Convergent refinement processes on Cartan-Hadamard manifolds

Svenja, Hüning
TU Graz, Austria

We are interested in the convergence analysis of refinement rules in Riemannian geometry which are algorithms producing limit curves by refining discrete sets of points. So far, convergence results could be proven for certain classes of subdivision schemes and/or special kinds of Riemannian manifolds. Examples are interpolatory subdivision schemes and schemes with nonnegative mask coefficients. Another approach to obtain convergence statements is to use so-called proximity conditions and show convergence for 'dense enough' input data.
To extend linear subdivision schemes to manifold valued data, we study the existence and uniqueness of the Riemannian center of mass on Cartan-Hadamard manifolds. We extend a known convergence result (which applies to all input data) to subdivision schemes without sign restrictions.

Checking for G^1- and G^2-Continuity using least squares methods

Florian, Lorenz
Siemens AG, Erlangen, Germany

The talk is about checking G^1- and G^2-Continuity of two surfaces sharing a common edge by using least squares methods. If one of the surfaces has a good parameterization by means of local isometry, the presented algorithms yield to known geometric error measurements like the angle between face normals (for G^1-Continuity) and the difference of the normal curvature in a specified tangent direction (for G^2-Continuity).

Matrix Recovery under Low-rankness and Sparsity Constraints

Johannes Maly
TU Munich, Germany

We consider the problem of reconstructing a matrix with combined low-rank and sparsity constraints from few linear measurements. The challenge lies in simultaneously exploiting the two different structures low-rankness and sparsity. Mere convex combination of convex regularizers like nuclear norm and l1-norm cannot best one of the two structures alone in number of measurements. Therefore, we approximate via minimisation of a non-convex multi-penalty functional.

Convergence of multiple subdivision schemes: matrix approach

Thomas Mejstrik
University of Vienna, Austria

Subdivision schemes are methods for generation of smooth curves and surfaces from given data. Convergent subdivision schemes iteratively refine a sequence of points and converge to a continuous limit. In multiple subdivision schemes, every refinement step is based on a different uniformly bounded refinement mask and on a different dilation matrix. In this talk, we characterise the convergence of multiple subdivision scheme in terms of the spectral properties of a finite set of square matrices derived from these refinement masks and dilation matrices. In particular, we show how to define these square matrices, prove the existence of their common invariant subspace, and present an algorithm which may decide in reasonable time if the multiple subdivision scheme is convergent.

How to increase the smoothness of subdivision schemes

Caroline Moosmüller
University of Passau, Germany

In this talk we study the regularity of curves generated by subdivision schemes. In particular, we are interested in increasing their regularity. In scalar subdivision, it is well known that a scheme which produces C^ℓ limit curves can be transformed to a new scheme producing $C^{\ell+1}$ limit curves by multiplying the scheme's symbol with the *smoothing factor* $\frac{z+1}{2}$. We give a short introduction to the scalar case and then present a similar procedure for Hermite subdivision schemes. Our approach is purely algebraic and works by manipulating the symbol of a given scheme. The algorithm presented in this talk allows to construct Hermite subdivision schemes of arbitrarily high regularity from a given Hermite schemes whose Taylor scheme is at least C^0.

Stability of Gabor Phase Retrieval

Martin Rathmair
University of Vienna, Austria

We consider the Gabor phase retrieval problem, i.e., the problem of reconstructing a signal f from the magnitudes $|V_\varphi f|$ of its Gabor transform

$$V_\varphi f(x,y) := \int_{\mathbb{R}} f(t) e^{-\pi(t-x)^2} e^{-2\pi \text{ß} y t} dt, \quad x, y \in \mathbb{R}.$$

Such problems occur in a wide range of applications, from optical imaging to audio processing.
While it is well-known that the solution of the Gabor phase retrieval problem is unique up to natural identifications, the stability of the reconstruction has remained wide open. It is not hard to construct instabilities by adding two (or more) functions whose Gabor transforms are concentrated on disjoint sets in the time-frequency plane. A converse result will be presented in the talk: The stability of a given Gabor phase retrieval problem will be related to the *Cheeger constant* w.r.t. the weight $|V_\varphi f|$ – a quantity from spectral geometry. Since the *Cheeger constant* quantifies the connectedness of the Gabor measurements we will be able to conclude that Gabor phase retrieval can in theory be done stably whenever the measurements are connected.

Mathematical aspects of digitized cultural heritage

Florian Schlenker
University of Passau, Germany

The digitalization of cultural heritage is of increasing importance, especially since historic sites and artifacts got destroyed during the last years. Moreover, digitized objects are useful for research purposes and for the creation of virtual exhibitions. But the process of digitizing objects incorporates mathematical and technical challenges. The talk will treat mathematical aspects of widely used digitalization methods such as photogrammetry and computerized tomography. Some of these techniques produce point clouds representing only a part of the whole object. To align several of these point clouds to each other registration algorithms can be used, which we will also introduce.

Multigrid and anisotropic, optimal, interpolatory subdivision

Valentina Turati
University of Insubria, Italy

Multigrid is an iterative solver for ill-conditioned systems of linear equations. We consider such linear systems of equations e.g. arising from finite difference discretization of anisotropic Laplacian boundary value problems. Anisotropic subdivision (grid transfer operator) deals with the anisotropic nature of the problem reducing appropriately the size of the original system matrix. Additionally, optimality and interpolatory properties of anisotropic subdivision lead to multigrid methods with optimal computational complexity. In this talk, we present a family of anisotropic, interpolary subdivision schemes with dilation matrix $\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$ and prove their optimality.

Smoothness of semi-regular subdivision via frames

Alberto Viscardi
University of Milano-Bicocca, Italy

Univariate frames are special function families $\{\psi_j\}_{j\in J} \subset L_2(\mathbb{R})$ (J is a countable index set) that are used for decomposition and analysis of functions in $L_2(\mathbb{R})$. In the shift-invariant setting, if the frame $\{\psi_j\}_{j\in J} \subset C^q(\mathbb{R})$, $q \in \mathbb{N}$, has q vanishing moments, i.e.

$$(x^k, \psi_j)_{L_2} = 0 \quad \text{for all} \quad k \in \{0, \ldots, q-1\},$$

then the decay rate of the frame coefficients $(\cdot, \psi_j)_{L_2}$, $j \in J$, characterizes the Sobolev spaces $H^p(\mathbb{R})$ for $p \in (0, q)$. We are interested in the analysis of smoothness of limits of semi-regular subdivision schemes, finitely many of whose basic limit functions are not integer shifts of the other ones. The first step towards similar characterizations of $H^p(\mathbb{R})$, $p \in (0, q)$, in the semi-regular setting, is our construction of the appropriate tight wavelet frame based on Dubuc-Deslauriers 4-point semi-regular interpolatory subdivision. In this talk, we present this construction, which is done using a modified Unitary Extension Principle, and study the properties of the corresponding frame elements.