

Horizontal Product Differentiation: Disclosure and Competition *

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Abstract

This paper studies the incentives to disclose horizontal product attributes in an environment where firms compete. With competition, two elements may play an important role, namely whether (i) firms can disclose only their own product characteristics or also those of their competitors, and whether (ii) competitors can react with their pricing decisions to the type of information disclosed. In all the possible cases that can arise, full revelation is an equilibrium outcome. More importantly, full disclosure is generically the unique equilibrium outcome when (i) advertising is comparative and (ii) prices are also advertised, that is, announced simultaneously with the information about product characteristics. When firms either do not engage in comparative advertising or do not advertise prices, many other nondisclosure equilibria exist.

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1 Introduction

In a large number of markets, sellers have important information about product attributes that are not publicly observable. In many instances, however, firms have the option of voluntarily disclosing this information in a credible and verifiable manner through a variety of means such as independent third party certification, labeling, rating by industry associations (or government agencies) and through informative advertising. Whether or not firms reveal their private information to consumers may have important consequences for the market outcomes in terms of prices and characteristics of the products that are produced and exchanged.

There is a large literature dealing with the question of whether firms have appropriate incentives to disclose information about their products. Most of this literature deals with this issue in the context of vertical product differentiation, where different firms sell different qualities. In this context, the well-known unraveling argument¹ establishes that a firm whose product quality is actually better than the average quality in the market has a positive incentive to voluntarily disclose the quality of its product to buyers. This then induces every firm whose quality is above the average *undisclosed* quality to also disclose. The unraveling argument results in a situation where all private information about quality should be revealed through voluntary disclosure. Observed nondisclosure is then explained in terms of "disclosure frictions", such as disclosure costs, consumers not understanding the information that is disclosed, etc. (see, e.g., Grossman and Hart, 1980, Jovanovic, 1982, and Fishman and Hagerty, 2003). Alternatively, Janssen and Roy (2014) show that nondisclosure can also be explained by a combination of *market competition* and the availability of *signaling* as an alternative means (to disclosure) of communicating private information.

Recently, Sun (2011), Koessler and Renault (2012) and Celik (2014) have analyzed the incentives of firms to disclose their product characteristics when *horizontal product differentiation* is the only or main dimension of differentiation. All these papers are set in a monopoly context. In a very general model, that includes vertical product differentiation as a special case, Koessler and Renault (2012) generalize the above unravelling result. They provide a necessary and sufficient condition on the structure of the match between consumer's tastes and firm's product characteristic for the fully revealing equilibrium to be the unique equilibrium outcome. They also show that when this condition does not hold, as is the case in the Hotelling model of this paper, a fully revealing equilibrium still exists as long as the firm type is not correlated with the consumer type, but that in that case the equilibrium is typically not unique. Sun (2011) also considers the setting that allows for both vertical

¹See, Viscusi (1978), Grossman (1981), Milgrom (1981), Jovanovic (1982).

and horizontal dimension of product differentiation. She finds that seller types with unfavorable horizontal attributes (towards the extreme points of the product line) do not have an incentive to disclose. More generally, her results imply that if either full disclosure of both horizontal and vertical attributes or no disclosure at all are the only possible reporting strategies, a seller with private information about both attributes may not want to disclose quality even if it is high. Celik (2014) considers horizontal product differentiation alone and shows that the amount of information disclosure critically depends on the strength of the buyers' preference for their ideal attribute. If buyers have very strong preferences for particular product varieties, then there exists an equilibrium in which the seller fully reveals the variety that he produces. Otherwise, the seller only partially reveals the variety. Moreover, the set of fully revealed varieties monotonically shrinks from all to (almost) none as the buyer's preference for her ideal attribute becomes weaker.

In this paper, we study the extent to which the findings of possible non-disclosure of horizontal product attributes generalize to an environment where firms compete. With competition, two additional elements may play an important role. First, with more than one firm in the market, firms can disclose only their own product characteristics or also those of their competitors. In the advertising literature, the former refers to the case of noncomparative advertising, while the latter refers to the case of comparative advertising. Second, with more than one firm in the market, competitors may or may not have the ability to react with their pricing decisions to the type of information disclosed. In the advertising literature, this refers to the situation where advertising includes information about price or where prices are set at a later stage. Thus, under competition, four different cases are possible: (i) noncomparative price advertising, (ii) noncomparative nonprice advertising, (iii) comparative and price advertising, and (iv) comparative nonprice advertising.

In all these cases, full revelation is an equilibrium outcome. Our main result, however, is that when advertising is comparative and includes price (comparative and price advertising), and only in that case, full disclosure is essentially the *unique* equilibrium outcome as it occurs with probability one. That is, competition stimulates firms to reveal their private information when firms have the ability to inform consumers not only about their own product characteristics but also about those of their competitors and when price is advertised simultaneously with product information. In all the other cases, there are, however, equilibria where large sets of firm types do not fully disclose their product characteristics. Under comparative and price advertising, nondisclosure equilibria also exist, but they require that the locations of the two firms have a very special relation to each other, which – from an ex ante perspective – occurs with zero probability.

The model we consider has two firms located on a Hotelling line, where each particular location represents the variety of the product. Location is known to both firms, but not to consumers.² The case where rival firms know each others' *vertical* characteristic is studied by, e.g., Board (2009) and Hotz and Xiao (2013). This type of literature, and our paper, are relevant for markets where firms have been active for some time and have the ability (and due to the frequent interaction also the incentives) to learn the features of the product produced by a competitor.

The timing of events is different in case of nonprice and price advertising. Under nonprice advertising, the two firms first simultaneously choose a message informing consumers about their own product characteristic (noncomparative advertising) or about product characteristics of both firms (comparative advertising), and then they simultaneously choose prices. Under price advertising, the message and price decisions are made simultaneously. We assume that firms cannot lie. That is, the true location(s) should be consistent with the messages that are chosen. One way to think about this *grain-of-truth* assumption is that information is verifiable and that there is a large fine for providing information that turns out to be false.³ Firms can either send a rather vague message, indicating that their location(s) is somewhere on the product line, as one extreme, or a much more precise message, indicating the exact location(s), as the other extreme, or anything in between. As regards prices, the key difference between the case of nonprice and price advertising is that under nonprice advertising, the pricing strategy of each firm can depend on the message sent by the competitor, while under price advertising, it cannot. Consumers observe the messages and the prices of both firms and decide where to buy the product. Given the information received, they update their beliefs about the locations of the two firms and buy from the firm that they expect to have the best fit with their preferences.

Having described the set-up, we can now discuss the results and explain the intuition as follows. The fact that full disclosure is an equilibrium outcome in all settings, follows from the independence of firms' locations and consumers' tastes. Indeed, as in Proposition 1 of Koessler and Renault (2012), when firms and consumers' types are independent, a firm's profit does not depend on its true location, but merely on price and consumer *beliefs* about the location. Thus, given the grain-of-truth assumption one can always construct out-of-equilibrium beliefs of consumers that "punish" a deviation from full disclosure. In fact, we find that full disclosure is generically the *unique* equilibrium outcome when advertising is comparative and includes price. To understand this result, consider that

²The competitive disclosure literature has also considered markets where firms do not know each others' type (see, e.g., Daughety and Reinganum, 2007; Caldieraro, Shin and Stivers, 2008, and Janssen and Roy, 2011).

³In fact, the assumption is in line with regulations concerning advertising and other disclosure mechanisms that require truthful information provision.

two conditions must hold for a nondisclosure equilibrium to exist in this case. First, as the competitor cannot react to a firm's deviation to full disclosure, it should be the case that where two types send the same equilibrium message, they cannot increase their market share by fully disclosing and keeping the same price. Then, as an increase in market share for one firm implies a decrease in market share for the other, and vice versa, this requirement implies that the locations of the nondisclosing firms should be such that if they fully disclose and keep prices at the equilibrium levels, their resulting market shares are exactly the same. Second, as firms can also deviate from the equilibrium prices, the requirement that this type of deviation should not increase profits either imposes a set of further constraints. Together, these constraints are such that they can only be fulfilled for a set of locations that is nongeneric in the full set of possible locations.

This result highlights the role of competition in the incentives of firms to disclose their horizontal product attributes. Indeed, in a monopoly setting, the analogous model of Celik (2014) reveals the existence of a large set of full and partial nondisclosure equilibria. However, even with competition, the essential uniqueness of a fully revealing equilibrium outcome turns out to be not robust to alternative model specifications. First, if prices are set not simultaneously with the messages but at a later stage (comparative *nonprice* advertising), then a competitor can choose its pricing strategy to "punish" the firm that deviates from a nondisclosing message. This sustains nondisclosure in equilibrium. In fact, using an argument similar to that in Proposition 2 of the monopoly analysis in Koessler and Renault (2012), we characterize a broad variety of (non)disclosure equilibria that emerge in this case. We find that any strategy profile that provides each firm with a pay-off that is at least as high as its full disclosure pay-off, and that does not allow imitation between firm types can be sustained in equilibrium. Second, if firms do not have the ability to reveal the location of the competitor (*noncomparative* advertising), then any deviation from a nondisclosure strategy can be discouraged as consumer out-of-equilibrium beliefs about the location of the rival firm continue to play a role and these beliefs can be chosen so as to "punish" the deviation. For example, if consumers believe that the (undisclosed) location of the competitor is better than the location of the deviating firm for most consumers, then no firm has an incentive to deviate from a nondisclosure strategy. In this case of noncomparative advertising (both price and nonprice), we show by construction that a large range of outcomes – from full disclosure to full nondisclosure – can be sustained in equilibrium.

In a series of papers, Anderson and Renault (2006, 2009) consider a similar framework and study the incentives of firms to disclose their product characteristics through advertising. They find that if products have both horizontal and vertical attributes and if qualities of firms' products are known

and sufficiently different from one another, only the firm with the lowest quality reveals its horizontal characteristic. The better quality firm remains silent, as disclosure would induce it to set a lower price in order to retain the consumers who like its rival more. If firms' product qualities are identical, both firms reveal their horizontal characteristics even under noncomparative advertising, and being able to make comparative statements does not change the equilibrium outcomes in their setting. Our paper differs from Anderson and Renault (2006, 2009) in two important respects. First, in Anderson and Renault (2009), firms can only fully disclose their horizontal characteristic or stay silent. In contrast, we consider a model where firms can send *any* message concerning their product characteristics that satisfies the grain-of-truth assumption. Second, and more importantly, Anderson and Renault (2006, 2009) do not analyze their model as a game with private information where out-of-equilibrium beliefs are important, while we do. This explains why our results also differ and full disclosure is the unique equilibrium outcome under comparative and price advertising, but not in the other cases. To see where out-of-equilibrium beliefs are important, consider a potential equilibrium where no type of firm discloses and all types set a high price. In a model that is not a fully-fledged signaling game, Anderson and Renault (2009) argue that this cannot be an equilibrium because of a standard Bertrand-like undercutting argument. However, in a fully-fledged signaling game, undercutting the candidate equilibrium price is formally an out-of-equilibrium action and therefore, consumers should form beliefs about the firms' product characteristics. If consumers believe that the undercutting firm has relatively disadvantageous product characteristics, the firm's demand would be lower than if it had not undercut, and therefore, the firm would not have an incentive to undercut in the first place. Therefore, by considering consumer out-of-equilibrium beliefs, in all but one case of our model we obtain a much larger set of equilibria than Anderson and Renault (2009), and comparative advertising makes a difference in case when price is also advertised in our model.

Standard refinements such as the Intuitive Criterion (Cho and Kreps, 1987) or D1 (Cho and Sobel, 1990) do not rule out these multiple nondisclosure equilibria as the profits of all firm types only depend on prices and consumer beliefs about firms' locations rather than on the actual locations. In the discussion section, we also briefly point out that if one restricts attention to special out-of-equilibrium beliefs, where consumers do not discriminate between different types of firms as soon as after observing the out-of-equilibrium action they have no reason to do so, then only the fully revealing outcome can be sustained in equilibrium. We do not necessarily argue that such restrictive out-of-equilibrium beliefs are natural to impose, particularly given that firms are informed about product characteristics of each other, and hence, can, in principle, use deviations to hide or signal some

information about their relative locations. By briefly discussing these restrictive out-of-equilibrium beliefs, we mainly want to highlight the role of out-of-equilibrium beliefs in creating nondisclosure equilibria and the fact that in this disclosure game, firms' pay-offs do not depend on true locations, but only on consumer beliefs about these locations.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 shows that fully revealing equilibria exist in all cases. Section 4 discusses the results on comparative and price advertising where the full disclosure equilibrium is essentially unique. Then Section 5 shows why this uniqueness result is not robust and many nondisclosure equilibria exist in the other cases. Section 6 concludes with a discussion, while proofs are contained in the Appendix.

2 Model

Consider a horizontally differentiated duopoly, where the variety of the good produced by each firm is represented by a particular location on the unit interval. Let x_i denote the variety produced by firm i and $x_i \in [0, 1]$, $i = 1, 2$. We focus on firms' disclosure policy and regard these varieties as given for both firms. We consider markets where firms know each others' variety, but consumers are unaware of the specific varieties produced by the firms. One way to think about this is that it requires resources to research the product characteristics of a firm, and that rival firms are better equipped or have more incentives to do this than consumers. In the following, we refer to x_1 and x_2 as locations of firms 1 and 2, respectively, and we refer to the pair of locations, $(x_1, x_2) \in [0, 1] \times [0, 1]$, as a type of each firm. Notice that because each firm knows its own location and the location of the competitor, the type of a firm is a two dimensional object. Throughout the paper, the first position in the pair (x_1, x_2) stands for the location of firm 1, and the second position – for the location of firm 2. Production costs do not depend on firms' locations and without loss of generality are set equal to zero.

The demand side of the economy is represented by a continuum of consumers. Each consumer has a preference for the ideal variety of the good that she would like to buy, denoted by λ . The value of λ , or consumers' location on $[0, 1]$, follows a uniform distribution.⁴ A consumer's net utility from buying variety x_i at price P_i , $i = 1, 2$, is $v - t(\lambda - x_i)^2 - P_i$, where v is the gross utility of a consumer when the variety of the good, x_i , matches with her ideal variety, λ , perfectly (i.e., when $x_i = \lambda$) and t measures the degree of disutility a consumer incurs when x_i and λ differ from each other.⁵ We

⁴This specification with a continuum of consumers whose preferences for variety are uniformly distributed over the unit interval, is identical to the specification with a single consumer who has a privately known taste for a variety drawn from the uniform density function defined over $[0, 1]$.

⁵Using common terminology, $t(\lambda - x_i)^2$ is the *transportation cost* of a consumer located at λ associated with buying

assume that v is sufficiently large so that the market is fully covered. Each consumer then chooses to buy the good from the firm where her expected utility is maximized. The consumer has unit demand and if she buys from firm i , then firm i 's payoff from the transaction is P_i ; otherwise, the payoff of firm i is zero.

The timing of the *nonprice* advertising games is as follows.⁶ At stage 0, Nature *independently* selects location x_1 for firm 1 and x_2 for firm 2 from a density function $f(x)$ that is non-atomic on $[0, 1]$.⁷ The locations are known to both firms, but not to consumers. At stage 1, firms send a costless message $M_i \subseteq [0, 1]$, $i = 1, 2$, about their own location or a message $M_i \subseteq [0, 1] \times [0, 1]$ about the locations of both firms; M_i is assumed to be compact. The former refers to the case of *noncomparative advertising*, where a firm can only provide information about its own variety. The latter refers to the case of *comparative advertising*, where a firm provides information about the varieties of both firms. We will examine each of these two cases separately, but unless explicitly stated otherwise, the same notation and definitions apply throughout. Notice that in the different cases $M_i = [0, 1]$ or $M_i = [0, 1] \times [0, 1]$ can be interpreted as "no message at all", or full nondisclosure of information by firm i . Messages have to contain a grain of truth in the sense that $x_i \in M_i$ for $i = 1, 2$ if advertising is noncomparative and $(x_1, x_2) \in M_i$ for $i = 1, 2$ if advertising is comparative. This means that firms cannot lie about their locations, but they can be more or less specific about the true locations. In the following we will refer to this assumption as the *grain-of-truth assumption*. After the messages have been sent, at stage 2, firms simultaneously set prices. Finally, at stage 3, consumers observe the messages and the prices of the two firms and decide where to buy. At the end of the game, the payoffs of all players – firms and consumers – are realized. All aspects of the game are common knowledge.

The only difference of the *price* advertising games is that prices are chosen simultaneously with the messages about locations, and the separate pricing stage is eliminated. The rest is identical to the nonprice advertising games.

As a solution concept, we use a perfect Bayesian equilibrium where beliefs of consumers off-the-equilibrium path, albeit arbitrary, are identical across consumers. Following Fudenberg and Tirole (1991), we will refer to this equilibrium as a *strong perfect Bayesian equilibrium*. To define it formally for the nonprice advertising games, we specify the strategy spaces as follows.⁸ The reporting

from a firm located at x_i .

⁶There are two such games in this model: with comparative and noncomparative messages about firms' locations.

⁷The probability measure function is called non-atomic if it has no *atoms*, i.e., measurable sets which have positive probability measure and contain no set of smaller but positive measure.

⁸For the *price* advertising games, all strategies are defined in the same way, apart from the pricing strategy. The pricing strategy of firm i is $p_i(x_i, x_j)$ and it is *not* conditional on the messages.

strategy of firm i is denoted by $m_i(x_i, x_j)$, where the image of m_i belongs to all subsets of $[0, 1]$ such that $x_i \in m_i$ (for noncomparative advertising), or to all subsets of $[0, 1] \times [0, 1]$ s.t. $(x_1, x_2) \in m_i$ (for comparative advertising). The pricing strategy of firm i is denoted by $p_i(x_i, x_j | M_i, M_j)$, where the messages sent by the two firms are M_i and M_j , respectively. Similarly, let the vector $b(\lambda, M_i, M_j, P_i, P_j)$ describe the buying strategy of a consumer with preferred variety λ , where $b = (1, 0)$ if the consumer buys the good from firm 1 and $b = (0, 1)$ if she buys from firm 2. Finally, let $\mu_i(z | M_i, M_j, P_i, P_j)$ be the probability density that consumers assign to $x_i = z$ when the firms send messages M_i, M_j and set prices P_i, P_j . A strong perfect Bayesian equilibrium is then defined as follows.

Definition A strong perfect Bayesian equilibrium of the nonprice advertising games⁹ is a set of reporting and pricing strategies $m_1^*, m_2^*, p_1^*, p_2^*$ of the two firms, strategy b^* of a consumer, and the probability density functions μ_1^*, μ_2^* which satisfy the following conditions:

- (1) For all M_1, M_2, P_1 and P_2 , b^* is a consumer's best buying decision as defined below:

$$b(\lambda, M_1, M_2, P_1, P_2) = \begin{cases} (1, 0) & \text{if } E(v - t(\lambda - x_1)^2 - P_1 | \mu_1(x_1 | M_1, M_2, P_1, P_2)) \geq \\ & E(v - t(\lambda - x_2)^2 - P_2 | \mu_2(x_2 | M_1, M_2, P_1, P_2)) \\ (0, 1) & \text{if } E(v - t(\lambda - x_2)^2 - P_2 | \mu_2(x_2 | M_1, M_2, P_1, P_2)) \geq \\ & E(v - t(\lambda - x_1)^2 - P_1 | \mu_1(x_1 | M_1, M_2, P_1, P_2)) \end{cases} \quad (1)$$

- (2) Given (1) and given the messages sent by the two firms and the price set by the competitor, p_i^* is the price that maximizes the profit of firm i , $i = 1, 2$.

- (3) Given (1), (2) and given the message sent by the competitor, m_i^* is the message that maximizes the profit of firm i , $i = 1, 2$, subject to the grain-of-truth assumption.

- (4) For all M_1, M_2, P_1 and P_2 , a consumer updates his or her beliefs, μ_i , regarding the locations of the firms in the following way:¹⁰

⁹For the *price* advertising games, parts (2) and (3) of this definition should be merged into one.

¹⁰Note that Bayes' rule cannot be applied when M_i is discrete. For example, in case of noncomparative advertising $M_i = \{y, z\}$, then both events, $x_i = y$ and $x_i = z$ have ex-ante zero probability. In this case, updating proceeds as follows:

$$\mu_i(z | M_1, M_2, P_1, P_2) = \lim_{\varepsilon \rightarrow 0} \frac{F(z + \varepsilon) - F(z)}{F(z + \varepsilon) - F(z) + F(y + \varepsilon) - F(y)}$$

Using l'Hôpital's rule,

$$\mu_i(z | M_1, M_2, P_1, P_2) = \lim_{\varepsilon \rightarrow 0} \frac{f(z + \varepsilon)}{f(z + \varepsilon) + f(y + \varepsilon)} = \frac{f(z)}{f(z) + f(y)}$$

- (i) according to Bayes' rule on the equilibrium path,
- (ii) arbitrarily off the equilibrium path.

All consumers have identical beliefs on and off the equilibrium path.

Part (1) of the definition states that for any observed messages and prices, a consumer buys a unit of the product from the firm, where her expected net utility, given the updated beliefs, is maximized. Each firm rationally anticipates the best response of consumers to any given messages and prices, and chooses the price and message that maximize its profit. This is stated in parts (2) and (3). Finally, part (4) claims that consumers update beliefs about locations of the two firms using Bayes' rule for any M_1, M_2, P_1 and P_2 that occur with positive density along the equilibrium path, and that beliefs off the equilibrium path are arbitrary but identical across consumers. Note that the grain-of-truth assumption implies that in the case of noncomparative advertising, consumers should only assign positive probability to the locations of a firm that are included in its message. Similarly, in the case of comparative advertising, a positive density should only be put on the locations that belong to the intersection of the two messages. This intersection is not empty due to the same assumption.

In this model, the *indifferent* consumer – with the ideal variety $\hat{\lambda}$ – obtains the same expected net utility when buying from either of the two firms, given the observed set of messages and prices. Consumers with preferred varieties below $\hat{\lambda}$ buy from the firm that is perceived to be located furthest to the left and all the others buy from the other firm. Thus, the indifferent consumer is defined by

$$v - tE\left((\hat{\lambda} - x_1)^2|\mu_1\right) - P_1 = v - tE\left((\hat{\lambda} - x_2)^2|\mu_2\right) - P_2. \quad (2)$$

In this expression, $tE\left((\hat{\lambda} - x_i)^2|\mu_i\right)$, $i = 1, 2$, is the expectation of the transportation cost of the indifferent consumer associated with buying from firm i , conditional on consumer beliefs.

We solve this equality for $\hat{\lambda}$. Note that

$$\begin{aligned} E\left((\hat{\lambda} - x_i)^2|\mu_i\right) &= \hat{\lambda}^2 + E(x_i^2|\mu_i) - 2\hat{\lambda}E(x_i|\mu_i) \\ &= \hat{\lambda}^2 + \text{var}(x_i|\mu_i) + E^2(x_i|\mu_i) - 2\hat{\lambda}E(x_i|\mu_i) \\ &= \left[\hat{\lambda} - E(x_i|\mu_i)\right]^2 + \text{var}(x_i|\mu_i). \end{aligned} \quad (3)$$

Thus, the quadratic term in the utility function of consumers (like any convex transportation costs) implies that a consumer dislikes uncertainty about the variety of the good and given two messages with the same conditional mean, favors the one with smaller variance. The quadratic formulation is used in order to simplify the analysis and to guarantee the existence of a pricing equilibrium. Using

(3) in (2) we obtain:

$$\begin{aligned}\widehat{\lambda}^2 + \text{var}(x_1|\mu_1) &+ E^2(x_1|\mu_1) - 2\widehat{\lambda}E(x_1|\mu_1) + \frac{P_1}{t} = \\ &= \widehat{\lambda}^2 + \text{var}(x_2|\mu_2) + E^2(x_2|\mu_2) - 2\widehat{\lambda}E(x_2|\mu_2) + \frac{P_2}{t},\end{aligned}$$

and so, the ideal variety of the indifferent consumer is equal to:

$$\widehat{\lambda} = \frac{1}{2} \frac{\frac{P_2 - P_1}{t} + \text{var}(x_2|\mu_2) - \text{var}(x_1|\mu_1)}{E(x_2|\mu_2) - E(x_1|\mu_1)} + \frac{1}{2} (E(x_1|\mu_1) + E(x_2|\mu_2)). \quad (4)$$

In (4) the expected locations of the two firms are assumed to be different. When $\widehat{\lambda} \notin [0, 1]$, the indifferent consumer is not well defined and either all consumers are indifferent or none is.

For convenience, below we use subscripts L and R for the firm with the furthest left and furthest right perceived location, respectively, so that $E(x_L|\mu_L) < E(x_R|\mu_R)$. Then given that $\widehat{\lambda}$ is actually the consumer demand of the firm that is perceived as furthest left, the profits of the two firms can be written as

$$\pi_L = P_L \left(\frac{1}{2} \frac{\frac{P_R - P_L}{t} + \text{var}(x_R|\mu_R) - \text{var}(x_L|\mu_L)}{E(x_R|\mu_R) - E(x_L|\mu_L)} + \frac{1}{2} (E(x_L|\mu_L) + E(x_R|\mu_R)) \right), \quad (5)$$

$$\pi_R = P_R \left(1 - \frac{1}{2} \frac{\frac{P_R - P_L}{t} + \text{var}(x_R|\mu_R) - \text{var}(x_L|\mu_L)}{E(x_R|\mu_R) - E(x_L|\mu_L)} - \frac{1}{2} (E(x_L|\mu_L) + E(x_R|\mu_R)) \right), \quad (6)$$

where π_L (π_R) is the profit of the firm with the furthest left (right) perceived location.

Note that apart from the prices, firms' expected demands and profits only depend on *expected* locations and on the precision of the messages about these locations, but – and this is important – *not* on actual locations. This implies that consumer beliefs can be used to "punish" deviating firms.

3 Full disclosure equilibria

We first discuss the nature of market outcomes under full disclosure and show that with comparative and noncomparative advertising and with price and nonprice advertising, full disclosure is an equilibrium outcome. This result is interesting in its own right, but we also use the full disclosure outcome in later sections to analyze the gains from deviating.

Full disclosure implies that in all expressions above $E(x_i|\mu_i) = x_i$, $\text{var}(x_i|\mu_i) = 0$ and profits of firm 1 and 2 are functions of exact locations x_1 , x_2 . In particular, following the notation that $x_L < x_R$ (the analogue of $E(x_L|\mu_L) < E(x_R|\mu_R)$), firms' profits are given by:

$$\pi_L(x_L, x_R) = P_L \left(\frac{1}{2t} \frac{P_R - P_L}{x_R - x_L} + \frac{1}{2} (x_L + x_R) \right), \quad (7)$$

$$\pi_R(x_L, x_R) = P_R \left(1 - \frac{1}{2t} \frac{P_R - P_L}{x_R - x_L} - \frac{1}{2} (x_L + x_R) \right). \quad (8)$$

The function π_i , $i = L, R$, is a strictly concave, quadratic function of P_i . Therefore, if firms reveal their locations *precisely*, so that consumers have correct beliefs about the locations irrespective of price deviations, the profit-maximization problem of each firm is well-defined and the first-order conditions determine the prices at which the profits are maximized:

$$\begin{aligned} -\frac{P_L}{t(x_R - x_L)} + \frac{1}{2} \left(\frac{P_R}{t(x_R - x_L)} + x_L + x_R \right) &= 0, \\ 1 - \frac{P_R}{t(x_R - x_L)} - \frac{1}{2} \left(-\frac{P_L}{t(x_R - x_L)} + x_L + x_R \right) &= 0. \end{aligned}$$

Solving these equations yields equilibrium prices under full disclosure with precise messages:

$$P_L(x_L, x_R) = \frac{1}{3}t(x_R - x_L)(2 + x_L + x_R), \quad (9)$$

$$P_R(x_L, x_R) = \frac{1}{3}t(x_R - x_L)(4 - x_L - x_R). \quad (10)$$

Plugging expressions (9) – (10) for prices into the profit functions of the two firms leads to:

$$\pi_L(x_L, x_R) = \frac{t}{18}(x_R - x_L)(2 + x_L + x_R)^2, \quad (11)$$

$$\pi_R(x_L, x_R) = \frac{t}{18}(x_R - x_L)(4 - x_L - x_R)^2. \quad (12)$$

In what follows, we will sometimes refer to prices in (9) – (10) as *full revelation prices* and to profits in (11) – (12) as *full revelation profits* based on (x_L, x_R) . Note that full revelation prices and profits are strictly positive for any $x_L < x_R$ and they are zero for $x_L = x_R$.¹¹

Note that under comparative advertising, these full revelation prices and profits are actually the only possible prices and profits in *any* full disclosure equilibrium. Indeed, as any type can reveal the locations of both firms, and prices in (9) – (10) are equilibrium prices given true locations, the only way to rule out profitable deviations by either firm is to have equilibrium prices as defined in (9) – (10). Under noncomparative advertising, this argument does not hold as any firm can only fully reveal its own location but not that of the other firm. This leads to the possibility of other prices in a full disclosure equilibrium for types that send imprecise messages about their locations. In either case though, full disclosure equilibria exist, and one of them is an equilibrium with precise messages and full revelation prices as in (9) – (10).

Proposition 1. *Whether or not advertising is comparative and whether or not prices are advertised, full disclosure is always an equilibrium outcome.*

The proof is a straightforward adaptation of Proposition 1 in Koessler and Renault (2012) and is available upon request. In a nutshell, the idea is the following. Suppose that both firms of any

¹¹The zero price and profit outcome in case when $x_L = x_R$ is a result of a standard Bertrand-type argument, as when the varieties produced by two firms are exactly the same, consumers buy from the firm with the lowest price.

type disclose their location (or type) precisely and choose full revelation prices.¹² If advertising is comparative, then the precise message of each firm indicates the locations of both firms, and hence, unilateral deviations to a nondisclosing strategy do not change consumer beliefs about the locations. Then as prices in (9) – (10) are equilibrium prices given true locations, no firm can benefit from deviation. If advertising is noncomparative, then the precise message of each firm indicates only its own location but not the rival’s location. Then if a firm deviates from the fully disclosing strategy and sends an imprecise message about its location, consumer out-of-equilibrium beliefs become important. Given the grain-of-truth assumption, that messages have to contain the true location, and the fact that firms’ pay-offs depend on expected locations and not on true locations, one can construct consumer out-of-equilibrium beliefs in such a way that given a certain message, the deviating pay-off of a firm is not larger than its equilibrium pay-off. For example, one can have out-of-equilibrium beliefs that assign probability one to such location in the deviating message, where the deviating firm’s full revelation profit, given the location and price of the competitor, is minimized. As the true location of the deviating firm is one of the locations in the message and as prices in (9) – (10) are equilibrium prices given true locations, it is clear that no firm has an incentive to deviate. This argument does not depend on whether the the prices are chosen after or simultaneously with the messages.

Note that the profit of firm L in (11) is *decreasing* in x_L , while the profit of firm R in (12) is *increasing* in x_R . Indeed,

$$\begin{aligned}\frac{\partial \pi_L}{\partial x_L} &= (2 + x_L + x_R) \left(\frac{t}{18} x_R - \frac{3t}{18} x_L - \frac{t}{9} \right) < 0, \\ \frac{\partial \pi_R}{\partial x_R} &= (4 - x_L - x_R) \left(\frac{t}{18} x_L - \frac{3t}{18} x_R + \frac{2t}{9} \right) > 0,\end{aligned}$$

where the signs of the derivatives are implied by the fact that $0 \leq x_L, x_R \leq 1$. It is also easy to see that π_L is increasing in x_R , while π_R is decreasing in x_L . These findings are consistent with the argument in the standard Hotelling model of location choice. Essentially, firms want to be located as far away as possible from each other due to the maximal differentiation principle.

In the analysis that follows we consider the possibility of other, not disclosing and partially disclosing equilibria, focusing on the conditions that can lead to nondisclosure.

¹²If prices are chosen after the messages, then suppose that full revelation prices are chosen *on the equilibrium path*.

4 Comparative and price advertising: unique full disclosure equilibrium

Let us begin our characterization of the full equilibrium set by considering the case of *comparative and price advertising*. In this case, firms choose prices simultaneously with the content of their messages about locations, and the message of each firm specifies not just the set of its own possible locations, but the set of possible locations of both firms.

Our key finding for this scenario is that full disclosure is generically the unique equilibrium outcome. That is, even though nondisclosure equilibria exist, in any such nondisclosure equilibrium, the union of all sets of types for which there is pooling has zero measure. Therefore, firms' product information is always fully revealed with probability 1.

Below we will formally state and discuss this uniqueness result and after that provide an example that proves the existence of a partially revealing equilibrium. To this end, we first introduce some new notation. Denote by d_U an upward sloping diagonal in the $[0, 1] \times [0, 1]$ square (the 45-degree line) and by d_D – the downward sloping diagonal, perpendicular to d_U . Thus, $x_1 = x_2$ along d_U , while $x_1 = 1 - x_2$ along d_D . Using this notation, Proposition 2 states the generic uniqueness of a fully disclosing equilibrium outcome.

Proposition 2. *Under comparative and price advertising, any equilibrium is either fully revealing or such that for every not fully revealing message and price combination in the range of the equilibrium strategy, the set of types that select this combination is a subset of $d_U \cup d_D$. Therefore, in any equilibrium, the union of all sets of types that pool has zero measure, so that nondisclosure is a zero probability event.*

The proof centers around two core ideas. First, if types (x_1, x_2) and (y_1, y_2) send the same equilibrium message so that consumers do not know whether firm 1 is at x_1 or y_1 and whether firm 2 is at x_2 or y_2 (but they may or may not know that if firm 1 is of type x_1 , firm 2 is of type x_2), then the requirement that a deviation to full disclosure, keeping the prices fixed, was not gainful implies that the indifferent consumer does not change as otherwise firm 1 or firm 2 of at least one of the types would have an incentive to deviate. Second, as firms can also deviate from the equilibrium prices, any marginal deviation from the equilibrium prices in any direction should not be gainful either. Together, these requirements impose so many conditions on the locations of the firms (that should not want to deviate) that any *generic* choice of two locations would violate these conditions. In fact, as it becomes clear from the proof, the only types in $[0, 1] \times [0, 1]$ that may have incentives to *not* fully disclose are all located on the two diagonals of the $[0, 1] \times [0, 1]$ square, $d_U \cup d_D$.

The proposition reveals the role of competition in the incentives of firms to disclose horizontal characteristics. A monopoly firm does not have these incentives to disclose (see, e.g., Koessler and Renault, 2011, and Celik, 2014). For example, in the monopoly context of an otherwise identical model, Celik (2014) finds that a large set of full and partial nondisclosure equilibrium outcomes exists. It is important, however, to note that even with competition, the result on generic uniqueness of fully revealing equilibrium outcome is not robust to changes in the exact specification of the advertising game. In fact, our next important finding is that two conditions are critical to this result. First, if the timing of pricing decisions is different and prices are chosen not simultaneously with the messages but at a later stage, a large variety of nondisclosure equilibrium outcomes results. This finding is demonstrated in the next section, where we address the scenario of comparative *nonprice* advertising. In that case, a competitor can choose its pricing strategy to punish the firm that deviates from a nondisclosure strategy, thus sustaining nondisclosure in equilibrium. Second, if firms do not have the ability to disclose the location of the competitor, and the message of each firm may contain information only about its own location, a wide range of nondisclosure equilibria emerges. In that case, deviations from a nondisclosure equilibrium can be punished through consumer beliefs about the location of the competing firm. Moreover, in this case of noncomparative advertising, a large number of equilibria exist irrespective of whether firms choose prices after or simultaneously with the messages about the locations. These findings are also demonstrated in the next section, where we examine the case of *noncomparative* price and nonprice advertising.

Turning back to the result of Proposition 2, one should note that even if firms engage in comparative and price advertising, there do exist equilibria where some types do not fully disclose. The example below provides an illustration of such equilibria. It shows that the types that do not disclose are all located on the upward and downward sloping diagonals of the $[0, 1] \times [0, 1]$ type space.

Example 1. Consider a compact set of types Ω such that $\Omega \subseteq d_U \cup \{(y, 1 - y), (1 - y, y)\}$ for some $y \in [0, 1]$ and $y \neq 0.5$. So, $\Omega \subseteq d_U \cup d_D$ and has exactly two symmetric points on d_D , apart from $(0.5, 0.5)$ (see Figure 1).

Consider the strategy profile, where types in Ω do not fully disclose, while types outside Ω fully disclose. Moreover, in Ω we distinguish between the types on d_U and types in $\{(y, 1 - y), (1 - y, y)\}$. The former pool among themselves and set $P^* = 0$, while the latter pool and set $P^* = 0.5t(1 - 2y)$, which is the full revelation price based on the locations y and $1 - y$. More precisely, the strategies prescribe firms 1 and 2 of any type $(x_1, x_2) \in d_U$ to send message $M^* = \Omega \cap d_U$ and to set price $P^* = 0$, they prescribe firms 1 and 2 of any type $(x_1, x_2) \in \{(y, 1 - y), (1 - y, y)\}$ to send message

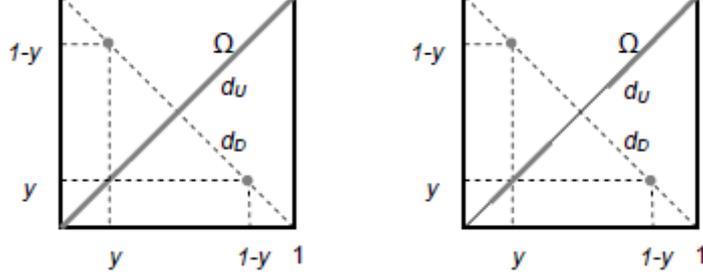


Figure 1: Examples of set Ω in $[0, 1] \times [0, 1]$ type space.

$M^* = \{(y, 1 - y), (1 - y, y)\}$ and to set price $P^* = 0.5t(1 - 2y)$ and they prescribe any other type $(x_1, x_2) \notin \Omega$ to fully disclose the type by sending the precise message and to set the full revelation prices based on (x_1, x_2) . Note that the choice of the equilibrium price $P^* = \frac{1}{2}t(1 - 2y)$ for types in $\{(y, 1 - y), (1 - y, y)\}$ guarantees that the candidate equilibrium pay-off of these types, $0.5P^*$, is equal to the full revelation profit based on $(y, 1 - y)$ (or $(1 - y, y)$).

It is easy to see that given this strategy profile, no type can imitate the behavior of other types. Thus, one only needs to construct a system of consumer out-of-equilibrium beliefs such that given these beliefs, no type has incentives to deviate. One way of constructing these beliefs is the following. Suppose that after observing $M_i = \Omega \cap d_U$ and $M_j \neq \Omega \cap d_U$ or $M_i = \{(y, 1 - y), (1 - y, y)\}$ and $M_j \neq \{(y, 1 - y), (1 - y, y)\}$, consumers assign probability one to any type $(x_1, x_2) \in M_i \cap M_j$, which exists, given the strategies and the grain-of-truth assumption. If one of the observed messages is precise and the other is not, $M_i = \{(x_1, x_2)\}$ and $M_j \neq \{(x_1, x_2)\}$, then consumers are bound to believe that the type is (x_1, x_2) since this is the only type that can send the fully revealing message M_i truthfully. Finally, if the observed deviation is only in price but not in message, then consumer out-of-equilibrium beliefs need to be specified only for the case when $M_i = M_j = \{(y, 1 - y), (1 - y, y)\}$. Suppose that in this case, consumers assign probability one to one of the two types in the message. A straightforward argument then confirms that given these out-of-equilibrium beliefs, no firm of either type has incentives to deviate. \square

The example shows that in the case of comparative and price advertising, nondisclosure may actually occur, but in accordance with Proposition 2, it can only occur if firms' locations have a very special relationship to each other. For all generic combinations of locations, firms fully disclose their types.

5 Nondisclosure equilibria

We next consider why both dimensions present in the advertising game of the previous section, namely, the comparative nature of the messages and the simultaneous timing of reporting and pricing decisions, are crucial for the result that disclosure is essentially the unique equilibrium outcome. To that end, we will consider the arguments for the necessity of both dimensions separately. The importance of the timing of the pricing decision can be intuitively explained as follows. Consider the situation where firms choose prices after they have sent their (possibly imprecise) messages about both locations. In this sequential setting, a competitor can punish a deviation of the firm from a proposed equilibrium strategy in the pricing stage, thus making this deviation unprofitable. In the simultaneous setting, such punishments were not possible, and many deviations were in fact gainful. To see at an intuitive level why we also need firms to have the ability to use comparative advertising in order to obtain the essential uniqueness of the full disclosure result, note that under noncomparative advertising, by deviating from a nondisclosure equilibrium, firms can only reveal their own location but not the location of the competitor. In this case, consumer out-of-equilibrium beliefs about the location of the competitor continue to play a role and these beliefs can be chosen so as to punish a deviation from a nondisclosure strategy. For example, as consumers know that firms are informed about both locations, they may believe that a firm deviates from a nondisclosure equilibrium only if the competitor has a location that is much better for most consumers. Given such beliefs, most consumers will buy from the competitor, making the deviation unprofitable. We will now make both intuitive arguments more precise.

Consider first the case of *comparative nonprice advertising*, where the message of each firm includes information about the locations of both firms and prices are chosen *after* the messages. We first characterize the multiple equilibria that emerge in this case, using an argument similar to that of Proposition 2 in Koessler and Renault (2012). We then provide an example that illustrates this result.

Koessler and Renault (2012) define what they call canonical disclosure strategies. Simply put, in the case of comparative advertising (where messages are two-dimensional), a strategy is canonical if all *different* messages in the range of this strategy form a partition of the whole type space. More formally, a strategy is called canonical if, for every price and message (p_i, m_i) in the range of that strategy, if firm i of type (x_i, x_{-i}) does not choose (p_i, m_i) , then the message m_i cannot be sent by type (x_i, x_{-i}) as, in our current case of comparative advertising, $(x_i, x_{-i}) \notin m_i$.

To illustrate the notion of canonical strategies, it is easiest to provide an example of what is *not* a

canonical strategy. Suppose a strategy has the following elements: firms of type $(x_1, x_2) \in [0, \frac{1}{2}] \times [0, \frac{1}{2}]$ send the message $[0, \frac{1}{2}] \times [0, \frac{1}{2}]$, while firms of any other type send the message $[0, 1] \times [0, 1]$. A strategy with such elements is not canonical as a firm of type $(x_1, x_2) \in [0, \frac{1}{2}] \times [0, \frac{1}{2}]$ may well imitate the message that is chosen by other types without violating the grain-of-truth assumption. To make this strategy canonical, we should modify the strategy's elements, for example, as follows: firms of type $(x_1, x_2) \in [0, \frac{1}{2}] \times [0, \frac{1}{2}]$ send the message $[0, \frac{1}{2}] \times [0, \frac{1}{2}]$ and firms of any other type fully disclose themselves. Then, and in general always when firms choose canonical strategies, no type can imitate the equilibrium message of another type.

Our next proposition provides an easy way to describe a specific but large class of equilibria, where firms use canonical strategies. We focus on canonical strategies that are sequentially rational in that the pricing strategy $p_i(x_i, x_j | M_i, M_j)$ chosen by each firm in the pricing stage is optimal given the messages sent by the two firms, consumer beliefs consistent with these messages and the competitor's price p_j^* . In other words, we focus on pricing strategies that satisfy the second requirement of our equilibrium definition in Section 2.

Proposition 3. *Under comparative nonprice advertising, any canonical (non)disclosure strategy profile that is sequentially rational induces an equilibrium if, and only if, the pay-off to any type of firm with this strategy profile is at least as high as the pay-off of that type in the fully disclosing equilibrium.*

Proof. With appropriate adaptation in notation to accommodate the competition in our model, the proof is identical to Koessler and Renault (2012, Proposition 2).

Note that Proposition 3 as such does not imply multiplicity of nondisclosure equilibria. However, in the example below we use this result to show that, in fact, a wide variety of nondisclosure equilibria exist. To understand the result of Proposition 3, let us briefly discuss the necessary and sufficient part separately.

The necessary part argues that under comparative nonprice advertising, a nondisclosure equilibrium can never yield a pay-off that is smaller than the full disclosure equilibrium pay-off. Indeed, as any firm can unilaterally fully disclose both locations, and in that case consumer beliefs are irrelevant and firms will set the full revelation prices, the non-deviation principle requires that equilibrium pay-offs must be at least as high as the full revelation pay-offs. This argument is quite general and does not depend on the specific Hotelling setting. It is easy to show, however, that under noncomparative advertising this is no longer true and pay-offs as low as 0 can be supported in equilibrium. For example, suppose that all types pool, so that both firms of any type send the same message

$[0, 1]$ and irrespective of the messages sent, set the same price $P^* = 0$. With this strategy, if after a deviation consumers believe that the two firms are located at exactly the same point, then given such beliefs and the zero price of the competitor, no firm has an incentive to deviate. The sufficiency part of the proposition says that all it takes to have a nondisclosure equilibrium with canonical, sequentially rational strategies is to check that deviations to full disclosure are not profitable. This simplifies the analysis of equilibria as one does not need to check all possible deviations to partial information revelation. The argument is simple. Suppose that after observing a deviation from a canonical, sequentially rational strategy, consumers believe that the two firms are located at such points in their messages where the full revelation profit of the deviating firm is minimized. Then as soon as the profit of each firm under the canonical strategy is at least as high as its full revelation profit based on *true* locations, no firm has an incentive to deviate. This argument holds equally for the case of noncomparative advertising.

We now use Proposition 3 to illustrate how large the set of equilibrium outcomes can be under comparative nonprice advertising. This illustration is provided in Example 2 that deals with a subset of these outcomes.

Example 2. Consider an arbitrary compact set of types Ω that is either empty or satisfies two conditions: (i) it is symmetric around d_U , i.e., for any type $(x_1, x_2) \in \Omega$ we have that $(x_2, x_1) \in \Omega$, and (ii) there exists a type $(y, 1 - y) \in \Omega \cap d_D$ that is different from $(0.5, 0.5)$ and represents an "extreme" point of Ω in the sense that $y(1 - y)$ is the furthest left (right) location of either firm among the types in set Ω .¹³ Figure 2 presents some examples of sets that satisfy these conditions.

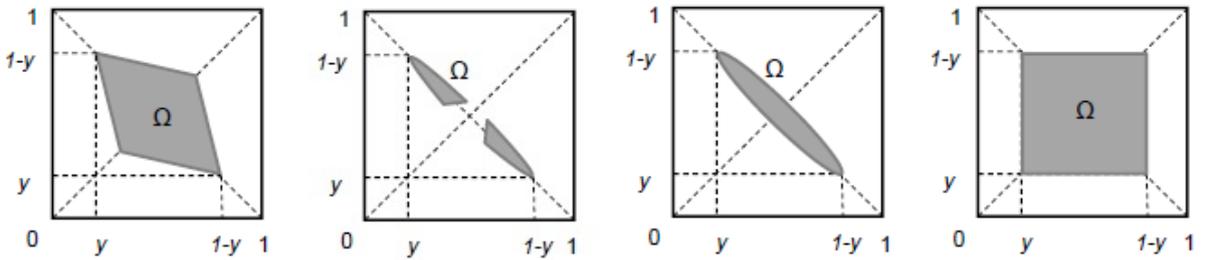


Figure 2: Examples of set Ω in $[0, 1] \times [0, 1]$ type space.

The equilibrium strategy profile that we propose below is such that all types in Ω choose the same nondisclosing message and all types outside Ω choose to precisely disclose themselves. It is easy to see that this example covers an infinite set of equilibrium outcomes, ranging from full disclosure

¹³In fact, $(y, 1 - y)$ only needs to be a focal point of $\Omega \setminus d_U$, that is, there may exist points in $\Omega \cap d_U$ with locations that are smaller than y or larger than $1 - y$. The same argument applies, leading to an even larger set of non-fully disclosing equilibria.

– when the set Ω is empty, – to full nondisclosure – when the set Ω is the whole type space.

Let the strategy of firms 1 and 2 of any type $(x_1, x_2) \in \Omega$ be to send the same message $M^* = \Omega$ and to set the same price $P^* = \frac{1}{2}t(1 - 2y)$ if both firms send the prescribed messages, and to set the full revelation prices based on any $(\tilde{x}_1, \tilde{x}_2)$ from the intersection of the firms' messages if at least one of the firms deviates to a different message. Note that as in Example 1, price $P^* = \frac{1}{2}t(1 - 2y)$ is actually the full revelation price based on the locations y and $1 - y$. Therefore, when all types $(x_1, x_2) \in \Omega$ send message Ω and set price $P^* = \frac{1}{2}t(1 - 2y)$, their candidate equilibrium profit, $0.5P^*$, turns out to be equal to the full revelation profit based on $(y, 1 - y)$. Firms 1 and 2 of any other type $(x_1, x_2) \notin \Omega$ fully disclose both locations by truthfully announcing them and set the full revelation prices based on (x_1, x_2) if at least one of the firms perfectly reveals its type.

Clearly, the described strategies are canonical in the definition of Koessler and Renault (2012). They also generate the pay-off for any type that is at least as high as the pay-off of that type in the fully disclosing equilibrium. Indeed, the pay-off is exactly equal to the full revelation pay-off for the types outside Ω and for the types $(y, 1 - y)$ and $(1 - y, y)$, and it is strictly higher than the full revelation pay-off for all other types $(x_1, x_2) \in \Omega$ that are different from $(y, 1 - y)$ and $(1 - y, y)$. The latter follows from the observation that (a) the pay-off for types $(x_1, x_2) \in \Omega$, given the strategies, is equal to the full revelation profit based on $(y, 1 - y)$, and (b) $(y, 1 - y)$ is the "extreme" point of Ω . Consistently with our remark in the end of Section 3, (b) implies that the full revelation profit of each firm based on $(y, 1 - y)$ is strictly larger than the full revelation profit based on any other type $(x_1, x_2) \in \Omega$, where the locations are not as far apart as y and $1 - y$. Then (a) implies that the pay-off of types $(x_1, x_2) \in \Omega$, given the considered strategies, is strictly larger than their full revelation pay-off.

To see why the proposed strategies induce equilibria, first note that as these strategies are canonical, no type can imitate the strategy of another type. It then suffices to construct a system of consumer out-of-equilibrium beliefs that rules out profitable deviations. Suppose that after observing $M_i = \Omega$ and $M_j \neq \Omega$, consumers assign probability one to the type $(\tilde{x}_1, \tilde{x}_2) \in M_i \cap M_j$ that is consistent with firms' pricing strategy.¹⁴ After observing $M_i = \{(x_1, x_2)\}$ and $M_j \neq \{(x_1, x_2)\}$, consumers believe the message of firm i . Finally, after a deviation only in price but not in message, consumers assign probability one to type $(y, 1 - y)$ when $M_i = M_j = \Omega$ and they believe the reported type when $M_i = M_j = \{(x_1, x_2)\}$. It is straightforward to check that given such out-of-equilibrium beliefs of consumers, no type can benefit from a deviation. \square

¹⁴Given the strategies and the grain-of-truth assumption, the intersection $M_i \cap M_j$ is not empty, so that such a type at the intersection exists.

We now turn to the case of noncomparative advertising, where firms are not able to reveal the location of the competitor. In this case, we also find that nondisclosure is a common equilibrium outcome, but for a different reason than when firms engage in comparative nonprice advertising. In fact, with noncomparative advertising messages, the timing of the pricing decision does not play a substantial role and the range of nondisclosure outcomes is very large irrespective of whether the prices are chosen after or simultaneously with the messages.

Proposition 4. *In the case of noncomparative advertising, multiple nondisclosure equilibria exist, where the set of types that pool has a positive measure.*

The proof of the proposition is provided by Example 3 below. In this example, we construct an infinite set of equilibria, where each equilibrium induces nondisclosure in a set of types $\Omega \times \Omega$ with Ω being an arbitrary compact subset of $[0, 1]$. As Ω can be empty or coincide with the whole $[0, 1]$ interval, full disclosure and full nondisclosure are two particular equilibrium outcomes.

In the example, we also note an interesting issue related to the possibility of types imitating the equilibrium action of other types that does not appear in many asymmetric information models with Senders and Receivers. In most models, where the type space and the action space are one-dimensional, if a Sender type imitates the equilibrium action of another Sender type, the Receiver believes this imitating type to be that other type. This, however, is not necessarily the case under noncomparative advertising in our paper, where the actions (messages) are one-dimensional, while the types are two-dimensional. Then a firm of a certain type can still choose the one-dimensional equilibrium action of some other type. However, this does not imply that the consumers believe the imitating type to be that other type, as consumer beliefs depend on the messages of both firms. We will clarify this issue below.

Example 3. Consider an arbitrary compact set of types $\Omega \times \Omega$, where Ω is a compact subset of $[0, 1]$. Let (y, z) be an "extreme" point of $\Omega \times \Omega$ in the sense that y is the furthest left and z is the furthest right location of each firm among the types in set $\Omega \times \Omega$. Note that unlike in Example 2, (y, z) may not lie on the diagonal d_D , that is, z may not be equal to $1 - y$. Figure 3 illustrates two examples of such a set.

Consider a strategy profile that prescribes all types in $\Omega \times \Omega$ to choose the same nondisclosing (one-dimensional) message and prescribes all types outside $\Omega \times \Omega$ to precisely disclose their locations. The pricing strategy of the types relies on an assumption about the timing of the pricing decision, and is therefore, different in the two cases. However, the difference is inessential and simply accounts for the fact that the price of each firm in the sequential setting is a function of the firms' messages,

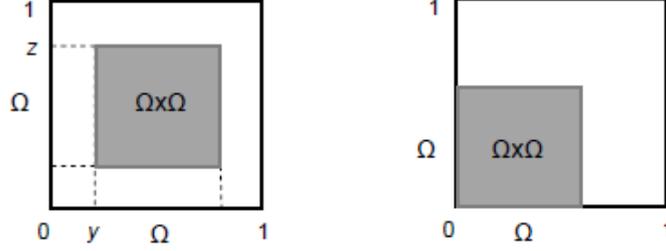


Figure 3: An example of set $\Omega \times \Omega$ in $[0, 1] \times [0, 1]$ type space.

while in the simultaneous setting it is not. Then the proof of the equilibrium is very similar in the two cases. Therefore, for concreteness, let us assume here that prices are chosen after the messages.¹⁵

So, let the strategy of firms 1 and 2 of any type (x_1, x_2) with $x_1 \in \Omega$ and $x_2 \in \Omega$ be to send the same message $M^* = \Omega$ and to adopt the following pricing rule. If both firms send message Ω , then they choose the same price $P^* = \max\{P_L(y, z), P_R(y, z)\}$, where $P_L(y, z)$ and $P_R(y, z)$ are the full revelation prices based on (y, z) (cf., (9) – (10)); and if at least one of the firms deviates to a different message, then both firms choose a zero price. Without loss of generality, let us assume that $P_L(y, z) \leq P_R(y, z)$, so that the equilibrium profit of all types in $\Omega \times \Omega$ is $0.5P_R(y, z)$. Firms 1 and 2 of any other type (x_1, x_2) (such that $x_i \notin \Omega$ for at least one of the locations) fully disclose their location by truthfully announcing it and adhere to the following pricing strategy. If both messages reveal the firms' locations precisely, then firms set the full revelation prices based on (x_1, x_2) ; and if at least one of the messages, M_1, M_2 is not precise, then both firms set the full revelation prices based on $(x'_1(M_1), x'_2(M_2))$, where $x'_1(M_1) \in \arg \min_{x_1 \in M_1} \pi_1(x_1, x_2)$, $x'_2(M_2) \in \arg \min_{x_2 \in M_2} \pi_2(x_1, x_2)$ and $\pi_i(x_1, x_2)$ is the full revelation profit of firm i based on (x_1, x_2) .¹⁶

Note that the described strategy profile makes imitation possible unlike the canonical strategies introduced in the discussion of comparative advertising. For example, if $x_1 \in \Omega$, but $x_2 \notin \Omega$, firm 1 could imitate a type $(\tilde{x}_1, \tilde{x}_2)$ with $\tilde{x}_1 \in \Omega$ and $\tilde{x}_2 \in \Omega$ by sending the message Ω . Also, a firm of a type whose locations are both in Ω can choose to fully disclose its own location, imitating the equilibrium action of a type whose own location (but not that of its rival) is in Ω . However, as consumers also observe the message sent by the rival firm, under unilateral deviations, the proposed equilibrium strategies are constructed so that a consumer can always deduce who has deviated.

¹⁵A full description of equilibrium strategies and the proof of equilibrium in the simultaneous setting are available from the authors upon request.

¹⁶Using our notation in (11) – (12),

$$\min_{x_i \in M_i} \pi_i(x_1, x_2) = \min\left\{ \min_{x_i \in M_i \text{ s.t. } x_i \leq x_j} \pi_L(x_i, x_j), \min_{x_i \in M_i \text{ s.t. } x_i > x_j} \pi_R(x_j, x_i) \right\}.$$

The proof of the equilibrium is then straightforward. We show that there exists a system of consumer out-of-equilibrium beliefs that rules out incentives for deviation. Suppose that after observing $M_i = \Omega$ and $M_j \neq \Omega$, consumers assign probability one to both firms having the same location in $M_i \cap M_j$.¹⁷ After observing $M_i = \{x_i\}$ and $M_j \neq \{x_j\}$, consumers assign probability one to firm j being located at $x'_j(M_j) \in M_j$, where firm j 's full revelation profit based on locations x_i, x_j is minimal across all possible locations $x_j \in M_j$.¹⁸ Finally, after a deviation only in price but not in message, consumer out-of-equilibrium beliefs need to be specified only for the case when $M_i = M_j = \Omega$. Suppose that in this case, consumers assign probability one to the location of a deviating firm being the extreme left location y and the location of the other firm being the extreme right location z .¹⁹

It is easy to check that given such out-of-equilibrium beliefs of consumers, no type can benefit from deviation. For example, whenever $M_i = M_j = \Omega$ and a firm deviates only in price, the best deviation profit is equal to $\pi_L(y, z)$, i.e., the full revelation profit of a firm with the extreme left location y , when the other firm is at the extreme right location z and the price of the other firm is $P_R(y, z)$. But $\pi_L(y, z) \leq 0.5P_R(y, z)$ due to the assumed relation $P_L(y, z) \leq P_R(y, z)$, so that the deviation is not gainful.²⁰ \square

Example 3 does not provide a full characterization of possible equilibrium outcomes, but the class that is encompassed comprises a rich variety of symmetric equilibrium outcomes.²¹

To complete the discussion of noncomparative advertising, note that with noncomparative messages, a nondisclosure strategy can induce an equilibrium even if the pay-off for some type given this strategy is actually *smaller* than the pay-off of that type in a fully disclosing equilibrium. Indeed, given the equilibrium strategy profile of Example 3, it is easy to show that $\pi_R(y, z)$ is greater than the equilibrium profit $0.5P_R(y, z)$, where $\pi_R(y, z)$ is the full revelation profit based on (y, z) that a firm located at z obtains when both firms make precise statements.²² The main difference with the case

¹⁷ $M_i \cap M_j \neq \emptyset$ due to the grain-of-truth assumption.

¹⁸If there is more than one location $x'_j(M_j)$ at which $\pi_j(x_i, x_j)$ is minimized, then suppose that consumers assign probability one to the location that is consistent with firms' candidate equilibrium pricing strategy.

¹⁹If the assumed relation between the full revelation prices was reversed, i.e., $P_L(y, z) \geq P_R(y, z)$, then consumers would assign probability one to the location of a deviating firm being the extreme right location z and the location of the other firm being the extreme left location y .

²⁰By definition of the full revelation prices in (9) – (10), $P_L(y, z) \leq P_R(y, z)$ is equivalent to $y + z \leq 1$. This, in turn, implies that $\pi_L(y, z)$ defined in (11) does not exceed the equilibrium profit, $0.5P_R(y, z)$.

²¹In fact, Example 3 easily generalizes to allow for a broader variety of nondisclosure equilibrium outcomes. For example, firms 1 and 2 of type (x_1, x_2) with $x_1 \in \Omega$ and $x_2 \in \Omega$ do not need to pool with *all* the other types whose both locations are in Ω . Instead, they can only pool with types in *any compact* subset $\tilde{\Omega}$ of Ω such that $x_1 \in \tilde{\Omega}$ and $x_2 \in \tilde{\Omega}$.

²²Thus, as we have already noted earlier, in the discussion of Proposition 3, under noncomparative advertising, the "only if" part of the proposition does not hold.

of comparative nonprice advertising considered in the beginning of this section is that under noncomparative advertising, by unilaterally deviating from a proposed nondisclosing equilibrium strategy, a firm can never obtain a full revelation equilibrium pay-off as it only reveals its own location but not that of the competitors. Consumer beliefs may then be used to punish deviations from the candidate equilibrium strategies, thereby also sustaining equilibria with low profits.

Thus, by providing different arguments to show why noncomparative advertising and nonprice advertising result in the existence of many nondisclosure equilibria, we have illustrated the necessity of both features to produce the essential uniqueness result in section 4.

6 Discussion and conclusions

In this paper we developed a duopoly model of horizontal product differentiation. We studied the incentives of a firm to disclose its horizontal product characteristic when this characteristic is known to both firms, but not to consumers. In the model, firms simultaneously choose a message about their location(s), such that this message is *truthful*, that is, the true location(s) of a firm(s) is consistent with the message. The messages can range from being very precise (indicating the exact location(s)) to very vague. We considered four different cases, dependent on whether or not firms engage in comparative or noncomparative advertising and on whether or not prices are announced simultaneously with the messages. Given the messages and the prices, consumers update their beliefs about firms' locations and decide where to buy.

The main result of the paper is that generically (that is except if firms' locations have a very particular relationship to each other) under comparative and price advertising there exists a unique equilibrium outcome where firms fully disclose their product characteristics. In all other cases, a full disclosure equilibrium also exists, but it is one among many other equilibria where firms do not (fully) disclose their information. These findings demonstrate, on the one hand, the importance of competition in providing incentives for firms to fully disclose their horizontal attribute, as in the monopoly setting, a broad variety of nondisclosure equilibria exist (Koessler and Renault, 2012; Celik, 2014). On the other hand, they illustrate the non-robustness of the essential uniqueness result to alternative model specifications, where firms choose prices *after* the advertising messages or where the messages are noncomparative.

The results for comparative and price advertising are different from the results for comparative nonprice advertising, as in the first case, optimally deviating from a nondisclosure situation is more beneficial since the other firm cannot react to a deviating message by setting a different price (as it

can do under comparative nonprice advertising). Noncomparative advertising – with prices chosen either simultaneously or after the messages – leads to the existence of many nondisclosure equilibria, as even if a firm discloses its own location, consumer out-of-equilibrium beliefs about the location of the competing firm continue to play a role and these beliefs can be chosen so as to "punish" any deviation from the equilibrium nondisclosure strategy.

None of the standard refinement notions such as the Intuitive Criterion (Cho and Kreps, 1987) or D1 (Cho and Sobel, 1990) can be relied upon to reduce the set of equilibria in this model due to the fact that the profits of all types of a firm depend only on prices and on consumer beliefs about firms' locations, but not on actual locations.²³ One way to get rid of the multiplicity of equilibria would be to invoke a restriction on consumer out-of-equilibrium beliefs. For example, under noncomparative advertising, one could argue that given that all types sending identical equilibrium messages concerning location have the same incentives to deviate to a non-equilibrium price, price cannot reasonably act as a signal of a firm's location. A similar argument could then be used to argue for the inability of a firm to signal the location of its competitor. In the first working paper version of this paper (Janssen and Teteryatnikova, 2012), we used these considerations to define an equilibrium where consumer out-of-equilibrium beliefs concerning a firm's location depend only on the firm's own message and not on its pricing decision or the message of the competitor. We showed that under these restrictive beliefs, all equilibria must be fully revealing. However, as we consider an environment where firms are informed about both firms' product characteristics, no interpretation of a deviation can be excluded, and this results in multiple nondisclosure outcomes. In their paper, Anderson and Renault (2009) conclude that there exists a unique full disclosure equilibrium where prices are equal to marginal cost, even when the advertising is noncomparative. However, they do not analyze their model as a fully-fledged signaling game and therefore do not consider consumer out-of-equilibrium beliefs. Our results show that in a signaling game with no ad-hoc restrictions on the out-of-equilibrium beliefs, full disclosure is just one of many possible equilibrium outcomes.

This paper has considered a simple framework where consumers are uniformly distributed over the unit interval and have quadratic transportation costs. Moreover, disclosure is completely costless and firms know not only their own location, but also that of the competitor. We have considered this simple framework to focus on the role of different information transmission processes in providing incentives for firms to fully disclose their private information in a competitive environment. Future

²³It is an interesting question for further research to investigate whether the notion of "undefeated" equilibrium (Mailath et al., 1993) can be used to reduce the number of equilibria. In his monopoly context, Celik (2014) uses the undefeated equilibrium as an equilibrium refinement concept to obtain a unique equilibrium. As we have infinitely many possible equilibria, it is not immediately clear how to use this refinement in the current duopoly environment.

work should focus on whether similar conclusions hold when some of these assumptions are replaced by others. For example, the case where firms have purely private information about their product characteristics could be of considerable interest. It is not difficult to see that under purely private information, full disclosure is also an equilibrium outcome and in this respect the result of the current paper easily generalizes. The main challenge is whether other equilibria also exist. This is not an easy task as without knowing the location of the competitor, disclosing one's own location has advantages for some, but not all locations of the rival firm.

Appendix

Proof of Theorem 2. Consider a set of types $\Phi \subseteq [0, 1] \times [0, 1]$ such that Φ is *not* a subset of $d_U \cup d_D$. In the following, we show that for *any* such set, there exists a type $(x_1, x_2) \in \Phi$ such that it does *not* have incentives to pool with the other types in Φ . This will then suggest that the only types in the $[0, 1] \times [0, 1]$ square that may have incentives to pool with other types are all located on either of the two diagonals, d_U or d_D , and therefore, in any equilibrium, the union of all sets of types that pool with each other has zero measure.

Suppose, on the contrary, that there exists an equilibrium in which all types in set Φ pool. Notice that the mere possibility of pooling requires the existence of at least two different types in Φ . So, let (x_1, x_2) and (y_1, y_2) be a pair of different types in Φ . Moreover, since Φ is not a subset of $d_U \cup d_D$, there exists at least one type in Φ that does not belong to either of the two diagonals. Suppose that $(y_1, y_2) \notin d_U \cup d_D$. Below we consider the two options: 1) $(x_1, x_2) \in d_U$ and 2) $(x_1, x_2) \notin d_U$. We show that in each case, a firm of at least one of the two types, (x_1, x_2) or (y_1, y_2) , has incentives to send a fully-revealing message rather than pool with the other types in Φ , so that pooling is *not* an equilibrium strategy.

To begin with, consider that in both cases, a firm of type (y_1, y_2) (the type that does not belong to d_U) clearly has an incentive to deviate to full disclosure if its equilibrium price is zero. Indeed, by deviating to full disclosure and setting its price marginally above zero, this firm raises its own profit from zero to positive.

Therefore, it remains to study the two cases under the condition that the equilibrium prices of both firms are positive. Let us consider each case in turn.

1. Suppose first that $(x_1, x_2) \in d_U$ (and $(y_1, y_2) \notin d_U \cup d_D$, $P_1^* > 0$, $P_2^* > 0$).

In this case, at least one of the firms of type (x_1, x_2) , where $x_1 = x_2$, has incentives to deviate to full disclosure. First, if $P_1^* = P_2^*$, then the firm whose equilibrium demand is lower or

equal to the demand of the competitor (hence, not larger than 0.5) gains from deviating to full disclosure at a slightly lower price. Indeed, such deviation discontinuously increases the demand of the firm while leaving the price essentially unchanged. Secondly, if $P_1^* \neq P_2^*$, then two cases can be distinguished. If the equilibrium demand of one of the firms is zero, then this firm of type (x_1, x_2) strictly increases its profit by deviating to full disclosure at any price that is lower or equal to the price of the competitor. On the other hand, if the equilibrium demand of each firm is strictly between zero and one, then the firm with the lower price can benefit from full disclosure: by revealing its type precisely and keeping the price unchanged, it attracts all consumers at the same price.

2. Suppose that $(x_1, x_2) \notin d_U$ (and $(y_1, y_2) \notin d_U \cup d_D$, $P_1^* > 0$, $P_2^* > 0$).

To prove the incentives for full revelation among types in Φ , we first observe that fully revealing one's own type, while keeping the price fixed, is always gainful for a firm as soon as it increases the firm's demand. Therefore, if there exists a type in Φ that, by fully revealing itself and keeping the price fixed, changes the equilibrium value of $\hat{\lambda}$, the location of the indifferent consumer, one of the firms of this type (the one that gains demand through the change in $\hat{\lambda}$) has incentives to deviate to full disclosure.

Thus, for pooling to be an equilibrium, it must be that for *any* type in Φ , revealing itself while keeping the price fixed does *not* change the location of the indifferent consumer. In particular, this means that the location of the indifferent consumer is the same no matter whether type (x_1, x_2) or type (y_1, y_2) fully discloses and it is equal to the location of the indifferent consumer in the supposed non-fully disclosing equilibrium (where types in Φ pool):²⁴

$$\hat{\lambda} = \frac{1}{2t} \frac{P_2^* - P_1^*}{x_2 - x_1} + \frac{1}{2} (x_1 + x_2) = \frac{1}{2t} \frac{P_2^* - P_1^*}{y_2 - y_1} + \frac{1}{2} (y_1 + y_2). \quad (13)$$

Recall that the location of the indifferent consumer represents the demand of the firm perceived

²⁴This argument relies on the assumption that the indifferent consumer in the supposed non-fully disclosing equilibrium is well defined. If this is not the case, the argument should be slightly different. The indifferent consumer is not well defined if either no one is indifferent or everyone is. In the first case, all consumers prefer the same firm and the other firm's equilibrium demand is zero. This firm (of any type) can benefit from deviation, as by fully disclosing its type and by setting its price equal to the price of the competitor, it gains positive profit. In the second case, equilibrium demand of each firm is equal to 0.5. This can only be an equilibrium if the demand of each firm remains equal to 0.5 after a deviation to full disclosure, while keeping the price fixed. This means that (13) holds with 0.5 instead of $\hat{\lambda}$. Moreover, since all consumers are indifferent between the two firms, it must be that $E(x_1|\mu_1) = E(x_2|\mu_2)$, which suggests that there must exist a pair of types in Φ such that firms' relative locations are different in the two types. For example, $x_1 < x_2$ and $y_1 > y_2$. Then the subsequent argument is almost identical to the one on pp. 27 – 28. Eventually, it leads to the conclusion that $P_1^* = P_2^*$, and then (13) implies that $x_1 + x_2 = y_1 + y_2 = 1$, which contradicts our choice of (y_1, y_2) .

to be furthest to the left. Without loss of generality, assume that $E(x_1|\mu_1) \leq E(x_2|\mu_2)$, that is, in equilibrium, firm 1 is perceived as furthest to the left. Then the equilibrium profits of firm 1 and 2, denoted by π_L^* and π_R^* , respectively, are given by:

$$\pi_L^* = P_1^* \left[\frac{1}{2t} \frac{P_2^* - P_1^*}{x_2 - x_1} + \frac{1}{2} (x_1 + x_2) \right] = P_1^* \left[\frac{1}{2t} \frac{P_2^* - P_1^*}{y_2 - y_1} + \frac{1}{2} (y_1 + y_2) \right] \quad (14)$$

$$\pi_R^* = P_2^* \left[1 - \frac{1}{2t} \frac{P_2^* - P_1^*}{x_2 - x_1} - \frac{1}{2} (x_1 + x_2) \right] = P_2^* \left[1 - \frac{1}{2t} \frac{P_2^* - P_1^*}{y_2 - y_1} - \frac{1}{2} (y_1 + y_2) \right]. \quad (15)$$

In the following, we show that irrespective of whether the *true* location of firm 1 of type (x_1, x_2) or (y_1, y_2) is indeed furthest to the left, – consistently with its *perceived* location due to a non-disclosing equilibrium message, – firm 1 or 2 of at least one of the two types has an incentive to deviate to full disclosure.

Suppose first that all types in Φ are such that for all types either firm 1 is located to the left of firm 2 or firm 2 is located to the left of firm 1. Notice, however, that since $E(x_1|\mu_1) \leq E(x_2|\mu_2)$, the latter cannot be the case. Therefore, if firms' location with respect to each other is the same for all types, then it must be that firm 1 is located to the left of firm 2. In particular, for types (x_1, x_2) and (y_1, y_2) , we have that $x_1 < x_2$ and $y_1 < y_2$. Consider a deviation by firm 1 of type (x_1, x_2) to full disclosure at a price $P' \neq P_1^*$. The deviation results in profit

$$P' \left[\frac{1}{2t} \frac{P_2^* - P'}{x_2 - x_1} + \frac{1}{2} (x_1 + x_2) \right].$$

For this deviation to *not* be gainful, the best deviating price must be exactly equal to P_1^* , that is, the partial derivative of this deviation profit with respect to P' must be zero at $P' = P_1^*$:

$$\frac{1}{2t} \frac{P_2^* - P_1^*}{x_2 - x_1} + \frac{1}{2} (x_1 + x_2) = \frac{1}{2t} \frac{P_1^*}{(x_2 - x_1)}.$$

The same argument must hold for the deviation of firm 1 of type (y_1, y_2) , so that we obtain:

$$\frac{1}{2t} \frac{P_2^* - P_1^*}{y_2 - y_1} + \frac{1}{2} (y_1 + y_2) = \frac{1}{2t} \frac{P_1^*}{(y_2 - y_1)}.$$

The left-hand sides of these two equations are equal according to (13). Therefore:

$$\frac{1}{2t} \frac{P_1^*}{(x_2 - x_1)} = \frac{1}{2t} \frac{P_1^*}{(y_2 - y_1)}.$$

Since $P_1^* > 0$, this suggests that $x_2 - x_1 = y_2 - y_1$. But then (13) can only hold if $x_1 + x_2 = y_1 + y_2$. Together, $x_2 - x_1 = y_2 - y_1$ and $x_1 + x_2 = y_1 + y_2$ imply that $x_1 = y_1$ and $x_2 = y_2$, that is, types (x_1, x_2) and (y_1, y_2) are the same. This contradicts our initial presumption. Hence, firm 1 of either of the two types can benefit from the deviation.

Next, suppose that there exists a pair of types in Φ such that firms' location with respect to each other is different for the two types. This can be true of types (x_1, x_2) and (y_1, y_2) or of some other pair of types. In any case, addressing just the first possibility is sufficient to complete the proof.²⁵ So, let types (x_1, x_2) and (y_1, y_2) be such that either $x_1 < x_2, y_1 > y_2$ or $x_1 > x_2, y_1 < y_2$. For concreteness, consider one of these symmetric cases, say, $x_1 < x_2, y_1 > y_2$.²⁶ If firm 1 of type (x_1, x_2) deviates to full disclosure at price P' , it obtains the deviation profit of

$$P' \left[\frac{1}{2t} \frac{P_2^* - P'}{x_2 - x_1} + \frac{1}{2} (x_1 + x_2) \right].$$

As before, for this deviation to *not* be gainful, the partial derivative of this function with respect to P' must be zero at $P' = P_1^*$:

$$\frac{1}{2t} \frac{P_2^* - P_1^*}{x_2 - x_1} + \frac{1}{2} (x_1 + x_2) = \frac{1}{2t} \frac{P_1^*}{(x_2 - x_1)}. \quad (16)$$

Similarly, for the deviation to full disclosure of firm 2 of type (x_1, x_2) to *not* be gainful, the derivative of its deviation profit with respect to the deviating price must be zero at the point where this price is equal to P_2^* . This results in

$$1 - \frac{1}{2t} \frac{P_2^* - P_1^*}{x_2 - x_1} - \frac{1}{2} (x_1 + x_2) = \frac{1}{2t} \frac{P_2^*}{(x_2 - x_1)}. \quad (17)$$

Now, consider the deviation to full disclosure by firms 1 and 2 of type (y_1, y_2) . Observe that for this type, the relation between the *true* locations of the two firms is the opposite of the relation between the *perceived* locations: $y_1 > y_2$ but $E(x_1|\mu_1) \leq E(x_2|\mu_2)$. Therefore, when firm 1 deviates to full disclosure, keeping the price unchanged, it obtains the profit of

$$P_1^* \left[1 - \frac{1}{2t} \frac{P_2^* - P_1^*}{y_2 - y_1} - \frac{1}{2} (y_1 + y_2) \right].$$

Using (15), this deviation profit can be expressed as $\frac{P_1^*}{P_2^*} \pi_R^*$. Hence, such deviation by firm 1 is unprofitable if and only if $\frac{P_1^*}{P_2^*} \pi_R^* \leq \pi_L^*$, or $P_1^* \pi_R^* \leq P_2^* \pi_L^*$.

On the other hand, when firm 2 of type (y_1, y_2) deviates to full disclosure, keeping the price fixed, it obtains the profit of

$$P_2^* \left[\frac{1}{2t} \frac{P_2^* - P_1^*}{y_2 - y_1} + \frac{1}{2} (y_1 + y_2) \right]$$

²⁵The second possibility implies that either (i) $x_1 < x_2$ and $y_1 < y_2$, in which case the argument considered above establishes the incentives for deviation, or (ii) $x_1 > x_2$ and $y_1 > y_2$, in which case there must exist another type $(z_1, z_2) \in \Phi$ such that $z_1 < z_2$ (as otherwise $E(x_1|\mu_1) \leq E(x_2|\mu_2)$ cannot hold) and then the argument identical to the one provided further in the proof confirms the incentives for deviation. Indeed, one would only need to substitute (z_1, z_2) for (x_1, x_2) as then we obtain a pair of types $(z_1, z_2), (y_1, y_2)$ with $z_1 < z_2$ and $y_1 > y_2$.

²⁶The argument for the other case is analogous.

which, given (14), is equal to $\frac{P_2^*}{P_1^*}\pi_L^*$. This deviation by firm 2 is unprofitable if and only if $\frac{P_2^*}{P_1^*}\pi_L^* \leq \pi_R^*$, or $P_2^*\pi_L^* \leq P_1^*\pi_R^*$.

Thus, in equilibrium we must have $P_2^*\pi_L^* = P_1^*\pi_R^*$. Using (14) – (15), this can be written as

$$P_1^*P_2^* \left[\frac{1}{2t} \frac{P_2^* - P_1^*}{x_2 - x_1} + \frac{1}{2} (x_1 + x_2) \right] = P_1^*P_2^* \left[1 - \frac{1}{2t} \frac{P_2^* - P_1^*}{x_2 - x_1} - \frac{1}{2} (x_1 + x_2) \right].$$

The expressions in square brackets are the equilibrium demands of firms 1 and 2. According to (16) and (17), each of them can be expressed more compactly, leading to

$$\frac{P_1^*}{(x_2 - x_1)} = \frac{P_2^*}{(x_2 - x_1)}.$$

This means that in equilibrium prices must be equal, i.e., $P_1^* = P_2^*$.

Now, given the equality of prices, condition (13) results in $x_1 + x_2 = y_1 + y_2$ and

$$\begin{aligned} \pi_L^* &= P^* \frac{1}{2} (x_1 + x_2) = P^* \frac{1}{2} (y_1 + y_2) \\ \pi_R^* &= P^* \left(1 - \frac{1}{2} (x_1 + x_2) \right) = P^* \left(1 - \frac{1}{2} (y_1 + y_2) \right) \end{aligned}$$

where $P^* = P_1^* = P_2^*$. Moreover, given that $x_1 < x_2$ and $y_1 > y_2$, this can only be an equilibrium when $\pi_L^* = \pi_R^*$. Indeed, suppose that $\pi_L^* \neq \pi_R^*$, say, $\pi_L^* < \pi_R^*$. Then firm 1 of type (y_1, y_2) , whose *true* location (as opposed to its *perceived* location) is to the right of the *true* location of firm 2, has incentives to fully disclose its type, keeping the price at P^* . Such deviation results in profit that is exactly equal to π_R^* , and hence, is larger than firm 1's equilibrium profit.²⁷ Therefore, we obtain that $\pi_L^* = \pi_R^*$. But this suggests that $x_1 + x_2 = y_1 + y_2 = 1$, which means that both, (x_1, x_2) and (y_1, y_2) belong to d_D , and this is a contradiction to our choice of (y_1, y_2) .

Thus, also in this case we obtain that a firm of at least one of the two types, (x_1, x_2) or (y_1, y_2) , can benefit from deviation to full disclosure.

□

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²⁷If $\pi_L^* > \pi_R^*$, then by the analogous argument, firm 2 of type (y_1, y_2) has incentives to fully disclose.

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