there exists a point $T_B^{BB} \in (0, T_B^*(2, A))$ such that $D_B^{BB}\left(T_B^{BB}\right) = 0$ and that the following result holds:

Claim 2 *In any SPNE, in any BB subgame starting at time $\tau < T_B^*(3)$ there is a unique equilibrium outcome, such that:*
(i) entries take place at $t_2 = \max\left\{\tau, T_B^{BB}\right\}$ and $t_3 = T_B^*(3)$.
(ii) If $\tau \leq T_B^{BB}$, both B firms achieve payoff $F_B^{BB}\left(T_B^*(3)\right)$, while if $\tau > T_B^{BB}$ payoffs for the early and late entrant are $L_B^{BB}(\tau)$ and $F_B^{BB}(\tau) < L_B^{BB}(\tau)$ respectively. ■

Proof of Lemma 3. We present a more general result that characterizes the equilibrium outcome of the AB subgames starting at any time $\tau < T_B^*(3)$ and implies Lemma 3. Consider the function $D_B^{AB}(t)$. It is strictly quasiconcave and admits a unique global maximum in $t = T_B^*(2, B) \in \left(T_A^*(2), T_B^*(3)\right)$. It takes negative value at zero by assumptions 1 and 3(i), and in $t = T_A^*(3)$ by definition of $T_B^*(3)$. Hence, in the interval $t \in [0, T_A^*(3)]$ the following cases are possible:

Case 1 The function is negative everywhere

Case 2 The function has two (possibly coinciding) intersections with zero, $\left\{\underline{T}_B^{2,B}, \overline{T}_B^{2,B}\right\}$ such that $\underline{T}_B^{2,B} \leq \overline{T}_B^{2,B}$, and $T_A^*(2) \leq \underline{T}_B^{2,B}$

Case 3 The function has two (possibly coinciding) intersections with zero, $\left\{\underline{T}_B^{2,B}, \overline{T}_B^{2,B}\right\}$ such that $\underline{T}_B^{2,B} \leq \overline{T}_B^{2,B}$, and $\underline{T}_B^{2,B} < T_A^*(2)$.

Therefore, the definition of T_B^{AB} in the text is equivalent to the following: In case (1), $T_B^{AB} \equiv +\infty$; in cases 2 and 3, $T_B^{AB} \equiv \underline{T}_B^{2,B}$. The following result holds:

Claim 3 *In any SPNE, in any AB subgame starting at time* $\tau < T_B^*(3)$*, it holds that:*
(i)If $\tau \leq \min\left\{T_A^*(2), T_B^{AB}\right\}$*, firm A enters first in the subgame, at* $t_2 = \min\left\{T_A^*(2), T_B^{AB}\right\}$ *and firm B enters later, at* $t_3 = T_B^*(3)$
(ii)If $\tau > \min\left\{T_A^*(2), T_B^{AB}\right\}$ *:*
- in case 1, firm A enters first in the subgame, at $t_2 = \tau$ *and firm B enters later, at* $t_3 = T_B^*(3)$.
- in cases 2 and 3, for $\tau \notin \left[\underline{T}_B^{2,B}, \overline{T}_B^{2,B}\right]$*, firm A enters first in the subgame, at* $t_2 = \tau$ *and firm B enters later, at* $t_3 = T_B^*(3)$*, while for* $\tau \in \left[\underline{T}_B^{2,B}, \overline{T}_B^{2,B}\right]$ *either firm A enters first in the subgame, at* $t_2 = \tau$ *and firm B enters later, at* $t_3 = T_B^*(3)$*, or firm B enters first in the subgame, at* $t_2 = \tau$ *and firm A enters later, at* $t_3 = T_A^*(3)$.

Proof. To prove this Claim, we first show that in our model, in an AB subgame, the condition $\overline{\Delta}_j\left(y_j, z_j, z_i\right) < \overline{\Delta}_i\left(y_i, z_i, z_j\right)$ in Theorem (1) in Riordan (1992) is satisfied, with the interpretation that $i = A$ and $j = B$. The equivalent of the condition $\overline{\Delta}_j\left(y_j, z_j, z_i\right) < \overline{\Delta}_i\left(y_i, z_i, z_j\right)$ in our model, for an AB subgame, is that $T_A^{AB} < T_B^{AB}$, where T_A^{AB} is defined as the smallest value of t such that $D_A^{AB}(t)$ is null. The function $D_A^{AB}(t)$ is strictly quasi-concave in t, strictly negative for $t = 0$, it has strictly positive value for $t = T_A^*(3)$, and admits a unique global maximum in $t = T_A^*(2) < T_A^*(3)$. Hence, T_A^{AB} is well defined and belongs to the interval $(0, T_A^*(2))$. For $T_A^{AB} < T_B^{AB}$ to hold, it is sufficient that $D_A^{AB}(t) - D_B^{AB}(t) > 0$ for every $t < T_A^*(3)$. To see that this condition holds, notice that $D_A^{AB}(t) - D_B^{AB}(t)$ can be rewritten as

$$\left[\pi_A(2) - \pi_B(2, B)\right] \int_t^{T_A^*(3)} e^{-rs} ds + \left[\pi_A(2) - \pi_A(3)\right] \int_{T_A^*(3)}^{T_B^*(3)} e^{-rs} ds$$
$$-\pi_B(2, B) \int_{T_A^*(3)}^{T_B^*(3)} e^{-rs} ds + c\left(T_A^*(3)\right) - c\left(T_B^*(3)\right).$$

The first two terms are positive by assumption 1(ii) and the last one by definition of $T_B^*(3)$.

Given that $T_A^{AB} < T_B^{AB}$, condition $\overline{\Delta}_j\left(y_j, z_j, z_i\right) < \overline{\Delta}_i\left(y_i, z_i, z_j\right)$ in Theorem (1) in Riordan (1992) is satisfied, and part (i) of the Lemma follows immediately from part (i) of Riordan's theorem. Moreover, part (ii) of the Lemma follows from the analysis in the Appendix of Riordan (1992), in particular from Lemma A3 and from the proof of Lemma A4, where $\widehat{\Delta_1}\left(z_1\right) \geq \widehat{\Delta_2}\left(z_2\right)$ is equivalent to $T_A^*(3) \leq T_B^*(3)$, $t\left(\widehat{\Delta_1}\left(x_1\right)\right)$ is equal to $T_A^*(2)$, $\overline{\Delta}_2\left(y_2, z_2, z_1\right) < \widehat{\Delta_1}\left(x_1\right) \leq \overline{\Delta}_1\left(y_1, z_1, z_2\right)$ is equivalent to $T_A^{AB} < T_A^*(2) \leq T_B^{AB}$, $\overline{\Delta}_2\left(y_2, z_2, z_1\right) = \widehat{\Delta_1}\left(x_1\right) < \overline{\Delta}_1\left(y_1, z_1, z_2\right)$ is equivalent to $T_A^{AB} < T_A^*(2) = T_B^{AB}$ and

$\overline{\Delta}_2(y_2, z_2, z_1) > \widehat{\Delta_1}(x_1)$ is equivalent to$T_A^*(2) > T_B^{AB}$. ■

Proof of Proposition 1.

Outline of the proof.

Lemma 1 established that all firms must enter by $T_B^*(3)$. Claim 2 characterized the equilibrium outcome of subgames with two B firms active starting before $T_B^*(3)$. Similarly, Claim 3 characterized the equilibrium outcome of subgames with A and one of the B firms active starting before $T_B^*(3)$. Building on these results, Claim 4 proves an important necessary condition: If $T_B^1 < T_A^1$, then it has to be the case that second entry takes place earlier if the first entrant is A, than if it is a type B firm. Finally, Claims 5, 6 and 7 complete the result establishing the equilibrium outcome of subgames with all three firms active, starting late in the game (Claim 5), early in the game (Claim 6), and at time zero (Claim 7).

Consider subgames with three active firms starting at $\tau < \min\left\{T_A^*(2), T_B^{AB}, T_B^{BB}\right\}$. The functions $D_A(\cdot)$ and $D_B(\cdot)$ as defined in section 3.3 are negative at zero by assumptions 1 and 3(i), are strictly quasiconcave, and maximized at $T_A^*(1)$ and $T_B^*(1) > T_A^*(1)$ respectively. The following Claim holds:

Claim 4 *If $T_B^1 < T_A^1$, then it has to be the case that $T_B^{BB} < \min\left\{T_A^*(2), T_B^{AB}\right\}$.*

Proof. We prove the result by contradiction. Suppose $\min\left\{T_A^*(2), T_B^{AB}\right\} \leq T_B^{BB}$ and consider the functions $D_A(t)$ and $D_B(t)$. Both functions are strictly quasiconcave and negative at zero. $D_A\left(\min\left\{T_A^*(2), T_B^{AB}\right\}\right)$ is strictly positive, so it has to be the case that $T_A^1 < \min\left\{T_A^*(2), T_B^{AB}\right\}$. Moreover, $D_B\left(\min\left\{T_A^*(2), T_B^{AB}\right\}\right)$ is strictly negative because the function $\left[\pi_B(2,A)\int_{\min\left\{T_A^*(2),T_B^{AB}\right\}}^{+\infty} e^{-rs}ds - c(t)\right]$ is strictly quasiconcave and maximized at $T_B^*(2,A) > T_B^{BB}$, hence it is strictly increasing in $\left[\min\left\{T_A^*(2), T_B^{AB}\right\}, T_B^{BB}\right]$. It follows that either $T_B^1 > \min\left\{T_A^*(2), T_B^{AB}\right\} > T_A^1$, in which case the assumption $T_B^1 < T_A^1$ is contradicted, or $T_B^1 \leq \min\left\{T_A^*(2), T_B^{AB}\right\}$. For the latter case, we show that $D_A(t) > D_B(t)$ for any $t < \min\left\{T_A^*(2), T_B^{AB}\right\}$ which in turn implies that the assumption $T_B^1 < T_A^1$ is contradicted.

First, notice that by Assumption (1) the first term in $D_A(t)$ is greater than the first term in $D_B(t)$, and the second term in $D_A(t)$ is positive. Moreover,

$$-\pi_B(2,A)\int_{\min\left\{T_A^*(2),T_B^{AB}\right\}}^{T_B^{BB}} e^{-rs}ds + c\left(\min\left\{T_A^*(2), T_B^{AB}\right\}\right) - c\left(T_B^{BB}\right) > 0$$

by definition of $T_B^*(2,A)$. We can therefore conclude that even if $T_B^1 \leq \min\left\{T_A^*(2), T_B^{AB}\right\}$, $D_A(t) > D_B(t)$ for any $t < \min\left\{T_A^*(2), T_B^{AB}\right\}$, which in turn implies it cannot be the case that $T_B^1 < T_A^1$. ■

We continue the proof of Proposition 1 considering subgames with three active firms starting at $\tau \in [T_B^{BB}, T_B^*(3))$.

At time τ, if all three firms are active and A enters first, it follows from Claim 2 that the two B firms follow at τ and $T_B^*(3)$ respectively. Payoffs are $L_A^{AB}(\tau)$ for A and a lottery between $L_B^{BB}(\tau)$ and $F_B^{BB}(\tau)$ for both B firms, with $L_B^{BB}(\tau) > F_B^{BB}(\tau)$.

If instead one of the B firms enters at τ, it follows from Claim 3 that if $\tau \in [T_B^{BB}, \min\{T_A^*(2), T_B^{AB}\})$, then the entry order is $B-A-B$, entry times are $(\tau, \min\{T_A^*(2), T_B^{AB}\}, T_B^*(3))$ and payoffs are $L_B(\tau)$, $L_A^{AB}(\min\{T_A^*(2), T_B^{AB}\})$ and $F_B^{AB}(\tau)$ for the first, second and third entrant, respectively.

If instead $\tau \in [\min\{T_A^*(2), T_B^{AB}\}, T_B^*(3)]$, then:

- in case 1, entry order is $B-A-B$, entry times are $(\tau, \tau, T_B^*(3))$ and payoffs are $L_B^{BB}(\tau)$, $L_A^{AB}(\tau)$ and $F_B^{AB}(\tau)$ for the first, second and third entrant, respectively.

- in cases 2 and 3, for $\tau \notin \left[\underline{T}_B^{2,B}, \overline{T}_B^{2,B}\right]$, entry order, entry times and payoffs are those described for case 1, while for $\tau \in \left[\underline{T}_B^{2,B}, \overline{T}_B^{2,B}\right]$ either entry order, entry times and payoffs are those described for case 1, or entry order is $B-B-A$, entry times are $(\tau, \tau, T_A^*(3))$ and the payoffs are $L_B^{AB}(\tau)$ for the first two entrants and $F_A^{AB}(\tau)$ for A.

The following Claim holds:

Claim 5 *If $T_B^1 < T_A^1$, in any SPNE of the game the outcome of subgames with three active firms starting at $\tau \in [T_B^{BB}, T_B^*(3))$ is as follows:*
(i) If $\tau \in [T_B^{BB}, \min\{T_A^*(2), T_B^{AB}\})$, one of the B firms enters at $t_1 = \tau$, the A firm enters at $t_2 = \min\{T_A^*(2), T_B^{AB}\}$ and the remaining B firm enters at $t_3 = T_B^*(3)$;
(ii) If $\tau \in [\min\{T_B^{AB}, T_A^*(2\}, T_B^*(3))$:
(iia) for any τ in the interval in case 1, and for any τ in the interval such that $\tau \notin \left[\underline{T}_B^{2,B}, \overline{T}_B^{2,B}\right]$ in cases 2 and 3, the unique outcome is that firm A and one of the B firms enter at $t_1 = t_2 = \tau$ and the remaining B firm enters at $t_3 = T_B^*(3)$;
(iib) moreover, in cases 2 and 3, for $\tau \in \left[\underline{T}_B^{2,B}, \overline{T}_B^{2,B}\right]$ the outcome is either that firm A and one of the B firms enter at $t_1 = t_2 = \tau$ and the remaining B firm enters at $t_3 = T_B^*(3)$, or that both B firms enter at $t_1 = t_2 = \tau$ and the A firm enters at $t_3 = T_A^*(3)$.

Proof. For simplicity, we develop the proof of this Claim under the following assumption: Suppose that at any time t, if a firm is indifferent between being the m-th investor at t and the $(m+1)$-th investor, then it invests at t. It is immediate to verify that even without this assumption the result still holds.

We divide subgames with three active firms starting at $\tau \in \left[T_B^{BB}, T_B^*(3)\right)$ into four classes, and derive the equilibrium outcome for each class.

- First, consider subgames with three active firms starting at $\tau \in [T_A^*(2), T_B^*(3))$ for case 1, or $\tau \in \left(\overline{T}_B^{2,B}, T_B^*(3)\right)$ for case 2. We prove that in equilibrium, at τ, it has to be the case that both B firms play Enter and A plays either Enter or Wait. Assumption 4 and Claims 2 and 3 then guarantee the result.

This is an equilibrium because if firms play either of these action profiles, payoffs are $L_A^{AB}(\tau)$ for the A firm, and a lottery between $L_B^{BB}(\tau)$ and $F_B^{BB}(\tau)$ for the B firms. By Claim 2, in this interval $L_B^{BB}(\tau) > F_B^{BB}(\tau)$ so no B firm has an incentive to deviate and receive $F_B^{BB}(\tau)$ with probability one. By the same argument, there cannot be an equilibrium in which at τ only one of the type B firms plays Enter, regardless of A's action, because the other one would rather deviate and play Enter. Consider now firm A. Given that both B firms play Enter, A's action does not affect its payoff, so A has no profitable deviation from either profile described above.

Next, we prove that there are no other action profiles at τ compatible with equilibrium. There cannot be an equilibrium in which only firm A plays Enter at τ, because each of the B firms would receive a lottery between $L_B^{BB}(\tau)$ and $F_B^{BB}(\tau)$ and would rather deviate and play Enter at τ as well, thus receiving a similar lottery but with higher probability to obtain $L_B^2(\tau)$. Moreover, there cannot be an equilibrium in which all three firms play Wait at τ. In such an equilibrium, the first entry would happen at some time t later than τ. By Lemma 1 first entry would happen at some later $t \in (\tau, T_B^*(3)]$. From the arguments presented so far in the proof of part (ii) of this Claim, it could only be the case that the A firm and one of the B firms enter simultaneously at t and the remaining B firm follows at $T_B^*(3)$. Since the function $L_A^{AB}(\tau)$ is strictly quasiconcave and maximized at $T_A^*(2) \leq \tau$, A would then have an incentive to deviate and preempt the rivals playing Enter at $(t-\varepsilon)$. So, in case 1, for $\tau \in [T_A^*(2), T_B^*(3))$ there cannot be an equilibrium in which all three firms play Wait at τ. Hence, we can conclude that for any τ in this interval the unique equilibrium outcome is the one described in part (iia) of the Claim.

- Second, consider subgames with three active firms starting at $\tau \in \left[\underline{T}_B^{2,B}, \overline{T}_B^{2,B}\right]$ We prove that in equilibrium all firms play Enter at τ. Assumption 4, together with Claims 2 and 3 then guarantee the result.

This is an equilibrium because if firms play the above profile, A receives $L_A^{AB}(\tau)$ with probability $\frac{2}{3}$ and $F_A^{AB}(\tau)$ with probability $\frac{1}{3}$. By the proof of Claim 3, $D_A^{AB}(\tau) = L_A^{AB}(\tau) - F_A^{AB}(\tau)$ is weakly positive in $[T_A^*(2), T_A^*(3)]$, hence in the interval we are considering. It follows that A has no incentive to deviate because it would then receive $L_A^{AB}(\tau)$ or $F_A^{AB}(\tau)$ with probabilities $\frac{1}{2}, \frac{1}{2}$. (By an analogous argument, there cannot be an equilibrium in which at τ A plays Wait and either one or both B firms play Enter). As for the B firms, if the above profile is played, each B firm receives $L_B^{AB}(\tau)$, $L_B^{BB}(\tau)$ and $F_B^{BB}(\tau)$ with probabilities $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$ while by deviating it would receive a similar lottery with probabilities $\left(\frac{1}{4}, \frac{1}{4}, \frac{1}{2}\right)$. Since in this interval $D_B^{AB}(\tau) = L_B^{AB}(\tau) - F_B^{AB}(\tau) > 0$ and $L_B^{BB}(\tau) > F_B^{BB}(\tau)$, the deviation is not profitable. (By an analogous argument, there cannot be an equilibrium in which at τ A plays Enter, and one or both of the B firms play Wait).

Finally, there cannot be an equilibrium in which all three firms play Wait at τ. In such an equilibrium, by the argument presented above, the first entry would take place at some later time $t \leq \overline{T}_B^{2,B}$. If in t only one or two firms plays Enter, any firm who plays Wait has an incentive to deviate and play Enter at $(t - \varepsilon)$. Similarly, if in t all three firms play Enter, each of them has an incentive to deviate and play Enter at $(t - \varepsilon)$. Hence, we can conclude that for any τ in this interval the unique equilibrium outcome is the one described in part (iib) of the Claim.

- Third, for case 2, consider subgames with three active firms starting at $\tau \in \left[T_A^*(2), \underline{T}_B^{2,B}\right]$. Given part (iib) of this Claim, the equilibrium outcome of any such subgame must be that first entry happens weakly before $\underline{T}_B^{2,B}$. Then, the same arguments presented in the first part of this proof guarantee that in equilibrium, at τ both B firms play Enter and A plays either Enter or Wait, which in turn guarantees that for any τ in this interval the unique equilibrium outcome is the one described in part (iia) of the Claim.

- Finally, consider subgames with three active firms starting at $\tau \in \left[T_B^{BB}, \min\left\{T_A^*(2), T_B^{AB}\right\}\right)$. We prove that in equilibrium, at τ, it has to be the case that the B firms play Enter and the A firm plays Wait. Then, Assumption 4, together with Claim 3, guarantees the result.

This is an equilibrium because if firms play the prescribed actions, A's payoff is

$$\pi_A(2)\int_{\min\left\{T_A^*(2),T_B^{AB}\right\}}^{T_B^*(3)} e^{-rs}ds+\pi_A(3)\int_{T_B^*(3)}^{\infty} e^{-rs}ds-c(\min\left\{T_A^*(2),T_B^{AB}\right\}) \tag{3}$$

and the B firms receive a lottery between

$$\pi_B(1)\int_{\tau}^{\min\left\{T_A^*(2),T_B^{AB}\right\}} e^{-rs}ds+\pi_B(2,A)\int_{\min\left\{T_A^*(2),T_B^{AB}\right\}}^{T_B^*(3)} e^{-rs}ds+\pi_B(3)\int_{T_B^*(3)}^{\infty} e^{-rs}ds-c(\tau). \tag{4}$$

and $F_B^{BB}(\tau)=F_B^{AB}(\tau)$ with probabilities $\left(\frac{1}{2},\frac{1}{2}\right)$ By Assumption 1, expression (4) is larger than $L_B^{BB}(\tau)$ which is turn larger that $F_B^{BB}(\tau)$. Therefore, no B firm has an incentive to deviate and receive $F_B^{BB}(\tau)$ with probability 1. By the same argument, there cannot be an equilibrium in which at τ only one B firm plays Enter.

As for firm A, by deviating it would receive a lottery between expression (3) and $L_A^{AB}(\tau)$. It is easy to verify that this deviation is not profitable, using the fact that the function

$$\pi_A(2)\int_{t}^{T_B^*(3)} e^{-rs}ds+\pi_A(3)\int_{T_B^*(3)}^{+\infty} e^{-rs}ds-c(t)$$

is strictly quasiconcave and maximized at $T_A^*(2)>\tau$. By an identical argument, a strategy profile in which the A firm and one or two B firms play Enter at τ, cannot be an equilibrium, since A would want to deviate and play Wait.

Finally, we prove that a profile in which all three firms play Wait at τ cannot be part of an equilibrium. By part (ii), first entry would then take place at some later time $t\leq \min\left\{T_A^*(2),T_B^{AB}\right\}$. In t, it holds that

$$\pi_B(1)\int_{t}^{\min\left\{T_A^*(2),T_B^{AB}\right\}} e^{-rs}ds+\pi_B(2,A)\int_{\min\left\{T_A^*(2),T_B^{AB}\right\}}^{T_B^*(3)} e^{-rs}ds$$

$$+\pi_B(3)\int_{T_B^*(3)}^{\infty} e^{-rs}ds-c(t)> L_B^{BB}(t)>F_B^{BB}(t).$$

Suppose at time t only the two B firms play Enter. By continuity, regardless of what A plays at t, each of the B firms has a strict incentive to preempt the rival and enter at time $(t-\varepsilon)$. Similarly, if at time t only one of the B firm plays Enter, then the other B firm has an incentive to preempt and enter at time $(t-\varepsilon)$. Finally, if at time t only the A firm plays Enter, then each B firm has an incentive to preempt and enter at time $(t-\varepsilon)$. Therefore, there cannot be an equilibrium of the subgame starting at τ in which the first entry happens later than τ. ■

The next Claim analyzes subgames with three active firms starting at $\tau \in [0, T_B^{BB})$. Consider again the functions $D_A(t)$ and $D_B(t)$. Evaluated at T_B^{BB}, $D_B(\cdot)$ is positive, because

$$D_B(T_B^{BB}) = [\pi_B(1) - \pi_B(2, A)] \int_{T_B^{BB}}^{\min\left\{T_A^*(2), T_B^{AB}\right\}} e^{-rs} ds > 0$$

by Assumption (1). It follows that there exists one and only one point $T_B^1 \in \left(0, T_B^{BB}\right)$ such that $D_B(T_B^1) = 0$. Conversely, $D_A(t)$ evaluated at T_B^{BB} is equal to

$$D_A(T_B^{BB}) = \pi_A(2) \int_{T_B^{BB}}^{\min\left\{T_A^*(2), T_B^{AB}\right\}} e^{-rs} ds - c(T_B^{BB}) + c\left(\min\left\{T_A^*(2), T_B^{AB}\right\}\right) < 0$$

which is negative because the function

$$\pi_A(2) \int_t^{\min\left\{T_A^*(2), T_B^{AB}\right\}} e^{-rs} ds - c(t)$$

is strictly quasiconcave, maximized at $T_A^*(2)$, hence strictly increasing for $t \in \left[T_B^{BB}, \min\left\{T_A^*(2), T_B^{AB}\right\}\right]$. It follows that two cases are possible:

Case a $D_A(t) < 0 \; \forall t \in \left[0, T_B^{BB}\right]$, and $T_A^1 = +\infty$;

Case b There exist two points, $\underline{T}_A^1$ and $\overline{T}_A^1$, with $0 < \underline{T}_A^1 \leq \overline{T}_A^1 < T_B^{BB}$, in which $D_A(t)$ is null, and $T_A^1 = \underline{T}_A^1$.

Given the assumption $T_B^1 \leq T_A^1$, the following Claim holds:

Claim 6 *If $T_B^1 < T_A^1$, in any SPNE of the game the outcome of subgames with three active firms starting at $\tau \in [0, T_B^{BB})$ is as follows:*

(i) If $\tau \leq T_B^1$ one of the B firms enters at $t_1 = T_B^1$, the A firm enters at $t_2 = \min\left\{T_A^*(2), T_B^{AB}\right\}$ and the remaining B firm enters at $t_3 = T_B^*(3)$

(ii) If $\tau \in (T_B^1, T_B^{BB})$:

(iia) for any τ in the interval in case a, and for any τ in the interval such that $\tau \notin \left[\underline{T}_A^1, \overline{T}_A^1\right]$ in case b, the unique outcome is that one of the B firms enters at $t_1 = \tau$, the A firm enters at $t_2 = \min\left\{T_A^*(2), T_B^{AB}\right\}$ and the remaining B firm enters at $t_3 = T_B^*(3)$

(iib) moreover, in case b, for $\tau \in \left[\underline{T}_A^1, \overline{T}_A^1\right]$ the outcome is either as in (iia), or that firm A enters at $t_1 = \tau$ and the B firms enter at $t_2 = T_B^{BB}$ and $t_3 = T_B^*(3)$ respectively.

Proof. We divide subgames with three active firms starting at $\tau \in [0, T_B^{BB})$ into four classes, and derive the equilibrium outcome for each class.

- First, consider subgames with three active firms starting at $\tau \in [T_B^1, T_B^{BB})$ for case a, or $\tau \in (\overline{T}_A^1, T_B^{BB})$ for case b. We prove that in equilibrium, at τ, it has to be the case that A plays Wait, and the B firms play Enter.

This is an equilibrium because if firms play the above profile, A receives $F_A(\tau)$ and each B firm a lottery between $L_B(\tau)$ and $F_B(\tau)$. By deviating, A would receive $L_A(\tau)$ with positive probability and a B firm would receive $F_B(\tau)$. Then, the fact that $D_A(\tau) < 0$ and $D_B(\tau) > 0$ guarantees that no firm has an incentive to deviate. There cannot be an equilibrium in which firm A and one of the B firms play Wait, and the other B firm plays Enter, because the B firm which plays Wait would rather deviate and play Enter, thus exchanging $F_B(\tau)$ for a lottery between $L_B(\tau)$ and $F_B(\tau)$. There cannot be an equilibrium in which the A firm and at least one of the B firms play Enter, because the A firm would then receive a lottery between $L_A(\tau)$ and $F_A(\tau)$ and would rather deviate and receive $F_A(\tau)$. There cannot be an equilibrium in which the A firm plays Enter and both B firms play Wait, because both B firms would receive $F_B(\tau)$ and would rather deviate and receive a lottery between $L_B(\tau)$ and $F_B(\tau)$. Finally, there cannot be an equilibrium in which all three firms play Wait at τ. In such an equilibrium, by Claim 5 first entry would take place at some later time $t \leq T_B^{BB}$. But this cannot be part of an equilibrium, because at t one of the following action profiles would have to be played:

- A plays Enter and either one or both B firms play Enter: then the A firm would rather deviate and play Wait.~

- A plays Enter and both B firms play Wait: then each B firm would rather deviate and play Enter

- A plays Wait and either one or both B firms play Enter: then each B firm would rather deviate and play Enter at $(t - \varepsilon)$.

Hence, we can conclude that for any τ in this interval the unique equilibrium outcome is the one described in part (iia) of the Claim.

- Second for case b, consider any subgame with three active firms starting at $\tau \in \left[\underline{T}_A^1, \overline{T}_A^1\right]$. We prove that in equilibrium all firms play Enter at τ.

This is an equilibrium because if firms play the above profile, each firm i receives a lottery between $L_i(\tau)$ and $F_i(\tau)$, and the fact that in this interval $D_A(\tau) > 0$ and $D_B(\tau) > 0$ guarantees that there are no profitable deviations. Similarly, this fact guarantees that there cannot be an equilibrium in which either one or two firms only play Enter at τ, because in that case there is at least one firm which plays Wait and has an incentive to deviate and play Enter. Finally, there cannot be an equilibrium in which all three firms play Wait at τ. In such an equilibrium, by the argument presented above first entry would happen at some later time $t \leq \overline{T}_A^1$. If at t only one or two firms plays Enter, any firm who plays Wait has an incentive to deviate and play Enter at $(t - \varepsilon)$. Similarly, if in t all three firms play Enter, each of them has an incentive to deviate and play Enter at $(t - \varepsilon)$. Hence, we can conclude that for any τ in this interval the unique equilibrium outcome is the one described in part (iiib) of the Claim.

- Third, for case b, consider any subgame with three active firms starting at $\tau \in \left[T_B^1, \underline{T}_A^1\right)$. Given part (iib), the equilibrium outcome of any such subgame must be that first entry happens weakly before $\underline{T}_A^1$, then the same arguments presented in the first part of this proof guarantee that in equilibrium, at τ, the A firm plays Wait and the B firms play Enter, which in turn guarantees that for any τ in this interval the unique equilibrium outcome is the one described in part (iiia) of the Claim.

- Finally, consider subgames with three active firms starting at $\tau \leq T_B^1$. We prove that in equilibrium all firms play Wait for any $t \in [\tau, T_B^1)$.

This is an equilibrium because if they do so, the outcome is the one described in part (i) of the Claim and firm i receives payoff $F_i(\tau)$. (Notice that each B firm receives a lottery between $L_B(T_B^1) = F_B(T_B^1)$, and by definition of T_B^1, and $F_B(\tau) = F_B(T_B^1)$). The fact that in this interval $D_A(\tau) < 0$ and $D_B(\tau) < 0$ guarantees that there are no profitable deviations. By the same argument, there cannot be an equilibrium in which any number of firms plays Enter at τ, because then there would be at least one firm receiving $L_i(\tau)$ with positive probability, and this firm would rather deviate and receive $F_i(\tau)$ with probability one. Hence, we can conclude that for any τ in this interval the unique equilibrium outcome is the one described in part (i) of the Claim. ■

The following Claim, which is an immediate implication of Claim 6, concludes the proof of Proposition 1

Claim 7 *The unique SPNE outcome of the game is that one of the B firms enters at* $t_1 = T_B^1$*, the A firm enters at* $t_2 = \min\left\{T_A^*(2), T_B^{AB}\right\}$ *and the remaining B firm enters at* $t_3 = T_B^*(3)$*.*■

Proof of Proposition 2

Part (a).

The proof consists of 3 steps. Step 1 proves that in the limit case with symmetric duopoly profits and asymmetric triopoly profits, $T_B^{BB} < \min\left\{T_A^*(2), T_B^{AB}\right\}$. Notice that symmetric duopoly profits violate assumption 1(ii). Step 2 proves that by continuity there exist sets of duopoly and triopoly profits satisfying assumption 1(ii) such that $T_B^{BB} < \min\left\{T_A^*(2), T_B^{AB}\right\}$. Step 3 proves that for any set of duopoly and triopoly profits such that $T_B^{BB} < \min\left\{T_A^*(2), T_B^{AB}\right\}$ there also exist monopoly profits satisfying assumption 1(ii) such that $T_B^1 < T_A^1$. Given these three steps, we can conclude that there exist profit structures satisfying assumption 1(ii) and such that $T_B^1 < T_A^1$. Proposition 1 then implies that for these profit structures the equilibrium entry order is $B - A - B$.

Step 1. For any vector of duopoly and triopoly profits $(\pi_A(2), \pi_B(2,B), \pi_B(2,A), \pi_A(3), \pi_B(3))$ such that: $\pi_A(2) = \pi_B(2,B) = \pi_B(2,A)$, $\pi_A(3) > \pi_B(3)$, $\pi_A(2) > \pi_A(3)$ and $\pi_B(2,B) = \pi_B(2,A) > \pi_B(3)$, it holds that $T_B^{BB} < \min\left\{T_A^*(2), T_B^{AB}\right\}$. Notice that these profits violate assumption 1(ii). We prove that $T_B^{BB} < \min\left\{T_A^*(2), T_B^{AB}\right\}$ starting from the observation that for any such vector $D_B^{BB}(t) > D_B^{AB}(t)$, hence $T_B^{BB} < T_B^{AB}$. Moreover, it follows from Fudenberg and Tirole (1985) that $T_B^{BB} < T_B^*(2,A)$. Since $\pi_A(2) = \pi_B(2,A)$ implies $T_A^*(2) = T_B^*(2,A)$, it follows that $T_B^{BB} < T_A^*(2)$. Therefore, it holds that $T_B^{BB} < \min\left\{T_A^*(2), T_B^{AB}\right\}$.

Step 2. There exist profits $(\pi_A(2), \pi_B(2,B), \pi_B(2,A), \pi_A(3), \pi_B(3))$ that satisfy: $\pi_A(2) > \pi_B(2,B) > \pi_B(2,A)$, $\pi_A(3) > \pi_B(3)$, $\pi_A(2) > \pi_A(3)$ and $\pi_B(2,B) > \pi_B(2,A) > \pi_B(3)$ and are such that $T_B^{BB} < \min\left\{T_A^*(2), T_B^{AB}\right\}$. We prove this starting from the observation that T_B^{BB} is a continuous function of $\pi_B(2,A)$ and $T_A^*(2)$ is continuous in $\pi_A(2)$. Moreover, if T_B^{AB} is finite, it is continuous in $\pi_B(2,B)$, otherwise, it is unaffected by sufficiently small changes in $\pi_B(2,B)$. The result then holds by continuity.

Step 3. Given any set of duopoly and triopoly profits such that $T_B^{BB} < \min\left\{T_A^*(2), T_B^{AB}\right\}$, there also exist monopoly profits satisfying assumption 1, and in particular such that $\pi_A(1) > \pi_B(1)$, $\pi_A(1) > \pi_A(2)$ and $\pi_B(1) > \pi_B(2,B)$, for which $T_B^1 < T_A^1$. To prove this, consider a profit structure that satisfies assumption 1 and for which $T_B^{BB} < \min\left\{T_A^*(2), T_B^{AB}\right\}$. Existence of such a $\boldsymbol{\pi}$ is guaranteed by step 2. Observe that $D_B(t)$ has at least one intersection with zero because it is negative at $t = 0$ and it is positive at T_B^{BB}. Next, consider the difference between the functions $D_B(t)$ and $D_A(t)$. We show that for $\pi_B(1) - \pi_A(1)$ positive but sufficiently small, it is positive.

$$
\begin{aligned}
&D_B(t) - D_A(t) = \\
&[\pi_B(1) - \pi_A(1)] \int_t^{T_B^{BB}} e^{-rs} ds + [\pi_B(1) - \pi_B(2, A)] \int_{T_B^{BB}}^{\min\left\{T_A^*(2), T_B^{AB}\right\}} e^{-rs} ds \\
&-\pi_A(2) \int_{T_B^{BB}}^{\min\left\{T_A^*(2), T_B^{AB}\right\}} e^{-rs} ds + \left[c\left(T_B^{BB}\right) - c\left(\min\left\{T_A^*(2), T_B^{AB}\right\}\right)\right].
\end{aligned}
$$

The first term is negative, and vanishes for $\pi_A(1) - \pi_B(1)$ positive but sufficiently small. The second term is strictly positive. The expression in the third line is strictly positive by definition of $T_A^*(2)$. To see this, notice that

$$
\pi_A(2) \int_{T_B^{BB}}^{\min\left\{T_A^*(2), T_B^{AB}\right\}} e^{-rs} ds - \left[c\left(T_B^{BB}\right) - c\left(\min\left\{T_A^*(2), T_B^{AB}\right\}\right)\right]
$$

$$
= \pi_A(2) \int_{T_B^{BB}}^{+\infty} e^{-rs} ds - c\left(T_B^{BB}\right) - \left[\pi_A(2) \int_{T_B^{BB}}^{+\infty} e^{-rs} ds - c\left(\min\left\{T_A^*(2), T_B^{AB}\right\}\right)\right]
$$

and the function $\pi_A(2) \int_t^{+\infty} e^{-rs} ds - c(t))$ is strictly increasing for $t < T_A^*(2)$, hence also in the interval from T_B^{BB} to $\min\left\{T_A^*(2), T_B^{AB}\right\}$. Therefore, we can conclude that for $\pi_A(1) - \pi_B(1)$ positive but sufficiently small, it holds that $D_B(t) - D_A(t) > 0$, hence $T_B^1 < T_A^1$.

Part (b).

Let $\widehat{\boldsymbol{\pi}}$ be a profit structure that satisfies the model assumptions and induces $B - A - B$ as the unique equilibrium entry order. Denote by $\widehat{T}_B^1$ and $\widehat{T}_A^1$ the associated values of T_B^1 and T_A^1. The proof consists of two steps. Step 1 proves that $\widehat{T}_B^1 < \widehat{T}_A^1$. Step 2 proves that for any profit structure satisfying inequalities *(i)* to *(vi)*, it holds that $T_B^1 < \widehat{T}_B^1 < \widehat{T}_A^1 \leq T_A^1$, and hence by Proposition 1 the equilibrium entry order $B - A - B$ is preserved.

Step 1. It has to be the case that $\widehat{T}_B^1 < \widehat{T}_A^1$ because for any profit structure satisfying the model assumptions, $T_B^1 < T_A^1$ is a necessary condition to guarantee that the unique equilibrium entry order is $B - A - B$. We prove the latter statement by contradiction. If $T_A^1 < T_B^1$, a type B firm cannot be the first entrant in equilibrium. If that were the case, it would have to be true that $D_B(t_1) \geq 0$. But this implies that A could profitably deviate by entering at $t_1 - \varepsilon$. If $T_A^1 = T_B^1 < \infty$, it is an immediate extension of the proof of Proposition 1 that there are two possible equilibrium outcomes, one with entry order $B - A - B$ and entry times $t_1 = T_B^1$, $t_2 =$min $\{T_A^*(2), T_B^{AB}\}$, $t_3 = T_B^*(3)$, and one with entry order $A - B - B$ and entry times $t_1 = T_A^1$, $t_2 = T_B^{BB}$, $t_3 = T_B^*(3)$. Finally, the case $T_A^1 = T_B^1 = \infty$ cannot occur for any profit structure. If $\min\{T_A^*(2), T_B^{AB}\} \leq T_B^{BB}$, then $D_A\left(\min\{T_A^*(2), T_B^{AB}\}\right)$ is strictly positive, hence T_A^1 is finite. If instead $\min\{T_A^*(2), T_B^{AB}\} > T_B^{BB}$, then $D_B\left(T_B^{BB}\right) > 0$ and T_B^1 is finite.

Step 2. Step 1 proves that $\widehat{T}_B^1 < \widehat{T}_A^1$. Consider a profit structure $\boldsymbol{\pi}$ that satisfies the model assumptions as well as inequalities *(i)* to *(vi)*. We now prove that the values of T_B^1 and T_A^1 associated to this profit structure satisfy $T_B^1 \leq \widehat{T}_B^1$ and $T_A^1 \geq \widehat{T}_A^1$, hence $T_B^1 < T_A^1$ and by Proposition 1 the equilibrium entry order $B - A - B$ is preserved. We first prove that profit inequalities *(i)* to *(vi)* imply $T_B^1 \leq \widehat{T}_B^1$. Then, we prove that they imply $T_A^1 \geq \widehat{T}_A^1$. The roman numbers *(i)* to *(vi)* below refer to the conditions of the proposition.

For any finite T_B^1, the sign of $\frac{dT_B^1}{d\pi_i(m,-i)}$ is given by the implicit function theorem:

$$\frac{dT_B^1}{d\pi_i(m,-i)} = -\frac{dD_B(\cdot)/d\pi_i(m,-i)}{dD_B(\cdot)/dt} = -\frac{dD_B(\cdot)/d\pi_i(m,-i)}{-\pi_B(1)\, e^{-rT_B^1} - c'\left(T_B^1\right)}. \tag{5}$$

The denominator is positive because $T_B^1 < T^*_B(1)$. For the numerator, observe that $\frac{dD_B(\cdot)}{d\pi_A(1)} = 0$ and $\frac{dD_B(\cdot)}{d\pi_B(1)} = \int_{T_B^1}^{\min\{T_A^*(2),T_B^{AB}\}} e^{-rs}ds \geq 0$. Hence:

(**i**) expression (5) is null for $\pi_i(m,-i) = \pi_A(1)$ and

(**iv**) nonpositive for $\pi_i(m,-i) = \pi_B(1)$. Moreover:

$$\frac{dD_B(\cdot)}{d\pi_i(m,-i)} = \frac{\partial D_B(\cdot)}{\partial \pi_i(m,-i)} + \frac{\partial D_B(\cdot)}{\partial \min\{T_A^*(2), T_B^{AB}\}} \frac{\partial \min\{T_A^*(2), T_B^{AB}\}}{\partial \pi_i(m,-i)} + \frac{\partial D_B(\cdot)}{\partial T_B^{BB}} \frac{\partial T_B^{BB}}{\partial \pi_i(m,-i)}. \tag{6}$$

Notice that $\frac{\partial D_B(\cdot)}{\partial \min\{T_A^*(2),T_B^{AB}\}} = [\pi_B(1) - \pi_B(2,A)]\, e^{-r\min\{T_A^*(2),T_B^{AB}\}} > 0$ by assumption 1 and $\frac{\partial D_B(\cdot)}{\partial T_B^{BB}} = \pi_B(2,A)\, e^{-rT_B^{BB}} + c'\left(T_B^{BB}\right) < 0$ because $T_B^{BB} < T_B^*(2,A)$.

(**ii**) For $\pi_i(m,-i) = \pi_B(2,A)$, expression (5) is negative because expression (6) is equal to: $\int_{\min\{T_A^*(2),T_B^{AB}\}}^{T_B^{BB}} e^{-rs}ds + \left[\pi_B(2,A)\, e^{-rT_B^{BB}} + c'\left(T_B^{BB}\right)\right] \frac{\int_{T_B^{BB}}^{T_B^*(3)} e^{-rs}ds}{\pi_B(2,A)e^{-rT_B^{BB}} + c'\left(T_B^{BB}\right)}$ which can be simplified as $\int_{\min\{T_A^*(2),T_B^{AB}\}}^{T_B^*(3)} e^{-rs}ds > 0$.

(**iii**) For $\pi_i(m,-i) = \pi_B(3)$, expression (5) is nonnegative because expression (6) is nonpositive. In particular, the first term in expression (6) null. The second term is nonpositive because $\frac{\partial T_A^*(2)}{\partial \pi_B(3)}$ is null and $\frac{\partial T_B^{AB}}{\partial \pi_B(3)} = -\frac{\left[e^{-rT_A^*(3)} - e^{-rT_B^*(3)}\right]/r + \left[\pi_B(3) \cdot e^{-rT_B^*(3)} + c'(T_B^*(3)\right] \cdot \frac{\partial T_B^*(3)}{\partial \pi_B(3)}}{-\pi_B(2,B)e^{-rT_B^{AB}} - c'\left(T_B^{AB}\right)} < 0$.

The inequality holds because the first term in the numerator is positive, as $T_A^*(3) < T_B^*(3)$, and the second term is null by definition of $T_B^*(3)$.

Finally, the last term in expression (6) is negative because $\frac{\partial T_B^{BB}}{\partial \pi_B(3)} = \frac{\partial T_B^{BB}}{\partial T_B^*(3)} \cdot \frac{\partial T_B^*(3)}{\partial \pi_B(3)} = -\frac{\pi_B(2,A)e^{-rT_B^*(3)}+c'\left(T_B^*(3)\right)}{-\pi_B(2,A)e^{-rT_B^{BB}}-c'\left(T_B^{BB}\right)} \cdot \frac{\partial T_B^*(3)}{\partial \pi_B(3)} > 0$ where the inequality holds because $\frac{\partial T_B^*(3)}{\partial \pi_B(3)} < 0$, and both the numerator and the denominator of $\frac{\partial T_B^{BB}}{\partial T_B^*(3)}$ are positive because $T_B^*(3) > T_B^*(2, A)$ and $T_B^{BB} < T_B^*(2, A)$.

(**v**) For $\pi_i(m, -i) = \pi_A(2)$, expression (5) is nonnegative because expression (6) is nonpositive. In particular, the first term in expression (6) null. The second term is nonpositive because $\frac{\partial T_A^*(2)}{\partial \pi_A(2)} < 0$ and $\frac{\partial T_B^{AB}}{\partial \pi_A(2)}$ is null. Finally, the last term is null because $\frac{\partial T_B^{BB}}{\partial \pi_A(2)} = 0$, since $\frac{\partial D_B^{BB}(\cdot)}{\partial \pi_A(2)} = 0$.

(**vi**) For $\pi_i(m, -i) = \pi_B(2, B)$, expression (5) is nonnegative because expression (6) is nonpositive. In particular, the first term and the last term in expression (6) are null. The second term is nonpositive because $\frac{\partial T_A^*(2)}{\partial \pi_B(2,B)}$ is null and $\frac{\partial T_B^{AB}}{\partial \pi_B(2,B)} = -\frac{\left[e^{-rT_B^{AB}}-e^{-rT_A^*(3)}\right]/r}{-\pi_B(2,B)e^{-rT_B^{AB}}-c'\left(T_B^{AB}\right)} < 0$ because both the numerator and the denominator are positive, as $T_B^{AB} \leq T_A^*(3)$ and $T_B^{AB} < T_B^*(2, B)$.

We conclude that since $\widehat{T}_B^1$ is finite and $\frac{dT_B^1}{d\pi_i(m,-i)}$ is null for $\pi_i(m, -i) = \pi_A(1)$, nonpositive for $\pi_i(m, -i) \in \{\pi_B(1), \pi_B(2, A)\}$ and nonnegative for $\pi_i(m, -i) \in \{\pi_B(3), \pi_A(2), \pi_B(2, B)\}$, then for any profit structure that satisfies inequalities *(i)* to *(vi)* it holds that $T_B^1 \leq \widehat{T}_B^1$.

Next, consider T_A^1. If it is finite, we can find the sign of $\frac{dT_A^1}{d\pi_i(m,-i)}$ using the implicit function theorem:

$$\frac{dT_A^1}{d\pi_i(m, -i)} = -\frac{dD_A(\cdot)/d\pi_i(m, -i)}{dD_A(\cdot)/dt} = -\frac{dD_A(\cdot)/d\pi_i(m, -i)}{-\pi_A(1)e^{-rT_A^1} - c'\left(T_A^1\right)}. \tag{7}$$

The denominator of expression (7) is positive, because $T_A^1 < T_A^*(1)$. For the numerator, observe that $\frac{dD_A(\cdot)}{d\pi_B(1)} = 0$ and $\frac{dD_A(\cdot)}{d\pi_A(1)} = \int_{T_A^1}^{T_B^{BB}} e^{-rs}ds \geq 0$. Hence:

(**i**) expression (7) is nonpositive for $\pi_i(m, -i) = \pi_A(1)$ and

(**iv**) null for $\pi_i(m, -i) = \pi_B(1)$. Moreover:

$$\frac{dD_A(\cdot)}{d\pi_i(m, -i)} = \frac{\partial D_A(\cdot)}{\partial \pi_i(m, -i)} + \frac{\partial D_A(\cdot)}{\partial \min\left\{T_A^*(2), T_B^{AB}\right\}} \frac{\partial \min\left\{T_A^*(2), T_B^{AB}\right\}}{\partial \pi_i(m, -i)} + \frac{\partial D_A(\cdot)}{\partial T_B^{BB}} \frac{\partial T_B^{BB}}{\partial \pi_i(m, -i)}. \tag{8}$$

Notice that $\frac{\partial D_A}{\partial \min\left\{T_A^*(2), T_B^{AB}\right\}} = \pi_A(2)\, e^{-r\min\left\{T_A^*(2),T_B^{AB}\right\}} + c'\left(\min\left\{T_A^*(2), T_B^{AB}\right\}\right) \leq 0$ because $\min\left\{T_A^*(2), T_B^{AB}\right\} \leq T_A^*(2)$, and $\frac{\partial D_A(\cdot)}{\partial T_B^{BB}} = \left[\pi_A(1) - \pi_A(2)\right] e^{-rT_B^{BB}} > 0$.

(**ii**) For $\pi_i(m,-i) = \pi_B(2,A)$, expression (7) is positive because expression (8) is negative. In particular, the first term in expression (8) is null. The second is also null because $\frac{\partial \min\left\{T_A^*(2), T_B^{AB}\right\}}{\partial \pi_B(2,A)}$ is null. The third term in (8) is negative because $\frac{\partial T_B^{BB}}{\partial \pi_B(2,A)} < 0$ as shown above.

(**iii**) For $\pi_i(m,-i) = \pi_B(3)$, expression (7) is nonpositive because expression (8) is nonnegative. In particular, the first term expression (8) is null. The second term is nonnegative because $\frac{\partial T_A^*(2)}{\partial \pi_B(3)}$ is null and $\frac{\partial T_B^{AB}}{\partial \pi_B(3)} = -\frac{\left[e^{-rT_A^*(3)} - e^{-rT_B^*(3)}\right]/r + \left[\pi_B(3)\cdot e^{-rT_B^*(3)} + c'(T_B^*(3))\right]\cdot \frac{\partial T_B^*(3)}{\partial \pi_B(3)}}{-\pi_B(2,B)e^{-rT_B^{AB}} - c'\left(T_B^{AB}\right)} < 0$. The inequality holds because the first term in the numerator is positive, as $T_A^*(3) < T_B^*(3)$, and the second term is null by definition of $T_B^*(3)$. Finally, the last term of expression (8) is nonnegative because $\frac{\partial T_B^{BB}}{\partial \pi_B(3)} > 0$ as shown above.

(**v**) For $\pi_i(m,-i) = \pi_A(2)$, expression (7) is nonpositive because expression (8) is nonnegative. In particular, the first term in (8) is $\frac{\partial D_A(\cdot)}{\partial \pi_A(2)} = \int_{T_B^{BB}}^{\min\left\{T_A^*(2),T_B^{AB}\right\}} e^{-rs} ds > 0$.

The second term is nonnegative because $\frac{\partial D_A}{\partial \min\left\{T_A^*(2), T_B^{AB}\right\}} < 0$ and $\frac{\partial \min\left\{T_A^*(2), T_B^{AB}\right\}}{\partial \pi_A(2)} \leq 0$ as shown above. The last term is null because $\frac{\partial T_B^{BB}}{\partial \pi_A(2)} = 0$ as shown above.

(**vi**) For $\pi_i(m,-i) = \pi_B(2,B)$, expression (7) is nonpositive because expression (8) is nonnegative. In particular, the first term in expression (8) is null. The second term is nonnegative because $\frac{\partial \min\left\{T_A^*(2), T_B^{AB}\right\}}{\partial \pi_B(2,B)} \leq 0$ as shown above.

We conclude that if $\widehat{T}_A^1$ is finite, then for any profit structure that satisfies inequalities *(i)* to *(vi)* it holds that $T_A^1 \geq \widehat{T}_A^1$, because $\frac{dT_A^1}{d\pi_i(m,-i)}$ is null for $\pi_i(m,-i) = \pi_B(1)$, nonpositive for $\pi_i(m,-i) \in \left\{\pi_A(1), \pi_A(2), \pi_B(2,B), \pi_B(3)\right\}$ and nonnegative for $\pi_i(m,-i) = \pi_B(2,A)$.

Suppose instead that $\widehat{T}_A^1$ is not finite. We observe that by definition of T_A^1 any change in a profit parameter that decreases $D_A(\cdot)$ leaves $T_A^1 = \infty$. Then, for any profit structure that satisfies inequalities *(i)* to *(vi)* it holds that $T_A^1 = \infty$, because $\frac{dD_A(\cdot)}{d\pi_i(m,-i)}$ is null for $\pi_i(m,-i) = \pi_B(1)$, nonnegative for $\pi_i(m,-i) \in \left\{\pi_A(1), \pi_A(2), \pi_B(2,B), \pi_B(3)\right\}$ and nonpositive for $\pi_i(m,-i) = \pi_B(2,A)$.$\blacksquare$

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