Competition and Disclosure of Comparative or Non-comparative Horizontal Product Information

Maarten C.W. Janssen†  Mariya Teteryanikova‡
University of Vienna, Austria. University of Vienna, Austria.

August 30, 2013

Abstract

This paper studies the incentives to disclose horizontal product attributes in an environment where firms compete. With competition two elements may play an important role, namely whether (i) firms can only disclose their own product characteristics or also those of their competitors, and whether (ii) competitors can react with their pricing decisions on the type of information that is disclosed. We study each of these four different cases that can arise: (i) non-comparative non-price advertising, (ii) non-comparative price advertising, (iii) comparative price advertising, and (iv) comparative non-price advertising. In all these cases, full revelation is an equilibrium outcome. More importantly, full disclosure is generically the unique equilibrium outcome under comparative price advertising, but not in the other cases considered.

JEL Classification: D43, D82, D83, M37

Keywords: Information disclosure, advertising, horizontal differentiation, price competition, asymmetric information

---

*We would like to thank the editor (Volker Nocke), an anonymous referee and Simon Anderson, Levent Celik, Regis Renault, and Santanu Roy for useful suggestions and feedback. We also thank participants of the VGSE microeconomic seminar, seminars at the University of Frankfurt, University of Grenoble 2, University of Innsbruck, EARIE 2012 and UECE 2011 conferences for helpful comments. Janssen acknowledges financial support from the Vienna Science and Technology Fund (WWTF) under project fund MA 09-017 and from the Basic Research Program of the National Research University Higher School of Economics.

†Department of Economics, University of Vienna, Oskar-Morgenstern-Platz 1, 1090 Vienna, Austria, and Higher School of Economics, Moscow. Tel: (+43)-1-427737438; E-mail: maarten.janssen@univie.ac.at.

‡Department of Economics, University of Vienna, Oskar-Morgenstern-Platz 1, 1090 Vienna, Austria. Tel: (+43)-1-427737427; E-mail: mariya.teteryatnikova@univie.ac.at.


1 Introduction

In a large number of markets, sellers have important information about product attributes that are not publicly observable. In many instances, however, firms have the option of voluntarily disclosing this information in a credible and verifiable manner through a variety of means such as independent third party certification, labeling, rating by industry associations (or government agencies) and through informative advertising. Whether or not firms reveal their private information to consumers may have important consequences for the market outcome in terms of price and products that are produced and exchanged.

There is a large literature dealing with the question whether firms have appropriate incentives to disclose information about the product they produce. Most of this literature deals with this issue in the context of vertical product differentiation, where different firms sell different qualities. In this context, the well-known unraveling argument\textsuperscript{1} establishes that a firm whose product is actually better than the average quality produced by firms who do not disclose product quality has a positive incentive to voluntarily disclose the quality of its product to buyers. This then induces every firm whose quality is above the average undisclosed quality to also disclose. The unraveling argument results in a situation where all private information about quality should be revealed through voluntary disclosure. Observed nondisclosure is then explained in terms of ”disclosure frictions”, such as disclosure costs, consumers not understanding the information that is disclosed, etc. (see, e.g., Grossman and Hart (1980), Jovanovic (1982) and Fishman and Hagerty (2003)). Alternatively, Janssen and Roy (2010) show that nondisclosure can also be explained by a combination of market competition and the availability of signaling as an alternative means (to disclosure) of communicating private information.

Recently, Sun (2011), Koessler and Renault (2012) and Celik (2013) have analyzed the incentives of firms to disclose their product characteristics when horizontal product differentiation is the only or main dimension of differentiation. All these papers are set in a monopoly context. Sun (2011) shows that seller types with unfavorable horizontal attributes (towards the extreme points of the product line) do not have an incentive to disclose. In combination with vertical differentiation, her results imply that if either full disclosure of both attributes or no disclosure at all are the only possible reporting strategies, a seller with private information about both horizontal and vertical attributes may not want to disclose quality even if it is high. Celik (2011) shows that the amount of information disclosure critically depends on the strength of the buyers’ preference for their ideal

attribute. If buyers have very strong preferences for particular product varieties, then there exists an equilibrium in which the seller fully reveals variety. Otherwise, the seller only partially reveals the variety he produces. Moreover, the set of fully revealed locations monotonically shrinks from all to (almost) none as the buyer’s preference for her ideal taste becomes weaker. Koessler and Renault (2012) provide a necessary and sufficient condition for the fully revealing equilibrium to be the unique equilibrium outcome. They also show that when this condition does not hold, a fully revealing equilibrium still exists as long as the firm type is not correlated with the consumer type, but that in that case the equilibrium is typically not unique.

In this paper, we study the extent to which the findings of possible non-disclosure of horizontal product attributes generalize to an environment where firms compete. With competition two additional elements may play an important role. First, with more than one firm in the market, the question is whether firms can only disclose their own product characteristics or also those of their competitors. In the advertising literature, this translates into the question whether advertising is comparative (where information about competitor’s products is also – partially – revealed) or non-comparative. Second, with more than one firm in the market, the question is whether competitors can react with their pricing decisions on the type of information that is disclosed. In the advertising literature, this translates into the question whether advertising includes information about price or whether prices are set at a later moment. Thus, under competition, we study these four different cases: (i) non-comparative non-price advertising, (ii) non-comparative price advertising, (iii) comparative price advertising, and (iv) comparative non-price advertising.

In all these cases, full revelation is an equilibrium outcome. Our main result, however, is that under comparative price advertising, and only in that case, nondisclosure is a probability zero event. That is, in all the other cases, there are equilibria where large sets of firm types do not fully disclose their product characteristics. Under comparative price advertising, nondisclosure equilibria also exist, but they require that the locations of the two firms have a very special relation to each other, which – from an ex ante perspective – occurs with zero probability.

The model we consider has two firms located on a Hotelling line, where each particular location represents the variety of the product. Location is known to both firms, but not to consumers. The case where rival firms know each others’ vertical characteristic is studied by, e.g., Board (2009) and Hotz and Xiao (2011). This type of literature, and thus our paper, is relevant for markets where firms have been active for some time and have the ability (and due to the frequent interaction also the

Footnote 2: The competitive disclosure literature has also considered markets where firms do not know each others’ type (see, e.g., Daughety and Reinganum (2007), Calderaro, Shin and Stivers (2008) and Janssen and Roy (2011)).
incentives) to learn the features of the product produced by a competitor. Under non-comparative non-price advertising, the two firms first simultaneously choose a message informing the market only about their own product characteristics. We assume that firms cannot lie. That is, the true location should be consistent with the message that is chosen. One way to think about this grain-of-truth assumption is that information is verifiable and that there is a large fine for providing information that turns out to be false. The assumption is in line with regulations concerning advertisement or other disclosure mechanisms requiring that firms provide truthful information. Firms can either send a rather vague message, indicating that their location is somewhere on the product line, as one extreme, or a much more precise message, indicating the exact location, as the other extreme, or anything in between. After firms have sent their messages, they both simultaneously choose prices. In this case of non-price advertising, the prices chosen can depend on the message chosen by the competitor. Consumers decide where to buy the product after observing the messages and the prices. Given the information they receive, consumers update their beliefs about the locations of the two firms and buy from the firm that they expect to have the best fit with their preferences (where we assume quadratic transportation cost). The other three cases are very similar in nature. Under non-comparative price advertising, firms choose price and message simultaneously; while under comparative advertising, the message sent is a two dimensional object indicating the own location and the location of the competitor. In that case we impose the grain-of-truth assumption on both dimensions.

Having described the set-up, we can explain the intuition for our results as follows. The fact that full disclosure is an equilibrium outcome in all settings, follows from the independence of firms’ locations with respect to the consumers’ types. Like in Proposition 1 in Koessler and Renault (2012) a firm’s profit does not depend on its true location, but merely on its price and the consumer beliefs about the location. Thus, given the grain-of-truth assumption one can always ”punish” a deviation from full disclosure by constructing out-of-equilibrium beliefs such that the deviation pay-off is lower than the equilibrium pay-off. To understand whether or not there are other types of equilibria, note that our case of comparative non-price advertising is very similar in nature to the monopoly analysis in Proposition 2 of Koessler and Renault (2012). Comparative advertising allows a firm to fully disclose both locations and as both firms react to such a full disclosure message in the price setting stage, firms can achieve the full disclosure pay-offs by deviating. One can construct many nondisclosure equilibria that give the firms a pay-off at least equal to the full disclosure pay-off by ”punishing” deviations with appropriate out-of-equilibrium beliefs (more on this below).

Comparative price advertising is different in that even if a firm fully discloses both locations, the
competitor will not react to this disclosure by setting a different price. As the competitor does not react to a deviation, to have a nondisclosure equilibrium it should be the case that for any two types that send the same message they cannot increase their market share by disclosing and keeping the same price. As an increase in market share for one means a decrease in market share for the other, this requirement implies that the locations of the nondisclosing firms should be such that if they fully disclose and keep prices at their equilibrium levels, they get the same market share. Firms can also disclose and marginally change prices. Then the requirement that this type of deviation should also not increase profits imposes a set of other constraints. Together these constraints are such that they can only be fulfilled for a set of locations that is nongeneric in the full set of possible locations.

When firms do not engage in comparative advertising, they are not able to fully disclose both locations and the above argument does not work. In particular, under non-comparative non-price advertising, in contrast to comparative non-price advertising, the firms cannot guarantee themselves the full disclosure pay-off if they deviate from a nondisclosure equilibrium. In these cases of non-comparative advertising, we show by construction that a large range of outcomes – from full disclosure to full nondisclosure – can be sustained in equilibrium.

In a series of papers, Anderson and Renault (2006, 2009) consider a similar framework and study the incentives of firms to disclose their product characteristics through advertising. They find that if products have both horizontal and vertical attributes and if qualities of firms’ products are known and sufficiently different from one another, only the firm with the lowest quality reveals its horizontal characteristic. The better quality firm remains silent as disclosure would induce it to set a lower price in order to retain the consumers who like its rival more. If firms’ product qualities are identical, both firms reveal their horizontal characteristics even under non-comparative advertising and being able to make comparative advertisement does not change the equilibrium outcomes in their setting. Our paper differs from Anderson and Renault (2006, 2009) in two important respects. First, in Anderson and Renault (2009) firms can only fully disclose their horizontal characteristic or stay silent. In contrast, we consider a model where firms can send any message concerning their product characteristics that satisfies the grain-of-truth assumption. Second, and more importantly, Anderson and Renault (2006, 2009) do not analyze their model as a game with private information where out-of-equilibrium beliefs are important, while we do. This explains why our results also differ and full disclosure is the unique equilibrium outcome under comparative price advertising, but not under non-comparative price advertising. To see where out-of-equilibrium beliefs are important, consider a potential equilibrium where no type of firm discloses and all types set a high price. Anderson
and Renault (2009) argue that this cannot be an equilibrium under non-comparative advertising because of a standard Bertrand-like undercutting argument. However, undercutting the candidate equilibrium price is formally an out-of-equilibrium action and therefore, consumers should form beliefs about the firms’ product characteristics. If consumers believe that the undercutting firm has relatively disadvantageous product characteristics, the firm’s demand would be lower than if it had not undercut, and therefore, the firm would not have an incentive to undercut in the first place. That is, under non-comparative advertising, we get a much larger set of equilibria than Anderson and Renault (2009) and comparative advertising makes a difference in case when price is also advertised in our model.

Standard refinements like the Intuitive Criterion (Cho and Kreps, 1987) or D1 (Cho and Sobel, 1990) do not rule out these multiple nondisclosure equilibria as the profits of all firms’ types only depend on consumer beliefs about firms’ locations and price rather than on the actual locations. In the discussion section we briefly point out that if one restricts attention to out-of-equilibrium beliefs that do not discriminate between different types of firms as soon as after observing the out-of-equilibrium action consumers have no reason to discriminate between different types, then only the fully revealing outcome can be sustained in equilibrium. We do not necessarily argue that such restrictive out-of-equilibrium beliefs are natural to impose. As we consider the case where firms are informed about both firms’ product characteristics, deviating from the equilibrium actions, may be interpreted by consumers that the deviating firm is trying to hide that it has the same location as its competitor or that it is signaling that it has a different product from its competitor. None of these interpretations can be excluded. By briefly discussing these restrictive out-of-equilibrium beliefs, we mainly want to point at the role of out-of-equilibrium beliefs in creating nondisclosure equilibria and the fact that in this disclosure game firms’ pay-offs do not depend on true locations, but only on perceived locations.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 shows that fully revealing equilibria exist in all cases. Section 4 then discusses the two cases where firms engage in comparative advertising and section 5 deals with the cases of non-comparative advertising. Section 6 concludes with a discussion, while proofs are contained in the Appendix.

2 Model

Consider a horizontally differentiated duopoly, where the variety produced by each firm is represented by a particular location on the unit interval. Let $x_i$ denote the variety produced by firm $i$ and
\( x_i \in [0, 1], i = 1, 2 \). We focus on the disclosure policy of the firms and consider these varieties to be given for the firms. Also, we consider markets where firms know each others’ variety, but consumers are unaware of the specific varieties of firms. One way to think about this is that it requires resources to research the product characteristics of a firm and that rival firms are better equipped or have more incentives to do this than consumers. In the following, we refer to \( x_1 \) and \( x_2 \) as locations of firms 1 and 2, respectively, and we refer to the pair of locations, \((x_1, x_2)\), as a type of each firm. Notice that because each firm knows its own location and the location of the competitor, the type of a firm is a two dimensional object. Throughout the paper, the first position in the pair \((x_1, x_2)\) stands for the location of firm 1, and the second position – for the location of firm 2. Production costs do not depend on firms’ locations and without loss of generality are set equal to zero.

The demand side of the economy is represented by a continuum of consumers. Each consumer has a preference for the ideal variety of the good that she would like to buy, denoted by \( \lambda \). The value of \( \lambda \), or consumers’ location on \([0, 1]\), follows a uniform distribution.\(^3\) A consumer’s net utility from buying variety \( x_i \) at price \( P_i \), \( i = 1, 2 \), is \( v - t(\lambda - x_i)^2 - P_i \), where \( v \) is the gross utility of a consumer when the variety of the good, \( x_i \), matches with her ideal variety, \( \lambda \), perfectly (i.e., when \( x_i = \lambda \)) and \( t \) measures the degree of disutility a consumer incurs when \( x_i \) and \( \lambda \) differ from each other. We assume that \( v \) is sufficiently large so that the market is fully covered. Each consumer then chooses to buy the good from the firm where her expected utility is maximized. The consumer has unit demand and if she buys from firm \( i \), then firm \( i \)'s payoff from the transaction is \( P_i \); otherwise, the payoff of firm \( i \) is zero.

The timing of the non-price advertising games is as follows. At stage 0, Nature independently selects location \( x_1 \) for firm 1 and \( x_2 \) for firm 2 from a strictly positive density function \( f(x) \). The locations are known to both firms, but not to consumers. At stage 1, firms send a costless message \( M_i \subseteq [0, 1], i = 1, 2 \), about their own location or a message \( M_i \subseteq [0, 1] \times [0, 1] \) about the locations of both firms. The former refers to the case of non-comparative advertising, where a firm can only provide information about own variety. The latter refers to the case of comparative advertising, where a firm provides information about varieties of both firms. In sections 4 and 5 we will examine each of these two cases separately, but unless explicitly stated otherwise, the same notation and definitions apply throughout. Notice that in the different cases \( M_i = [0, 1] \) or \( M_i = [0, 1] \times [0, 1] \) can be interpreted as ”no message at all”, or full nondisclosure of information by firm \( i \). Messages have to contain a grain of truth in the sense that \( x_i \in M_i \) for \( i = 1, 2 \) if advertising is non-comparative.

\(^3\)This specification with a continuum of consumers whose preferences for variety are uniformly distributed over the unit interval, is identical to the specification with a single consumer who has a privately known taste for a variety drawn from the uniform density function defined over \([0, 1]\).
and \((x_1, x_2) \in M_i\) for \(i = 1, 2\) if advertising is comparative. This means that firms cannot lie about their locations, but they can be more or less specific about the true locations. In the following we will refer to this assumption as the *grain-of-truth assumption*. At stage 2, firms simultaneously set prices. Finally, at stage 3, consumers observe the messages and the prices of the two firms and decide where to buy. At the end of the game, the payoffs of all players – firms and consumers – are realized. All aspects of the game are common knowledge. Under *price advertising*, firms set prices together with sending their message on locations, and the separate pricing stage is eliminated. The rest of the game is identical to the respective non-price advertising games.

As a solution concept we use perfect Bayesian equilibrium where beliefs of consumers off-the-equilibrium path, albeit arbitrary, are identical across consumers. Following Fudenberg and Tirole (1991) we will refer to this equilibrium as a **strong perfect Bayesian equilibrium**. To define it formally for the non-price advertising games, we specify the strategy spaces as follows.\(^4\) The reporting strategy of firm \(i\) is denoted by \(m_i(x_i, x_j)\), where the image of \(m_i\) belongs to all subsets of \([0, 1]\) such that \(x_i \in m_i\) (for non-comparative advertising), or to all subsets of \([0, 1] \times [0, 1]\) s.t. \((x_1, x_2) \in m_i\) (for comparative advertising). The pricing strategy of firm \(i\) is denoted by \(p_i(x_i, x_j|M_i, M_j)\), where the messages sent by the two firms are \(M_i\) and \(M_j\), respectively. Similarly, let the vector \(b(\lambda, M_i, M_j, P_i, P_j)\) describe the buying strategy of a consumer with preferred variety \(\lambda\), where \(b = (1, 0)\) if the consumer buys the good from firm 1 and \(b = (0, 1)\) if she buys from firm 2. Finally, let \(\mu_i(z|M_i, M_j, P_i, P_j)\) be the probability density that consumers assign to \(x_i = z\) when the firms send messages \(M_i, M_j\) and set prices \(P_i, P_j\). A strong perfect Bayesian equilibrium is then defined as follows.

**Definition** A strong perfect Bayesian equilibrium of the non-price advertising games\(^5\) is a set of reporting and pricing strategies \(m^*_1, m^*_2, p^*_1, p^*_2\) of the two firms, strategy \(b^*\) of a consumer, and the probability density functions \(\mu^*_1, \mu^*_2\) which satisfy the following conditions:

1. For all \(M_1, M_2, P_1\) and \(P_2\), \(b^*\) is a consumer’s best buying decision as defined below:

\[
\begin{align*}
b(\lambda, M_1, M_2, P_1, P_2) = \\
&= \begin{cases} 
(1, 0) & \text{if } \int_{0}^{1} (v - t(\lambda - x_1)^2 - P_1)\mu_1(x_1|M_1, M_2, P_1, P_2)dx_1 \geq 0 \\
& \int_{0}^{1} (v - t(\lambda - x_2)^2 - P_2)\mu_2(x_2|M_1, M_2, P_1, P_2)dx_2 \\
(0, 1) & \text{if } \int_{0}^{1} (v - t(\lambda - x_2)^2 - P_2)\mu_2(x_2|M_1, M_2, P_1, P_2)dx_2 \geq 0 \\
& \int_{0}^{1} (v - t(\lambda - x_1)^2 - P_1)\mu_1(x_1|M_1, M_2, P_1, P_2)dx_1 
\end{cases}
\end{align*}
\]

\(^4\)For the *price* advertising games, all strategies are defined in the same way, apart from the pricing strategy. The pricing strategy of firm \(i\) is \(p_i(x_i, x_j)\) and it is not conditional on the messages.

\(^5\)For the *price* advertising games, parts (2) and (3) of this definition should be merged into one.
(2) Given (1) and given the messages sent by the two firms and the price set by the competitor, \( p^*_i \) is the price that maximizes the expected profit of firm \( i, i = 1, 2 \).

(3) Given (1), (2) and given the message sent by the competitor, \( m^*_i \) is the message that maximizes the expected profit of firm \( i, i = 1, 2 \), subject to the grain-of-truth assumption.

(4) For all \( M_1, M_2, P_1 \) and \( P_2 \), a consumer updates his or her beliefs, \( \mu_i \), regarding the locations of the firms in the following way:

- according to Bayes’ rule on the equilibrium path,
- arbitrarily off the equilibrium path.

All consumers have identical beliefs on and off the equilibrium path.

Part (1) of the definition states that for any observed messages and prices, a consumer buys a unit of the product from the firm, where her expected net utility, given the updated beliefs, is maximized. Each firm rationally anticipates the best response of consumers to any given messages and prices, and chooses the price and message that maximizes its expected profit. This is stated in parts (2) and (3). Finally, part (4) claims that consumers update beliefs about the locations using Bayes’ rule for any \( M_1, M_2, P_1 \) and \( P_2 \) that occur with positive density along the equilibrium path and that beliefs off the equilibrium path are arbitrary but identical across consumers. Note that the grain-of-truth assumption implies that in case of non-comparative advertising, consumers should only assign positive probability to the locations of a firm that are included in its message. Similarly, in case of comparative advertising, a positive density should only be put on the locations that are in the intersection of the two messages. This intersection is not empty due to the same assumption.

In this model, the *indifferent* consumer – with the ideal variety \( \hat{\lambda} \) – obtains the same expected net utility of buying from either of the two firms, given the observed set of messages and prices. Consumers with preferred varieties below \( \hat{\lambda} \) buy from the firm with the most left perceived location and all others buy from the other firm. Thus, the indifferent consumer is defined by

\[
 v - t E \left( (\hat{\lambda} - x_1)^2 | \mu_1 \right) - P_1 = v - t E \left( (\hat{\lambda} - x_2)^2 | \mu_2 \right) - P_2. \tag{2}
\]

---

Note that Bayes’ rule cannot be applied when \( M_i \) or a subset of \( M_i \) is discrete. For example, if in case of non-comparative advertising \( M_i = \{ y, z \} \), then both events, \( x_i = y \) and \( x_i = z \) have ex-ante zero probability. In this case, updating proceeds as follows:

\[
\mu_i(z|M_1, M_2, P_1, P_2) = \lim_{\varepsilon \to 0} \frac{F(z + \varepsilon) - F(z)}{F(z + \varepsilon) - F(z) + F(y + \varepsilon) - F(y)}
\]

Using l’Hôpital’s rule,

\[
\mu_i(z|M_1, M_2, P_1, P_2) = \lim_{\varepsilon \to 0} \frac{f(z + \varepsilon)}{f(z + \varepsilon) + f(y + \varepsilon)} = \frac{f(z)}{f(z) + f(y)}
\]
In this expression, \( tE\left( (\hat{\lambda} - x_i)^2|\mu_i \right) \), \( i = 1, 2 \), is the expectation of the transportation costs of the indifferent consumer associated with buying from firm \( i \), conditional on consumers’ beliefs.

We solve this equality for \( \hat{\lambda} \). Note that

\[
E \left( (\hat{\lambda} - x_i)^2|\mu_i \right) = \hat{\lambda}^2 + E \left( x_i^2|\mu_i \right) - 2\hat{\lambda}E \left( x_i|\mu_i \right) = \hat{\lambda}^2 + \text{var} \left( x_i|\mu_i \right) + E^2 \left( x_i|\mu_i \right) - 2\hat{\lambda}E \left( x_i|\mu_i \right) = \left[ \hat{\lambda} - E \left( x_i|\mu_i \right) \right]^2 + \text{var} \left( x_i|\mu_i \right)
\]

Thus, the quadratic term in the utility function of consumers (like any convex transportation costs) implies that a consumer dislikes uncertainty about the variety of the good and given two messages with the same conditional mean, favors the one with the smaller variance. The quadratic formulation is used to simplify the analysis and guarantee the existence of a pricing equilibrium. Using (5) in (2) we obtain

\[
\hat{\lambda}^2 + \text{var} \left( x_1|\mu_1 \right) + E^2 \left( x_1|\mu_1 \right) - 2\hat{\lambda}E \left( x_1|\mu_1 \right) + \frac{P_1}{t} = \hat{\lambda}^2 + \text{var} \left( x_2|\mu_2 \right) + E^2 \left( x_2|\mu_2 \right) - 2\hat{\lambda}E \left( x_2|\mu_2 \right) + \frac{P_2}{t},
\]

and so, the ideal variety of the indifferent consumer is equal to:

\[
\hat{\lambda} = \frac{1}{2} \left[ \frac{P_2 - P_1}{E(x_2|\mu_2) - E(x_1|\mu_1)} + \frac{1}{2} \left( E(x_1|\mu_1) + E(x_2|\mu_2) \right) \right].
\]

In (6) the expected locations of the two firms are assumed to be different. When \( \hat{\lambda} \notin [0, 1] \), the indifferent consumer is not well defined and either all consumers are indifferent or none is.

For convenience, below we use subscripts \( L \) and \( R \) for the firm with the most left and most right perceived location, respectively, so that \( E(x_L|\mu_L) < E(x_R|\mu_R) \). Then given that \( \hat{\lambda} \) is actually the consumer demand of the firm that is perceived most left, the profits of the two firms can be written as

\[
\pi_L = P_L \left( \frac{1}{2} \frac{P_R - P_L}{E(x_R|\mu_R) - E(x_L|\mu_L)} + \frac{1}{2} \left( E(x_L|\mu_L) + E(x_R|\mu_R) \right) \right)
\]

\[
\pi_R = P_R \left( 1 - \frac{1}{2} \frac{P_L - P_R}{E(x_R|\mu_R) - E(x_L|\mu_L)} - \frac{1}{2} \left( E(x_L|\mu_L) + E(x_R|\mu_R) \right) \right)
\]

where \( \pi_L \) (or \( \pi_R \)) is the profit of the firm with the most left (or right) perceived location.

Notice that apart from the prices, firms’ expected demand and profits only depend on expected locations and on the precision of the messages about these locations, but (and this is important) not
on actual locations. This implies that, subject to the grain-of-truth assumption, consumer beliefs can be used to "punish" deviating firms.

3 Full disclosure equilibria

We first discuss the nature of market outcomes under full disclosure and show that in all the four cases considered full disclosure is an equilibrium outcome. This result is interesting in its own right, but we also bring attention to it here as the full disclosure outcome will be used in later sections to analyze the gains from deviating.

Full disclosure implies that in all expressions above $E(x_i|\mu_i) = x_i$, $\text{var}(x_i|\mu_i) = 0$ and profits of firm 1 and 2 are functions of exact locations $x_1, x_2$. In particular, following the notation that $x_L < x_R$ (the analogue of $E(x_L|\mu_L) < E(x_R|\mu_R)$), the firms’ profits are given by:

$$\pi_L(x_L, x_R) = P_L \left( \frac{1}{2t} \frac{P_R - P_L}{x_R - x_L} + \frac{1}{2} (x_L + x_R) \right)$$

$$\pi_R(x_L, x_R) = P_R \left( 1 - \frac{1}{2t} \frac{P_R - P_L}{x_R - x_L} - \frac{1}{2} (x_L + x_R) \right).$$

The function $\pi_i, i = L, R$, is a strictly concave, quadratic function of $P_i$. Hence, given full disclosure, the profit-maximization problem of each firm is well-defined and the first-order conditions yield the price at which the profit is maximized:

$$-\frac{P_L}{t (x_R - x_L)} + \frac{1}{2} \left( \frac{P_R}{t (x_R - x_L)} + x_L + x_R \right) = 0$$

$$1 - \frac{P_R}{t (x_R - x_L)} - \frac{1}{2} \left( \frac{P_L}{t (x_R - x_L)} + x_L + x_R \right) = 0.$$

Solving these equations results in the equilibrium prices under full disclosure:

$$P_L(x_L, x_R) = \frac{1}{3} t (x_R - x_L) (2 + x_L + x_R)$$

$$P_R(x_L, x_R) = \frac{1}{3} t (x_R - x_L) (4 - x_L - x_R).$$

Plugging expressions (11) – (12) for prices into the profit functions of the two firms yields:

$$\pi_L(x_L, x_R) = \frac{t}{18} (x_R - x_L)(2 + x_L + x_R)^2$$

$$\pi_R(x_L, x_R) = \frac{t}{18} (x_R - x_L)(4 - x_L - x_R)^2.$$

In what follows we will sometimes refer to prices in (11) – (12) as full-revelation prices and to profits in (13) – (14) as full-revelation profits based on $(x_L, x_R)$. Both expressions in (13) – (14) are strictly
positive for any \( x_L < x_R \). Furthermore, when \( x_L = x_R \), consumers buy from the firm with the lowest price. The usual Bertrand-type argument then establishes that \( P_L = P_R = 0 \) and so, \( \pi_L = \pi_R = 0 \).

**Proposition 1.** Whether or not advertising is comparative and whether or not prices are advertised, full disclosure is always an equilibrium outcome.

The proof is a straightforward adaptation of Proposition 1 in Koessler and Renault (2012) and available upon request. Essentially, the idea is the following. If a firm deviates from a fully disclosing strategy, then it does not (precisely) reveal its location (or type), and consumer out-of-equilibrium beliefs are important. Given the grain-of-truth assumption (that messages have to contain the true location (or type)) and the fact that pay-offs depend on expected locations and not on true locations, one can construct beliefs such that given a certain message, the deviating pay-off is not larger than the equilibrium pay-off. This argument does not depend on the particular case.

Note that the profit of firm L in (13) is **decreasing** in \( x_L \), while the profit of firm R in (14) is **increasing** in \( x_R \). Indeed,

\[
\frac{\partial \pi_L}{\partial x_L} = (2 + x_L + x_R) \left( \frac{t}{18} x_R - \frac{3t}{18} x_L - \frac{t}{9} \right) < 0
\]

\[
\frac{\partial \pi_R}{\partial x_R} = (4 - x_L - x_R) \left( \frac{t}{18} x_L - \frac{3t}{18} x_R + \frac{2t}{9} \right) > 0
\]

where the signs of the derivatives are implied by the fact that \( 0 \leq x_L, x_R \leq 1 \). Also, it is easy to see that \( \pi_L \) is increasing in \( x_R \), while \( \pi_R \) is decreasing in \( x_L \). These findings are consistent with the argument in the standard Hotelling model of location choice. Firms want to be located maximally far from each other as differentiation allows them to charge higher prices, which turns out to outweigh the adverse effect of a decline in demand.

### 4 Comparative advertising

Consider first the case of **comparative non-price advertising** where reporting and pricing decisions of firms are made sequentially and the message of each firm specifies not just the set of own possible locations but the set of possible locations of firms 1 and 2. We first characterize the multiple equilibria that emerge in this case using an argument similar to that of Proposition 2 in Koessler and Renault (2012).

Koessler and Renault (2012) define what they call canonical disclosure strategies. They call a strategy canonical if, for every price and message \((p_i, m_i)\) in the range of that strategy, if type \((x_i, x_{-i})\) does not choose \((p_i, m_i)\), then the message \(m_i\) cannot be sent by type \((x_i, x_{-i})\) as, in our current case
of comparative advertising, \((x_i, x_{-i}) \notin m_i\). If firms choose canonical strategies, no type can imitate the equilibrium message of another type. As we will illustrate below, canonical strategies induce a very large set of equilibria and cover a large set of nondisclosure outcomes.

**Proposition 2.** Under comparative non-price advertising, any canonical (non)disclosure strategy profile induces an equilibrium if, and only if, the pay-off to any type of firm given this strategy profile is at least as high as the pay-off of that type in the fully disclosing equilibrium.

**Proof.** With appropriate adaptation in notation to accommodate the competition in our model, the proof is identical to Koessler and Renault (2012, Proposition 2).

To illustrate how large the set of equilibrium outcomes under Proposition 2 is, we will provide an example dealing with a subset of these outcomes. We first introduce some new notation. Denote by \(d_U\) an upward sloping diagonal in the \([0, 1] \times [0, 1]\) square (the 45-degree line) and by \(d_D\) – the downward sloping diagonal, perpendicular to \(d_U\). Thus, \(x_1 = x_2\) along \(d_U\), while \(x_1 = 1 - x_2\) along \(d_D\). Also, let us call a set of types \(\Omega\) symmetric around \(d_U\) if for any \((x_1, x_2) \in \Omega\) we have that \((x_2, x_1) \in \Omega\). Finally, let \(\overline{\Omega}\) denote the complement of set \(\Omega\) in \([0, 1] \times [0, 1]\), that is, the set of types that do not belong to \(\Omega\). Using this notation, Example 1 describes a broad class of canonical strategy profiles that induce fully disclosing and not fully disclosing equilibria.

**Example 1.** Consider an arbitrary compact set of types \(\Omega\) such that it is symmetric around \(d_U\) and there exists a type \((y, 1 - y) \in \Omega \cap d_D\) that is different from \((0.5, 0.5)\) and represents an ”extremal” point of \(\Omega\) in the sense that \(y (1 - y)\) is the most left (most right) location of either firm among the types in set \(\Omega\). \(^7\) Figure 1 presents some examples of sets that satisfy these conditions.

![Figure 1: Examples of set \(\Omega\) in \([0, 1] \times [0, 1]\) type space.](image)

The equilibrium strategy profile that we propose below is such that all types in \(\Omega\) choose the same nondisclosing message and all types outside \(\Omega\) choose to precisely disclose themselves. It is easy to see that the example covers an infinite set of equilibrium outcomes, ranging from full disclosure –

\(^7\)In fact, \((y, 1 - y)\) only needs to be a focal point of \(\Omega \setminus d_U\), that is, there may exist points in \(\Omega \cap d_U\) with locations that are smaller than \(y\) or larger than \(1 - y\). The same argument applies, leading to even larger set of non-fully disclosing equilibria.
when the set $\Omega$ is empty, to full nondisclosure when the set $\Omega$ is the whole types’ space. Let the strategy of firms 1 and 2 of any type $(x_1, x_2) \in \Omega$ be to send the same message $M^* = \Omega$ and to set the same price $P^* = \frac{1}{2}t(1 - 2y)$, i.e., the full-revelation price in case locations are $y$ and $1 - y$, if both firms sent the prescribed messages, and to set the full-revelation prices based on any $(\tilde{x}_1, \tilde{x}_2)$ from the intersection of the firms’ messages if at least one of the firms deviated to a different message. Notice that when all types $(x_1, x_2) \in \Omega$ send message $\Omega$ and set price $P^* = \frac{1}{2}t(1 - 2y)$, their equilibrium profit, $0.5P^*$, turns out to be equal to the full-revelation profit based on $(y, 1 - y)$. Firms 1 and 2 of any other type $(x_1, x_2) \in \Omega$ fully disclose both locations by truthfully announcing them and set the full-revelation prices based on $(x_1, x_2)$ if at least one of the firms perfectly revealed its type.

Clearly, the described strategies are canonical in the definition of Koessler and Renault (2012). They also generate the pay-off to any type that is at least as high as the pay-off of that type in the fully disclosing equilibrium. The pay-off is strictly higher than the full-revelation pay-off for types $(x_1, x_2) \in \Omega$ that are different from $(y, 1 - y)$ and $(1 - y, y)$. The latter follows from the observation that (a) the pay-off of these types, given the strategies, is equal to the full-revelation profit based on $(y, 1 - y)$ and (b) $(y, 1 - y)$ is the ”extremal” point of $\Omega$. Then consistently with our remark in Section 3, the full-revelation profit of each firm based on $(y, 1 - y)$ is strictly larger than the full-revelation profit based on any other type, where the most left location is larger than $y$ and the most right location is smaller than $1 - y$.

To see why the proposed strategies induce equilibria, first note that as these strategies are canonical, no type can imitate the strategy of another type. Thus, one only needs to construct a system of out-of-equilibrium beliefs such that given these beliefs, no type wants to deviate. One way of constructing this system of beliefs is the following. After observing $M_i = \Omega$ and $M_j \neq \Omega$, consumers assign probability one to the type $(\tilde{x}_1, \tilde{x}_2) \in M_i \cap M_j$ that is consistent with firms’ pricing strategy. After observing $M_i = \{(x_1, x_2)\}$ and $M_j \neq \{(x_1, x_2)\}$, consumers are bound to believe that the type is $(x_1, x_2)$ since this is the only type that can send the fully revealing message $M_i$ truthfully. Finally, after a deviation only in price but not in message, consumers assign probability one to type $(y, 1 - y)$ when $M_i = M_j = \Omega$ and they believe the reported type when $M_i = M_j = \{(x_1, x_2)\}$. It is straightforward to check that given such out-of-equilibrium beliefs of consumers, no type can benefit from a deviation. □

Next, consider the case of comparative price advertising, where firms choose prices simultaneously with the content of their message concerning both firms’ locations. The difference that simultaneous

---

*Given the strategies and the grain-of-truth assumption, the intersection $M_i \cap M_j$ is not empty, so that such type in the intersection exists.*
decision making introduces to the analysis concerns the fact that the pricing strategy of each firm is independent of the message sent by the competitor. We find that in this case, the results are very different and we should expect full disclosure in any equilibrium.

**Theorem 3.** In case of comparative price advertising, there does not exist an equilibrium where the set of types that do not fully disclose has a positive measure. Therefore, in any equilibrium, nondisclosure is a zero probability event.

The main difference with the case of non-price advertising discussed before in this section is that in the simultaneous setting a deviating firm can take advantage of the inability of the competitor to adjust its price after the firm has sent a deviating message. Therefore, many deviations from the equilibrium strategies that were not profitable in the sequential setting become profitable in this simultaneous setting. In particular, under comparative price advertising, the fact that a canonical strategy profile generates a pay-off that is not lower than the full-revelation pay-off is not sufficient to guarantee that this strategy profile is an equilibrium.

The proof centers around two core ideas. First, if types \((x_1, x_2)\) and \((y_1, y_2)\) send the same message so that consumers do not know whether firm 1 is at \(x_1\) or \(y_1\) and whether firm 2 is at \(x_2\) or \(y_2\) (but they may or may not know that if firm 1 is of type \(x_1\), firm 2 is of type \(x_2\)), then the requirement that a deviation to full disclosure keeping the prices fixed was not gainful implies that the indifferent consumer does not change as otherwise firm 1 or firm 2 of at least one of the types would have an incentive to deviate. Second, as firms can also deviate from the equilibrium prices, any marginal deviation from the equilibrium prices in any direction should also not be gainful. Together, these requirements impose so many conditions on the locations of firms (that should not want to deviate) that any generic choice of two types of firms would violate these restrictions.\(^9\)

There do exist, however, equilibria where some types do not fully disclose. The example below provides an illustration.

**Example 2.** Consider the set of types \(\Omega\) such that \(\Omega \subseteq d_U \cup \{(y, 1 - y), (1 - y, y)\}\) for some \(y \in [0, 1]\) and \((0.5, 0.5) \notin \Omega\). So, \(\Omega \subseteq d_U \cup d_D\) and has exactly two symmetric points on \(d_D\) (see Figure 2).

Consider the strategy profile, where types in \(\Omega\) do not fully disclose, while types outside \(\Omega\) fully disclose. Moreover, in \(\Omega\) we distinguish between the types on \(d_U\) and types in \(\{(y, 1 - y), (1 - y, y)\}\). The former pool among themselves and set \(P^* = 0\), while the latter pool and set \(P^* = 0.5t(1 - 2y)\). More precisely, the strategies prescribe firms 1 and 2 of any type \((x_1, x_2) \in d_U\) to send message \(M^* =\)

\(^9\)In fact, as it becomes clear from the proof, the only types in \([0, 1] \times [0, 1]\) that may have incentives to not fully disclose are all located on the two diagonals of the \([0, 1] \times [0, 1]\) square, \(d_U \cup d_D\).
Ω \cap d_U and set price \( P^* = 0 \), they prescribe firms 1 and 2 of any type \( (x_1, x_2) \in \{(y, 1-y), (1-y, y)\} \) to send message \( M^* = \{(y, 1-y), (1-y, y)\} \) and set price \( P^* = 0.5t(1-2y) \) and they prescribe any other type \( (x_1, x_2) \in \Omega \) to fully disclose the type by sending the precise message and to set the full-revelation prices based on \( (x_1, x_2) \). As in Example 1, price \( P^* = 0.5t(1-2y) \) is actually the full-revelation price in case locations are \( y \) and \( 1-y \) and therefore, the candidate equilibrium pay-off to any firm of type \( (x_1, x_2) \in \{(y, 1-y), (1-y, y)\} \) is equal to the full-revelation profit based on \( (y, 1-y) \) (or \( (1-y, y) \)).

As in Example 1, any type cannot imitate the behavior of other types. We can then consider the following system of out-of-equilibrium beliefs to show that deviations are unprofitable. Suppose that after observing \( M_i = \Omega \cap d_U \) and \( M_j \neq \Omega \cap d_U \) or \( M_i = \{(y, 1-y), (1-y, y)\} \) and \( M_j \neq \{(y, 1-y), (1-y, y)\} \), consumers assign probability one to any type \( (x_1, x_2) \in M_i \cap M_j \), which exists, given the strategies and the grain-of-truth assumption. If one of the observed messages is precise and the other is not, \( M_i = \{(x_1, x_2)\} \) and \( M_j \neq \{(x_1, x_2)\} \), then consumer beliefs are dictated by the message of firm \( i \). Finally, if the observed deviation is only in price but not in message, then consumer out-of-equilibrium beliefs need to be specified only for the case when \( M_i = M_j = \{(y, 1-y), (1-y, y)\} \). Suppose that in this case, consumers assign probability one to one of the two types in the message. A straightforward argument confirms that given these out-of-equilibrium beliefs, no incentives for deviation exist for any type. □

The example shows that nondisclosure may actually occur, but in accordance with Theorem 3, it can only occur if firms’ locations have a very special relationship to each other. For all generic combinations of locations, firms fully disclose under comparative price advertising.

5 Non-comparative advertising

Now, consider the case of non-comparative advertising, where each firm sends a message about its own location only. There is an interesting issue here related to the possibility of types imitating
the equilibrium action of other types that does not appear in many asymmetric information models with Senders and Receivers. In most models, where the type space and the action space is one dimensional, if a Sender type imitates the equilibrium action of another Sender type, the Receiver believes this imitating type to be of another type. This is, however, not necessarily the case under non-comparative advertising in our paper. In our case, these two elements have to be separated. A certain type can still choose the one-dimensional equilibrium action of some other type. However, this does not imply that the consumers believe this imitating type to be that other type as consumer beliefs depend on both messages. We will clarify this issue below.

To begin with, suppose that the prices are chosen after the messages, so that the advertising is what we call non-comparative non-price advertising. We find that there exists an infinite set of equilibria and a wide range of nondisclosure outcomes can be obtained. For example, for any symmetric set \( \Omega \times \Omega \), where \( \Omega \) is an arbitrary compact subset of \([0, 1]\), there exists an equilibrium in which all types in \( \Omega \times \Omega \) (with both locations in \( \Omega \)) do not disclose. As \( \Omega \) can be empty or coincide with the whole \([0, 1]\) interval, full disclosure and full nondisclosure are two particular equilibrium outcomes.

Further, as we demonstrate below, under non-comparative non-price advertising, a nondisclosure strategy can induce an equilibrium even if the pay-off to any type given this strategy is actually smaller than the pay-off of that type in the fully disclosing equilibrium. The main difference with the previous section here is that under non-comparative advertising, by unilaterally deviating from a proposed nondisclosing equilibrium strategy, a firm can never obtain the fully revealing equilibrium pay-off as it only reveals its own location and not the competitor’s location.

The next proposition describes a set of equilibrium strategy profiles that induce nondisclosure in any (compact) set of types \( \Omega \times \Omega \).

**Proposition 4.** For any compact subset \( \Omega \subseteq [0, 1] \) there exists an equilibrium where

- firms 1 and 2 of any type \((x_1, x_2)\) with \(x_1 \in \Omega\) and \(x_2 \in \Omega\) send the same message \(M^* = \Omega\); irrerespectively of the messages sent, both firms set the same price \(P^* = 0\);

- firms 1 and 2 of any other type \((x_1, x_2)\) (such that \(x_i \notin \Omega\) for at least one of the locations) fully disclose their location by truthfully announcing it, i.e., \(M_i^* = \{x_i\}, i = 1, 2\); the pricing strategy is as follows:

  - if \(M_i = \{x_i\}, i = 1, 2\), then both firms set full-revelation prices based on \((x_1, x_2)\) (cf., (11))
  - (12) if \(x_1 \neq x_2\) and \(P_1^* = P_2^* = 0\) if \(x_1 = x_2\);
if $M_i \neq \{x_i\}$ for at least one of the two firms, then both firms set full-revelation prices based on $(x'_1(M_1), x'_2(M_2))$, where $x'_1(M_1) \in \arg \min_{x_1 \in M_1} \pi_1(x_1, x_2)$, $x'_2(M_2) \in \arg \min_{x_2 \in M_2} \pi_2(x_1, x_2)$ and $\pi_i(x_1, x_2)$ is the full-revelation profit of firm $i$ based on $(x_1, x_2)$.

The proposition does not provide a full characterization of possible equilibrium outcomes, but the class that is encompassed comprises a rich variety of symmetric equilibrium outcomes. In that, the equilibrium price of zero used in Proposition 4 is not the only price that can be sustained in equilibrium, but it is the only price that is consistent with simple out-of-equilibrium beliefs of consumers which we use in our proof.

Note that the definition of firms’ equilibrium strategies in Proposition 4 makes imitating possible unlike under the canonical strategies introduced earlier in the discussion of comparative non-price advertising. For example, if $x_1 \in \Omega$, but $x_2 \notin \Omega$, firm 1 could imitate a type $(\tilde{x}_1, \tilde{x}_2)$ with $\tilde{x}_1 \in \Omega$ and $\tilde{x}_2 \in \Omega$ by sending the message $\Omega$. Also, a firm of a type whose both locations are in $\Omega$ can choose to fully disclose its own location, imitating the equilibrium action of a type whose own location (but not that of its rival) is in $\Omega$. However, as consumers also observe the message sent by the rival firm, under unilateral deviations the proposed equilibrium strategies are constructed so that a consumer can still deduce who has deviated.

The proof of Proposition 4 is then straightforward. We show that any deviation by a firm is actually not profitable as we can construct a system of consumer out-of-equilibrium beliefs that rules out incentives for deviation. For example, since the message of each firm is one dimensional, whenever the out-of-equilibrium messages of the two firms intersect, consumers can believe that both firms are located at the same point within the intersection. Then the best deviation profit of a firm is bound to be zero.

The statement of Proposition 4 easily generalizes to allow for a broader variety of nondisclosure equilibrium outcomes. For example, firms 1 and 2 of type $(x_1, x_2)$ with $x_1 \in \Omega$ and $x_2 \in \Omega$ do not need to pool with all the other types whose both locations are in $\Omega$. Instead, they can only pool with types in any compact subset $\tilde{\Omega}$ of $\Omega$ such that $x_1 \in \tilde{\Omega}$ and $x_2 \in \tilde{\Omega}$.

Let us now turn to the last case of non-comparative price advertising, with simultaneous choice.
of messages and prices. Unlike in case of comparative advertising, the change in timing of firms’
decisions does now not affect the set of equilibria described for the sequential setting. It follows that
full and partial nondisclosure is an equilibrium outcome in both cases. In particular, the spirit of
Proposition 4 remains valid; its formulation should only be corrected for the fact that the price of
each firm cannot be a function of messages. The appropriate statement is then:

**Proposition 5.** For any compact subset $\Omega \subseteq [0,1]$ there exists an equilibrium where

- firms 1 and 2 of any type $(x_1,x_2)$ with $x_1 \in \Omega$ and $x_2 \in \Omega$ send the same message $M^* = \Omega$ and
  set the same price $P^* = 0$;

- firms 1 and 2 of any other type $(x_1,x_2)$ (such that $x_i \notin \Omega$ for at least one of the locations)
  fully disclose their location by truthfully announcing it, i.e. $M^*_i = \{x_i\}$, $i = 1,2$, and set
  full-revelation prices based on $(x_1,x_2)$.

The proof of this proposition is essentially identical to the proof of Proposition 4. The only
difference concerns a deviation in the message sent by one of the firms. With sequential decision
making, both firms react to this deviation by ”adjusting” their price in an individually rational
manner. With simultaneous decision making, only the deviating firm can adjust its price, while the
other firm has its price fixed. This affects the specification of consumer out-of-equilibrium beliefs
that we use in the proof, namely, the beliefs after having observed a pair of messages where one
message reveals the location of a firm by naming it precisely and the other does not.\footnote{Formally, the way in which the beliefs supporting the set of equilibria change is described in the Appendix.}

Thus, if advertising is non-comparative, then irrespective of whether reporting and pricing strate-
gies of firms are chosen sequentially or simultaneously, an infinite set of disclosing and not fully
disclosing equilibria exists and nondisclosure is a common equilibrium outcome.

6 Discussion and conclusions

In this paper we developed a duopoly model of horizontal product differentiation. We studied the
incentives of a firm to disclose its horizontal product characteristic when this characteristic is known to
both firms, but not to consumers. Firms first simultaneously choose a message about their location(s),
such that this message is truthful, that is, the true location(s) of a firm(s) is consistent with the
message. The messages can range from being very precise (indicating the exact location(s)) to very
vague. We considered four different cases, dependent on whether or not firms engage in comparative
or non-comparative advertising and on whether or not price is announced simultaneously with the
advertisement message. Given the messages and the prices, consumers update their beliefs about firms’ locations and decide where to buy.

The main result of the paper is that generically (that is except if firms’ locations have a very particular relationship to each other) under comparative price advertising there exists a unique equilibrium outcome where firms fully disclose their product characteristics. In all other cases, a full disclosure equilibrium also exists, but it is one among many other equilibria where firms do not (fully) disclose their information. The results for comparative price advertising are different from the results for comparative non-price advertising as in the first case optimally deviating from a nondisclosure situation is more beneficial since the other firm cannot react to a full disclosure message by setting a different price (as it can do under comparative non-price advertising). Non-comparative advertising – with prices chosen either simultaneously or after the messages – leads to the existence of many nondisclosure equilibria as even if a firm discloses its own location, consumer out-of-equilibrium beliefs about the location of the competing firm continue to play a role and these beliefs can be chosen so as to ”punish” any deviation from the equilibrium nondisclosure strategy.

None of the standard refinement notions like the Intuitive Criterion (Cho and Kreps (1987)) or D1 (Cho and Sobel (1990)) can be relied upon to reduce the set of equilibria in this model due to the fact that the profits of all types of a firm depend only on consumer beliefs about firms’ locations and price, but not on actual locations. The only way one could get rid of the multiplicity of equilibria is to invoke a restriction on consumer out-of-equilibrium beliefs. For example, under non-comparative advertising, one could argue that given that all types sending identical equilibrium message concerning location have the same incentives to deviate to a non-equilibrium price, price cannot reasonably act as a signal of firm’s location. A similar argument could then be used to argue for the inability of a firm to signal the location of its competitor. In the first working paper version of this paper (Janssen and Teteryatnikova (2012)) we used these considerations to define an equilibrium where consumer out-of-equilibrium beliefs concerning a firm’s location only depend on firm’s own message and not on its pricing decision or the message of the competitor. To indicate the role of out-of-equilibrium beliefs in sustaining multiple nondisclosure equilibria, we showed that under these restrictive beliefs all equilibria must be fully revealing. However, as we consider an environment where firms are informed about both firms’ product characteristics, any interpretation of a deviation cannot be excluded. Therefore, in order to conclude in the non-comparative advertising case that there exists a unique full disclosure equilibrium where prices are equal to marginal cost, Anderson and Renault (2009) implicitly have to assume this type of restrictive out-of-equilibrium beliefs. Our
results show that without this assumption, full disclosure is just one of possible out-of-equilibrium outcomes.

The present paper has considered a simple framework where consumers are uniformly distributed over the unit interval and have quadratic transportation costs. Moreover, disclosure is completely costless and firms know not only their own location, but also the location of the competitor. We have considered this simple framework to focus on the role of different information transmission processes in providing incentives for firms to fully disclose their private information in a competitive environment. Future work should focus on whether similar conclusions hold when some of these assumptions are replaced by others. For example, the case where firms have purely private information about their product characteristics could be of considerable interest. It is not difficult to see that under purely private information, full disclosure is also an equilibrium outcome and in this respect the result of the current paper easily generalizes. The main challenge is whether other equilibria also exist. This is not an easy task as without knowing the location of the competitor, disclosing own location has advantages for certain, but not for all locations of the rival firm.

Appendix

Proof of Theorem 3. Consider a set of types $\Phi \subseteq [0,1] \times [0,1]$ such that $\Phi$ is not a subset of $d_U \cup d_D$. In the following we show that for any set, there exists a type $(x_1, x_2) \in \Phi$ among the types that do not belong to $d_U \cup d_D$ such that it does not have incentives to pool with the other types in $\Phi$. This will then suggest that the only types in the $[0,1] \times [0,1]$ square that may have incentives to pool with other types are all located on either of the two diagonals, $d_U$ or $d_D$, and therefore, there does not exist an equilibrium where the set of types that do not fully disclose has a positive measure.

Suppose, on the contrary, that there exists an equilibrium in which all types in set $\Phi$ pool. Notice that the mere possibility of pooling requires the existence of at least two different types in $\Phi$. So, let $(x_1, x_2)$ and $(y_1, y_2)$ be a pair of different types in $\Phi$. Moreover, since $\Phi$ is not a subset of $d_U \cup d_D$, there exists at least one type in $\Phi$ that does not belong to either of the two diagonals. Suppose that $(y_1, y_2) \notin d_U \cup d_D$. Below we consider the two options: 1) $(x_1, x_2) \in d_U$ and 2) $(x_1, x_2) \notin d_U$. We show that in each case, a firm of at least one of the two types, $(x_1, x_2)$ or $(y_1, y_2)$, has incentives to send a fully-revealing message rather than pool with the other types in $\Phi$, so that pooling is not an equilibrium strategy.

To begin with, consider that in both cases, a firm of type $(y_1, y_2)$ (the type that does not belong to $d_U$) clearly has an incentive to deviate to full disclosure if its equilibrium price is zero. Indeed,
by deviating to full disclosure and setting its price marginally above zero, this firm raises own profit from zero to positive.

Therefore, it remains to study the two cases under the condition that the equilibrium prices of both firms are positive. Consider each case in turn.

1. Suppose first that \((x_1, x_2) \in d_U \) (and \((y_1, y_2) \notin d_U \cup d_D, P_1^* > 0, P_2^* > 0)\).

In this case, at least one of the firms of type \((x_1, x_2)\), where \(x_1 = x_2\), has incentives to deviate to full disclosure. First, if \(P_1^* = P_2^*\), then the firm whose equilibrium demand is lower or equal to the demand of the competitor (hence, not larger than 0.5) gains from deviating to full disclosure at a slightly lower price. Indeed, such deviation discontinuously increases the demand of the firm while leaving the price essentially unchanged. Secondly, if \(P_1^* \neq P_2^*\), then two cases can be distinguished. If equilibrium demand of one of the firms is zero, then this firm of type \((x_1, x_2)\) strictly increases its profit by deviating to full disclosure at any price that is lower or equal to the price of the competitor. On the other hand, if equilibrium demand of each firm is strictly between zero and one, then the firm with the lower price can benefit from full disclosure: by revealing its type precisely and keeping the price unchanged, it attracts all consumers at the same price.

2. Suppose that \((x_1, x_2) \notin d_U \) (and \((y_1, y_2) \notin d_U \cup d_D, P_1^* > 0, P_2^* > 0)\).

To prove the incentives for full revelation among types in \(\Phi\), we first observe that fully revealing own type, while keeping the price fixed, is always gainful for a firm as soon as it increases the firm’s demand. Therefore, if there exists a type in \(\Phi\) that, by fully revealing itself and keeping the price fixed, changes the equilibrium value of \(\hat{\lambda}\), the location of the indifferent consumer, one of the firms of this type (the one that gains demand through the change in \(\hat{\lambda}\)) has incentives to deviate to full disclosure.

Thus, for pooling to be an equilibrium, it must be that for any type in \(\Phi\), revealing itself while keeping the price fixed does not change the location of the indifferent consumer. In particular, this means that the location of the indifferent consumer is the same no matter whether type \((x_1, x_2)\) or type \((y_1, y_2)\) fully discloses and it is equal to the location of the indifferent consumer in the supposed non-fully disclosing equilibrium (where types in \(\Phi\) pool):

\[
\hat{\lambda} = \frac{1}{2t} \left( P_2^* - P_1^* \right) x_2 - x_1 + \frac{1}{2} (x_1 + x_2) = \frac{1}{2t} \left( P_2^* - P_1^* \right) y_2 - y_1 + \frac{1}{2} (y_1 + y_2). \tag{15}
\]

Recall that the location of the indifferent consumer represents the demand of the firm with the
most left perceived location. Without loss of generality, assume that \( E(x_1|\mu_1) \leq E(x_2|\mu_2) \), that is, in equilibrium, firm 1 is perceived as most left. Then the equilibrium profits of firm 1 and 2, denoted by \( \pi^*_L \) and \( \pi^*_R \), respectively, are given by:

\[
\pi^*_L = P^*_1 \left[ \frac{1}{2t} \frac{P^*_2 - P^*_1}{x_2 - x_1} + \frac{1}{2} (x_1 + x_2) \right] = P^*_1 \left[ \frac{1}{2t} \frac{P^*_2 - P^*_1}{y_2 - y_1} + \frac{1}{2} (y_1 + y_2) \right] \quad (16)
\]

\[
\pi^*_R = P^*_2 \left[ 1 - \frac{1}{2t} \frac{P^*_2 - P^*_1}{x_2 - x_1} - \frac{1}{2} (x_1 + x_2) \right] = P^*_2 \left[ 1 - \frac{1}{2t} \frac{P^*_2 - P^*_1}{y_2 - y_1} - \frac{1}{2} (y_1 + y_2) \right]. \quad (17)
\]

In the following we show that irrespective of whether the true location of firm 1 of type \((x_1, x_2)\) or \((y_1, y_2)\) is indeed most left, – consistently with its perceived location due to a non-disclosing equilibrium message, – firm 1 or 2 of at least one of the two types has an incentive to deviate to full disclosure.

Suppose first that all types in \( \Phi \) are such that for all types either firm 1 is located to the left of firm 2 or firm 2 is located to the left of firm 1. Notice, however, that since \( E(x_1|\mu_1) \leq E(x_2|\mu_2) \), the latter cannot be the case. Therefore, if firms’ location with respect to each other is the same for all types, then it must be that firm 1 is located to the left of firm 2. In particular, for types \((x_1, x_2)\) and \((y_1, y_2)\), we have that \( x_1 < x_2 \) and \( y_1 < y_2 \). Consider a deviation by firm 1 of type \((x_1, x_2)\) to full disclosure at a price \( P' \neq P^*_1 \). The deviation results in profit

\[
P' \left[ \frac{1}{2t} \frac{P^*_2 - P'}{x_2 - x_1} + \frac{1}{2} (x_1 + x_2) \right].
\]

For this deviation to not be gainful, the best deviating price must be exactly equal to \( P^*_1 \), that is, the partial derivative of this deviation profit with respect to \( P' \) must be zero at \( P' = P^*_1 \):

\[
\frac{1}{2t} \frac{P^*_2 - P^*_1}{x_2 - x_1} + \frac{1}{2} (x_1 + x_2) = \frac{1}{2t} \frac{P^*_1}{x_2 - x_1}.
\]

The same argument must hold for the deviation of firm 1 of type \((y_1, y_2)\), so that we obtain:

\[
\frac{1}{2t} \frac{P^*_2 - P^*_1}{y_2 - y_1} + \frac{1}{2} (y_1 + y_2) = \frac{1}{2t} \frac{P^*_1}{y_2 - y_1}.
\]

The left-hand sides of these two equations are equal according to (15). Therefore:

\[
\frac{1}{2t} \frac{P^*_1}{x_2 - x_1} = \frac{1}{2t} \frac{P^*_1}{y_2 - y_1}.
\]

Since \( P^*_1 > 0 \), this suggests that \( x_2 - x_1 = y_2 - y_1 \). But then (15) can only hold if \( x_1 + x_2 = y_1 + y_2 \).

Together, \( x_2 - x_1 = y_2 - y_1 \) and \( x_1 + x_2 = y_1 + y_2 \) imply that \( x_1 = y_1 \) and \( x_2 = y_2 \), that is,
types \((x_1, x_2)\) and \((y_1, y_2)\) are the same. This contradicts our initial presumption. Hence, firm 1 of either of the two types can benefit from the deviation.

Next, suppose that there exists a pair of types in \(\Phi\) such that firms’ location with respect to each other is different for the two types. This can be true about types \((x_1, x_2)\) and \((y_1, y_2)\) or about some other pair of types. In any case, addressing just the first possibility is sufficient to complete the proof. So, let types \((x_1, x_2)\) and \((y_1, y_2)\) be such that either \(x_1 < x_2, y_1 > y_2\) or \(x_1 > x_2, y_1 < y_2\). For concreteness, consider one of these symmetric cases, say, \(x_1 < x_2, y_1 > y_2\). If firm 1 of type \((x_1, x_2)\) deviates to full disclosure at price \(P'\), it obtains the deviation profit of

\[
P' \left[ \frac{1}{2t} \frac{P^t_2 - P'}{x_2 - x_1} + \frac{1}{2} (x_1 + x_2) \right].
\]

As before, for this deviation to not be gainful, the partial derivative of this function with respect to \(P'\) must be zero at \(P' = P^*_1\):

\[
\frac{1}{2t} \frac{P^t_2 - P^*_1}{x_2 - x_1} + \frac{1}{2} (x_1 + x_2) = \frac{1}{2t} \frac{P^*_2}{x_2 - x_1}.
\]

Similarly, for the deviation to full disclosure of firm 2 of type \((x_1, x_2)\) to not be gainful, the derivative of its deviation profit with respect to the deviating price must be zero at the point where this price is equal to \(P^*_2\). This results in

\[
1 - \frac{1}{2t} \frac{P^t_2 - P^*_1}{x_2 - x_1} - \frac{1}{2} (x_1 + x_2) = \frac{1}{2t} \frac{P^*_2}{x_2 - x_1}.
\]

Now, consider the deviation to full disclosure by firms 1 and 2 of type \((y_1, y_2)\). Observe that for this type, the relation between the true locations of the two firms is the opposite of the relation between the perceived locations: \(y_1 > y_2\) but \(E(x_1 | \mu_1) \leq E(x_2 | \mu_2)\). Therefore, when firm 1 deviates to full disclosure, keeping the price unchanged, it obtains the profit of

\[
P^*_1 \left[ 1 - \frac{1}{2t} \frac{P^*_2 - P^*_1}{y_2 - y_1} - \frac{1}{2} (y_1 + y_2) \right].
\]

Using (17), this deviation profit can be expressed as \(\frac{P^*_1}{H_1^1} \pi^*_R\). Hence, such deviation by firm 1 is unprofitable if and only if \(\frac{P^*_1}{H_1^1} \pi^*_R \leq \pi^*_L\), or \(P^*_1 \pi^*_R \leq P^*_1 \pi^*_L\).

\textsuperscript{13} The second possibility implies that either (i) \(x_1 < x_2\) and \(y_1 < y_2\), in which case the argument considered above establishes the incentives for deviation, or (ii) \(x_1 > x_2\) and \(y_1 > y_2\), in which case there must exist another type \((z_1, z_2)\) in \(\Phi\) such that \(z_1 < z_2\) (as otherwise \(E(x_1 | \mu_1) \leq E(x_2 | \mu_2)\) cannot hold) and then the argument identical to the one provided further in the proof confirms the incentives for deviation. Indeed, one would only need to substitute \((z_1, z_2)\) for \((x_1, x_2)\) as then we obtain a pair of types \((z_1, z_2), (y_1, y_2)\) with \(z_1 < z_2\) and \(y_1 > y_2\).

\textsuperscript{14} The argument for the other case is analogous.
On the other hand, when firm 2 of type \((y_1, y_2)\) deviates to full disclosure, keeping the price fixed, it obtains the profit of

\[
P_2^* \left[ \frac{1}{2t} \frac{P_2^* - P_1^*}{y_2 - y_1} + \frac{1}{2} (y_1 + y_2) \right]
\]

which, given (16), is equal to \(\frac{c}{t} \pi_L^*\). This deviation by firm 2 is unprofitable if and only if \(\frac{c}{t} \pi_L^* \leq \pi_R^*\), or \(\pi_L^* \leq \frac{t}{c} \pi_R^*\).

Thus, in equilibrium we must have \(\pi_L^* = \pi_R^*\). Using (16) – (17), this can be written as

\[
(P_1^*)^2 \left[ \frac{1}{2t} \frac{P_2^* - P_1^*}{x_2 - x_1} + \frac{1}{2} (x_1 + x_2) \right] = (P_2^*)^2 \left[ 1 - \frac{1}{2t} \frac{P_2^* - P_1^*}{x_2 - x_1} - \frac{1}{2} (x_1 + x_2) \right].
\]

The expressions in square brackets are equilibrium demands of firms 1 and 2. According to (18) and (19), each of them can be expressed more compactly, leading to

\[
\frac{(P_1^*)^3}{(x_2 - x_1)} = \frac{(P_2^*)^3}{(x_2 - x_1)}.
\]

This means that in equilibrium prices must be equal, i.e., \(P_1^* = P_2^*\).

Now, given the equality of prices, condition (15) results in \(x_1 + x_2 = y_1 + y_2\) and

\[
\pi_L^* = P^* \frac{1}{2} (x_1 + x_2) = P^* \frac{1}{2} (y_1 + y_2)
\]

\[
\pi_R^* = P^* \left( 1 - \frac{1}{2} (x_1 + x_2) \right) = P^* \left( 1 - \frac{1}{2} (y_1 + y_2) \right)
\]

where \(P^* = P_1^* = P_2^*\). Moreover, given that \(x_1 < x_2\) and \(y_1 > y_2\), this can only be an equilibrium when \(\pi_L^* = \pi_R^*\). Indeed, suppose that \(\pi_L^* \neq \pi_R^*\), say, \(\pi_L^* < \pi_R^*\). Then firm 1 of type \((y_1, y_2)\), whose true location (as opposed to its perceived location) is to the right of the true location of firm 2, has incentives to fully disclose its type, keeping the price at \(P^*\). Such deviation results in profit that is exactly equal to \(\pi_R^*\), and hence, is larger than firm 1’s equilibrium profit.\(^{15}\) Therefore, we obtain that \(\pi_L^* = \pi_R^*\). But this suggest that \(x_1 + x_2 = y_1 + y_2 = 1\), which means that both, \((x_1, x_2)\) and \((y_1, y_2)\) belong to \(d_D\), and this is a contradiction to our choice of \((y_1, y_2)\).

Thus, also in this case we obtain that a firm of at least one of the two types, \((x_1, x_2)\) or \((y_1, y_2)\), can benefit from the deviation to full disclosure.

\[\text{\[15\]If \(\pi_L^* > \pi_R^*\), then by the analogous argument, firm 2 of type \((y_1, y_2)\) has incentives to fully disclose.}\]

24
Proof of Proposition 4. To begin with, notice that no type of a firm can or has incentives to imitate the strategy of another type. Indeed, even when the grain-of-truth assumption allows a firm to send an equilibrium message of another type, this imitation will be “spotted” by consumers since their beliefs about the type depend on messages of both firms and the other firm still sends an equilibrium message. For example, if \( x_1 \in \Omega \), but \( x_2 \notin \Omega \), firm 1 could imitate a type \((\tilde{x}_1, \tilde{x}_2)\) with \( \tilde{x}_1 \in \Omega \) and \( \tilde{x}_2 \in \Omega \) by sending the message \( \Omega \). However, since firm 2 of type \((x_1, x_2)\) reveals its own location precisely, and this location is outside \( \Omega \), consumers, who know the equilibrium strategies, deduce that firm 1 has deviated. Similarly, a firm of a type whose both locations are in \( \Omega \) can choose to fully disclose its own location, imitating the equilibrium action of a type whose own location (but not that of its rival) is in \( \Omega \). But having observed the message \( \Omega \) of the other firm, consumers deduce that both firms’ locations are in \( \Omega \) and hence, the first firm has deviated from its equilibrium strategy. Finally, a firm of a type with at least one of the locations not in \( \Omega \) can imitate the strategy (the message and the price) of another such type — if this firm’s location is the same for both types. But given that the rival firm fully discloses its true location, the imitating firm cannot succeed in pretending to be of the other type.

Then, as the deviating firm can always be detected, it remains to construct a set of out-of-equilibrium beliefs of consumers such that given these beliefs, a firm has no incentives to deviate. Consider the following out-of-equilibrium beliefs. After observing \( M_i = \Omega \) and \( M_j \neq \Omega \), consumers assign probability one to both firms having the same location in \( M_i \cap M_j \).\(^{16}\) After observing \( M_i = \{x_i\} \) and \( M_j \neq \{x_j\} \), consumers assign probability one to firm \( j \) being located at \( x_j'(M_j) \in M_j \) such that

\[
x_j'(M_j) \in \arg \min_{x_j \in M_j} \pi_j(x_i, x_j)
\]

where \( \pi_j(x_i, x_j) \) denotes the full-revelation profit of firm \( j \) based on locations \( x_i, x_j \). Using our notation in (13) – (14),

\[
\min_{x_j \in M_j} \pi_j(x_i, x_j) = \min \{ \min_{x_j \in M_j \text{ s.t. } x_i \leq x_j} \pi_L(x_i, x_j), \min_{x_j \in M_j \text{ s.t. } x_i > x_j} \pi_R(x_j, x_i) \}.
\]

Moreover, if there is more than one location \( x_j'(M_j) \) at which \( \pi_j(x_i, x_j) \) is minimized, then suppose that consumers assign probability one to the location that is consistent with firms’ candidate equilibrium pricing strategy. Finally, after a deviation only in price but not in message, consumers assign probability one to both firms having the same location in \( \Omega \) if \( M_i = M_j = \Omega \) and they assign probability one to the reported locations if \( M_i = \{x_i\} \) and \( M_j = \{x_j\} \).

\(^{16}M_i \cap M_j \neq \emptyset \) due to the grain-of-truth assumption.
First, it is easy to see that given such consumer out-of-equilibrium beliefs, the prescribed pricing strategy of each type induces equilibrium of the price-setting stage, following the deviation in message. Second, as we demonstrate next, no type of a firm has incentives to deviate from the candidate equilibrium strategy.

Indeed, a firm of type \((x_1, x_2)\) such that \(x_1 \in \Omega\) and \(x_2 \in \Omega\) has no incentives to deviate neither only in price nor in message and price since after any such deviation consumers believe that both firms are located at the same point in \(\Omega\) and given that the price of the rival firm is zero, the best deviation profit of the firm is zero. Next, consider a deviation by a firm of type \((x_1, x_2)\) such that at least one of the locations \(x_1, x_2\) does not belong to \(\Omega\). Clearly, the deviation only in price but not in message is not profitable for either firm, given the definition of the full-revelation prices. On the other hand, if firm \(j\) deviates to some admissible message \(M_j \neq \{x_j\}\), then given the consumer beliefs in this case, the subsequent choice of consumers will be as if the true location of firm \(j\) is \(x_j'(M_j)\) for sure, so that the deviation pay-off of firm \(j\) is exactly equal to \(\pi_j(x_i, x_j'(M_j))\). As \(x_j \in M_j\), it follows that \(\pi_j(x_i, x_j'(M_j)) \leq \pi_j(x_i, x_j)\). Therefore, the deviation is not gainful. 

Proof of Proposition 5. The proof is identical to the proof of Proposition 4 apart from the specification of consumer out-of-equilibrium beliefs after the deviation where \(M_i = \{x_i\}\) and \(M_j \neq \{x_j\}\). Let in this case consumers assign probability one to firm \(j\) being located at \(x_j'(M_j) \in M_j\) such that

\[ x_j'(M_j) \in \arg \min_{x_j \in M_j} \pi'_j(x_i, x_j) \]

where \(\pi'_j(x_i, x_j)\) denotes the profit of firm \(j\) resulting from fully revealed locations \(x_i, x_j\), equilibrium price \(P^*_i\) of firm \(i\) and optimally adjusted (best-response) price of firm \(j\). The only difference between \(\pi'_j(x_i, x_j)\) and \(\pi_j(x_i, x_j)\) (from the proof of Theorem 4) is that the specification of \(\pi_j\) assumes that both firms set their prices optimally, given the fully revealed locations \(x_i\) and \(x_j\), while the definition of \(\pi'_j\) assumes that only the deviating firm \(j\) sets its price optimally and the price of the other firm is fixed. Notice that this difference reflects the distinction between the firms’ pricing strategies in case of simultaneous and sequential decision making. However, and this is important, when \(x_j\) is the true location of firm \(j\), then by definition of the equilibrium price \(P^*_i\), \(\pi_j(x_i, x_j) = \pi_j(x_i, x_j)\) and that is the equilibrium profit of firm \(j\). Therefore, the consumer out-of-equilibrium beliefs specified above still result in the situation where the best deviation profit of a firm of type \((x_1, x_2)\) such that at least one of the locations \(x_1, x_2\) does not belong to \(\Omega\), does not exceed its equilibrium profit. Indeed, given the beliefs, the best deviation profit of firm \(j\) is equal to \(\pi'_j(x_i, x_j'(M_j))\) and since \(x_j \in M_j\), it follows
that $\pi'_j(x_i, x_j(M_j)) \leq \pi'_j(x_i, x_j) = \pi_j(x_i, x_j)$. □

References


