On the Positive Role of Negative Political Campaigning

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Abstract

This paper studies the incentives of parties in political campaigns to disclose their true, intended policies to voters. Parties compete for the vote share that determines their political power or percentage of seats won in the election. We consider two cases: one in which parties can only disclose their own policy (no negative political campaigning) and the other, in which they can also disclose the policy of their adversary (negative political campaigning). In both cases, full revelation is one of the equilibrium outcomes. More importantly, in case of negative campaigning, all equilibrium outcomes, with full and partial disclosure, are such that all voters make choices that they would have also made under full disclosure. If parties do not or are not allowed to engage in negative campaigning, a large variety of nondisclosure equilibria exist where voters’ choices are different from those under full information.

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1 Introduction

In political campaigns, politicians try to convince the electorate on the policy or reforms that they intend to implement once their party is in power. Politicians can decide how specific they want to be in announcing their intended polices, making statements that vary from being very specific to very ambiguous.\(^1\) Politicians can also make statements about the intended policies of their adversaries, the case that we will refer to as negative campaigning. This paper shows the positive role of negative campaigning. It demonstrates that negative campaigning provides politicians with incentives to disclose their policies in a way that allows voters to deduce correctly which of the parties they like best and vote accordingly.\(^2\) In contrast, without negative campaigning politicians may deliver statements that are so fuzzy that voters cannot decipher which of the parties has an intended policy that is closest to their ideal policy. In this case, they may vote for the party that they would not have chosen under full information. We show that under negative campaigning non-precise statements are also possible, but the nature of fuzziness in these statements is such that voters can always make out which of the parties they prefer.\(^3\)

The issue of information disclosure in political campaigns is voiced most sharply in public debates over the necessity of law-enforced disclosure rules, requiring that politicians reveal the nature and identity of their financial donors.\(^4\) The main idea behind these practical disclosure rules is that politicians’ intended, true policies are usually in line with the interests of the institutions and private parties that financially support their campaign. Voters have then the right to know who is supporting which political party to assist them in voting for the party they prefer most. Our paper analyzes the incentives political parties have to voluntarily disclose their intended policies (which may be thought of as the interests of their financial contributors) in the absence of such disclosure rules. We find that if parties can also disclose the true, intended policies of their adversaries and if lying is too costly but remaining fuzzy is not,\(^5\) then there is no need for mandatory disclosure.

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\(^1\)The fact that politicians’ talks can be and often are ambiguous is well known and documented in the literature (Downs, 1957; Kelley, 1960; Page, 1976; Campbell, 1983; Laslier, 2006). It is sometimes attributed to rational seeking of support, approval or votes. A recent example of political ambiguity is a statement made by the Minister of Finance of Greece, Yanis Varoufakis, who declared that the government was proud of the “degree of creative ambiguity” used in drafting reforms set as the condition of prolonging the country’s bailout program (source: http://www.theguardian.com/world/2015/feb/28/greece-poll-surge-syriza-alexis-tsipras).

\(^2\)The fact that information disclosure improves voters’ choices is demonstrated, for example, by the seminal work of Lupia (1994). It finds that more precise and specific information on an insurance reform in California enabled voters to vote as if they knew more than they actually did.

\(^3\)This conclusion is at odds with a common perception of negative campaigning as being detrimental to the political process and “information environment” of elections. For example, Geer (2006) shows that such an opinion is widely expressed in the political science literature.


\(^5\)Plain lying in political campaigns can be costly either because of the possibility of being sued, or because of future retaliation by voters or party dissonants.
rules as private incentives are aligned with public interests. This complements the arguments of
the opponents of disclosure laws who view mandatory disclosure as burdensome and unnecessary. Of
course, if parties can costlessly lie or if they are not able or not allowed to disclose the policy of
their adversaries, then there may be good reasons for imposing disclosure rules.

We analyze a spatial model of elections where political parties have a position on a line segment
representing the different policies or reforms that a party can advocate. To focus on what politicians
disclose about their positions, we consider the positions themselves to be fixed. We analyze
the situation where political parties do not only know their own position, but also that of their
adversary. This reflects the fact that political parties usually have a strong incentive to learn
their chances of success in the elections and have therefore a strong incentive to find out the true,
intended policy of their adversary. The electorate does not know the positions of the parties and
has to rely on the statements that are disclosed. Voters do not believe these statements at face
value, and consider which party has an incentive to deliver which statement. Voters derive utility
from voting for the party whose true position is closest to theirs and only vote if their idiosyncratic
cost of voting does not exceed the utility. Politicians are assumed to make statements that are
consistent with their true, intended policies, that is, they cannot lie. This is represented by the
requirement that the statements made should include the party’s true, intended policy. The choice
political parties have is about the precision of these statements. This choice is driven by the
parties’ incentive to maximize their share in political power, say, a percentage of parliamentary
seats won. We focus on political systems with proportional representation where the share of a
party’s political power is determined by the share of supporting votes in the total voter turnout.

The popular view on politicians is that they frequently lie and break their promises. We are
agnostic on whether this is true and whether actual policy decision may be different from intended
policy, for example, because of new information arriving when politicians have taken up office,
or because parties have to form coalition governments and compromise on some of their intended
policies. What is important for our arguments to go through is that politicians have true, intended

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6 For reference on this debate, see research of Clyde Wilcox, for example, Wilcox (2001).
7 Thus, our paper studies the mirror image of traditional spatial models of elections, going back to Hotelling (1929)
and Black (1948). In that literature, political parties choose a position and once the position is taken, the electorate
is immediately informed about that position. Here, positions cannot be chosen, but what is disclosed about these
positions is the result of strategic competition between the parties.
8 Such assumption, where people receive utility directly from voting, is common in the literature on “expressive”
voting (Schneider, 2000; Glaeser et al., 2005).
9 In footnotes we comment on how our results would change for presidential elections where candidates only care
about whether or not they win the election.
10 Proportional representation is used in the majority of countries. Examples include Italy, Finland, Sweden, Netherlands, Portugal, Israel, Colombia, South Africa, Tunisia. For other examples and detailed information on
policies (that may, for example, be derived from interests of the organizations that finance a party) and that these intended policies can be verified by some court. In fact, it does seem plausible that deviating from announced intended policies, which give voters the best indication available of what actual policies may look like once a party is in office, can be very costly to politicians’ future careers and parties’ popularity. What is also important is that this paper makes a distinction between fuzziness and lying, a distinction that is often not made in more popular discussions. Downs (1957) and the literature that follows, including recent papers by Demange and Van der Straeten (2013) and Schipper and Woo (2015), also assume that politicians do not bluntly lie, but may make statements that are ambiguous.

We analyze two different environments: one in which parties do not, or are not allowed to, engage in negative campaigning, and one where they do. Our results can then be understood as follows. Without negative campaigning, there is a continuum of equilibria with disclosure statements ranging from full disclosure to full nondisclosure. To see why parties may create maximum fuzziness, it is important that voters know that candidates are aware not only about their own position, but also about the position of their adversary. If one party unexpectedly discloses its own position, voters may believe that the party does so only if it knows that the position of the adversary is closer to the median voter than its own. Given such beliefs, parties are better off under maximal fuzziness. We also show that a nondisclosure equilibrium in this case is often ex-post inefficient as some voters may regret the choices they have made after the true intended policies of the parties become transparent. Under negative campaigning, the situation is different. In that case, either one of the parties can always guarantee itself and the other party the vote shares belonging to full disclosure. If these vote shares are distorted, it then has to be the case that at least one of the parties wants to disclose both its own and the adversary’s position. This leaves open the possibility of existence of some special types of nondisclosure equilibria where parties’ vote shares are the same as under full disclosure. We show that if such equilibria exist, then it must be the case that for a generic combination of parties’ positions, the voters will still be able to deduce their most preferred party and hence, vote as they would do given the full information.

There appears to have been little research on the strategy of political disclosure, particularly on the role of negative campaigning. Polborn and Yi (2006), Demange and Van der Straeten
In a rich model of voters’ limited awareness about candidates’ political positions, the main finding of Schipper and Woo (2015) is an unraveling result: all issues that voters may not have been aware of are raised, and all information on candidates’ positions (on all issues) is revealed to voters. The possibility of negative campaigning reinforces, but is not necessary for, this result. In contrast, in our model, the absence of negative campaigning leads to the existence of multiple nondisclosure equilibria where voters make choices that are different from those they would have made under full disclosure. Negative political campaigning is necessary to rule out these equilibria. The main difference between the results of Schipper and Woo (2015) and ours lies in the fact that they allow for micro targeting of specific voters and consider the possibility of a few voters, whereas we consider situations where microtargeting is not possible and each voter’s influence on the elections is negligibly small. Negative campaigning is also examined in Polborn and Yi (2006). Consistently with our results, they find that negative campaigning, on average, facilitates a more informed choice by the electorate. However, the choice of disclosure strategies available to candidates is much more limited than in our setting: candidates each “own” an issue and can either remain silent or provide correct, precise information on their own issue or the opponent’s issue. In Demange and Van der Straeten (2013) information about the opponent is “leaked unvoluntarily” rather than chosen strategically. More generally, the element of strategic interaction between parties, which is key in our analysis, is omitted from their model, as parties choose their disclosure strategies taking into account the effect on voters of their own strategy only.

There is also a large literature on the necessity of disclosure laws forcing firms to disclose the quality and/or ingredients of the products they produce or the production technologies they adopt (see, e.g., Grossman and Hart, 1980; Grossman, 1981; Jovanovic, 1981, and Milgrom, 1982). This literature has inspired a further literature on incentives to disclose, including papers by Daughety and Reinganum (1995), Board (2003), Anderson and Renault (2006) and Koessler and Renault (2012). Recently, this literature has been extended to include the question whether firms have incentive to disclose horizontal product attributes, rather than vertical attributes such as quality (see, e.g., Sun, 2011; Celik, 2014, and Janssen and Teteryatnikova, 2013). Our paper builds on the analysis of Janssen and Teteryatnikova (2013), but excludes the price setting stage, that is key in competition between firms, and adds the cost of voting, that is essential in voting behaviour.


12The question of political disclosure is also studied empirically, for example, by Djankow et al. (2010).
absence of price competition makes the analysis cleaner and allows us to focus on the essential mechanism underlying the incentives to disclose, and to study welfare implications of nondisclosure.

The rest of the paper is organized as follows. The next section describes the model. Section 3 describes a full disclosure equilibrium that exists with and without negative campaigning, and introduces the notion of equilibrium ex-post efficiency. Sections 4 and 5 provide the analysis of the two cases considered (with and without negative campaigning). Section 6 concludes with a brief discussion.

2 Model

Consider elections where two parties compete for votes by making statements about their policy or policy platform. The policy platform of each party is represented by a position on the unit interval. Policy platforms with values close to 0 can be regarded as left-wing positions, while policy platforms with values close to 1 as right-wing positions. Voters have preferences over policies and the ideal policy of a voter is also represented by a position on the unit interval. We denote by $x_1, x_2 \in [0, 1]$ the positions of the two parties and by $\lambda \in [0, 1]$ the position of a voter. We focus on disclosure decisions of the political parties and consider policy positions themselves as exogenously given. Political parties do not only know their own position, but also the position of the adversary. This reflects the fact that politicians usually have a strong interest in learning their prospects for success in the elections. Therefore, they or people in their environment find ways to learn the true position of their adversary. On the other hand, voters do not know the true positions of the political parties and have to rely on the statements that are disclosed. Notice that since parties know their own position and the position of the adversary, the type of each party is the pair of positions, $(x_1, x_2)$. In what follows, the first element in the pair $(x_1, x_2)$ stands for the position of party 1, and the second element for the position of party 2.

The value of a voter’s favourite policy, $\lambda$, follows a continuous distribution with full support on $[0, 1]$, symmetric around the middle point 0.5. We denote the probability density function of this distribution by $g$ and the cumulative distribution function by $G$, so that $G(0) = 0, G(1) = 1$ and $G(0.5) = \int_0^{0.5} g(\lambda) d\lambda = 0.5$. Voters derive utility from voting for the party whose true position

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13 An alternative approach to representing a policy platform, which may include more than one issue of interest to voters, would be to consider a position (and disclosure decision) of a party on each issue independently. Such approach is, in fact, equivalent to our simpler single-position approach here as soon as we assume that in order to maximize the percentage of seats won, political parties need to get the most favorable outcome on each issue. This assumption is reasonable when politicians do not know precisely how voters make their final decision based on the combination of most favored positions on the individual issues.

14 The specification with a continuum of voters whose ideal policies are drawn from a certain distribution on the unit interval, is identical to the specification with a single voter who has a privately known ideal policy drawn from the same distribution.
is close to theirs. Namely, let voter’s utility, or benefit, from voting for party \( i \) located at \( x_i \) be
\[ b_i = a - t(\lambda - x_i)^2, \]
where \( t(\lambda - x_i)^2 \) is a cost of mismatch between the policy of party \( i \), \( x_i \), and voter’s ideal policy, \( \lambda \), and \( a \) measures the psychological gain from expressing support for the voter’s ideal policy. By the same token, utility from voting against party \( j \) located at \( x_j \) is equal to \(-b_j\). Thus, the overall utility from voting for party \( 1 \) and against party \( 2 \) is given by
\[ u_1 = t(\lambda - x_2)^2 - t(\lambda - x_1)^2 \]
and the overall utility from voting for party \( 2 \) and against party \( 1 \) is \( u_2 = t(\lambda - x_1)^2 - t(\lambda - x_2)^2 = -u_1 \). The maximum of these two utilities does not exceed the upper bound \( \bar{u} \), which is the largest value that either of \( u_1, u_2 \) can reach across all \( \lambda \in [0, 1] \) and all \( x_1, x_2 \in [0, 1] \). It is easy to see, that in our framework, \( \bar{u} = t \).

This definition of voters’ preferences, where people receive utility directly from voting is often referred to as “expressive” voting (Schuessler, 2000; Glaeser et al., 2005). Deriving utility from voting for the closest party may also reflect voters’ preference for the policy regime where the favourite party’s influence is stronger. However, this latter interpretation of utility requires citizens to overestimate the relevance of their vote to political outcomes.

Given such preferences and given the information about positions of the two parties, voters support the party whose policy platform is perceived as closest to their ideal policy. Voting is costly, and therefore, people vote only if their cost of voting, \( c \), does not exceed the utility gain. Namely, a voter with position \( \lambda \) votes when
\[ c \leq \max\{tE((\lambda - x_2)^2|\mu_2) - tE((\lambda - x_1)^2|\mu_1), tE((\lambda - x_1)^2|\mu_1) - tE((\lambda - x_2)^2|\mu_2)\}, \tag{1} \]
where \( tE((\lambda - x_i)^2|\mu_i) \) is the expectation of the mismatch cost associated with the position of party \( i \), conditional on voter’s belief about this party’s position.

The cost of voting \( c \) is independent of voters’ political preferences and drawn from the uniform distribution with full support on \([0, \bar{u}]\). We consider \( \bar{u} \geq \bar{u} \), so that for every voter position \( \lambda \in [0, 1] \), there is a positive probability that the voter will abstain and a positive probability that she will vote. A voter’s (realized) cost of voting is known to the voter but not known to the political parties who only know the cost distribution.

We denote by \( t_1 \) and \( t_2 \) the voter turnout for party \( 1 \) and party \( 2 \), respectively. Then \( \pi_1 = \frac{t_1}{t_1 + t_2} \) and \( \pi_2 = \frac{t_2}{t_1 + t_2} \) can be interpreted as respective shares of party \( 1 \) and party \( 2 \) in political power, or a percentage of parliamentary seats won by each party. Such a rule by which divisions in an electorate are reflected proportionately in the elected body is typical for electoral systems with proportional representation. In what follows we consider \( \pi_1 \) and \( \pi_2 \) as the pay-offs of party \( 1 \) and party \( 2 \), respectively.
The timing of the game is the following. At stage 0, Nature independently selects position \( x_1 \) for party 1 and \( x_2 \) for party 2 from a non-atomic density function \( f(x) \). Parties learn both positions but voters do not. At stage 1, the political campaign begins with both parties making statements about their positions. Two cases are possible. Without negative political campaigning, politicians can only provide information about the position of their own party, and then a statement of each party is a subset of the unit segment, \( S_i \subseteq [0,1] \), \( i = 1, 2 \). With negative political campaigning, politicians can provide information about both positions, and then a statement of each party is a subset of the unit square, \( S_i \subseteq [0,1] \times [0,1] \). Notice that in the different cases \( S_i = [0,1] \) or \( S_i = [0,1] \times [0,1] \) can be interpreted as full nondisclosure of information by party \( i \). Statements must be consistent with parties’ true positions in the sense that \( x_i \in S_i \) for \( i = 1, 2 \) when negative campaigning is not possible and \( (x_1, x_2) \in S_i \) for \( i = 1, 2 \) in case it is. This means that even though the statements of the two politicians about their proposed policies can be very fuzzy, plain lying (which is reporting a statement that does not contain the true position) about the parties’ actual, intended policies is not allowed, or is so costly in terms of future careers that it is never optimal to do so. In the following, the two cases will be examined separately, but unless stated otherwise, the same notation and definitions apply throughout. At stage 2, voters observe the statements of the two parties and, given their cost of voting \( c \), decide whether to vote and, if so, for which party. Voting decisions determine the pay-offs of the political parties and ex-ante, expected pay-offs/utility of voters. Ex-post pay-offs of voters are realized at the end of the game, when the outcomes of elections are implemented and uncertainty about the parties’ true, intended policies is resolved. All aspects of the game are common knowledge.

To solve the game, we apply the concept of a strong perfect Bayesian equilibrium (Fudenberg and Tirole, 1991), that is, a perfect Bayesian equilibrium where voters’ beliefs off-the-equilibrium path, albeit arbitrary, are identical across voters. The formal definition relies on the following specification of the strategy spaces. The strategy of party \( i \) is denoted by \( s_i(x_i, x_j) \), where the image of \( s_i \) belongs to all subsets of \([0,1]\) such that \( x_i \in s_i \) (for non-negative campaigning), or to all subsets of \([0,1] \times [0,1]\) such that \((x_1, x_2) \in s_i \) (for negative campaigning). The vector \( v(\lambda, c, S_i, S_j) \) denotes the voting strategy of a voter with position \( \lambda \) and cost of voting \( c \), when the parties’ statements are \( S_i \) and \( S_j \), respectively. We say that \( v = \emptyset \) if the voter abstains, \( v = (1,0) \) if the voter votes for party 1 and \( v = (0,1) \) if she votes for party 2. Finally, \( \mu_i(z|S_i, S_j) \) is the probability density that voters assign to \( x_i = z \) when the parties announce \( S_i \) and \( S_j \). Given this notation, we

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\(^{15}\)The probability measure function is called non-atomic if it has no atoms, i.e., measurable sets which have positive probability measure and contain no set of smaller but positive measure.
can now state the definition of a strong perfect Bayesian equilibrium as follows.

**Definition** A strong perfect Bayesian equilibrium of the game is a set of strategies $s^*_1, s^*_2$ of the two parties, strategy $v^*$ of a voter, and the probability density functions $\mu^*_1, \mu^*_2$ that satisfy the following conditions:

(1) For all $S_1$ and $S_2$, $v^*$ is a voter’s best voting decision as defined below:

$$v^*(\lambda, c, S_1, S_2) = \begin{cases} (1, 0) & \text{if } tE \left( (\lambda - x_2)^2 | \mu^*_2 \right) - tE \left( (\lambda - x_1)^2 | \mu^*_1 \right) \geq \max \{ c, tE \left( (\lambda - x_1)^2 | \mu^*_1 \right) - tE \left( (\lambda - x_2)^2 | \mu^*_2 \right) \} \\
(0, 1) & \text{if } tE \left( (\lambda - x_1)^2 | \mu^*_1 \right) - tE \left( (\lambda - x_2)^2 | \mu^*_2 \right) \geq \max \{ c, tE \left( (\lambda - x_2)^2 | \mu^*_2 \right) - tE \left( (\lambda - x_1)^2 | \mu^*_1 \right) \} \\
\emptyset & \text{if } c > \max \{ tE \left( (\lambda - x_2)^2 | \mu^*_2 \right) - tE \left( (\lambda - x_1)^2 | \mu^*_1 \right), \\
& \quad tE \left( (\lambda - x_1)^2 | \mu^*_1 \right) - tE \left( (\lambda - x_2)^2 | \mu^*_2 \right) \} 
\end{cases}$$

(2) Given (1) and given the statement made by the adversary, $s^*_i$ is the statement that maximizes the pay-off of party $i$, $i = 1, 2$, subject to the assumption about the statements’ consistency with true positions.

(3) For all $S_1$ and $S_2$, a voter updates her beliefs, $\mu^*_1, \mu^*_2$, regarding the positions of the parties in the following way:\(^{16}\)

(i) according to Bayes’ rule on the equilibrium path,

(ii) arbitrary off the equilibrium path.

All voters have identical beliefs on and off the equilibrium path.

This definition implies that (1) for any observed statements of the two political parties, people either abstain or vote for the party, whose perceived position, given the updated beliefs, maximizes their ex-ante, expected utility; (2) parties anticipate the best response choices of the electorate to any pair of the parties’ statements and choose the statements that maximize their share in political power; (3) voters update beliefs about parties’ positions using Bayes’ rule for any statements that occur with positive probability along the equilibrium path, and beliefs off the equilibrium path are arbitrary but identical across voters. Importantly, the requirement of statements’ consistency with statements made by the adversary is satisfied.

\(^{16}\)Note that Bayes’ rule cannot be applied when $S_i$ is discrete. For example, if in case without negative campaigning $S_i = \{ y, z \}$, then both events, $x_i = y$ and $x_i = z$ have ex-ante zero probability. In this case, updating proceeds as follows:

$$\mu^*_i(z|S_1, S_2) = \lim_{\varepsilon \to 0} \frac{F(z + \varepsilon) - F(z)}{F(z + \varepsilon) - F(z) + F(y + \varepsilon) - F(y)}$$

Using l’Hôpital’s rule,

$$\mu^*_i(z|S_1, S_2) = \lim_{\varepsilon \to 0} \frac{f(z + \varepsilon)}{f(z + \varepsilon) + f(y + \varepsilon)} = \frac{f(z)}{f(z) + f(y)}$$
true positions implies that voters should only assign positive probability to those positions of the
different parties that are a part of the statements. In other words, when there is no negative campaigning,
voters should only assign positive probability to the positions of a party that are included in its
statement. And similarly, when campaigning is negative, only the pairs of positions that belong
to the intersection of the two statements should receive a positive probability. Note that the
intersection of the two parties’ statements in case of negative campaigning is never empty due to
the same consistency assumption.

Next, we define the indifferent voter as a voter whose overall utility from voting for either of
the two parties is the same, given the information that is disclosed through parties’ statements.
Denote the position of this voter by \( \hat{\lambda} \). Note that depending on the realization of the cost of voting
for the indifferent voter, she may actually prefer to abstain. Nevertheless, the position of this voter
marks the important threshold between voters who never vote for a given party and those who –
conditional on voting – always vote for this party. For example, voters to the left of \( \hat{\lambda} \) never vote
for the party that is perceived to have the furthest right position, even if their cost of voting is
low. On the other hand, voters to the right of \( \hat{\lambda} \) always vote for this party, provided they vote at
all. Formally, \( \hat{\lambda} \) is defined by

\[
E \left( (\hat{\lambda} - x_2)^2 | \mu_2 \right) - E \left( (\hat{\lambda} - x_1)^2 | \mu_1 \right) = tE \left( (\hat{\lambda} - x_1)^2 | \mu_1 \right) - tE \left( (\hat{\lambda} - x_2)^2 | \mu_2 \right)
\]

or

\[
E \left( (\hat{\lambda} - x_1)^2 | \mu_1 \right) = E \left( (\hat{\lambda} - x_2)^2 | \mu_2 \right).
\]

The solution of this equation is given by:

\[
\hat{\lambda} = \frac{E \left( x_2^2 | \mu_2 \right) - E \left( x_1^2 | \mu_1 \right)}{2 \left( E \left( x_2^2 | \mu_2 \right) - E \left( x_1^2 | \mu_1 \right) \right)}.
\]

Note that since

\[
E \left( x_2^2 | \mu_2 \right) - E \left( x_1^2 | \mu_1 \right) = \text{var} \left( x_2 | \mu_2 \right) - \text{var} \left( x_1 | \mu_1 \right) + \text{var} \left( x_1 | \mu_1 \right) - \text{var} \left( x_1 | \mu_1 \right) + E^2 \left( x_2 | \mu_2 \right) - E^2 \left( x_1 | \mu_1 \right),
\]

equation (4) suggests that voters dislike uncertainty about political positions of the parties. Indeed, for any
\( E \left( x_2 | \mu_2 \right) > E \left( x_1 | \mu_1 \right) \), larger variance of the second party’s statement relative to that of the first
moves the position of the indifferent voter to the right, closer to the position of the second party,
so that more people vote for party 1. In fact, such outcome is typical for preferences based on
convex mismatch costs. The specific, quadratic formulation is used to simplify the analysis and to
make sure that(4) defines a unique value of \( \hat{\lambda} \). The indifferent voter’s position is not well-defined
only when (i) \( \hat{\lambda} \) lies outside the (0,1) interval, in which case no voter is indifferent and everyone
prefers the same party, or (ii) equality (3) holds for any \( \hat{\lambda} \), in which case all voters are indifferent.
between the two parties.

For convenience, we will employ subscripts $L$ and $R$ for the party with the furthest left and furthest right perceived position, respectively, so that $E(x_L|\mu_L) < E(x_R|\mu_R)$. Then, as soon as $\hat{\lambda}$ is well-defined, the turnout for the party that is perceived as furthest left, $t_L$, is equal to the expected share of voters to the left of $\hat{\lambda}$ whose cost of voting does not exceed the overall utility from voting for their preferred, left party. Similarly, the turnout for the party that is perceived as furthest right, $t_R$, is equal to the expected share of voters to the right of $\hat{\lambda}$ whose cost of voting does not exceed the overall utility from voting for their preferred, right party. Since the costs of voting for all voters are i.i.d. and the distribution of costs is uniform on $[0, c]$, the turnout for each party is given by:

$$t_L = \int_0^{\hat{\lambda}} \frac{E(u_L|\mu_L, \mu_R)}{c} g(\lambda) d\lambda, \quad (5)$$

$$t_R = \int_{\hat{\lambda}}^1 \frac{E(u_R|\mu_L, \mu_R)}{c} g(\lambda) d\lambda. \quad (6)$$

Here $E(u_L|\mu_L, \mu_R)$ and $E(u_R|\mu_L, \mu_R)$ denote the expected overall utility of a voter with position $\lambda$ from voting for the party with the furthest left and furthest right perceived position, respectively, and $\frac{E(u_i|\mu_L, \mu_R)}{c}$ is the probability that the cost of voting $c$ does not exceed the expected overall utility from voting for party $i$, given the parties’ statements and voters’ beliefs.

Further, we observe that using (4), the expected overall utility of a voter $\lambda$ from voting for the party with the furthest left and furthest right perceived position can be expressed in terms of $\hat{\lambda}$:

$$E(u_L|\mu_L, \mu_R) = t E((\lambda - x_R)^2|\mu_R) - t E((\lambda - x_L)^2|\mu_L) =$$

$$= t \left(2\lambda (E(x_L|\mu_L) - E(x_R|\mu_R)) + E(x_R^2|\mu_R) - E(x_L^2|\mu_L)\right)$$

$$= 2t \left((\hat{\lambda} - \lambda) (E(x_R|\mu_R) - E(x_L|\mu_L))\right),$$

$$E(u_R|\mu_L, \mu_R) = t E((\lambda - x_L)^2|\mu_L) - t E((\lambda - x_R)^2|\mu_R) =$$

$$= 2t \left((\lambda - \hat{\lambda}) (E(x_R|\mu_R) - E(x_L|\mu_L))\right) = -E(u_L|\mu_L, \mu_R).$$

Thus, the turnouts of the two parties in (5) – (6) become:

$$t_L = \frac{2t (E(x_R|\mu_R) - E(x_L|\mu_L))}{c} \int_0^{\hat{\lambda}} (\hat{\lambda} - \lambda) g(\lambda) d\lambda, \quad (7)$$

$$t_R = \frac{2t (E(x_R|\mu_R) - E(x_L|\mu_L))}{c} \int_{\hat{\lambda}}^1 (\lambda - \hat{\lambda}) g(\lambda) d\lambda. \quad (8)$$
This leads to the following pay-offs of the two parties:

\[
\pi_L = \frac{t_L}{t_L + t_R} = \frac{\int_0^\lambda (\hat{\lambda} - \lambda)g(\lambda)d\lambda}{\int_0^\lambda (\hat{\lambda} - \lambda)g(\lambda)d\lambda + \int_\lambda^1 (\lambda - \hat{\lambda})g(\lambda)d\lambda}, \tag{9}
\]

\[
\pi_R = \frac{t_R}{t_L + t_R} = \frac{\int_0^1 (\lambda - \hat{\lambda})g(\lambda)d\lambda + \int_\lambda^1 (\lambda - \hat{\lambda})g(\lambda)d\lambda}{\int_0^\lambda (\hat{\lambda} - \lambda)g(\lambda)d\lambda + \int_\lambda^1 (\lambda - \hat{\lambda})g(\lambda)d\lambda} = 1 - \pi_L. \tag{10}
\]

Note that these pay-off functions are uniquely determined by \(\hat{\lambda}\), and \(\pi_L\) is strictly increasing in \(\hat{\lambda}\), while \(\pi_R\) is strictly decreasing in \(\hat{\lambda}\). This is confirmed by a simple comparative statics exercise that uses the Leibniz rule of integral differentiation. Another important property of these pay-off functions is that the sum of the parties’ pay-offs, \(\pi_L + \pi_R\), is always equal to 1, irrespective of the parties’ statements and voters’ beliefs. Moreover, it is easy to see that for any probability density function \(g\) that is symmetric around 0.5, the pay-offs of the two parties are the same if the indifferent voter is located in the middle of the interval: \(\hat{\lambda} = 0.5\) implies that \(\pi_L = \pi_R = 0.5\).

In cases when \(\hat{\lambda}\) is not well-defined, the pay-offs of both parties are either the same if all voters are indifferent, or the pay-off of the party that is strictly preferred by all voters is one, while the pay-off of the other party is zero. In the former case \(\pi_L = \pi_R = 0.5\), while in the latter \(\pi_L = 1\), \(\pi_R = 0\) or \(\pi_L = 0\), \(\pi_R = 1\).

We also observe that the representation of turnout rates in (7) – (8) implies that \(t_L > t_R\) if and only if \(\hat{\lambda} > 0.5\). Then since \(\pi_L > \pi_R\) is equivalent to \(t_L > t_R\), as suggested by (9) – (10), it follows that \(\pi_L > \pi_R\) if and only if \(\hat{\lambda} > 0.5\). Intuitively, the further right the location of the indifferent voter, the larger the pool of voters who favour the left-wing policy, and the larger the relative turnout and the gain for the party that is perceived as furthest left.

The final observation about the parties’ pay-offs in (9) – (10) is that both pay-offs are fully determined by \(\hat{\lambda}\) and hence, they only depend on expected or perceived positions of the two parties but not on their actual positions. This observation is key as it implies that, subject to the assumption of consistency of the political statements with true positions, voter beliefs can be used to “punish” a deviating party.

\[17\] The same is not necessarily true for the turnout rates \(t_L\) and \(t_R\). Indeed, while larger \(\hat{\lambda}\) increases the pool of voters who prefer the left party, the probability of voting by these voters might decline. For example, if an increase in \(\lambda\) is associated with the fact that the perceived location of the left party moves to the right, closer to the location of the right party, so that the difference \(E(x_L|x_R) - E(x_L|x_L)\) declines, then some of the voters to the left of \(\hat{\lambda}\) may lose interest in voting at all, as their preference for the left party is not strong enough to outweigh the cost of voting. This is also suggested by the representation of \(t_L\) and \(t_R\) in (7) – (8).

\[18\] See Appendix.

\[19\] Indeed, \(t_L > t_R\) means that \(\int_0^\lambda (\hat{\lambda} - \lambda)g(\lambda)d\lambda > \int_\lambda^1 (\lambda - \hat{\lambda})g(\lambda)d\lambda\). Simple algebra suggests that the latter is equivalent to \(\hat{\lambda} > \int_0^\lambda \lambda g(\lambda)d\lambda\), where the right-hand side is the mean ideal policy of voters. This mean ideal policy is equal to 0.5 as distribution \(g\) is symmetric around 0.5.
3 Full disclosure equilibria and ex-post efficiency

Let us first consider the situation where the true, intended policies of both parties are fully revealed. In this section we show that irrespective of whether political campaigning is negative or not, full disclosure is an equilibrium outcome.

Under full disclosure, the position of the indifferent voter is determined according to (4) by the middle point of the segment connecting $x_L$ and $x_R$. It is well-defined for any $x_L \neq x_R$:

$$\lambda = \frac{x_L + x_R}{2}. \tag{11}$$

A straightforward implication of this representation of $\lambda$ is that under full disclosure the party that is located closer to 0 obtains a strictly larger pay-off. Indeed, using the observation made in the previous section, that $\pi_L > \pi_R$ if and only if $\lambda > 0$, we obtain that under full disclosure $\pi_L > \pi_R$ if and only if $x_L + x_R > 1$. Note that the sum of two positions is greater than 1 in two cases: either when both $x_L \geq 0.5$ and $x_R > 0.5$, or when $x_L \leq 0.5 < x_R$ and positions $x_L$, $x_R$ are not exactly symmetric around 0.5 but such that $x_L$ is closer to 0.5 than $x_R$. In both cases, the position of the left party is closer to 0.5 than the position of the right party. Similar considerations apply in the analogous cases, where both $x_L < 0.5$ and $x_R \leq 0.5$, or $x_L < 0.5 \leq x_R$ and positions $x_L$, $x_R$ are not exactly symmetric but such that $x_R$ is closer to 0.5 than $x_L$. The remaining case (with unequal positions) is when $x_L$ and $x_R$ are exactly symmetric around 0.5. In this case, $x_L + x_R = 1$, that is, the indifferent voter is located precisely in the middle of the unit interval, and the pay-offs of both parties are the same, $\pi_L = \pi_R = 0.5$.

If, on the other hand, positions of the two parties are identical, then $\lambda$ is not well-defined, as all voters are indifferent. In this case, again, the pay-offs of the parties are the same, $\pi_L = \pi_R = 0.5$.

The following proposition establishes the first simple result:

**Proposition 1.** Whether or not negative political campaigning is used, full disclosure is always an equilibrium outcome.

The proof of Proposition 1 is straightforward.\textsuperscript{20} Suppose that both parties of any type disclose their true position (or type) precisely. If negative campaigning is used, then the precise statement of each party indicates the positions of both parties, and hence, unilateral deviations to nondisclosing statements do not change voter beliefs about the positions, and so, no party can benefit from deviating. If no negative campaigning is used, then the precise statement of each party indicates only its own position and not the adversary’s position. Then if a party deviates from the fully

\textsuperscript{20}The details are available from the authors.
disclosing strategy and makes a statement that does not reveal its position precisely, voter out-
of-equilibrium beliefs become important. Given the statements’ consistency assumption, that
statements have to contain the true positions, and the fact that parties’ pay-offs depend on expected
rather than actual positions, these beliefs can be constructed so that for any not fully revealing
statement, the deviating party’s pay-off is not larger than its equilibrium pay-off.

Note that when parties’ true, intended policies are fully revealed, voters are able to make
fully-informed choices and thereby, maximize not only their ex-ante but also \textit{ex-post} utility from
voting, the two being the same in this case. In this sense, any fully disclosing equilibrium is
\textit{ex-post efficient}. Similarly, a not fully revealing equilibrium is \textit{ex-post} efficient whenever voters’
equilibrium choices are not distorted by uncertainty. That is, for any actual positions of the
two parties, voters’ \textit{ex-post} utility, obtained after the uncertainty about these positions has been
resolved, is maximized. This is the case when given not fully disclosing equilibrium statements,
two conditions hold: (i) the position of the indifferent voter (if it is well-defined) is the same as
under full disclosure, and (ii) for all types that can make the equilibrium statements consistently
with their true positions, parties’ relative positions with respect to each other are the same as
their relative \textit{perceived} positions.\footnote{Formally, condition (ii) means that for all \((x_1, x_2)\) that can make the equilibrium non-fully revealing statements, either \(x_1 < x_2\) (and hence, perceived position of party 1 is smaller than perceived position of party 2) or \(x_2 < x_1\) (and hence, perceived position of party 2 is smaller than perceived position of party 1).}

Other not fully revealing equilibria are inefficient because there exists at least one pair of parties’ positions for which with positive probability some or all voters make “wrong” choices, so that their \textit{ex-post} utility from voting is lower than under full
disclosure.\footnote{Given the probabilistic nature of voting in this model, choices of voters in such inefficient equilibria may happen to be the same as under full disclosure, but only for a specific realization of random voting costs or for a specific allocation of votes in case when nonrevealing equilibrium leaves all or some voters indifferent between the two parties. For example, if the position of the indifferent voter in nonrevealing equilibrium is larger than under full disclosure but the costs of voting for all people with positions between the “two indifferent voters” appear to be too high, these voters abstain and hence, do not make “wrong” choices.}

In the next two sections we consider the scenarios with and without negative political cam-
paigning and show that only in the former case, (generically) all equilibria are \textit{ex-post} efficient.

4 Negative political campaigning

In case of \textit{negative political campaigning} parties reveal information not only about their own policy
or policy platform but also about the policy of their adversary. This means that the statement of
each party specifies not just the set of own possible positions but the set of possible positions of
both parties. We find that in this case, nondisclosure may be an equilibrium outcome. However,
any nondisclosure equilibrium has an important property that generally speaking, uncertainty
associated with nondisclosure does not affect optimal voting behavior that one should expect under full disclosure. That is, for any generic combination of parties’ positions, nondisclosure does not distort voters’ decisions and the equilibrium is ex-post efficient.\textsuperscript{23}

**Proposition 2.** In case of negative political campaigning, there does not exist an equilibrium where the set of types that (a) do not fully disclose and (b) make a statement inducing inefficient voters’ choices has a positive measure. Thus, for all generic pairs of positions of the two parties, the equilibrium is ex-post efficient.

The idea of the proof is simple. Clearly, if the equilibrium is fully disclosing, then it is efficient. If equilibrium is not fully disclosing, then there exist at least two types \((x_1, x_2)\) and \((y_1, y_2)\) that pool by making the same statement, and voters cannot distinguish between the two types. In any such equilibrium, the pay-off of each party must be the same as its full disclosure pay-off (cf. (9) – (10) with \(\lambda = \frac{x_L + x_R}{2}\)). This follows from two observations. First, since parties know positions of each other, each of them can guarantee itself a full disclosure pay-off by revealing both positions precisely. Therefore, in any not fully revealing equilibrium, the pay-off to any type of a party must be at least as high as the pay-off to that type in the full disclosure equilibrium. Second, the pay-off to any type of a party cannot be strictly larger than the full disclosure pay-off since the sum of the parties’ pay-offs is always equal to one. Indeed, if one party would get a pay-off in a nondisclosing equilibrium that is larger than its full disclosure pay-off, then the other party would have an incentive to fully disclose. This means that in case of negative political campaigning, all equilibria are pay-off equivalent, and the pay-off of each type of a party is equal to the full-revelation pay-off.

Now, given that parties’ pay-offs are uniquely determined by \(\lambda\) (whenever it is well-defined), the equality of any equilibrium pay-off and full-revelation pay-off implies that for every generic pooling type two conditions should hold: (i) the indifferent voter is the same in a given not fully disclosing equilibrium and under full disclosure, and (ii) relative positions of parties with respect to each other are the same as their relative perceived positions.\textsuperscript{24}

\textsuperscript{23}The outcome is different in case of presidential elections, where the candidate gaining the largest vote share wins (pay-off is 1), and the other candidate looses (pay-off is 0). In this case, many nondisclosure equilibria are ex-post inefficient according to our definition. For example, there exists an equilibrium where all types \((x_1, x_2)\) such that \(x_2 < x_1 < 0.5\) or \(0.5 < x_1 < x_2\) (candidate 1 is located closer to 0.5 than candidate 2) pool, and where all types \((x_1, x_2)\) such that \(0.5 < x_2 < x_1\) or \(x_1 < x_2 < 0.5\) (candidate 2 is located closer to 0.5 than candidate 1) pool. Such equilibrium is ex-post inefficient because given the pooling statements, the location of the indifferent voter is not the same as for any actual pooling type, so that some voter choices might be distorted. However, the nature of pooling in this equilibrium (and in fact, in any nondisclosure equilibrium) is such that the candidate that is perceived as located closer to 0.5 is, in fact, located closer to 0.5. Therefore, the candidate that wins the election is determined “correctly”, despite nondisclosure.

\textsuperscript{24}Condition (ii) is not necessary only when the full-revelation payoffs of both parties are the same, equal to 0.5, for any pooling type. From our discussion in the previous section, it follows that this is the case when all pooling types are such that parties’ positions are exactly symmetric around 0.5 (\(\lambda = 0.5\)). Whenever such specific dependence between the parties’ positions does not hold, condition (ii) must be satisfied.
Conditions (i) and (ii) immediately imply that any nondisclosure equilibrium is ex-post efficient. It is only when the indifferent voter is not well-defined, that this argument does not go through. In the proof we show that this can only be the case in equilibrium when all nondisclosing types are such that parties’ positions are either equal to each other or exactly symmetric around 0.5. If there exists at least one type with positions that do not have this special relationship to each other (and the indifferent voter under nondisclosure is not well-defined), then for one of the two parties of this type, the full-revelation pay-off is larger than the pay-off from nondisclosure. Hence, equilibrium nondisclosure with possible loss of efficiency can only occur if parties’ positions are equal or symmetric around 0.5.\footnote{These are the positions corresponding to types on the upward and downward sloping diagonals of the \([0, 1] \times [0, 1]\) square.} For any \textit{generic} combinations of positions in the \([0, 1] \times [0, 1]\) square, nondisclosure does not mislead voters, and the equilibrium is ex-post efficient.

Note that according to Proposition 2, it is the inefficiency and not nondisclosure that is non-generic. The nondisclosure of parties’ types is, in fact, common; it just does not distort voters’ choices.\footnote{From our discussion above it follows that any such not fully disclosing equilibrium is weak, in the sense that nondisclosure is never strictly preferred to full disclosure by either of the pooling types.} Consider one example of such equilibrium, where \textit{any} type in \([0, 1] \times [0, 1]\) pools with some other types but voters’ choices are the same as under full disclosure. In particular, suppose that any type \((x_1, x_2) \in [0, 1] \times [0, 1]\) such that \(x_1 < x_2\) pools with any other type \((y_1, y_2)\) such that \(y_1 < y_2\) and \(x_1 + x_2 = y_1 + y_2\). That is, all types on any downward sloping segment \(x_1 + x_2 = \text{const}\) above the 45-degree line pool with each other. Symmetrically, suppose that all types on any downward sloping segment \(x_1 + x_2 = \text{const}\) below the 45-degree line also pool. Finally, let all types exactly on the 45-degree line, where \(x_1 = x_2\) for any type, pool with each other. This results in the situation, where no type in the whole unit square fully reveals parties’ positions, as schematically shown on Figure 1 below. Yet, we claim that the induced voters’ choices are the same as under full disclosure, so that the equilibrium is ex-post efficient.

First, to see why the described strategy profile is an equilibrium, note that no type can imitate the strategy of another type because parties’ statements must be consistent with their true positions. Note also, that the full-revelation pay-off to any type is exactly the same as the nondisclosure pay-off: on the 45-degree line it is equal to 0.5 for both parties, and on any downward sloping segment the pay-offs to the parties are given by (9) and (10) with \(\hat{\lambda} = \frac{x_1 + x_2}{2}\), both pay-offs being constant along a given segment.\footnote{The position of the indifferent voter \(\hat{\lambda}\) is the same for any type on a segment, therefore, it is also equal to the position of the indifferent voter for the given nondisclosure statement (the whole segment).} Moreover, a simple set of voter out-of-equilibrium beliefs rules out incentives for deviation. For example, suppose that after a deviating statement voters are
Figure 1: An equilibrium where any type in \([0,1] \times [0,1]\) pools with some other types but voters’ choices are not distorted. Pooling types belong to all downward sloping segments above and below the 45-degree line and to the 45-degree line.

certain that the true type is a particular type in the intersection of the deviating statement and the equilibrium nondisclosure statement of the other party.\(^{28}\) As this type is one of the pooling types to which the deviating party’s type belongs, the resulting full-revelation pay-off to the deviating party is exactly equal to its equilibrium pay-off.

A further simple argument confirms that given this nondisclosure equilibrium, the voters’ choices are the same as under full disclosure. Indeed, the nature of pooling among types is such that the indifferent voter associated with a nondisclosing statement is the same as the indifferent voter associated with full disclosure of any pooling type. For nondisclosure on the downward sloping segments, both above and below the 45-degree line, the position of the indifferent voter is \(\hat{\lambda} = \frac{1}{2} (x_1 + x_2)\) (which is the same for all types on a given segment); for nondisclosure along the 45-degree line, \(all\) voters are indifferent between parties 1 and 2. Moreover, the pooling strategy is also such that the relative actual positions of the two parties at any pooling type are the same as their relative perceived positions due to a nondisclosing statement: on downward sloping segments above the 45-degree line party 1 is always located to the left of party 2, the opposite is true on downward sloping segments below the 45-degree line, and on the 45-degree line both parties have the same location. Therefore, nondisclosure in the described equilibrium does not mislead voters’ choices, even though it “involves” the whole type space, where no single type is fully revealed.

There do, however, exist nondisclosure equilibria, where pooling in a non-generic set of types may misguide voters. One example of such equilibrium is provided in the Appendix. According to Proposition 2, all equilibria of this kind are very peculiar in the sense that types that pool and mislead voters’ choices are such that parties’ positions have very special relationship to each

\(^{28}\)This intersection is not empty due to the assumption of consistency of the parties’ statements.
other. For all generic combinations of the parties’ positions, voters’ choices are not distorted and equilibria are ex-post efficient, even if they are not fully revealing.

5 No negative political campaigning

Let us now consider the case where parties do not engage themselves in negative political campaigning. Thus, a statement of each party includes information about its own policy only. Formally, this means that while the type space is two-dimensional, the action space (statements) is one-dimensional. This leads to the situation where a party of one type can imitate the one-dimensional equilibrium statement of another type but voters can still detect the deviation as their beliefs depend on statements of both parties. To see this, consider as an example a situation where types reveal $\Phi \subseteq [0, 1]$ if and only if both positions $x_1, x_2 \in \Phi$, whereas the other types fully disclose the parties’ positions. A party whose position is in $\Phi$ could then disclose $\Phi$ even if its adversary’s position is not in $\Phi$, thus imitating a type with both positions in $\Phi$. Similarly, a party of a type with both positions in $\Phi$ could fully reveal its own position, imitating the statement of a type whose own position (but not that of the adversary) is in $\Phi$. However, as voters observe the statements made by both parties, the proposed equilibrium strategies are constructed so that a voter can always deduce a unilateral deviation.

Without negative campaigning, we find that there exists an infinite set of not fully disclosing equilibria and a broad variety of nondisclosure outcomes are ex-post inefficient. For example, the strategy profile described above, is an equilibrium for any compact set $\Phi$. Such equilibrium induces nondisclosure in the symmetric set $\Phi \times \Phi$, which can be empty or coincide with the whole type space when $\Phi = [0, 1]$. All these nondisclosure outcomes are such that parties 1 and 2 of the types in $\Phi \times \Phi$ are perceived by voters as absolutely identical and hence, gain equal shares in political power. However, for most of the actual types in $\Phi \times \Phi$ (those where $x_1 \neq x_2$) this should not be the case as some voters should strictly prefer one party over the other, leading to different success rates of the two parties.

This last observation suggests that when no negative political campaigning is used, many equilibria are not pay-off equivalent. In fact, nondisclosure can be an equilibrium outcome even if the pay-off to one of the parties of a pooling type is actually lower than its pay-off from full disclosure. The reason why this is different from the case of negative political campaigning is that here a party can never guarantee itself the full-revelation pay-off by unilaterally revealing own position if the other party does not reveal its position. For example, voters may interpret a deviation to full disclosure as not only revealing the position of the deviating party itself but
also as signalling the relative position of the adversary. A non-favorable signal can “punish” the
deviation and sustain a large set of equilibrium outcomes, with pay-offs below the full disclosure
level.\textsuperscript{29}

We now formally characterize equilibrium strategies of the two parties that induce nondisclosure
in any (compact) set of types $\Phi \times \Phi$. As noted earlier, whenever $\Phi$ is not empty, these equilibrium
strategies may misguide voters and lead to inefficient choices.\textsuperscript{30}

**Proposition 3.** For any compact subset $\Phi \subseteq [0, 1]$ there exists an equilibrium where

- parties 1 and 2 of any type $(x_1, x_2)$ with $x_1, x_2 \in \Phi$ make the same statement $S^* = \Phi$;
- parties 1 and 2 of any other type $(x_1, x_2)$ (such that $x_i \notin \Phi$ for at least one of the positions) fully disclose their position by making a precise statement, i.e., $S_i^* = \{x_i\}$, $i = 1, 2$.

Clearly, the described symmetric equilibria are only a subset of all possible equilibria. But this
subset encompasses a rich variety of nondisclosure equilibria where inefficient voting is common.
The proof of Proposition 3 is based on the idea that one can construct a system of voter out-of-
equilibrium beliefs such that unilateral deviations of any type of a party are viewed as signalling an
even less popular intended policy of this party than under the equilibrium strategies. For example,
consider a deviation to a non-equilibrium statement by a party whose type has both positions in $\Phi$.
Since the statement of each party is one dimensional and consistent with true position, the
statements of the two parties intersect. Then voters can simply believe that the parties’ positions
in the intersection are such that the deviating party is at least as far from the middle of the unit
interval as its adversary. Given such beliefs, the deviation pay-off of the party is bound to be less
than or equal to 0.5, that is, less than or equal to its equilibrium pay-off.

Thus, in contrast to the case of negative political campaigning, an infinite set of not fully
disclosing equilibria exist where voters’ choices are inefficient.

6 Discussion and conclusions

This paper has considered the incentives of political parties to reveal their true, intended policy
to uninformed voters. The paper shows the importance of being able to reveal the adversary’s
position, which we call negative campaigning. Under negative campaigning, the generic equilibrium

\textsuperscript{29}In particular, this means that in contrast to the case of negative political campaigning, where all nondisclosure equilibria are weak, in case without negative campaigning, many unilateral deviations from nondisclosure are diverted since the deviation pay-off is strictly lower than the equilibrium pay-off.

\textsuperscript{30}The same strategies constitute an equilibrium in case of presidential elections, with 0-1 pay-offs. Hence, also in this case, many nondisclosure equilibria are ex-post inefficient.
outcome is such that all voters are able to infer their most preferred party and vote accordingly. In this sense, negative campaigning allows voters to maximize their ex-post utility from voting, given the revealed intended policies, and leads to efficient outcomes. Without negative campaigning, there are many equilibria where the parties’ disclosure strategies are such that voters cannot know what their most preferred party is and inefficient outcomes may result.

The main assumption underlying our analysis is that parties know each others’ positions. In a world where the stakes for winning the largest share in political power are high, this seems to be a realistic assumption to make. Different parties are supported by different interest groups and the information of these groups is typically leaked to the adversaries. Otherwise the politicians themselves will find it effective to spend resources on finding out the underlying motives of their adversary.

Given this assumption, we have analyzed the two cases separately: one where the parties engage in negative political campaigning, and one where they do not. We have not formally analyzed the game where the parties have a choice whether or not to engage in negative campaigning. It is, however, not difficult to see that the arguments leading to effectively full disclosure of positions and efficiency of voters’ choices under negative campaigning also lead to generic efficiency of any equilibrium, where parties can choose the type of campaigning.\textsuperscript{31}

Under presidential elections, where the candidate gaining the largest vote share wins, the results are somewhat different. In this case, ex-post inefficiency is a feature of many nondisclosure equilibria, irrespective of whether the political campaigning is negative or not. It results from the fact that candidates’ (0–1) pay-offs do not have a one-to-one monotonic relationship with the position of the indifferent voter, so that the indifferent voter given the equilibrium nondisclosure statement might be different from the indifferent voter for any actual pooling type.\textsuperscript{32} We observe, however, that any such nondisclosure equilibrium, while distorting the choices of some voters, leads to the same election outcome as under full disclosure. Indeed, as comparative statements allow the candidates to disclose both positions precisely, the candidate whose true position is closer to

\textsuperscript{31}Indeed, first, the fact that the sum of parties’ pay-offs is always equal to one implies that given a choice between the two types of campaigning, parties will only be “satisfied” with the equilibrium where their pay-offs are equal to the full-revelation pay-offs. As a result, any equilibrium, with or without negative political campaigning, will satisfy this property. Second, the monotonic functional dependence of parties’ pay-offs on the position of the indifferent voter suggests that in any nondisclosure equilibrium, the position of the indifferent voter for any nondisclosing type must be the same as under full disclosure. Then due to the same argument as before this will imply that generically any equilibrium outcome is ex-post efficient.

\textsuperscript{32}An example of such nondisclosure equilibrium is where all types $(x_1, x_2)$ such that $x_2 < x_1 < 0.5$ or $0.5 < x_1 < x_2$ (candidate 1 is located closer to 0.5 than candidate 2) pool, and where all types $(x_1, x_2)$ such that $0.5 < x_2 < x_1$ or $x_1 < x_2 < 0.5$ (candidate 2 is located closer to 0.5 than candidate 1) pool. In this equilibrium the nondisclosure pay-off of the candidate that is perceived as located closer to 0.5 is 1, just as under full disclosure, but the indifferent voter associated with any particular pooling type and with the nondisclosure equilibrium statement is not the same.
0.5 and who, therefore, wins the election under full disclosure, will only pool with other types if he also wins under nondisclosure. If negative political campaigning is not feasible under presidential elections, then many of the nondisclosure equilibria remain not only ex-post inefficient but also lead to distorted election outcomes.

**Appendix**

*Demonstration that $\pi_L$ is increasing in $\hat{\lambda}$. Using the definition of $\pi_L$ in (9) and applying the Leibniz rule of integral differentiation, we obtain:*

$$
\pi'_L = \frac{\int_0^{\hat{\lambda}} g(\lambda) d\lambda \cdot \left(\int_0^{\hat{\lambda}} (\hat{\lambda} - \lambda) g(\lambda) d\lambda + \int_1^{\hat{\lambda}} (\hat{\lambda} - \lambda) g(\lambda) d\lambda\right) - \int_0^{\hat{\lambda}} (\hat{\lambda} - \lambda) g(\lambda) d\lambda \cdot \left(\int_0^{\hat{\lambda}} g(\lambda) d\lambda - \int_1^{\hat{\lambda}} g(\lambda) d\lambda\right)}{\left(\int_0^{\hat{\lambda}} (\hat{\lambda} - \lambda) g(\lambda) d\lambda + \int_1^{\hat{\lambda}} (\hat{\lambda} - \lambda) g(\lambda) d\lambda\right)^2}
$$

$$= \frac{G(\hat{\lambda}) \left(\int_0^{\hat{\lambda}} (\hat{\lambda} - \lambda) g(\lambda) d\lambda + \int_1^{\hat{\lambda}} (\hat{\lambda} - \lambda) g(\lambda) d\lambda\right) - \int_0^{\hat{\lambda}} (\hat{\lambda} - \lambda) g(\lambda) d\lambda \cdot \left(2G(\hat{\lambda}) - 1\right)}{\left(\int_0^{\hat{\lambda}} (\hat{\lambda} - \lambda) g(\lambda) d\lambda + \int_1^{\hat{\lambda}} (\hat{\lambda} - \lambda) g(\lambda) d\lambda\right)^2}
$$

$$= \frac{\int_0^{\hat{\lambda}} (\hat{\lambda} - \lambda) g(\lambda) d\lambda \cdot \left(1 - G(\hat{\lambda})\right) + \int_1^{\hat{\lambda}} (\hat{\lambda} - \lambda) g(\lambda) d\lambda \cdot G(\hat{\lambda})}{\left(\int_0^{\hat{\lambda}} (\hat{\lambda} - \lambda) g(\lambda) d\lambda + \int_1^{\hat{\lambda}} (\hat{\lambda} - \lambda) g(\lambda) d\lambda\right)^2}.
$$

This expression is strictly positive for any $\hat{\lambda} \in (0, 1)$. □

*An example of equilibrium under negative campaigning where pooling in a non-generic set of types may misguide voters.* Consider the following strategy profile. An asymmetric selection of types on the downward sloping diagonal of the $[0, 1] \times [0, 1]$ square pool and all the other types fully reveal both positions.\footnote{Full disclosure of all the other types is considered for simplicity. Instead, we could, for example, consider that all types which are not on the downward sloping diagonal pool with each other in the same manner as in the example of section 4. As we have shown in that example, such kind of pooling does not distort voters’ choices, and in this sense, it is equivalent to full disclosure.} Note that types above the 45-degree line have party 1 located to the left of party 2 and types below the 45-degree line have party 2 located to the left of party 1. Suppose that the asymmetric selection of pooling types is such that it consists of all types on the diagonal above the 45-degree line and only one type below the 45-degree line. Figure 2 provides an illustration.

Note that all the pooling types have positions of the two parties symmetric around 0.5, i.e., $x_1 = 1 - x_2$. Therefore, the associated pay-off to both parties of any pooling type is 0.5 as the indifferent voter is located exactly in the middle of the unit interval.

It is easy to see that the proposed strategy profile is an equilibrium. First, just as in the example of section 4, no type can imitate the strategy of another type due to the assumption of consistency of the parties’ statements with actual positions. Second, by fully revealing both
Figure 2: An equilibrium where types on the downward sloping diagonal above the 45-degree line and one type below the 45-degree line pool. The induced voters’ choices can be different than under full disclosure.

positions, a party of any pooling type obtains the same pay-off as in the proposed equilibrium, equal to 0.5. Finally, one can easily construct voter out-of-equilibrium beliefs such that after any deviating statement, voters believe that the true type is one of the types in the statement where the associated full-revelation pay-offs to both parties are equal to their equilibrium pay-offs. Hence, a deviation is not gainful.

Clearly, the described strategies may lead to inefficient choices at least for some voters. Indeed, given the equilibrium strategies, and after observing the pooling statement, voters assume that it is infinitely more likely that party 1 is located to the left of party 2. Therefore, all voters with a position to the left of 0.5 vote for party 1, provided that their cost of voting is not too high. However, there is one type making the same statement that is located on the other side of the 45-degree line, and if that type materializes, all voters vote for the “wrong” party. That is, an inefficient outcome occurs.

Proof of Proposition 2. Consider a set of types $\Phi \subseteq [0,1] \times [0,1]$ such that $\Phi$ is not a subset of types on the upward and downward sloping diagonals. In the following we show that for any such set, as soon as there exists an equilibrium in which all types in $\Phi$ pool with each other, their nondisclosure does not mislead voters and equilibrium voting choices are the same as under full disclosure. This will then suggest that the only types in the $[0,1] \times [0,1]$ square that (a) may have incentives to pool with other types and (b) by pooling induce inefficient voters’ choices are all located on either of the two diagonals. Therefore, there does not exist an equilibrium where

$^{34}$Recall that $x_1 = x_2$ for all types along the upward sloping diagonal, and $x_1 = 1 - x_2$ for all types along the downward sloping diagonal.
the set of types that (a) do not fully disclose and (b) make a statement inducing inefficient voters’ choices has a positive measure.

So, suppose that there exists an equilibrium in which all types in set $\Phi$ pool. Pooling requires the existence of at least two different types in $\Phi$. Let us denote them by $(x_1, x_2)$ and $(y_1, y_2)$. Moreover, since $\Phi$ is not a subset of the upward and downward sloping diagonals, there exists at least one type in $\Phi$ – let it be $(y_1, y_2)$ – that does not belong to either of the diagonals, so that $y_1 \neq y_2$ and $y_1 \neq 1 - y_2$.

Notice that the pay-off of each party of any type in $\Phi$ is equal to the pay-off of this party in the full disclosure equilibrium. This follows from two observations. First, since parties know positions of each other, each of them can guarantee itself a full disclosure pay-off by revealing both positions precisely. Therefore, in any not fully revealing equilibrium, the pay-off to any type of a party must be at least as high as the pay-off to that type in the full disclosure equilibrium. Second, the pay-off to any type of a party cannot be strictly larger than the full disclosure pay-off since the sum of the parties’ pay-offs is always equal to one. Indeed, if one party would get a pay-off in a nondisclosure equilibrium that is larger than her full disclosure pay-off, then the other party would have an incentive to deviate to full disclosure. Thus, the pay-off to a party of any type in $\Phi$ is equal to its full-revelation pay-off.

The equality of equilibrium pay-offs and full disclosure pay-offs is equivalent to the following two conditions. First, as the pay-off to each type is uniquely determined by $\hat{\lambda}$ (whenever it is well-defined), we should have that for any type in $\Phi$, the position of the indifferent voter is the same for a given equilibrium pooling statement and under full disclosure. Second, as the pay-offs to parties 1 and 2 of a type in $\Phi$ are not the same (we will show this below), parties’ relative positions with respect to each other should be the same as their relative perceived positions due to the pooling equilibrium statement. The two conditions imply that nondisclosure by types in $\Phi$ does not mislead voters and the equilibrium is ex-post efficient.

The fact employed for the second condition – that the pay-offs to parties 1 and 2 of a type in $\Phi$ are not the same, – is an immediate implication of the first condition and the assumption that at least one type in $\Phi$, for example, $(y_1, y_2)$, has its two positions asymmetric around 0.5. Indeed, the first condition means that the value of $\hat{\lambda}$ due to the equilibrium pooling statement is equal to the position of the indifferent voter when type $(x_1, x_2)$ or $(y_1, y_2)$ fully discloses:

$$\hat{\lambda} = \frac{1}{2} (x_1 + x_2) = \frac{1}{2} (y_1 + y_2).$$

(12)

In this expression $y_1 + y_2 \neq 1$, so that $\hat{\lambda} \neq 0.5$. Therefore, the pay-offs of the two parties, as defined
by (9) – (10), are not the same.\footnote{Recall that from our discussion in the end of section 2 it follows that the pay-offs of the parties with the furthest left and right position are equal, \( \pi_L = \pi_R \), if and only if \( \hat{\lambda} = 0.5 \).}

It remains to consider the case where the indifferent voter is not well-defined, as this is the only case in which the above argument does not go through. Below we show that since our set of pooling types \( \Phi \) includes \((y_1, y_2)\), where parties’ positions are neither equal nor symmetric around 0.5, this situation is not an equilibrium. This would then contradict the definition of set \( \Phi \), and thereby, conclude the proof.

Suppose first that the indifferent voter is not well-defined for the equilibrium pooling statement. Then the equilibrium pay-off to a party of any type in \( \Phi \) is either 0.5 (when all voters are indifferent between the two parties) or 0 or 1 (if no voter is indifferent). In either case, it is easy to see that one of the parties of type \((y_1, y_2)\) would strictly benefit from deviating to full disclosure. Now, suppose that the indifferent voter is not well-defined when one of the pooling types in \( \Phi \) fully discloses. This can only occur when the positions of the two parties of that type are equal, so that all voters are indifferent between parties 1 and 2. Then the full-revelation pay-off of both parties of this type is 0.5 and given the equality of the full-revelation pay-off and equilibrium nondisclosure pay-off, the equilibrium nondisclosure pay-off of this – and any other type in \( \Phi \) – is also equal to 0.5.\footnote{Here we employed the fact that the equilibrium pay-off of a party is the same at any pooling type and does not depend on the type.} But this implies that the party of type \((y_1, y_2)\) whose location is closer to 0.5 can benefit from deviating to full disclosure. Hence, the situation where the indifferent voter is not well-defined, either for the equilibrium pooling statement or for one of the pooling types in \( \Phi \), is not an equilibrium.

Thus, we obtain that for any equilibrium set of pooling types \( \Phi \), nondisclosure does not mislead voters and their choices are the same as under full disclosure. \( \square \)

\textit{Proof of Proposition 3.} First, notice that no type of a party can or has incentives to imitate the strategy of another type. Indeed, even if the statements’ consistency assumption allows a party to make an equilibrium statement of another type, this imitation will be detected by voters. This is so as voter beliefs about the type are formed based on statements of both parties and the other party still makes an equilibrium statement. For example, if \( x_1 \in \Phi \), but \( x_2 \notin \Phi \), party 1 could imitate a type with both positions in \( \Phi \) by making a statement \( S^* = \Phi \). However, since party 2 of type \((x_1, x_2)\) reveals its own position precisely, and this position lies outside \( \Phi \), voters, who know the equilibrium strategies, deduce that party 1 has deviated. Similarly, a party of a type with both positions in \( \Phi \) can fully disclose its own position, imitating the equilibrium statement...
of a type where the position of that party (but not the position of the adversary) is in $\Phi$. But given that the statement of the other party is $\Phi$, voters deduce that both parties have positions in $\Phi$ and hence, the first party has deviated. Finally, a party of a type with at least one of the positions outside $\Phi$ can imitate the strategy of another such type – if this party’s position is the same for both types. But given that the other party fully reveals its position, the imitating party cannot succeed in pretending to be of the other type.

Since the deviating party can always be detected, we now need to construct a system of voter out-of-equilibrium beliefs such that given these beliefs, no incentives to deviate exist. Consider the following out-of-equilibrium beliefs. If voters observe $S_i = \Phi$ and $S_j \neq \Phi$, then they assign probability one to such positions of the two parties in $S_i \cap S_j$ where party $j$ is at least as far from 0.5 as her adversary, that is, where $|x_j - 0.5| \geq |x_i - 0.5|$. Recall that according to voters’ out-of-equilibrium beliefs, the deviation pay-off is smaller or equal to the pay-off of the competitor.

And if voters observe $S_i = \{x_i\}$ and $S_j \neq \{x_j\}$, then they assign probability one to party $j$ being located at $x'_j(S_j) \in S_j$ where the distance from party $j$ to 0.5 is the largest among all locations in $S_j$, that is, where the full-revelation payoff of party $j$, given position $x_i$ of the adversary, is minimized.

It is easy to see that given such voter out-of-equilibrium beliefs, no type of a party has incentives to deviate from the proposed equilibrium strategy. Indeed, a party of type $(x_1, x_2)$ such that $x_1 \in \Phi$ and $x_2 \in \Phi$ has no incentives to deviate since its equilibrium pay-off, given the symmetry of the statements, is 0.5, while any deviation pay-off does not exceed 0.5. Next, consider a deviation by a party of type $(x_1, x_2)$ such that either $x_1$ or $x_2$ or both positions do not belong to $\Phi$. If party $j$ deviates to some admissible statement $S_j \neq \{x_j\}$, then given the voter beliefs in this case, the subsequent choice of voters will be as if the true position of party $j$ is $x'_j(S_j)$ for sure. Then the deviation pay-off of party $j$ is equal to its full-revelation pay-off based on the pair of positions where its own position is $x'_j(M_j)$ and the position of the adversary is $x_i$. As $x_j \in S_j$, it follows that this pay-off does not exceed the party’s full revelation pay-off based on the true positions. Thus, the deviation is not gainful. 

References


\[37\] $S_i \cap S_j \neq \emptyset$ due to the assumption of consistency of the parties’ statements with true positions.

\[38\] Recall that according to voters’ out-of-equilibrium beliefs, the deviation pay-off is smaller or equal to the pay-off of the competitor.


[34] Schipper, B. C. and H. Y. Woo [2015], ”Political Awareness, Microtargeting of Voters, and Negative Electoral Campaigning,” mimeo.

