Noncommutative Quantum Field Theory

Application for a research project

to the

Fonds zur Förderung der Wissenschaftlichen Forschung

Applicant

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1 Scientific aspects

1.1 Aims of the project

1.1.1 Scientific background and relevance of the project

Four dimensional quantum field theory suffers from infrared and ultraviolet divergences as well as from the divergence of the renormalized perturbation expansion. Despite the impressive agreement between theory and experiments and despite many attempts, these problems are not settled and remain a big challenge for theoretical physics. Furthermore, attempts to formulate a quantum theory of gravity have not yet been fully successful. It is astonishing that the two pillars of modern physics, quantum field theory and general relativity, seem incompatible. This convinced physicists to look for more general descriptions: After the formulation of supersymmetry and supergravity, string theory was developed, and anomaly cancellation forced the introduction of six additional dimensions. On the other hand, loop gravity was formulated, and led to spin networks and space-time foam. Both approaches are not fully satisfactory. A third impulse came from noncommutative geometry developed by Alan Connes, providing a natural interpretation of the Higgs effect at the classical level. This finally led to noncommutative quantum field theory, which is the framework of this project. It allows to incorporate fluctuations of space into quantum field theory. There are of course relations among these three developments. In particular, the field theory limit of string theory leads to certain noncommutative field theory models, and some models defined over fuzzy spaces are related to spin networks.

The argument that space-time should be modified at very short distances goes back to Schrödinger and Heisenberg. Noncommutative coordinates appeared already in the work of Peierls for the magnetic field problem, and are obtained after projecting onto a particular Landau level. Pauli communicated this to Oppenheimer, whose student Snyder [1] wrote down the first deformed space-time algebra preserving Lorentz symmetry.

After the development of noncommutative geometry by Connes [2], it was first applied in physics to the integer quantum Hall effect [3]. Gauge models on the two-dimensional noncommutative tori were formulated, and the relevant projective modules over this space were classified. Through interactions with John Madore I realized that such Fuzzy geometries allow to obtain natural cutoffs for quantum field theory [4]. This line of work was developed further together with Peter Presnajder and Ctirad Klimcik [5]. At almost the same time, Filk [6] developed his Feynman rules for the canonical deformed four dimensional field theory, and Doplicher, Fredenhagen and Roberts [7] published their work on deformed spaces. The subject experienced a major boost after one realized that string theory leads to noncommutative field theory under certain conditions [8, 9], and the subject developed very rapidly; see e.g. [10, 11, 12].

1.1.2 State-of-the-art in Noncommutative Quantum Field Theory

The formulation of Noncommutative Quantum Field Theory (NCFT) follows a dictionary worked out by mathematicians. Starting from some manifold $\mathcal{M}$ one obtains the commutative algebra of smooth functions over $\mathcal{M}$, which is then quantized along with additional structure. Space itself then looks locally like a phase space in quantum mechanics. Fields are elements of the algebra resp. a finitely generated projective module, and integration is replaced by a suitable trace operation.

Following these lines, one obtains field theory on quantized (or deformed) spaces, and Feynman rules for a perturbative expansion can be worked out. However some unexpected features such as IR/UV mixing arise upon quantization, which are described below.

Renormalizability of Noncommutative Quantum Field Theory. In 2000 Minwalla, van Raamsdonk and Seiberg realized [13] that perturbation theory for field theories defined on the Moyal plane faces a serious problem. The planar contributions show the standard singularities which can be handled by a renormalization procedure. The nonplanar one loop contributions are finite for generic momenta, however they become singular at exceptional momenta. The usual UV divergences are then reflected in new singularities in the infrared, which is called IR/UV mixing. This spoils the usual renormalization procedure: Inserting many such loops to a higher order diagram generates singularities of any inverse power. Without imposing a special structure such as supersymmetry, the renormalizability seems lost; see also [14, 15].

However, crucial progress was made recently, when Raimar Wulkenhaar and the applicant were able to give a solution of this problem for the special case of a scalar four dimensional theory defined on the deformed Moyal space $\mathbb{R}^4_\theta$ [16]. The IR/UV mixing contributions were taken into account through a modification of the free Lagrangian by adding an oscillator term with parameter Ω, which modifies the spectrum of the free Hamiltonian. The harmonic oscillator term was obtained as a result of the renormalization proof. The model fulfills then the Langmann-Szabo duality [17] relating short distance and long distance behavior. Our proof followed ideas of Polchinski. There are indications that a constructive procedure might be possible and give a nontrivial ϕ^4 model, which is currently under investigation [18]. At $\Omega = 1$ the model becomes self-dual, and will be studied in greater details in this project.

Nonperturbative aspects. Nonperturbative aspects of NCFT have also been studied in recent years. The most significant and surprising result is that the IR/UV mixing can lead to a new phase denoted as "striped phase" [19], where translational symmetry is spontaneously broken. The existence of such a phase has indeed been confirmed in numerical studies [20, 21]. To understand better the properties of this phase and the phase transitions, further work and better analytical techniques are required, combining results from perturbative renormalization with nonperturbative techniques. Here a particular feature of scalar NCFT is very suggestive: the field can be described as a hermitian matrix, and the quantization is defined non-perturbatively by integrating over all such matrices. This provides a natural starting point for nonperturbative studies. In particular, it suggests and allows to apply ideas and techniques from random matrix theory.

This idea was realized recently by H. Steinacker in [22], by focusing on the eigenvalues of the scalar field in a matrix model formulation. This allowed to obtain an analytic description of a non-trivial phase diagram, in accordance with the expectations mentioned above. The generalization and combination of these techniques is one goal of this proposal.

Remarkably, gauge theories on quantized spaces can also be formulated in a similar way [23, 24, 25, 26]. The action can be written as multi-matrix models, where the gauge fields are encoded in terms of matrices which can be interpreted as "covariant coordinates". The field strength can be written as commutator, which induces the usual kinetic terms in the commutative limit. Again, this allows a natural non-perturbative quantization in terms of matrix integrals.

Numerical studies for gauge theories have also been published including the 4-dimensional case [27], which again show a very intriguing picture of nontrivial phases and spontaneous symmetry breaking. These studies also strongly suggest the non-perturbative stability and renormalizability of NC gauge theory, adding to the need of further theoretical work.

Spaces with additional structure, fuzzy spaces An important question in this context is to which extent the IR/UV problems and the renormalizability depends on the details of the space under consideration.

Quantum spaces with additional structure, such as covariance under a classical or quantum group, have been studied extensively. In particular, the so-called “fuzzy spaces” are very important and useful, and arise in many different contexts. They are typically quantizations of coadjoint orbits (such as $S^2, \mathbb{C}P^2$) in terms of a *finite-dimensional* Hilbert space. Fields are then simply $N \times N$ matrices. This leads to a very transparent and simple formulation of field theory with UV cutoff, which has been studied in great detail in both 2 and 4 dimensions; see e.g. [5, 28, 29, 30, 31, 32, 33, 34, 25, 35, 36]. However, renormalizability has not yet been established in these cases. A further variant of these fuzzy spaces are the so-called q-deformed fuzzy spaces [37, 38], which are covariant under a quantum group but are still finite. This is again related to string theory [39], and allows to link NC field theory with the representation theory of quantum groups. We also analyzed the question of IR/UV mixing for the κ - deformed space [40, 41, 42] in a recent work with Michael Wohlgenannt, by performing a one-loop computation. It turns out that again the typical divergences for exceptional external momenta arise and IR/UV mixing occurs. As a further example of similar mixing properties we may mention the work of my PhD student Matthias Kornexl, who analyzed field theories on the deformed tori. Again there was no obvious way to cure the occurence of mixing. A systematic analysis of the IR/UV mixing should therefore be developed. Related work on other quantum spaces can be found e.g. in [43, 44, 45, 46].

In summary, field theory on NC spaces has established itself in recent years as a sound, very active and rich new branch of theoretical physics, which is likely to play an important role in the context of fundamental theories of matter and fields. One major goal is the formulation of a renormalized deformed Standard Model with realistic interactions, taking into account quantum fluctuations of space-time.

1.1.3 Innovative aspects of the project

The aim of this research project is to generalize the results and techniques on perturbative renormalizability of NCFT discussed above [16, 47] to other models and spaces, and to study nonperturbative aspects of such field theories using a combination with the methods in [22].

The results of this project are expected to establish a broader class of models of NCFT to be renormalizable and accessible to systematic computational (perturbative) tools. This is very important for further development of the field, and in particular provides a firm ground for physical applications in the context of elementary particle theory. In addition, the development of nonperturbative techniques is crucial to understand important features such as phase transitions and collective phenomena of the models.

In more detail, we expect new results on the following topics:

1. *Renormalizability of scalar and gauge theories on various quantized space-times, focusing on the question of IR/UV mixing.*

   Up to now, the only known renormalizable model of NCFT in four dimensions without additional symmetry (such as supersymmetry) has been obtained by myself and Raimar Wulkenhaar; this will be described below. In this project, we intend to follow these lines of work and develop similar techniques for models with gauge fields. The major steps are a reformulation in terms of a dynamical matrix model, and a proof of the appropriate decay properties of the free propagator.

   We will also study renormalizability for certain other quantized spaces. In particular, renormalizability has not been settled even for scalar fields on the two-dimensional Fuzzy Sphere. We will also address this question for the κ - Poincare space continuing an ongoing work with Michael Wohlgenannt, and for the noncommutative torus.

2. *Application of nonperturbative methods to NC field theories, in particular matrix model techniques.*

   One promising innovative approach to NC field theory was proposed recently by Harold Steinacker [22]. The basic idea is to look at certain collective degrees of freedom, which carry enough information for the statistical and thermodynamical properties of the full model. This can be achieved by focusing on the eigenvalues of the scalar field in a matrix model formulation, leading to an analytic description of the phase diagram and the phase transition.

In this project, we plan to develop further this method by considering the eigenvalue sector of the models studied in [16, 48], and to other models such as complex scalar fields, sigma models and eventually gauge fields. This nonperturbative approach will be combined with the perturbative methods and results discussed above.

3. *Application of non-classical representations of $U_q(sl(2))$ to NC field theory*

   This third part concerns special structures of field theory which arise on the q-deformed Fuzzy Sphere. For q being a root of unity, there exist both "classical" as well as "non-classical" (e.g. indecomposable) representations of the relevant quantum group $U_q(sl(2))$. Up to now, almost exclusively the classical ones have been used in physical applications. However, the non-classical representations are relevant e.g. to field theory on non-compact quantum spaces with Lorentzian signature [49, 50, 51]. Furthermore, the quantum coadjoint action can then become nontrivial. The q-deformed Fuzzy sphere is the simplest toy model [37], where we plan to set up a field theory based on these non-classical representations.

1.2 Methods

1.2.1 Perturbative quantization on various deformed spaces

We briefly sketch the methods used by Raimar Wulkenhaar and the applicant [16] in the proof of renormalizability for scalar field theory defined on the 4-dimensional quantum plane $\mathbb{R}^4_\theta$, with commutation relations $[x_\mu, x_\nu] = i\theta_{\mu\nu}$. The IR / UV mixing was taken into account through a modification of the free Lagrangian, by adding an oscillator term which modifies the spectrum of the free Hamiltonian:

$$S = \int d^4x \Big(\frac{1}{2}\partial_\mu\phi \star \partial^\mu\phi + \frac{\Omega^2}{2}(\tilde{x}_\mu\phi) \star (\tilde{x}^\mu\phi) + \frac{\mu^2}{2}\phi \star \phi + \frac{\lambda}{4!}\phi \star \phi \star \phi \star \phi \Big)(x) \ . \qquad (1)$$

Here $\star$ is the Moyal star product

$$(a\star b)(x) := \int d^D y \frac{d^D k}{(2\pi)^D} a(x+\tfrac{1}{2}\theta\cdot k)b(x+y)\,\mathrm{e}^{\mathrm{i}ky}\;, \qquad \theta_{\mu\nu} = -\theta_{\nu\mu} \in \mathbb{R}\;. \tag{2}$$

The harmonic oscillator term in (1) was found as a result of the renormalization proof. The model is covariant under the Langmann-Szabo duality relating short distance and long distance behavior. At $\Omega = 1$ the model becomes self-dual, and will be studied in greater detail in this project. This leads to the hope that a constructive procedure around this particular case allows the construction of a nontrivial interacting Φ^4 model, which would be an extremely interesting and remarkable achievement.

The renormalization proof proceeds by using a matrix base, which leads to a dynamical matrix model of the type:

$$S = (2\pi\theta)^2 \sum_{m,n,k,l\in\mathbb{N}^2} \Big(\frac{1}{2}\phi_{mn}\Delta_{mn;kl}\phi_{kl} + \frac{\lambda}{4!}\phi_{mn}\phi_{nk}\phi_{kl}\phi_{lm}\Big)\;, \tag{3}$$

where

$$\begin{aligned}\Delta_{\substack{m^1\\m^2}\substack{n^1\\n^2};\substack{k^1\\k^2}\substack{l^1\\l^2}} &= \big(\mu^2+\tfrac{2+2\Omega^2}{\theta}(m^1+n^1+m^2+n^2+2)\big)\delta_{n^1k^1}\delta_{m^1l^1}\delta_{n^2k^2}\delta_{m^2l^2}\\ &\quad - \tfrac{2-2\Omega^2}{\theta}\big(\sqrt{k^1l^1}\,\delta_{n^1+1,k^1}\delta_{m^1+1,l^1} + \sqrt{m^1n^1}\,\delta_{n^1-1,k^1}\delta_{m^1-1,l^1}\big)\delta_{n^2k^2}\delta_{m^2l^2}\\ &\quad - \tfrac{2-2\Omega^2}{\theta}\big(\sqrt{k^2l^2}\,\delta_{n^2+1,k^2}\delta_{m^2+1,l^2} + \sqrt{m^2n^2}\,\delta_{n^2-1,k^2}\delta_{m^2-1,l^2}\big)\delta_{n^1k^1}\delta_{m^1l^1}\;.\end{aligned} \tag{4}$$

The interaction part becomes a trace of product of matrices, and no oscillations occur in this basis. The propagator obtained from the free part is quite complicated, and in 4 dimensions is:

$$\begin{aligned}G_{\substack{m^1\\m^2}\substack{n^1\\n^2};\substack{k^1\\k^2}\substack{l^1\\l^2}} &= \frac{\theta}{2(1+\Omega)^2} \sum_{v^1=\frac{|m^1-l^1|}{2}}^{\frac{m^1+l^1}{2}} \sum_{v^2=\frac{|m^2-l^2|}{2}}^{\frac{m^2+l^2}{2}} B\big(1+\tfrac{\mu^2\theta}{8\Omega}+\tfrac{1}{2}(m^1+k^1+m^2+k^2)-v^1-v^2, 1+2v^1+2v^2\big)\\ &\quad\times {}_2F_1\left(\begin{matrix}1+2v^1+2v^2\,,\ \frac{\mu^2\theta}{8\Omega}-\frac{1}{2}(m^1+k^1+m^2+k^2)+v^1+v^2\\ 2+\frac{\mu^2\theta}{8\Omega}+\frac{1}{2}(m^1+k^1+m^2+k^2)+v^1+v^2\end{matrix}\,\middle|\,\frac{(1-\Omega)^2}{(1+\Omega)^2}\right)\Big(\frac{1-\Omega}{1+\Omega}\Big)^{2v^1+2v^2}\\ &\quad\times\prod_{i=1}^{2}\delta_{m^i+k^i,n^i+l^i}\sqrt{\binom{n^i}{v^i+\frac{n^i-k^i}{2}}\binom{k^i}{v^i+\frac{k^i-n^i}{2}}\binom{m^i}{v^i+\frac{m^i-l^i}{2}}\binom{l^i}{v^i+\frac{l^i-m^i}{2}}}\;.\end{aligned} \tag{5}$$

These propagators (in 2 and 4 dimensions) show asymmetric decay properties: they decay exponentially on particular directions, but have power law decay in others. These decay properties are crucial for the perturbative renormalizability respectively nonrenormalizability of the models. Our proof in [47] then followed the ideas of Polchinski [52]. The integration of the Polchinski equation from some initial scale down to the renormalization scale leads to divergences after removing the cutoff. For relevant/marginal operators, one therefore has to fix certain initial conditions. The goal is then to find a procedure involving only a finite number of such operators. Through the invention of a mixed integration procedure and by proving a certain power counting theorem, we were able to reduce the divergences to only four relevant/marginal operators. This justifies a posteriori our starting point of adding one new term to the action (1), the oscillator term Ω. A somewhat long sequence of estimates and arguments then leads to the proof of renormalization. This being established, it was easy to derive beta functions for the coupling constant flow, which shows that the ratio of the coupling constants λ/Ω^2 remains bounded along the renormalization group flow up to first order. In particular, the beta function vanishes at the self-dual point $\Omega = 1$, indicating special properties of the model. One part of this project is to explore this special point in more detail.

Gauge field models. In this project, we will extend this renormalization method and apply it to other models, in particular to gauge fields. There are two natural approaches:

1. Gauge theories arise naturally in noncommutative geometry from fluctuations of a Dirac operator [53]. It is not difficult to write down two natural candidates for a four-dimensional Dirac operator, which are connected to an oscillator potential. One may then obtain an action for the gauge fields by using the spectral action [53, 54]. This requires a long calculation of the fluctuation spectrum.

2. A second approach starts from covariant coordinates $X_\mu = \theta^{-1}_{\mu\nu} x^\nu + A_\mu$, by writing down the most general quartic gauge invariant action functional using the star product. The resulting action can then be compared with the one obtained from the spectral action principle explained above, and might turn out to be identical.

The next step towards a quantized field theory is to add the ghost sector to the gauge field action and to promote the gauge symmetry to a nilpotent BRST symmetry. One then has to control possible violations of the Slavnov-Taylor identities, etc. We plan to collaborate on these highly nontrivial steps with Raimar Wulkenhaar at the MPI Leipzig. There are obvious relations in particular for the second approach with the matrix-model formulation of NC gauge theory, which is discussed in Section 1.2.2 in relation with matrix-model techniques. Combining these two points of view should allow to make substantial progress in the ambitious goal to establish renormalizable gauge theories on NC spaces.

Quantization on spaces with additional structure, fuzzy spaces. The extension of these methods to certain other spaces such as fuzzy spaces will be studied. Even for the case of the two-dimensional fuzzy sphere, the question of renormalizability for scalar fields has not been settled. A careful one-loop analysis was performed by H. Steinacker et.al. [31], establishing a number of useful techniques in this context. In this project, we will try to adapt the renormalization-group techniques developed for the Moyal case to the fuzzy sphere. In particular, the decay properties of the propagator must be established, and suitable free actions must be determined with or without oscillator-like term.Generalizations to 4 dimensions are also planned, in particular for fuzzy $\mathbb{C}P^2$ [55, 35] and fuzzy $S^2 \times S^2$ [26].

Furthermore, we are analyzing the question of IR/UV mixing for the κ - Poincare space in a current work with Michael Wohlgenannt, by performing a one-loop computation. It turns out that again the typical IR/UV mixing occurs. Therefore a systematic analysis of IR/UV mixing will be attempted, which should shed light on the general strategy of controlling them by adding suitable terms in the action.

1.2.2 Matrix-model techniques

A second method we will use is an extension and refinement of matrix-method techniques, in particular as developed in [22]. This will be combined with the perturbative methods explained above. It can be applied in principle to any quantum space which is finite or regularized by a finite-dimensional Hilbert space.

We briefly explain this method. Consider e.g. the scalar field theory defined by (3). Since

ϕ is a hermitian matrix, it can be diagonalized as $\phi = U^{-1} diag(\phi_i) U$ where ϕ_i are the real eigenvalues. Hence the field theory can be reformulated in terms of the eigenvalues ϕ_i and the unitary matrix U. The main idea is now the following: consider the probability measure for the (suitably rescaled) eigenvalues ϕ_i induced the path integral by integrating out U:

$$\begin{aligned} Z &= \int \mathcal{D}\phi \, \exp(-S(\phi))) = \int d\phi_i \Delta^2(\phi_i) \int dU \exp(-S(U^{-1}(\phi_i)U)) \\ &= \int d\phi_i \exp(-\tilde{\mathcal{F}}(\phi) - (2\pi\theta)^{d/2} \sum_i V(\phi_i) + \sum_{i \neq j} \log|\phi_i - \phi_j|), \end{aligned} \tag{6}$$

where the analytic function

$$e^{-\tilde{\mathcal{F}}(\phi)} := \int dU \exp(-S_{kin}(U^{-1}(\phi)U)) \tag{7}$$

is introduced, which depends only on the eigenvalues of ϕ. The crucial point is that the logarithmic terms in the effective action above implies a repulsion of the eigenvalues ϕ_i, which therefore arrange themselves according to some distribution similar as in the standard matrix models of the form $\tilde{S} = \int d\phi \exp(Tr\tilde{V}(\phi))$. This is related to the fact that nonplanar diagrams are suppressed. The presence of the unknown function $\tilde{\mathcal{F}}(\phi)$ in (6) cannot alter this conclusion qualitatively, since it is analytic. The function $\tilde{\mathcal{F}}(\phi)$ can be determined approximately by considering the weak coupling regime. For example, the effective action of the eigenvalue sector for the ϕ^4 model in the noncommutative regime $\frac{1}{\theta} \ll \Lambda^2$ becomes essentially

$$\tilde{S}(\phi) = f_0(m) + \frac{2N}{\alpha_0^2(m)} Tr\phi^2 + g\phi^4, \tag{8}$$

where $\alpha_0^2(m)$ depends on the degree of divergence of a basic diagram [22].

This effective action (8) can now be studied using standard results from random matrix theory. For example, this allows to study the renormalization of the effective potential using matrix model techniques. The basic mechanism is the following: In the free case, the eigenvalue sector follows Wigners semicircle law, where the size of the eigenvalue distribution depends on m via $\alpha_0(m)$. Turning on the coupling g alters that eigenvalue distribution. The effective or renormalized mass can be found by matching that distribution with the “closest” free distribution. To have a finite renormalized mass then requires a negative mass counterterm as usual.

This approach is particularly suitable to study the thermodynamical properties of the field theory. For the ϕ^4 model, the above effective action (8) implies a phase transition at strong coupling, to a phase which was identified with the striped or matrix phase in [22]. Based on the known universality properties of matrix models, these results on phase transitions are expected to be realistic, and should not depend on the details of the unknown function $\tilde{\mathcal{F}}(\phi)$. The method is applicable to 4 dimensions, where a critical line is found which terminates at a non-trivial point, with finite critical coupling. This can be seen as evidence for a new non-trivial fixed-point in the 4-dimensional NC ϕ^4 model. This is in accordance with results from the RG analysis of [56], which also point to the existence of nontrivial ϕ^4 model in 4 dimensions. If confirmed, this will be a very remarkable result with major impact.

Application and generalization. In this project, this method will be refined, and applied in particular to complex scalar and sigma models. The required adaptations will be worked out, and the corresponding phase diagrams will be obtained. In particular, we plan to determine thermodynamical quantities such as specific heat, critical exponents and susceptibilities, in particular near the critical point. It should be possible to obtain the critical exponents from the known results for matrix models, adapted to the present situation. These are expected to be reliable due to the known universality of matrix models, and can be compared with the results of numerical simulations.

Furthermore, we plan to generalize this analysis and apply it to the models of [16, 48] with oscillator term in the free action. This is a more substantial modification. We will focus first on the self-dual case $\Omega = 1$. Then the model is particularly simple, and we expect it to be accessible to an exact matrix-model analysis similar to the work of [57]. We have already conducted some preliminary work, which shows that even though the model is slightly more complicated than [57], it does admit similar matrix model techniques as discussed above, which pass nontrivial consistency checks. Therefore we expect to obtain a very good understanding of this model with $\Omega = 1$ using both perturbative and nonperturbative methods. The 4-point function will be computed, and analyzed in various scaling limits.

Possible application of this nonperturbative approach to gauge models will also be pursued, on the fuzzy spaces $\mathbb{C}P_N^2$ and $S_N^2 \times S_N^2$. While the corresponding multi-matrix models cannot be solved exactly, a similar analysis of suitable collective degrees of freedom in the large N limit should be possible. This is supported by the broad applicability of matrix models to chaotic quantum systems, and the structural similarity of NC gauge theories as in [25, 26] with NC scalar field theories.

1.2.3 Application of representation theory for spaces with additional structure

For some quantum spaces with additional structure such as fuzzy spaces and the q-deformed fuzzy sphere, one can apply the representation theory of ordinary groups resp. quantum groups. These spaces can be seen as "covariant lattices", combining the discreteness of a lattice with covariance under some symmetry. A particularly remarkable case is the q-deformed fuzzy sphere for q a root of unity, where indecomposable representations arise in the fusion structure. Certain truncation procedures are usually imposed to get rid of the non-standard representations, e.g. in the work of Alekseev, Recknagel and Schomerus [39].

We want to go beyond these truncations and use the non-classical representations as well. This opens up the possibility to obtain a nontrivial realization of the quantum coadjoint action [58] on these spaces. Different truncation schemes have also been proposed [59], which are suited for the non-compact case of e.g. fuzzy AdS^2. We would like to implement these possibilities in gauge models similar as those discussed in [37], where different choices of vacua are possible, in particular those with nontrivial realization of the of the quantum coadjoint action. The meaning of this additional symmetry structure can then be studied. We expect that the realization of these structures in the models is related to recent work done in collaboration with K-G. Schlesinger [60], which hints at special properties under renormalization.

1.3 Work plan

This project requires 2 postdocs with expertise in the area of NC field theory, working for 2 years each. Some initial investigations along these lines are already under way in collaboration with Dr. H. Steinacker, Dr. M. Wohlgenannt, and Dr. K-G. Schlesinger.

The following steps of the project will be addressed by the applicant H. Grosse, by the proposed collaborator H. Steinacker, and by a further postdoc not yet specified. Other collaborations on individual aspects will be mentioned explicitly. The main, ambitious goal of establishing new renormalizable field theories on NC spaces is divided into several different steps and approaches, which allows sufficient flexibility to take into account new developments.

1.3.1 Analysis of the self-dual point $\Omega = 1$ for scalar field theory on $\mathbb{R}^n_\theta$

The models (1) at the self-dual model with $\Omega = 1$ are fixed points of the RG flow, and therefore of particular interest. The model can then be formulated as a matrix model coupled to an external matrix. We have already conducted some preliminary work, which shows that the model does admit similar matrix model techniques as discussed above, which pass nontrivial consistency checks. In particular the eigenvalue distribution can be found, and thermodynamical properties will be established.

Furthermore, the 4-point function will be computed using similar techniques as in [57], and analyzed in various scaling limits. It is expected to be nontrivial in a suitable limit, which would establish the model to be a very simple but nontrivial field theory which can serve as a testing ground for various ideas in the context of NC field theory.

Once the self-dual point $\Omega = 1$ is fully under control, one can start to slightly perturb it and study it near $\Omega = 1$ using the perturbative methods established in [16].

Expected duration: 1 year.

Collaborations: E. Langmann from Stockholm University.

1.3.2 IR/UV mixing and renormalization of scalar field theory on other quantum space

The application of the methods explained in Section 1.2 to scalar fields on certain other spaces such as the fuzzy sphere will be studied. For a perturbative analysis, the propagator will be expressed in terms of suitable orthogonal polynomials, which are truncated Legendre polynomials [5] in this case. The decay properties will be analyzed, and suitable free actions must be determined with or without additional oscillator-like term such that the general results [47] can be applied. We expect no major difficulties in this case, however again the relation with the nonperturbative matrix model techniques should be particularly instructive here.

Depending on the time available for this project and on the progress, generalizations to 4 dimensions are also planned, in particular for fuzzy $\mathbb{C}P^2$ which is formally very similar to the fuzzy sphere. Furthermore, a more general study of IR/UV mixing is envisaged (following a study for the κ - Poincare space in a current work with Michael Wohlgenannt, and others). The aim is to find a general strategy of controlling the IR/UV mixing by adding suitable terms in the action.

Expected duration: 2 years.

Collaborations: R. Wulkenhaar, MPI Leipzig.

1.3.3 Extension of the matrix model analysis to scalar fields and sigma models, and the Higgs model

Parallel to the above steps, the matrix model approach of [22] to scalar NC field theory will be refined, and thermodynamic properties such as specific heat, critical exponents etc. will be computed near the critical point. These can be obtained from the known results for conventional matrix models, adapted to the present situation. These are expected to be realistic due to the known universality of matrix models near the critical point. Moreover, they can be compared with the results of numerical simulations.

In a further step, the necessary generalizations to complex scalar fields and e.g. the (linear) $U(2)$ sigma model will be worked out. Again, one can apply known results and techniques from random matrix theory for complex matrices, and adapt the techniques described above. These techniques are expected to be applicable also to the other parts of the project, providing consistency checks and new connections.

The next natural step is to apply this analysis to the NC Higgs effect. Using the known formulation of gauge theory in terms of multi-matrix models [26, 35, 24, 25], a (fundamental or adjoint) Higgs can be added, or is already part of the gauge multiplet for fuzzy spaces. The eigenvalue sector of such a Higgs field can be studied using the same methods as discussed above for the scalar case. The mechanism of symmetry breaking is expected to be the same as in the ϕ^4 case, and characterized by a split of the eigenvalue distribution into 2 separated pieces. This is quite different from the conventional mechanism, which should have profound physical implications. This shall be studied in detail.

Expected duration: 1 year for scalar case, 1 year for extensions to Higgs.

1.3.4 Renormalization of other models on $\mathbb{R}^4_\theta$, including gauge fields

As described in Section 1.2, there are two natural approaches to generalize the above perturbative methods to gauge fields:

1. Starting form a suitable Dirac operator. It is not difficult to write down two natural candidates for a four-dimensional Dirac operator, which are connected to an oscillator potential. One may then obtain an action for the gauge fields by using the spectral action [53, 54]. This requires a long calculation of the fluctuation spectrum, which will be initiated.

2. A second approach starts from covariant coordinates $X_\mu = \theta^{-1}_{\mu\nu} x^\nu + A_\mu$, by writing down the most general quartic gauge invariant action functional using the star product.

After some initial work on both approaches, the most suitable one to proceed will be identified. Adding ghosts and a nilpotent BRST symmetry is expected to be straightforward. A major step is then to work out the propagators in a suitable basis, and to determine their decay properties. The relations in particular for the second approach with the matrix-model formulation of NC gauge theory will be used. Combining these two points of view should allow to make substantial progress in the ambitions goal to establish renormalizable gauge theories on NC spaces.

Expected duration: ≥ 2 years, with partial results available earlier.

Collaborations: R. Wulkenhaar, MPI Leipzig.

1.3.5 Non-classical representations on the q-deformed fuzzy sphere, and the quantum coadjoint action

In previous work with J. Madore and H. Steinacker we formulated scalar and gauge models on q-deformed Fuzzy spaces, for generic values of the deformation parameter. New phenomena show up for q being root of unity. Going beyond the truncation used e.g. in work of Alekseev, Recknagel and Schomerus [39], indecomposable representations arise in the fusion structure. This opens up in particular the possibility to obtain a nontrivial realization of the quantum coadjoint action on these spaces. This additional symmetry structure may be used to single out special gauge models on these spaces. Our recent work [60] with K.-G. Schlesinger suggests that such models might have very special properties under renormalization. This part of the project will be pursued as time permits.

Expected duration: 1 year.

Collaborations: K-G. Schlesinger.

Dissemination. The results of this project will be published and communicated to the scientific community as usual through publications, international conferences and workshops. In particular, I can take advantage of numerous invitations to conferences and workshops as a result of recent publications.

1.4 Collaborations with other groups

There are ongoing collaborations with the following scientists, which are relevant to this project and expected to be continued:

- **Dr. R. Wulkenhaar**
  MPI für Mathematik in den Naturwissenschaften
  Leipzig

- **Prof. Dr. M. Schweda**
  Technische Universität Wien

- **Prof. Dr. P. Presnajder**
  Comenius University
  Bratislava

- **Prof. Dr. Edwin Langmann**
  KTH University Stockholm

2 Human ressources

The persons participating in the project are:

- **Prof. Dr. Harald Grosse** (Applicant)

  Institut für theoretische Physik der Universität Wien.

  I have been working since 1992 in noncommutative field theory. Together with collaborators we have established several important results in this subject, most notably the first proof of renormalizability for a ϕ^4 model on noncommutative $\mathbb{R}^4$. I consider the goals of this project as very important and crucial steps in promising area of research, towards realistic field theoretical models on quantized spaces.

- **Doz. Dr. Harold Steinacker** (requested postdoc)

  24 months, from 1.4. 2006 to 31.3.2008.

  Harold Steinacker is presently postdoc at the University of Vienna (his project ends March 2006). He obtained the Habilitation in theoretical physics from the University of Munich in 2003, and has an excellent scientific background acquired in leading international institutions. He has contributed many new ideas and results to noncommutative field theory. We have had ongoing collaborations for many years, with several joint publications. This project would provide a very important opportunity for him to continue his research on NC field theory. His international experience and contacts together with the active visitor program at the ESI Vienna should be an excellent basis for finding a permanent position in theoretical physics.

- **Dr. N.N.** (requested postdoc)

  24 months, from 1.10. 2005 - 30.9.2007

  One excellent candidate would be Dr. M. Wohlgenannt, presently Postdoc at the University of Vienna in my group (his project ends September 2005). He also has many years of experience in this subject, for example he was part of the collaboration proposing a deformed standard model based on the Seiberg-Witten map [61].

3 Further impact

One can expect that this work will lead to new methods and insights to quantum field theory (QFT), which would be highly welcome in the wider theoretical physics community, in view of our poor understanding of physics beyond the standard model. In particular, relations with quantum gravity and string theory are implicit. Applications in other contexts such as condensed matter in the presence of magnetic fields (quantum Hall systems) or incompressible fluids can also be expected.

Furthermore, the clean mathematical formulation of quantum field theory as dynamical (or multi-) matrix models is very suitable for rigorous investigations, and should lead to exact mathematical results in the context of QFT. I would like to point out again here the first renormalization proof for a NC field theory, which was appreciated also in the mathematical community as reflected by numerous invitations to conferences and workshops.

4 Financial aspects

The project will be carried out at the

Institut für theoretische Physik
Universität Wien
Boltzmanngasse 5
1090 Wien

The reserach group of the applicant has the following members

a.o.Prof. Harald Grosse	Member of Institute for Theoretical Physics
Doz. Dr. Harold Steinacker	financed by FWF project **P16779-NO2** until 3/2006
Dr. Karl-Georg Schlesinger	currently ESI fellow, previously DFG-Fellow
Dr. Michael Wohlgenannt	financed by FWF project **P16779-NO2** until 9/2005
Mag. Matthias Kornexl	Dissertation fellowship of the ÖAW
Paul Schreivogl	Diploma student
Daniela Klammer	Diploma student
Christoph Zauner	Diploma student

The institute will provide office space and infrastructure such as computing facilities and office equipment. The project can use excellent libraries for physics and mathematics.

The project leader is engaged at the Erwin Schrödinger Institute for Mathematical Physics since its foundation. There will be a follow-up programme on ”String theory in curved backgrounds and boundary conformal field theory” coorganized by H. Grosse (together with A. Recknagel and V. Schomerus) and a programme “Gerbes, groupoids and quantum field theory” in 2006 co-organised by H. Grosse during which many scientists with related research interests will be invited. I succeeded in getting Prof. Varghese Mathai as a Senior Fellow at ESI in spring 2006. He will deliver a lecture course. All these interactions will be very useful for the project.

Requested support

Since this research requires sophisticated techniques in a rapidly progressing field of contemporary quantum field theory, financial support is requested for 2 post-docs for 2 years each. This is necessary in order to pursue simultaneously the different approaches described, which are all interrelated and not linearly ordered.

Doz. Dr. Harold Steinacker	
all aspects, and in particular matrix model techniques	Dienstvertrag Postdoc $50.240,00\,\frac{€}{\text{year}} \times 2\,$years $=$ €100.480,00
Post-doc N.N.	
Perturbative computations, RG techniques, IR/UV mixing, ...	Dienstvertrag Postdoc $50.240,00\,\frac{€}{\text{year}} \times 2\,$years $=$ €100.480,00
total personal request:	€200.960,00
Travel support for conference visits	$1\,400,00\frac{€}{\text{year}}$ for postdoc
total requested non-personal costs:	€5 600,00
total request:	**€206.560,00**

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Curriculum Vitae, Harald Grosse

Personal data

1944	Born in Vienna, Austria
1963	A-Level(Matura) examen passed ‘With Distinction’.
1963	Military Service
1964-	Studies of Physics and Mathematics, University of Vienna
1969	Assistant at Institute of Theoretical Physics
1971	Ph.D. Doctor degree, ‘With Distinction’.
1973	Diploma examen (Magister), ‘With Distinction’.
1974	got married
1977	Two years fellowship at CERN-TH.
1977	daughter Barbara born
1979	daughters Alexandra and Claudia born
1980	Habilitation (Dozent) at the University of Vienna
1981	Award of Merit of the Austrian Minister of Science
1983	CERN
1984	Brookhaven National Lab.
1986	Extraordinary Professor of Theoretical Physics at University of Vienna
1992-	Project Leader at the Erwin Schrödinger Institute, Vienna
1993	Editor of ‘Texts and Monographs in Physics’, Springer.
1995	obtained title Ingenieur
1997-	Advisory Board of ESI
2001	Offer of a full Professorship at University of Graz
2001	Leibniz professor at the University of Leipzig
2002	Advisory Board of DFG
2003	University of Berkeley

Lecturing

1975	Theoretical Physics I, Mechanics
1976	Mathematical Methods in Physics 3; Group Theory
1981	Mathematical Methods in Physics 2; Spinsystems and Lattice Gauge Models
1982	Mathematical Methods in Physics 2; Theory of Phase Transitions
1983	Renormalization group, Structures of condensed matter
1984	Theoretical Physics 2, Electrodynamics; Solvable models of statistical physics and quantum field theory
1985	Theoretical Physics for Teachers: Mechanics and Electrodynamics; Theoretical Physics for Teachers: Quantum Mechanics and Statistical Physics
1986	Quantum Field Theory 1; Quantum Field Theory 2

1987	QFT and Critical Phenomena; QFT1
1988	QFT2;QFT and Critical Phenomena 1
1989	QFT and Critical Phenomena 2
1990	Conformal Field Theory in two dimensions; Quantum Groups
1991	Quantum Groups; Integrable Models
1992	Theoretical Physics 4, Statistical Physics; Theoretical Physics 8
1993	Mathematical Physics 3
1994	Models of Statistical Physics 1; 2
1995	Models of Statistical Physics 1; 2
1996	Quantum Field Theory
1997	Theoretical Physics 1, Mechanics
1998	Quantum Field Theory
1999	Mathematical Methods of Physics 2
2000	Models of QFT and Statistical Physics
2001	QFT and noncommutative Geometry; Theoretical Physics 2, Quantum Mechanics
2002	Mathematical Methods of Physics 1, 2; Noncommutative QFT
2003	Statistical Physics 2
2004	Stat. Ph. II, Pros. II, Priv., Lit. sem.,
2005	Pros. M1, Th. Ph. 2, QM, Prosem. T2, Lit.sem., Priv.

PHD- Diploma Students, Project collaborator

1984	Ludwik Dabrowski, Armin Scrinzi, Roman Tomaschitz
1985	Fritz Gesztesy (FWF), Gunther Karner (FWF), Gerald Opelt (FWF), Peter Falkensteiner
1985	Markus Kemmerling, F. Doppler
1986	Edwin Langmann (sub ausp.) (FWF), M. Konecny
1988	Bernd Thaller
1994	Walter Maderner (sub ausp.) (FWF), Christian Reitberger (sub ausp.) (FWF)
1995	E. Schörghofer
1996	Ernst Raschhofer (FWF)
1998	F. Leitenberger (DFG), Karl-Georg Schlesinger (DFG), Peter Presnajder (ESI), John Madore (ESI)
1997	Robert Pitzl
1998	Alexander Strohmaier
1999	Gert Reiter (FWF), Rudolf Hanel
2000	Hans Miglbauer (FWF)
2002	Christian Rupp (Grduiertenkolleg), Raimar Wulkenhaar (EU), Matthias Kornexl (Graduiertenkooeg and Academy), Harold Steinacker (ESI), Stefan Schraml (EU)
2004	Paul Schreivogl, Daniela Klammer (Fellowship), Michael Wohlgenannt (FWF), Harold Steinacker (FWF)

List of Publications during the past 5 years

1. **On second quantization of quantum groups**
   together with K.G. Schlesinger
   Jour. Math. Phys. 41 (2000) 7043

2. On a trialgebraic deformation of the Manin plane
   together with K.G. Schlesinger
   Lett. Math. Phys. **52** (2000) 263

3. Chiral Schwinger Models without gauge anomalies
   together with E. Langmann
   Nucl. Phys. **B587** (2000) 568

4. Fuzzy Projective Spaces
   together with G. Reiter
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   World Scientific, 2000, p 535.

5. Noncommutative Supergeometry of Graded Matrix Algebras
   together with G. Reiter
   Proceedings of the International Schladming School, Springer 2000

6. Simple Fuzzy Superspaces
   together with G. Reiter
   Proc. of Euroconference, Torino on NCG and Hopf Algebras in Field Theory and Particle Physics, World Schientific 2000

7. The Superfield formalism applied to the Wess-Zumino model
   together with A.A. Bichl, J.M. Grimstrup, L. Popp, M. Schweda, R. Wulkenhaar,
   JHEP **10** (2000) 046

8. **Field Theory on the q-deformed Fuzzy sphere I**
   together with J. Madore and H. Steinacker
   Journ. Geom. Phys 38 (2001) 308-342

9. **Renormalization of noncommutative Yang-Mills theory: a simple example**
   together with T. Krajewski and R. Wulkenhaar
   hep-th/0001182

10. The Energy-Momentum Tensor on noncommutative spaces
    together with A. Gerhold, J. Grimstrup, L. Popp, M. Schweda and R. Wulkenhaar
    hep-th/0012112

11. **Fuzzy Line Bundles, the Chern Character and Topological Charges over the Fuzzy Sphere**
    together with C. Rupp and A. Strohmaier
    Journal of Geometry and Physics 42 (2002)54

12. **Field Theory on the q-defomed Fuzzy Sphere II**
    together with J. Madore and H. Steinacker
    Journal of Geometry and Physics 800 (2002) 1-36

13. A suggestion for an integrability notion for two-dimensional spin-systems
    together with K.G. Schlesinger
    Lett. Math. Phys. **55** (2001) 161

14. Anomalous C-violating three photon decay of the neutral pion in noncommutative Quatum Electrodynamics
    together with Yi Liao
    Phys. Lett. B **520** (2001) 63-68

15. Pair production of neutral Higgs bosons through noncommutative QED interactions at linear colliders
    together with Yi Liao
    Phys. Rev. **D64** (2001) 115007

16. Strong Connections and Chern-Connes Pairing in the Hopf-Galois Theory
    together with L. Dabrowski and P. Hajak
    Comm. Math. Phys. **230** (2001) 301

17. Renormalization of the noncommutative photon self-energy to all orders via Seiberg-Witten map
    together with A. Bichl, J. Grimstrup, L. Popp, M. Schweda and R. Wulkenhaar,
    JHEP **0106** (2001) 013

18. Regularization and Renormalization of QFT from Noncommutative Geometry
    New Developments in Fundamental Interaction Theories, 37 th Karpacz Winter School, AIP Conference Proceedings, Volume 589, p. 232, 2001, Editors: J. Lukierski, J. Rembielinski

19. Noncommutative Spin-1/2 representations
    together with J. Grimstrup, E. Kraus, L. Popp, M. Schweda and R. Wulkenhaar
    Eur. Jour. Phys. bf C24 (2002)485

20. Fuzzy Instantons
    together with M. Maceda, J. Madore and H. Steinacker
    Int. Jour. of Mod. Phys **A17no15** (2002)2095

21. Deformation of conformal field theory to models with noncommutative world sheets
    together with K.G. Schlesinger
    ESI 931, (2001)

22. Noncommutative Lorentz Symmetry and the Origin of the Seiberg-Witten Map
    together with A. Bichl, J. Grimstrup, E. Kraus, L. Popp, M. Schweda and R. Wulkenhaar
    Eur. Phys. J. **C24** (2002)165

23. On a noncommutative deformation of the Connes-Kreimer algebra
together with K.G. Schlesinger
math.QA/0107105,

24. **Spinfoam models for M-theory**
together with K.G. Schlesinger
Phys. Lett. B528, (2002)106

25. IR-singularities in Noncommutative Perturbative Dynamics
together with J. Grimstrup, L. Popp, V. Putz, M. Schweda, M. Winckenhauser and R. Wulkenhaar,
hep-th/ 0202093 Europhys. Lett. **67** (2004) 186

26. Noncommutative U(1) Super-Yang-Mills Theory: Self-Energy Corrections
together with A. Bichl, M. Ertl, A. Gerhold, J. Grimstrup, L. Popp, V. Putz, M. Schweda and R. Wulkenhaar,
hep-th/ 0203141 Int.J.Mod.Phys. **A19** (2004) 4231

27. Space/time noncommutative field theories and causality
together with H. Bozkaya, P. Fischer, M. Pitschmann, V. Putz, M. Schweda, R. Wulkenhaar
hep-th/0209253 Eur. Phys. J. **C29** (2003) 133

28. The Landau-Problem on the Theta-Deformed Torus
together with M. Kornexl
hep-th/0210042 Lett. Math. Phys. **63** (2003) 73

29. **Power counting theorem for nonlocal matrix models and renormalization**
together with R. Wulkenhaar
hep-th/0305066, Comm. Math. Physics **254** (2005) 91

30. Renormalization of Phi**4 theory on noncommutative R**2 in the matrix base
together with R. Wulkenhaar
hep-th/0307017, JHEP 0312:019,2003

31. Exact solution of a one D many body system with momentum dependent interactions
together with E. Langmann
math.ph/0401003, J. Phys.**A37**:4579,2004

32. Renormalisation of Noncommutative Quantum Field Theories
together with R. Wulkenhaar
Czech.J.Phys. **54** (2004) 1305

33. Regularization and Renormalization of Quantum Field Theories on Noncommutative Spaces
together with R. Wulkenhaar
J.Nonlin.Math.Phys. **11S1** (2004) 9

34. **Renormalization of Phi**4 theory on noncommutative R**4 in the matrix base together with R. Wulkenhaar hep-th/0401128, Comm. Math. Phys. 256 (2005) 305**

35. **The Beta function in duality covariant noncommutative Phi**4 theory together with R. Wulkenhaar hep-th/0402093, Eur.Phys.J.C35:277-282,2004**

36. Renormalization of Phi**4 theory on noncommutative R**4 to all orders together with R. Wulkenhaar hep-th/0403232, Lett. Math. Phys. **71** (2005) 13

37. **Finite Gauge theory on Fuzzy CP**2 together with H. Steinacker hep-th/0407089 Nucl. Phys B707 (2005) 145**

38. The eigenfunctions of the q-deformed Harmonic Operator on the Quantum Line together with S. Schraml math qa/0410389

39. The Universal Envelope of the topological closed string BRST-complex together with K.-G. Schlesinger hep-th/0412161

40. A remark on the motivic Galois group and the coadjoint action together with K.-G. Schlesinger hep-th/0412162

41. Renormalisation of scalar quantum field theory on noncommutative R**4 together with R. Wulkenhaar Fortsch. Phys. **53** (2005) 634

Curriculum Vitae

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Personal Data

Born on Jan 15, 1968 in Innsbruck, Austria
Nationality: Austria
Marital Status: married, 1 child

Education

5/1986	Graduation, Akademisches Gymnasium Innsbruck, Austria
10/1986 – 6/1992	**Universität Innsbruck, Austria** Study of Physics and Mathematics
9/1990 – 5/1991	**University of Notre Dame, USA** Exchange stipend, work on master's thesis
21.3.1992	Master's degree in Physics, **Universität Innsbruck, Austria**
9/1992 – 5/1997	**University of California at Berkeley, USA** Postgraduate study in theoretical physics thesis advisor: Prof. Bruno Zumino
24.5.1997	Ph.D. in physics, **UC Berkeley**
25.6.2003	Habilitation (Dr. habil) in theoretical physics, **University of Munich**

Scientific Positions

5/1997 – 8/1997	**Lawrence Berkeley National Laboratory, USA** Research assistant
10/1997 – 9/2000	**University of Munich (LMU), Germany** Research Associate (C1), Institute for mathematical physics
10/2000 – 9/2001	**Université de Paris–Sud, Orsay, France** Postdoctoral fellow (DFG)
10/2001 – 3/2005	**University of Munich (LMU), Germany** Research Associate (C1), Institute for mathematical physics
since 4/2005	**University of Vienna, Austria** Postdoc

Research Interests

Theoretical High Energy Physics and Mathematical Physics.
Quantum Field Theory, Noncommutative Geometry, Quantum Groups, Conformal Field Theory, String Theory.

Fellowships

fall 94 – spring 95	The Regents Fellowship of the University of California, Berkeley
fall 90	Exchange fellowship with University of Notre Dame, USA
fall 88 - spring 92	several “Leistungsstipendien”, Universität Innsbruck

Other awards

International Mathematical Olympiad Finland 1985: third prize

International Mathematical Olympiad Poland 1986: second prize

Student supervision

Andrea Pollock-Narayanan (Diploma work, LMU Munich)

Frank Meyer (Diploma and Doctoral work, LMU Munich)

Andreas Sykora (Doctoral work)

Wolfgang Behr (Doctoral work)

Member of the doctoral thesis commission for Marco Maceda (Université de Paris-Sud, Orsay)

Organizational activities

Coorganizer for the workshop
Noncommutativity: From Mathematics to Phenomenology
Bayrischzell, 2.-5. May 2003 (Germany).

Coorganizer for
Bayrischzell workshop on noncommutativity and physics
Bayrischzell, 4.-7. June 2004 (Germany).

Director of
III Summer School in Modern Mathematical Physics
20-31 August 2004, Zlatibor, Serbia and Montenegro

Coorganizer for
Bayrischzell workshop on noncommutativity and physics
Bayrischzell, 6.-9. May 2005 (Germany).

List of publications during the past 5 years

Recent preprints

[1] J. Pawelczyk, H. Steinacker, R. Suszek, "Twisted WZW Branes from Twisted REA's.". [hep-th/0412146]

Publications in refereed journals

[2] **W. Behr, F. Meyer, H. Steinacker, "Gauge Theory on Fuzzy $S^2 \times S^2$ and Regularization on Noncommutative $\mathbb{R}^4$". accepted for publication in *JHEP*;** [hep-th/0503041]

[3] **H. Steinacker, "A non-perturbative approach to non-commutative scalar field theory." *JHEP* 0503 (2005) 075;** [hep-th/0501174]

[4] **H. Grosse, H. Steinacker, "Finite Gauge Theory on Fuzzy $\mathbb{C}P^2$". *Nucl. Phys.* B 707 vol.1-2 (2005) 145-198;** [hep-th/0407089]

[5] U. Carow-Watamura, H. Steinacker, S. Watamura, "Monopole Bundles over Fuzzy Complex Projective Spaces". *J. Geom. Phys.* **54**, Nr. 4 (2005) 373-399; [hep-th/0404130]

[6] F. Meyer, H. Steinacker, "Gauge Field Theory on the $E_q(2)$-covariant Plane". *Int.J.Mod.Phys.* **A19** (2004) 3349-3376; hep-th/0309053

[7] **H. Steinacker, "Quantized Gauge Theory on the Fuzzy Sphere as Random Matrix Model" *Nucl. Phys.* B679, vol 1-2 (2004) 66-98;** [hep-th/0307075]

[8] J. Pawelczyk, H. Steinacker, "Algebraic brane dynamics on SU(2): excitation spectra" *JHEP* **0312** (2003) 010 (20pp.); [hep-th/0305226]

[9] **J. Pawelczyk, H. Steinacker, "A quantum algebraic description of D-branes on group manifolds" *Nucl.Phys.* B638 (2002) 433-458;** [hep-th/0203110]

[10] J. Pawelczyk, H. Steinacker, "Matrix description of D-branes on 3-spheres." *JHEP* **0112** (2001) 018; [hep-th/0107068]

[11] H. Grosse, M. Maceda, J. Madore, Harold Steinacker, "Fuzzy Instantons". *Int. Jour. Mod. Phys. A*, **17 No. 15** (2002) 2095-2111; [hep-th/0107068]

[12] **C-S. Chu, J. Madore, H. Steinacker, "Scaling Limits of the Fuzzy Sphere at one Loop." *JHEP* 0108 (2001) 038.** [hep-th/0106205]

[13] H. Grosse, J. Madore, H. Steinacker, "Field Theory on the q-deformed Fuzzy Sphere II." *J. Geom. Phys.* **43**, (2002) 205-240; [hep-th/0103164]

[14] **H. Grosse, J. Madore, H. Steinacker, "Field Theory on the q-deformed Fuzzy Sphere I"** ***J. Geom. Phys*** **38 (2001) 308-342.** [hep-th/0005273]

[15] G. Fiore, H. Steinacker, J. Wess, "Unbraiding the braided tensor product." *J.Math.Phys.* **44** (2003) 1297-1321; [math.QA/0007174]

[16] **H. Steinacker, "Quantum Anti-de Sitter space and sphere at roots of unity."** ***Adv. Theor. Math. Phys.*** **4, Nr. 1 (2000)** [hep-th/9910037]

[17] J. Madore, H. Steinacker, "Propagator on the h-deformed Lobachevsky plane." *J. Phys. A: Math. Gen.* **33**, 327 (2000). [math-QA/9907023]

[18] **H. Steinacker, "Unitary Representations of Noncompact Quantum Groups at Roots of Unity."** ***Rev. Math. Phys.*** **13: 1035, 2001.** [math.QA/9907021]

[19] **S. Cho, R. Hinterding, J. Madore, H. Steinacker, "Finite Field Theory on Noncommutative Geometries."** ***Int.J.Mod.Phys. D9 (2000) 161-199.*** [hep-th/9903239].

Conference proceedings

[20] "Non-commutative gauge theory on fuzzy $\mathbb{C}P^2$." Proceedings to the III Summer school on modern mathematical physics, Zlatibor

[21] "Gauge Theory on the Fuzzy Sphere and Random Matrices." Proceedings to the 9th adriatic meeting on Particle Physics and the Universe, D

[22] "Fuzzy D-branes on Group Manifolds." To appear in Proceedings of Group 24, XXIV International Colloquium on Group Theoretical Methods in Physics, July 15-20, Paris.

[23] "Quantum Field Theory on the q-deformed Fuzzy Sphere" Proceedings of the XXXVII Karpacz Winter School "New Developments in Fundamental Interactions Theories" (February 6-15, 2000, Karpacz, Poland).

[24] "Aspects of the q-deformed Fuzzy Sphere" Proceedings to the Euroconference "Brane New World and Noncommutative Geometry" in Villa Gualino, Torino, Italy, October 2 - 7, 2000.

Habilitation Thesis:

[25] "Field theoretic models on covariant quantum spaces." Habilitationsschrift, Ludwig-Maximilians Universität München, Oktober 2002

Ph.D. Thesis:

[26] “Quantum Groups, Roots of Unity and Particles on quantized Anti-de Sitter Space” Dissertation, University of Berkeley, USA; Mai 1997.

Diploma Thesis:

[27] “Das Spin–bag modell der Hochtemperatur – Supraleitung.” Diplomarbeit, Universität Innsbruck; März 1992.