Consumer Search and Vertical Relations: The Triple Marginalization Problem*

Maarten Janssen† and Sandro Shelegia‡

October 9, 2012

Abstract

This paper shows that the double marginalization problem significantly underestimates the inefficiencies arising from vertical relations in markets where consumers who are uninformed about the wholesale arrangements between manufacturers and retailers search for the best retail price. Consumer search provides manufacturers an additional incentive to substantially increase wholesale prices. Consequently, all market participants are worse off and we call this phenomenon the triple marginalization problem. We also show that, when the wholesale price is unknown, retail prices decrease and industry profits and consumer surplus increase in search cost, whereas the opposite is true when the wholesale price is known.

JEL Classification: D40; D83; L13

Keywords: Vertical Relations, Consumer Search, Double Marginalization

---

*We thank Simon Anderson, Jean-Pierre Dube, Liang Guo, Marco Haan, Stephan Lauermann, Jose-Luis Moraga-Gonzalez, Andrew Rhodes, K. Sudhir, Yossi Spiegel, Mariano Tappata, Chris Wilson, and seminar participants at New Economic School (Moscow), Edinburgh, Groningen, III Workshop on Consumer Search and Switching Cost (Higher School of Economics, Moscow), V Workshop on the Economics of Advertising (Beijing), the 2012 European Meetings of the Econometric Society and the 2012 EARIE Meetings for helpful discussions and comments. We thank Anton Sobolev from the Laboratory of Strategic Behavior and Institutional Design (Higher School of Economics) for computational assistance. Support from the Basic Research Program of the National Research University Higher School of Economics is gratefully acknowledged. Janssen acknowledges financial support from the Vienna Science and Technology Fund (WWTF) under project fund MA 09-017.

†Department of Economics, University of Vienna. Email: maarten.janssen@univie.ac.at

‡Department of Economics, University of Vienna. Email: sandro.shelegia@univie.ac.at
1 Introduction

To understand the implications of consumers search in retail markets it is important to know whether or not consumers know retailers’ cost. Most of the literature assumes that consumers know these costs and therefore that they can well understand the distribution of a retailer’s (price) offer on the next search (see, e.g., Stahl (1989), Wolinsky (1986) and other papers in the consumer search literature). In most retail markets, however, consumers do not know retailers’ cost: as consumers have to search to get to know the prices they have to pay, it is unlikely they know the retailers’ cost underlying these price decisions. Upon observing a relatively high price, consumers are uncertain whether this high price is due to a high margin for the retailer, or whether it is due to a high underlying cost (which may be common to all retailers). This has lead Benabou and Gertner (1993), Dana (1994) and Fishman (1996), and more recently, Tappata (2009), Chandra and Tappata (2011) and Janssen et al. (2011) to incorporate cost uncertainty into the search literature by assuming retailers’ cost follows some random process.

In most markets, however, retailer’s cost is not random, but rather chosen by an upstream firm. In this paper we introduce vertical relations between a manufacturer and retailers in a model of sequential consumer search. As a reference point, we consider the case where consumers are fully informed about the wholesale price retailers have to pay to the manufacturer and show that the familiar double marginalization problem arises in this context. We then consider the often more realistic setting where consumers are uninformed about retailers’ cost. In that case consumers cannot condition their search rule (reservation price) on the wholesale price. We show that this results in a much more inelastic demand curve for the upstream firm, and hence, in even higher upstream and downstream prices. It follows that all market participants (manufacturer, retailers and consumers) are worse off. We call this the triple marginalization problem.

The triple marginalization problem provides some insight into why in the US car retail market, car dealers inform consumers about the wholesale price (and manufacturers provide this information in such a way that it can be used by the car dealers). In a recent empirical study Scott Morton et al. (2011) show that search is an important factor determining car dealer prices. They also show that if buyers are

---

1 See, e.g., http://www.carbuyingtips.com and http://beatthecarsalesman.com for more details on the information car dealers provide to consumers and on the bargaining process between car dealers and potential buyers. We do not explicitly model the full bargaining situation between car dealers and consumers. Retailers are supposed to make a binding offer that buyers can accept or reject. The price that is actually chosen by a car dealer is such that consumers with search cost do not find it optimal to continue to search. The forces that we uncover also hold true in more general bargaining situations as there, as in our model the outside option of consumers in the bargaining situation (which is the expected price they pay at another dealer plus their search cost) rules the bargaining outcome.
informed about the car dealer’s invoice amount he pays to the manufacturer, they pay on average $121 less for their car, providing evidence that car retail prices are indeed lower when the wholesale price is known. In many markets, it is, however, too costly to provide this information credibly. These markets suffer from the triple marginalization problem.

Our paper also provides an alternative perspective on the issue whether it should be mandatory for intermediaries, for example in the financial sector, to disclose their margins. Recent policy discussions in, for instance, the EU and the USA have lead to legislation mandating intermediaries to reveal this information. Our paper argues that this legislation may actually benefit all market participants as it provides (i) consumers with a better benchmark on what prices to expect in the market, (ii) upstream firms with a reduced incentive to squeeze retailers leading to overall price levels and quantities sold to be closer to the efficient outcomes. In a recent paper, Inderst and Ottaviani (2012) come to an opposite conclusion. Their framework focuses on the information an intermediary has on how well a product matches the current state of the economy and how competing manufacturers incentivize the intermediary to recommend their products to consumers. Our paper abstracts from these issues as it deals with a homogeneous product and instead focuses on the effect of knowing margins for the search behaviour of consumers and the incentives of the upstream firm to price its products.

Our base model has the following key ingredients. There is one manufacturer who sells its product to two retailers using a linear pricing scheme. Both retailers buy this product at the same wholesale price and sell to consumers in a retail market that has downward sloping demand. As the canonical model on double marginalization considers a homogeneous goods market, we also do so and therefore build on Stahl (1989)’s sequential search model. In this model there are two types of consumers. Some consumers, the shoppers, can search at no cost and simply buy at the lowest price. Other consumers, the non-shoppers, have to pay a positive search cost for each shop they visit after the first one. As explained before, we analyze two versions of this model; one where consumers are informed about the wholesale price, and one where they are not.

In the base line model where retailers’ cost is observed by consumers, we have the following characterization results. First, for any given level of retailers’ cost the downstream market is, as in Stahl (1989), characterized by a mixed strategy distribution of prices. If non-shoppers’ search cost is small, the upper bound of that distribution is given by non-shoppers’ reservation price (which depends on the wholesale price); for larger search cost it is given by the retailers’ monopoly price (for given wholesale price). The behavior of the downstream market creates a derived

---

2See the references given in footnote 4 of Inderst and Ottaviani (2012) for details.
expected demand for the upstream monopolist. We show that the manufacturer’s maximization problem is always well-defined and that the optimal wholesale price equals the monopoly price of a vertically integrated monopolist if the upper bound of the downstream market prices is given by the retailers’ monopoly price. If the search cost is smaller, the manufacturer’s optimal price is below this monopoly price of a vertically integrated monopolist. For all values of the parameters, there is a double marginalization problem in the sense that downstream expected price is larger than the price a vertically integrated monopolist would charge.

In the case where the wholesale price is unobserved by consumers, non-shoppers cannot condition their reservation price on the wholesale price. Instead, they form beliefs about the wholesale price, and in equilibrium, these beliefs are correct. For large search cost, where the upper bound of the retailers’ price distribution is given by the retail monopoly price, the equilibrium is exactly the same as before. The crucial difference is when the search cost is smaller. In case a reservation price equilibrium exists, we show that the wholesale price is larger than the monopoly price of a vertically integrated monopolist and (much) larger than when consumers observe the wholesale price. This is the triple marginalization problem alluded to above. The reason for this result is as follows. The non-shoppers’ reservation price is based on a conjectured level of the wholesale price. If the manufacturer would choose a wholesale price that is higher than this conjectured level, non-shoppers do not adjust their reservation price, and retailers are squeezed. They face higher cost, but cannot fully adjust their prices upwards as they would do if consumers knew that all of them face a higher wholesale price. The downstream price adjustment to an increase in the wholesale price is therefore smaller than in the case where consumers know the wholesale price. For the upstream manufacturer this means that its expected demand is less sensitive to price changes and, therefore, it has an incentive to charge higher prices. This effect is strongest when the search cost is small (as in that case the reservation price is always close to the conjectured level of retailers’ cost and retailers are maximally squeezed) and when the fraction of non-shoppers is large. In the latter case, the incentives to increase upstream price (for a given conjectured reservation price) can be so strong that a reservation price equilibrium fails to exist.\footnote{We argue that it is difficult to characterize what equilibrium may look like in this case. The parameter values for which equilibrium does not exist in our model have an overlap with the parameter values for which a reservation price equilibrium does not exist in Janssen et al. (2011). Thus, it seems unlikely that a reservation price equilibrium exists where the upstream firm randomizes its choice.}

We also show that the comparative static results crucially depend on whether or not consumers know the wholesale price. When search cost of non-shoppers increases from initially lower (but not very low) levels, the upstream price is increasing in the
observed case, whereas it is decreasing in the unobserved case. This pattern is even more pronounced for expected downstream prices: when search cost increases from initially low levels, downstream expected prices are decreasing when consumers do not know the wholesale price, but they are increasing when they do. Thus, paradoxically, when they do not know the wholesale price, consumers are better off with higher search cost! Moreover, total profits of upstream and downstream firms are also increasing in search cost. The underlying reason is that when search costs are higher, retailers are able to increase their margins and the upstream firm is not able to maximally squeeze them. The upstream firm takes these higher retail margins into account and lowers its price. As this brings retail prices closer to the monopoly price of a vertically integrated firm, total industry profits increase. The distribution of profits between upstream and downstream firms is, however, affected by the level of search cost as retail firms benefit from higher search cost, while the upstream firm does not.

Another interesting comparative statics result is to compare the different ways to converge to a market where search frictions are negligible: one can either let the fraction of fully informed consumers become close to one, or let the search cost approach zero. In the base line model where consumers know the wholesale price, the equilibrium outcome in both cases converges to the standard outcome where the upstream monopolist charges the monopoly price and there is Bertrand competition downstream. When consumers do not know the wholesale price, however, the situation is very different. When search cost becomes small, the triple-marginalization effect becomes very strong and expected upstream price increases to a level that is up to 36% above the monopoly price of a vertically integrated firm!

The issues we touch upon in this paper have a similar flavour to the issues recently discussed in the literature on recommended retail prices (see, e.g., Lubensky (2011) and Buehler and Gärtner (2012)). Lubensky suggests that by recommending retail prices manufacturers provide information to consumers on what reasonable retail prices to expect in an environment where the manufacturer’s marginal cost is random and only known to the manufacturer. Manufacturer thereby affects consumers’ search behaviour. In our framework, when informed about the wholesale price, consumers have a better notion of how large retail margins are and this benefits all market participants by reducing the manufacturer’s perverse incentive to increase its price. In Buehler and Gärtner (2012) consumers are not strategic and the recommended retail price is used by the manufacturer to communicate demand and cost information to the retailer.

The remainder of this paper is organized as follows. In Section 2 we discuss the model where consumers know the wholesale price. This model builds on results by Stahl (1989) and adds a stage to that model, where a manufacturer determines the
wholesale price. Section 3 then discusses the model where consumers do not know retailers’ cost. Section 4 compares the qualitative properties of the two models and performs a comparative statics analysis. Section 5 analyzes some extensions and shows that the qualitative features of our analysis are robust. We show that the triple marginalization problem does not disappear when the upstream firm chooses a two-part tariff. We also demonstrate that the upstream firm does not benefit from price discrimination between the two retailers by charging them different prices, or (in the extreme case) by foreclosing one of them. Finally, we numerically check the robustness of our analysis with respect to the number of retailers present in the market. Section 6 concludes. Proofs are provided in the Appendix.

2 The Model where retailers’ cost is observed

In the case where consumers observe the price set by the upstream firm, our model incorporates the sequential search model of Stahl (1989), but we add a price setting stage where the upstream firm sets the price it charge to the retailers. We modify Stahl’s model in two dimensions. First, for analytic tractability we consider duopoly (and relegate the numerical analysis of downstream oligopoly to Section 5), and, second, we explicitly consider retailers’ marginal costs. In particular, we consider a homogenous goods market where an upstream firm chooses a wholesale price $w$ for each unit the retailers sell. For simplicity, we abstract away from the issue of how the cost of the wholesale firm is determined and set this cost equal to 0.4 Two retailers take this wholesale price (their marginal cost) $w$ as given and compete in prices. Each firms’ objective is to maximize profits, taking the prices charged by other firms and the consumers’ behavior as given.

On the demand side of the market we have a continuum of risk-neutral consumers with identical preferences. A fraction $\lambda \in [0, 1]$ of consumers, the shoppers, have zero search cost. These consumers sample all prices and buy at the lowest price. The remaining fraction of $1 - \lambda$ non-shoppers have positive search costs $s > 0$ for every second search they make.5 These consumers face a non-trivial problem when searching for low prices, as they have to trade off the search cost with the (expected) benefit from search. Consumers can always come back to previously visited firms

---

4To focus on the new insights one derives from studying the vertical relation between retailers and manufacturers in a search environment, we assume all market participants know the manufacturer’s cost equals 0. Alternatively, the manufacturer’s cost could be chosen by a third party or could be uncertain. This would create, however, additional complexity that may obscure the results.

5Our analysis for small search cost is unaffected when consumers also have to incur a cost for the first search they make as the expected consumer surplus of making one search is larger than the search cost in that case. For higher search cost, the analysis would become more complicated as we then have to consider the participation decision of non-shoppers (see, Janssen et al. (2005) for an analysis of the Stahl model where the first search is not free).
incurring no additional cost. If a consumer buys at price $p$ she demands $D(p)$, and in most of the analysis we assume for simplicity that $D(p) = 1 - p$.

The retail monopoly price is denoted by $p^m(w)$ and with the simple linear demand function we consider the retail monopoly price equals $(1 + w)/2$.

The timing is as follows. First, the upstream firm chooses $w$, which is in this section observed by both retailers and all consumers. Then, given $w$, each of the retailers $i$ sets price $p_i$. Finally, consumers engage in optimal sequential search given the Nash equilibrium distribution of retail prices and retailer’s cost, not knowing the actual prices set by individual retailers.

### 2.1 Equilibrium

With the wholesale price known to consumers, there exists a unique symmetric subgame perfect equilibrium where consumer behavior satisfies a reservation price property. As consumers know $w$, consumers’ reservation prices are dependent on $w$ and denoted by $\rho(w)$. To characterize this equilibrium it is useful, to first characterize the behavior of retailers and consumers for given $w$. We use the following notation: $F(p|w)$ for the distribution of retail prices charged by the retailers (with density $f(p|w)$), and $b(w)$ and $\bar{p}(w)$ for the lower- and upper- bound of their support, respectively. This behavior is by now fairly standard and Proposition 1 below is stated without proof.

**Proposition 1.** For $\lambda \in (0,1)$, the equilibrium price distribution for the subgame starting with $w$ is given by

$$F(p|w) = 1 - \left( \frac{1 - \lambda (\bar{p}(w) - w)D(\bar{p}(w)) - (p - w)D(p)}{(p - w)D(p)} \right)$$

respectively

$$f(p|w) = \frac{1 - \lambda (\bar{p}(w) - w)D(\bar{p}(w))}{2\lambda [(p - w)D(p)]^2} \frac{[D(p) + (p - w)D'(p)]}{[D(p) + (p - w)D'(p)]}$$

with support on $[b(w), \bar{p}(w)]$ where $b(w)$ is the solution to:

$$(b(w) - w)(1 + \lambda)D(b(w)) = (\bar{p}(w) - w)(1 - \lambda)D(\bar{p}(w)).$$

---

6Alternatively, we could consider consumers with unit demand and their maximal willingness to pay being uniformly distributed over the interval $[0,1]$. This would yield a downward sloping demand at the market level (which is what matters). The interpretation of one of the results in Section 3 is easier when we consider this interpretation.
The upper bound \( \bar{p}(w) \) can be equal to the monopoly price \( p^m(w) \), or to the non-shoppers’ reservation price \( \rho(w) \). To define the reservation price that is consistent with utility maximization of non-shoppers we have to compare expected benefit and cost of search. Expected benefit from searching the other firm for a non-shoppers who has encountered price \( \rho(w) \) is given by

\[
ECS(\rho(w)) \equiv \int_{b(w)}^{\rho(w)} D(p)F(p) \, dp.
\]

This has to be equal to \( s \) so that all non-shoppers who encounter prices below \( \rho(w) \) do not search, whereas those that observe prices above \( \rho(w) \) do search. Thus, the condition that implicitly defines the reservation price \( \rho(w) \) is:

\[
ECS(\rho(w)) = s.
\]

The next lemma argues that for small \( s \), the upper bound of the price distribution \( \bar{p}(w) \) is determined by \( \rho(w) \), while for larger \( s \) it is given by the monopoly price \( p^m(w) \). There is a critical value of \( s \) that distinguishes these two cases and it is denoted by \( \hat{s}^o(\lambda; w) \), where the superscript \( o \) denotes the fact that consumers observe the retailers’ cost \( w \).

**Lemma 2.** For every \( w \in [0, 1) \) and every \( \lambda \in (0, 1) \), there exists a critical value \( \hat{s}^o(\lambda; w) \) such that for all \( s < \hat{s}^o(\lambda; w) \) \( \bar{p}(w) = \rho(w) \) and for all \( s > \hat{s}^o(\lambda; w) \) \( \bar{p}(w) = p^m(w) \).

This completes the description of the behavior of the downstream market for a given \( w \). Now we turn to the optimal behavior of the upstream manufacturer who determines \( w \). One can easily verify that by increasing \( w \), the upstream firm shifts the retailers’ price distribution to the right so that upstream expected demand is decreasing in \( w \). For a given \( w \) expected profit of the upstream firm is given by:

\[
\pi_u(w) = \left( (1 - \lambda) \int_{b(w)}^{\bar{p}(w)} D(p)f(p) \, dp + 2\lambda \int_{b(w)}^{\bar{p}(w)} D(p)f(p)(1 - F(p)) \, dp \right) w.
\]

The first integral is the expected profit earned from the non-shoppers; the second integral is expected profit stemming from the demand of shoppers who buy at the lowest retail price. For non-shoppers the density of prices is \( f(p) \) as they only sample one firm, whereas for the shoppers this density function is given by \( 2f(p)(1 - F(p)) \). As all functions are continuously differentiable, and as upstream profits equal 0 at \( w = 0, 1 \), it follows that there is an optimal value of \( w \in (0, 1) \), denoted by \( w^* \), and
that this $w^*$ solves\footnote{Note that this condition is necessary, but not sufficient. This will be important when discussing the issue of existence of equilibrium in Section 3.}

$$\frac{\partial \pi_u(w)}{\partial w} = 0.$$  

As $w^* > 0$ and at the retail level $p > b(w^*) > w^*$, both the upstream and downstream firms are able to charge a margin above their marginal cost. Unfortunately, as all expressions $\bar{p}(w)$, $b(w)$ and $F(p|w)$ depend on $w$ in a relatively complicated way, it is impossible to get an explicit expression for $w^*$. Nevertheless, we can show the following in the case where demand is linear.

**Proposition 3.** Suppose $D(p) = 1 - p$. For all $\lambda$, if $s > \bar{s}^o(\lambda) \equiv \bar{s}^o(\lambda; 0.5)$ so that $\bar{p}(w) = p^m(w)$, $w^* = 0.5$. Moreover, when $s$ approaches 0, then $w^*$ approaches 0.5.

The fact that for relatively large search cost, the optimal wholesale price is independent of $s$ follows from the fact that retailers’ pricing behaviour is independent of search cost when $s$ is large. That for high search cost the upstream firm chooses the monopoly price when demand is linear follows from the special relationship between the upper and lower bound of the price distribution in case the upper bound is the retailers’ monopoly price. The limiting result for search cost approaching zero can be easily understood as follows. If search cost approaches zero, the non-shoppers’ reservation price has to converge to the cost level of retailers. This implies that there is (almost) Bertrand competition at the retail level and retailers set retail prices almost equal to their marginal cost. As this cost is known to consumers, they effectively demand $1 - w$ and therefore, the wholesale profit function is simply $w(1 - w)$, which is maximized at 0.5.

It is difficult to solve the maximization problem of the wholesale firm for general $s$ as for $s$ values smaller than $\bar{s}^o(\lambda)$ the upper bound of the retailers’ price distribution equals the reservation price and this price depends in a non-trivial way on $w$. Figure 1 illustrates that for a given value of $\lambda$ (at 0.5) $w^* < 0.5$ for all values of $s$ such that $\bar{p}(w) < p^m(w)$. Thus, the optimal upstream price is non-monotonic in the search cost $s$. (This is further corroborated in Tables 1 and 3 presented in Section 4, where we also show that the upstream price is non-monotonic in $\lambda$). The non-monotonicity of $w$ in $s$ stems from a balance between two effects that $s$ has on the equilibrium value of $w$. First, the higher the search costs, the higher the retail margins. When $s$ increases from 0, the retail margins increase from a very low level and the upstream firm reacts by lowering the wholesale price. The reason for this can be seen by looking at the optimal wholesale price as a function of a fixed retail margin over $w$. If the margin is assumed to be equal to $s$ (so that downstream margins are equal to search cost), the optimal wholesale price is $\frac{1 - s}{2}$ and the corresponding retail price
Double Marginalization

$\tilde{p}^o$

Double Marginalization

$w^o$

Figure 1: Upstream price (solid) and weighted average retail price (dashed) as function of $s$.

is $\frac{1+s}{2}$. As can be seen from this simple calculation, higher search costs lead to higher retail margins and subsequently to a lower wholesale price and higher retail prices. Second, as $s$ keeps on increasing, the reservation price approaches $p^m$, and so the retail prices move closer to the retail monopoly price.\footnote{As $s$ increases the retail monopoly price is increasing, but so is $\rho$ which increases even faster.} The closer retail prices get to $p^m$, the less sensitive they become to $w$, which gives the upstream firm an incentive to increase $w$. For large enough $s$ this incentive outweighs the tendency of the upstream firm to accommodate increasing downstream margins by lowering the wholesale price, and the wholesale price starts increasing in $s$. Eventually $s$ reaches $\hat{s}^o(\lambda)$, the level at which $w$ reaches 0.5 and $w$ becomes independent of $s$.

We will say that there is a double marginalization problem if $\tilde{p}$ is above the monopoly price, i.e., if $\tilde{p} \equiv \lambda E \min(p_1, p_2) + (1 - \lambda) Ep > 0.5$. If this is the case all market participants can be made better off if they could agree on lowering prices. The Proposition says that there is certainly a double marginalization problem if the search cost $s$ is relatively high (as in that case $w^* = 0.5$ and for all prices $p$ at the retail level it is true that $p > b > w^* = 0.5$). For smaller values of the search cost, the situation is less clear. Numerically, we can establish, however, that also in this case the upstream firm sets $w^*$ such that $\tilde{p} > 0.5$. See Tables 1 and 3 in Section 4 for details.

\footnote{See Tables 1 and 3 in Section 4 for details.}
3 Unobserved wholesale price

We now turn to the analysis of the model where consumers do not know the wholesale price. All other aspects of the model remain the same. At a technical level, this implies that for a given $w$ the downstream market cannot be analyzed as a separate subgame anymore. In the spirit of the notion of subgame perfect equilibrium we have used so far, we wish to impose certain reasonable requirements on retailers’ response to any $w$, not only the equilibrium one.

To fully understand the implications of this change in the information structure, let us start the analysis by providing a formal definition of what we mean by a reservation price equilibrium in this case. First, as non-shoppers are not informed about $w$ the reservation price is now just a number $\rho$ that is independent of the wholesale price. They form expectations about this price, and in equilibrium, these expectations are correct. As retailers do know the wholesale price, however, their pricing remains dependent on it. As in the previous section, retailers will choose to randomize their prices, and thus we have:

**Definition 4.** A reservation price equilibrium\(^9\) is characterized by a price $w^*$ set by the manufacturer, a distribution of retail prices $F(p|w)$, one for each $w$, and non-shoppers’ reservation price $\rho^*$ such that

1. the manufacturer chooses $w^*$ to maximize its expected demand (which depends on $F(p|w)$ and $\rho^*$);

2. each retailer uses a price strategy $F(p|w)$ that maximizes expected profit, given the actual cost chosen by the manufacturer, the competing retailer’s price strategies, and non-shoppers’ reservation price $\rho^*$;

3. non-shoppers’ reservation price $\rho^*$ is such that they search optimally given their correct beliefs about $w$ and $F(p|w)$; shoppers observe all prices and then buy at the lowest retail price.

In the spirit of the notion of sequential equilibrium this definition guarantees that retailers maximize their profits for wholesale prices $w$ that are off-equilibrium. Any price above $\rho^*$ is interpreted by consumers as a deviation by the retailer and is punished by further search. This is necessary for a reservation price to keep its meaning. Otherwise, if consumers buy even if $p > \rho^*$, then retailer will charge such prices and $\rho^*$ will no longer be a reservation price. This belief by consumers is not a belief in an asymmetric information game, but rather one of the possible beliefs

\(^9\)Technically, this equilibrium definition is in the spirit of sequential equilibrium, requiring all players to choose optimally at all of their information sets.
a consumer has in a game where the interaction between different types of firms produces the final retail price.

There are different ways to justify these beliefs. First, a deviation by the upstream firm alone to a wholesale price \( w < \rho^* \) will not result in retailers choosing prices above the reservation price as given the consumers’ strategy, these retailers will make no profit. After observing a larger price than the reservation price, consumers could believe it is both the manufacturer and the retailer that have deviated, but this would invoke two deviations, whereas it is more natural to believe that only one player has deviated (if this single deviation by itself can explain the observed price). Thus, the only way the consumer can attribute the deviation to the upstream firm alone, is if the upstream firm would have chosen a wholesale price \( w > \rho^* \). In this case, (after the deviation of the upstream firm) the equilibrium strategies of the retailers would imply that \( p = w \). Hence, non-shoppers may want to buy rather than continue to search as the expected price at the next firm would also be higher than \( \rho^* \). We can show, however, in line with the economic ideas underlying the D1 refinement,\(^\text{10}\) that the retailers have more incentive to deviate to prices that are larger than the reservation price than the upstream firm, and therefore consumers should blame the retailer if they observe a price that is larger than the reservation price.\(^\text{11}\)

We first examine the properties of a reservation price equilibrium, assuming that such an equilibrium exists. We next consider the existence question. A first observation is that in a reservation price equilibrium, for a given \( w \) the upper bound of the price distribution has to be equal to the minimum of the monopoly price \( p^m(w) \) and the reservation price \( \rho^* \) of non-shoppers, i.e. \( \bar{p}(w) = \min(\rho^*, p^m(w)) \). It is clear that the retailers will never set a price larger than the monopoly price for given \( w \) as this can never be profit-maximizing. Consider then the case where \( \rho^* < p^m(w) \) and suppose that for some \( w \), \( \bar{p}(w) > \rho^* \). If a retailer would charge \( \bar{p}(w) \), it will not sell to shoppers as they observe lower prices with probability one. Furthermore, by setting \( \bar{p}(w) \) it will not sell to non-shoppers either, as these consumers will continue to search after observing a price larger than \( \rho^* \) and will find with probability one a

\(^{10}\)See Cho and Sobel (1990) for a statement of this refinement. Note, however, that the game we analyze here is not a signaling game, and therefore the formal D1 analysis does not apply here.

\(^{11}\)The informal argument goes as follows. Suppose that after observing a deviation price \( p > \rho^* \) a non-shopper buys with probability \( q(p) \). Clearly, if \( q(p) \) is sufficiently close to 1, a retailer has an incentive to deviate. Then consider the case where only the wholesale firm deviates. The only way a deviation by the upstream firm alone could account for retail prices above the reservation price is \( w > \rho^* \) resulting in a profit of \( w(1 - w) \). Given that, later on, we show the equilibrium wholesale price to be always larger than 0.5 (and the equilibrium reservation price even higher than that), this is never optimal for the wholesale firm (whatever the consumers do). As there are values of \( q(p) \) such that a retail firm has an incentive to deviate, whereas there are no values of \( q(p) \) such that the wholesale firm has an incentive to deviate, D1 requires non-shoppers to consider it infinitely more likely that the individual retailer has deviated.
lower price at the other retailer. Finally, a standard argument in the search literature can be invoked to show that it cannot be the case that \( \bar{p}(w) < \min(\rho^*, p^m(w)) \).

It follows that the behavior of downstream retailers in this case of unobserved retailers’ cost is very similar to what was described in Proposition (2.1) of the previous section. It is easy to see that also with unobserved \( w \), there is a critical value of \( s \), denoted by \( \hat{s}^{\alpha}(\lambda; w) \), such that below that value the upper bound of the price distribution is given by \( \rho^* \), whereas above it it is given by \( p^m(w) \). We present both facts in one proposition (without proof).

**Proposition 5.** For \( \lambda \in (0, 1) \), the equilibrium retail price distribution and density function for given \( w \) and \( \rho^* \) is represented by

\[
F(p|w) = 1 - \frac{1 - \lambda}{2\lambda} \left[ \frac{(\bar{p}(w) - w)D(\bar{p}(w))}{(p - w)D(p)} - 1 \right]
\]

respectively

\[
f(p|w) = \frac{1 - \lambda (\bar{p}(w) - w)D(\bar{p}(w))}{2\lambda [(p - w)D(p)]^2} [D(p) + (p - w)D'(p)]
\]

with support on \( [b(w), \bar{p}(w)] \), where \( b(w) \) is the solution to:

\[
(b(w) - w)(1 + \lambda)D(b(w)) = (\bar{p}(w) - w)(1 - \lambda)D(\bar{p}(w)).
\]

Moreover, for every \( w \) and \( \lambda \), there exists a critical value \( \hat{s}^{\alpha}(\lambda; w) \) such that for all \( s < \hat{s}^{\alpha}(\lambda; w) \) \( \bar{p}(w) = \rho^* \) and for all \( s > \hat{s}^{\alpha}(\lambda) \) \( \bar{p}(w) = p^m(w) \).

The equilibrium behavior of the upstream manufacturer is now more difficult to determine as we have to look for a value of \( w \) such that if non-shoppers predict that the upstream monopolist will set this price, the upstream firm does not have an incentive to set a different price. It remains true, however, that for linear demand when the search cost are high the upstream firm sets the monopoly price.

**Proposition 6.** In the case where consumers do not observe the upstream price \( w \) and demand is given by \( D(p) = 1 - p \), it is the case that for all \( \lambda \) there exists a \( \hat{s}^{\alpha}(\lambda) \) such that for all \( s > \hat{s}^{\alpha}(\lambda) \) \( w^* = 0.5 \).

Proposition 6 can be understood as follows. First, the only difference between the case where the wholesale price is observed and the case where it is not is that in the first case, the reservation price changes with \( w \), whereas it cannot change with

\[12\text{In equilibrium, the wholesale price } w \text{ is correctly anticipated by consumers and } \bar{p}(w) > w.\]
when consumers do not observe retail cost. It follows that if search cost is high enough so that the upper bound of retailers' price distribution is determined by the monopoly price, rather than by the reservation price, the two models should yield identical results.

The next result shows that when the search cost $s$ becomes small, the behavior of the upstream firm is very different from that studied in the previous section.

**Proposition 7.** In the case where consumers do not observe the upstream price $w$ and demand is given by $D(p) = 1 - p$, and a reservation price equilibrium exists when $s$ approaches 0, then $w^*$ approaches $1/(1 + \lambda) > 0.5$.

Thus, the model exhibits a discontinuity at $s = 0$ that is similar to the Diamond paradox (Diamond (1971) and Rhodes (2012) for a recent contribution). When $s = 0$, there is no distinction between shoppers and non-shoppers and everyone observes both prices before buying, creating Bertrand competition at the downstream level. Knowing this, the upstream firm sets the monopoly price just as in the case where the wholesale price is known. However, when $s$ is small, non-shoppers have a reservation price based on an expected wholesale price and given this expectation, the upstream firm pushes the wholesale price up.

Figure 2 shows in $(s, \lambda)$ space the different cases that can arise. The upward-sloping line separating the region where the equilibrium upper bound of the retailers’ price distribution is given by the reservation price and the region where it is given by the retailers’ monopoly price represents the curve $\hat{s}^{\text{no}}(\lambda)$. To the right of this curve $s$ is relatively large, the reservation price is not binding and the upstream firm’s optimal price is $w^* = 0.5$. To the left of the curve $\hat{s}^{\text{no}}(\lambda)$ the reservation price is binding and around the vertical axis, the upstream firm’s optimal price approaches $w^* = 1/(1 + \lambda)$.

Proposition 6 also alludes to the possibility that a reservation price equilibrium does not exist if $s$ is small. The reason for this potential non-existence can also be understood intuitively by looking at the profit maximization problem of the upstream firm. As the demand of non-shoppers is relatively inelastic, the manufacturer may have an incentive for a given reservation price to choose higher price levels, lose some demand from shoppers but squeeze retailers. This is especially true when the fraction of non-shoppers is relatively large. The next proposition establishes that there is a region of $(s, \lambda)$ values where a reservation price equilibrium does not exist.

**Proposition 8.** In the case where consumers do not observe the upstream price $w$ and demand is given by $D(p) = 1 - p$, there exists a critical value of $\lambda$, denoted by $\lambda^*$, such that for all $\lambda < \lambda^*$ there exists a $\hat{s}(\lambda)$ such that for all $s < \hat{s}(\lambda)$ a reservation price equilibrium does not exist, where $\lambda^* \approx 0.47$ solves $\frac{2(1 - \lambda(1 + \lambda))}{\lambda} - \frac{(1 - \lambda)^2}{\lambda^2} \log \left(\frac{1 + \lambda}{1 - \lambda}\right) = 0$. 
Figure 2 also shows the region where a reservation price equilibrium does not exist. One way to understand some aspects of this result is to relate it again to the Diamond Paradox. The Diamond paradox can arise in search models because for a given level of the (equilibrium) price that is expected by consumers, a firm will have an incentive to slightly increase the price. The Diamond paradox relies on the fact that in a search environment, before they are engage in search consumers are uninformed about the price at which they may buy, and after they find out the price the search cost is sunk. This effect is also present in our model, but it is softened by the fact that a fraction of consumers incurs no search cost - the Stahl (1989) “solution” to the Diamond paradox - and the existence of an intermediate retail level in the product chain. In case the search cost is small, the intermediate retail level does not provide much of a buffer, however, as retail prices are close to the wholesale price. In case the fraction of shoppers is small, the Stahl ‘solution’ to the Diamond paradox is also not effective. Therefore, when searches cost are small and the fraction of shoppers is small the Diamond paradox enters our model in full force.

Figure 3 provides more detail why for these values a reservation price equilibrium does not exist. In the first panel we see for $\lambda = 0.5$ and $s = 0.05$ the shape of the manufacturer’s profit function. For a given (belief about the wholesale price resulting in a given) expected reservation price, the optimal upstream price is given by the first-order condition. Given these values of the parameters, if the non-shoppers correctly anticipate the wholesale price, the (equilibrium) reservation price is approximately at 0.74. This is the point in the first panel where the profit function has a kink. As higher values of $w$ we simply plot $\pi_u(w) = w(1-w)$. In the second panel,
we see what happens to the upstream profit function for $\lambda = 0.2$ and $s = 0.02$. Given the non-shoppers correctly anticipate the wholesale price, the first-order condition for the upstream firm yields a value of $w$ that is represented in the figure given by the black point and the reservation price is again given by the hollow point where the profit function exhibits a kink. It is easy to see that given the expected level of the wholesale price, the upstream firm has an incentive to deviate, squeeze the retailer’s margins to zero and set price equal to the reservation price. This cannot be an equilibrium, however, as when this higher wholesale price would be expected, the reservation price would be even higher.

![Figure 3: Upstream profits for fixed equilibrium beliefs as function of $w$.](image)

One may wonder what type of equilibria do exist in case a reservation price equilibrium does not exist. This is, however, an inherently difficult question to answer. It turns out that the parameter values for which a reservation price equilibrium does not exist in this model, is similar to the area of parameter values for which an equilibrium does not exist in Janssen et al. (2011). This implies that one has to look for equilibria where consumers follow a search strategy that is not characterized by a reservation price. Such equilibria are, however, difficult to characterize as they will involve some non-shoppers searching more than once.

4 Comparison and Comparative Statics

In the previous two sections, it has become clear that if the upper bound of the retailers’ price distribution is given by the monopoly price, the upstream firm sets its price equal to the monopoly price of a 0.5. The lower bounds of the search cost for which this is the case were denoted by $\hat{s}^o(\lambda)$ and $\hat{s}^{no}(\lambda)$, respectively. The first lemma of this Section demonstrates that $\hat{s}^o(\lambda) = \hat{s}^{no}(\lambda)$, and therefore we will simply drop the superscript and write $\hat{s}(\lambda)$. 

16
Lemma 9. The critical boundaries $\hat{s}^o(\lambda)$ and $\hat{s}^{no}(\lambda)$ are equal: $\hat{s}^o(\lambda) = \hat{s}^{no}(\lambda)$.

We can thus conclude that for all large enough search costs, it is unimportant whether the wholesale price is known to the consumers or not. In both cases, there is a double marginalization problem.

More interesting is the comparison of the two cases when the search cost is smaller. The next proposition argues that the optimal wholesale price is higher when consumers do not observe the wholesale price.

Proposition 10. If (i) $s$ and $\lambda$ are such that a reservation price equilibrium exists in the case where wholesale price is unobserved, (ii) the upper bound of the price distribution in both the observed and the unobserved case is given by the reservation price and (iii) for all $p \in (0, p^m), \pi'_r(p)\pi_r(p) - (\pi'_r(p))^2 < 0$, where $\pi_r(p) = (p - w)D(p)$, then the optimal wholesale price in the case where wholesale prices are not observed by consumes is larger than the optimal wholesale price in the case where they are observed.

Thus, for demand functions satisfying a mild regularity condition guaranteeing that the reservation price $\rho(w)$ is increasing in $w$, the double marginalization problem is strengthened in case search cost $s$ is such that the reservation price is the binding upper bound for the retailers’ pricing decisions. The reasons is as follows. As the upper bound of the retail prices does not react to a change in the wholesale price $w$ in the case where it is not observed by consumers, the distribution of retail prices shifts more up with an increase in $w$ if $w$ is observed than when it is not observed (See Figure 4). This implies that the demand for the upstream firm is less sensitive to wholesale price changes when these are not observed by consumers. The upstream firm therefore has an incentive to set higher prices.

Figures 5 and 6 illustrate the effects of different parameter values on how much the wholesale price is higher in the case consumers do not observe it. In both figures the superscript $o$ refers to the observed case, while the superscript $no$ refers to the unobserved case. When $\lambda = 0.5$, Figure 5 confirms the fact that for high search cost $s$, the wholesale price $w$ is the same in both cases (Propositions 3 and 6), while for smaller values of $s$ the wholesale price is larger in the case where consumers do not observe it (Proposition 10). It also confirms the limiting results when $s$ becomes very small that are studied in Propositions 3 and 7. For these parameter values, the Figure shows that the wholesale price can be up to 33% higher in the case where consumers do not observe the wholesale price! Figure 6 shows a similar picture in case we fix $s$ at 0.03 and we vary $\lambda$. For small $\lambda$, $s > \hat{s}(\lambda)$ and the upstream

\[\text{For the linear demand case where } D(p) = 1 - p \text{ the second condition implies that } s < \hat{s}(\lambda), \text{ while the third requirement is satisfied for linear demand.}\]
Figure 4: Cumulative distribution of downstream prices for \( w = 0.45 \) and \( \rho = \rho(0.45) = 0.6615 \) (black curve), \( w = 0.48 \) and \( \rho = \rho(0.48) = 0.7026 \) (thin dashed blue), and \( w = 0.48 \) and \( \rho = \rho(0.45) = 0.6615 \) (thick dashed red).

price \( w \) equals the monopoly price of a vertically integrated monopolist. When \( \lambda \) approaches 1, we also have that the wholesale price \( w \) approaches this vertically integrated monopoly price. In between \( w^{\text{no}} > 0.5 > w^o \).

The two Figures also depict \( \hat{p}, \hat{p} = \lambda E \min(p_1, p_2) + (1 - \lambda)Ep \), in both cases. These are the prices consumers expect to pay before they “know” whether they are shoppers or non-shoppers. Figure 4 clearly shows the different effect of search cost \( s \) on \( \hat{p} \) in both cases. When the wholesale price is observed, we have the standard effect in the consumer search literature that expected prices are increasing in the search cost. However, in the often more realistic case where consumers do not observe the wholesale price, \( \hat{p} \) is decreasing in \( s \). The reason for this counterintuitive result is that there are two countervailing forces here. First, when search cost increases, retailers have more market power and can increase their retail margins for given cost. Second, the triple marginalization problem becomes less severe as retailers are able to pass on wholesale price increases and the upstream firm internalizes this effect and charges significantly lower prices in response (see the steep decrease in the \( w^{\text{no}} \) curve in Figure 5). As for linear demand the second effect dominates, the total effect is that retail prices are decreasing.

The result that \( \hat{p} \) is decreasing in search cost may be an interesting theoretical explanation for the finding in Moraga-González et al. (2012) concerning the automobile market in the Netherlands. Using their empirical analysis, they simulate the effect of search cost by considering how much car prices would change if search
costs were equal to 0. They find that depending on the brand, retail prices of new cars may be up to 12% higher if search cost are negligible. Our explanation for this finding relies on the interaction between upstream and downstream firms and the fact that typically the wholesale price is not observed. Also, our result sheds an alternative perspective on the effect of websites like www.edmunds.com that attempt to reduce search cost by making it easier for consumers to find lower car prices. Thus, websites that intend to help consumers in getting better deals may in the end lead to higher prices, unless they also provide wholesale prices.

An alternative way to consider the different comparative static effects of the search cost in the two cases considered is as follows. Changing $w$ has two main effects in both cases where wholesale prices are observed and unobserved by consumers. First, by increasing $w$ in the relevant range around or above 0.5 the wholesale firm decreases the total profits that are to be shared between wholesale and retail firms. Second, by increasing $w$ the wholesale firm claims a larger share of the total profit. Although both effects are present in both cases, the strength of the first effect depends on whether wholesale prices are observed and on the level of search cost $s$. As the distribution of retail prices shifts more up with an increase in $w$ if $w$ is observed, total industry profit is decreasing more in that case. Also, when search costs increase retailers’ share in total profits becomes larger and this effect is also stronger in case wholesale prices are observed. Thus, when wholesale prices are not observed, the upstream firm has a much larger incentive to reduce the wholesale price in a response to an increase in search cost than when wholesale prices are observed.
This incentive is so large that despite the larger margins retail firms make, the $\tilde{p}$ is inversely related to search cost.

The figures also clearly convey the idea that triple marginalization is of quantitative significance. For a large subset of the parameter space, $\tilde{p}$ in the case where consumers do not observe retailers’ cost is at least 20% higher than the $\tilde{p}$ in the case where consumers do observe this cost. Identifying the normal double marginalization problem with the extent to which this $\tilde{p}$ is larger than the monopoly price of 0.5, the figures also show that the strengthening of the double marginalization problem by the consumers’ inability to observe wholesale prices often outweighs the normal double marginalization effect.

Table 1: Equilibrium for observed $w$, $D(p) = 1 - p$ and $\lambda = 0.5$

<table>
<thead>
<tr>
<th>$s$</th>
<th>$w$</th>
<th>$b$</th>
<th>$\tilde{p}$</th>
<th>$\hat{p}$</th>
<th>$\pi_u$</th>
<th>$2\pi_r$</th>
<th>$\pi_u + 2\pi_r$</th>
<th>$CS$</th>
<th>$W$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001</td>
<td>0.498</td>
<td>0.499</td>
<td>0.502</td>
<td>0.500</td>
<td>0.249</td>
<td>0.001</td>
<td>0.250</td>
<td>0.125</td>
<td>0.375</td>
</tr>
<tr>
<td>0.02</td>
<td>0.466</td>
<td>0.491</td>
<td>0.552</td>
<td>0.505</td>
<td>0.231</td>
<td>0.019</td>
<td>0.250</td>
<td>0.122</td>
<td>0.372</td>
</tr>
<tr>
<td>0.05</td>
<td>0.469</td>
<td>0.516</td>
<td>0.688</td>
<td>0.547</td>
<td>0.213</td>
<td>0.034</td>
<td>0.247</td>
<td>0.103</td>
<td>0.350</td>
</tr>
<tr>
<td>0.07</td>
<td>0.500</td>
<td>0.546</td>
<td>0.750</td>
<td>0.577</td>
<td>0.212</td>
<td>0.031</td>
<td>0.243</td>
<td>0.090</td>
<td>0.333</td>
</tr>
</tbody>
</table>

Tables 1-4 summarize the findings of our numerical analysis where we also study the impact of changes in $s$ and $\lambda$ on upstream and downstream profits, (weighted) consumer surplus and total welfare (which here is measured as the sum of total industry profit and consumer surplus). As retail prices are chosen according to a mixed strategy distribution, the table reports expected values for profits, consumer surplus
and welfare. As the first search is for free and consumers do not search beyond the
first search, search cost is not incorporated into the measure of total welfare. Tables
1 and 2 provide the numerical analysis of the observed and unobserved case for a
fixed value of \( \lambda = 0.5 \) and different values of \( s \). Tables 3 and 4 provide the numerical
analysis of the observed and unobserved cost case for a fixed value of \( s = 0.05 \) and
different values of \( \lambda \). The tables convey two important messages.

The first important message is that when consumers do not observe the whole-
sale price, an increase in search cost is both good for total industry profit and for
consumers. The reason is that as (i) \( \hat{p} \) at which consumers buy are initially very
high (i.e., much above the monopoly price) and as (ii) an increase in search cost
decreases this average retail price, total surplus is much higher with higher search
cost and firms benefit from the increase in demand. As already explained above,
it is the retail firms that really benefit from this increased search cost (as they can
increase their margins) at the expense of the upstream firm. Again, the effects are
quantitatively significant: Table 2 shows that an increase in search cost from a level
close to 0 to 0.07 increases welfare by 20%. (The effects on consumer surplus and
welfare are so strong that even if we would assume the first search comes at a cost
\( s \), consumer surplus and welfare are still increasing in \( s \).)

\[14\] Note that when \( \lambda = 0.5 \) only half of the consumers pays a search cost and in that case we have

---

Table 2: Equilibrium for observed \( w \), \( D(p) = 1 - p \) and \( s = 0.05 \)

<table>
<thead>
<tr>
<th>( \lambda )</th>
<th>( w )</th>
<th>( b )</th>
<th>( \hat{p} )</th>
<th>( \hat{\pi} )</th>
<th>( \pi_u )</th>
<th>( 2\pi_r )</th>
<th>( \pi_u + 2\pi_r )</th>
<th>( CS )</th>
<th>( W )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.500</td>
<td>0.606</td>
<td>0.750</td>
<td>0.643</td>
<td>0.179</td>
<td>0.050</td>
<td>0.229</td>
<td>0.064</td>
<td>0.293</td>
</tr>
<tr>
<td>0.5</td>
<td>0.469</td>
<td>0.516</td>
<td>0.688</td>
<td>0.547</td>
<td>0.213</td>
<td>0.034</td>
<td>0.247</td>
<td>0.103</td>
<td>0.350</td>
</tr>
<tr>
<td>0.7</td>
<td>0.475</td>
<td>0.496</td>
<td>0.643</td>
<td>0.513</td>
<td>0.231</td>
<td>0.018</td>
<td>0.249</td>
<td>0.119</td>
<td>0.368</td>
</tr>
</tbody>
</table>

Table 3: Equilibrium for unobserved \( w \), \( D(p) = 1 - p \) and \( \lambda = 0.5 \)

<table>
<thead>
<tr>
<th>( s )</th>
<th>( w )</th>
<th>( b )</th>
<th>( \hat{p} )</th>
<th>( \hat{\pi} )</th>
<th>( \pi_u )</th>
<th>( 2\pi_r )</th>
<th>( \pi_u + 2\pi_r )</th>
<th>( CS )</th>
<th>( W )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001</td>
<td>0.661</td>
<td>0.663</td>
<td>0.667</td>
<td>0.664</td>
<td>0.222</td>
<td>0.001</td>
<td>0.223</td>
<td>0.057</td>
<td>0.280</td>
</tr>
<tr>
<td>0.02</td>
<td>0.578</td>
<td>0.607</td>
<td>0.688</td>
<td>0.624</td>
<td>0.217</td>
<td>0.017</td>
<td>0.234</td>
<td>0.071</td>
<td>0.305</td>
</tr>
<tr>
<td>0.05</td>
<td>0.507</td>
<td>0.552</td>
<td>0.740</td>
<td>0.582</td>
<td>0.212</td>
<td>0.030</td>
<td>0.242</td>
<td>0.088</td>
<td>0.330</td>
</tr>
<tr>
<td>0.07</td>
<td>0.500</td>
<td>0.546</td>
<td>0.750</td>
<td>0.577</td>
<td>0.212</td>
<td>0.031</td>
<td>0.243</td>
<td>0.090</td>
<td>0.333</td>
</tr>
</tbody>
</table>

Table 4: Equilibrium for unobserved \( w \), \( D(p) = 1 - p \) and \( s = 0.05 \)

<table>
<thead>
<tr>
<th>( \lambda )</th>
<th>( w )</th>
<th>( b )</th>
<th>( \hat{p} )</th>
<th>( \hat{\pi} )</th>
<th>( \pi_u )</th>
<th>( 2\pi_r )</th>
<th>( \pi_u + 2\pi_r )</th>
<th>( CS )</th>
<th>( W )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.500</td>
<td>0.606</td>
<td>0.750</td>
<td>0.643</td>
<td>0.179</td>
<td>0.050</td>
<td>0.229</td>
<td>0.064</td>
<td>0.293</td>
</tr>
<tr>
<td>0.5</td>
<td>0.507</td>
<td>0.552</td>
<td>0.740</td>
<td>0.582</td>
<td>0.212</td>
<td>0.030</td>
<td>0.242</td>
<td>0.088</td>
<td>0.330</td>
</tr>
<tr>
<td>0.7</td>
<td>0.516</td>
<td>0.537</td>
<td>0.701</td>
<td>0.555</td>
<td>0.230</td>
<td>0.017</td>
<td>0.246</td>
<td>0.099</td>
<td>0.346</td>
</tr>
</tbody>
</table>
The second important message is that comparing the observed cost case to the unobserved case (that is comparing Tables 1 and 2, and also Tables 3 and 4) reveals that all market participants benefit when the wholesale price is observed by consumers. Again, the main difference between the two cases is that the weighted average of retail prices is much lower in the observed cost case, and as these prices are above the monopoly price, total surplus generated in the observed cost case is much higher. In this case, all market participants benefit and the total effect on welfare can be as high as 35 percent! This may be a reason why in case market participants can credibly commit to publishing retailers’ cost in a credible way - as they try to do in the US automobile industry - they will do so (e.g. Scott Morton et al. (2011)).\textsuperscript{15} The result also shows the welfare loss in case market participants cannot credibly convey this information to consumers.

5 Extensions

In this section we consider several possible extensions. A natural question in the context of vertical relations is whether the problem of triple marginalization disappears with non-linear prices. It is well-know that the double marginalization problem disappears with two-part tariffs. In the first subsection we show that a triple marginalization effect continues to exist under two-part tariffs, but that the order of magnitude is much smaller. In the second subsection we consider the question whether triple marginalization remains to be an important issue if the upstream firm can price discriminate between the two retailers and charge them different prices. The result of the second subsection is that the upstream firm will never want to price discriminate as its profits will be lower than when it charges both retailers the same price. Finally, we numerically confirm that our results, established in the main body of the paper for a downstream duopoly market, can be generalized to allow for more retailers.

5.1 Two-part tariffs

When the upstream firm engages in two-part tariffs, it charges a price per unit and a fixed fee to maximize profits. As firms are risk neutral, the upstream firm can charge a fixed fee equal to the expected profits of the retailers such that the retailers to deduct half of the difference in search cost to see the effect of including a first costly search on welfare and consumer surplus.

\textsuperscript{15}Neither retailers nor the manufacturer can credibly reveal \( w \) if they are not required by law to tell the truth. The manufacturer has an incentive to deceive consumers by saying \( w \) is low, so that consumers continue to search and drive down retail prices. The retailers have the opposite incentive. They want consumers to believe that \( w \) is high, so that they accept high prices. Thus if there is no credible way to reveal \( w \), consumers will not believe any announcement by firms.
are still willing to participate. Thus, when charging an optimal two-part tariff, the upstream firm effectively maximizes total industry profits. In this case, the upstream firm has clearly less incentives to squeeze the retailers’ profits as it can recover these profits by charging a higher fixed fee. So, the question is whether two-part tariffs completely eliminate the triple marginalization problem (like it eliminates the double marginalization problem) or whether part of the problem remains.

First, note that when the wholesale price arrangement is observed by consumers, the upstream firm wants to set its upstream price per unit such that the retail price distribution centers around the integrated monopoly price of 0.5 to maximize total industry profit. As there is price dispersion at the retail level, the upstream firm will never charge a price per unit that is equal to its marginal cost. To do so, would imply that the upper bound of the retail price distribution is equal to the integrated monopoly price (for large search cost $s$) or below (for smaller values of $s$) so that almost surely the retail price is effectively too small to maximize profits. To have the retail price distribution center around the integrated monopoly price, the upstream firm sets a positive price per unit and a fixed fee. This is shown in Figure 7 for the case where $\lambda = 0.5$ and demand is linear and given by $D(p) = 1 - p$. Note that because of the price distribution at the retail level, the upstream firm cannot get the maximal profit of an integrated monopolist.

![Figure 7: Two-part tariffs. Upstream prices (solid) and weighted average downstream prices (dashed) for the observed (blue) and unobserved (red thick) retailers’ costs as functions of $s$ for $\lambda = 0.5$.](image)

Next, consider the case where the wholesale price arrangement is not observed by consumers. If the upper bound of the retail price distribution is given by the retail monopoly price, the same consideration apply as above and the outcome of
the two models coincide. However, when the search cost \( s \) is small, the upper bound of the retail price distribution is given by the non-shoppers’ reservation price, which now does not depend on the wholesale price per unit. Thus, like in the baseline model without fixed fee, the retailers’ price distribution (and thus the demand for the upstream firm) reacts less to an increase in the price per unit set by the upstream firm. thus, compared to the case where the wholesale price arrangement is observed by consumers, the upstream firm has (again) an incentive to increase the per unit wholesale price. As with two-part tariffs, the upstream firm effectively maximizes total industry profits and the downstream profits decrease with an increase in the per unit wholesale price, the incentives to increase the per unit wholesale price are dampened. The figure shows that the triple marginalization problem continues to exist, but the magnitude is much smaller.

Moreover, the effect analyzed in the previous section that the triple marginalization problem is most severe for small search cost, disappears. The reason why, as explained above, under two-part tariffs the triple marginalization problem continues to exist for positive search cost is because of the price dispersion at the retail level. When the search cost becomes arbitrarily small, however, the retail price dispersion disappears and the upstream firm can extract maximal profits by setting the wholesale price per unit equal to the integrated monopoly price. Thus, under two-part tariffs the two models converge for both large search cost and when the search cost is arbitrarily small.

5.2 Price discrimination between retailers

In the main body of the paper we considered environments where the upstream firm cannot price discriminate between different retailers. In this extension, we discuss the kind of considerations that apply when price discrimination is feasible and ask whether the symmetric equilibrium without price discrimination we characterized so far remains an equilibrium or not.

To consider this question, we have to distinguish two different cases, depending on whether or not rival retailers observe the wholesale price the other retailer pays. In the main part of the analysis where retailers know the upstream firm cannot price discriminate, these two cases coincide as knowing your own cost level (the wholesale price) you also know the cost of your competitor. Consider now first the case where retailers do not observe the wholesale price rival firms pay. In this case, whether or not it is optimal for the upstream firm to deviate depends on the beliefs retailers have about the type of deviation chosen by the upstream firm. Note that in this case of private information about their own wholesale price, after observing a deviation by the upstream firm, retailers have to form beliefs about the wholesale...
price the deviating upstream firm charges to the rival retailer, and they have to have beliefs about the rival’s beliefs about their own wholesale price, and so on. There is no standard refinement notion in game theory that restricts these type of beliefs, and one can easily construct beliefs such that the retailers react in such a way that the upstream firm’s deviation is not profitable. For example, if after observing a deviation, a retailer believes that the upstream firm has deviated in such a way that both retailers have the same cost and believes that the other retailer has similar beliefs, then our analysis of no price discrimination applies.\(^\text{16}\) Thus, one can support the symmetric equilibrium we have concentrated on so far, also in case the upstream firm can price discriminate between retailers by appropriate beliefs of the retailers after a deviation of the upstream firm.

Next consider, the case where the rivals observe each others’ costs. In that case we can show that local deviations are not profitable, but to show that global deviations are also unprofitable is beyond analytical tractability, and it is not clear that this case is worthwhile to pursue (as retailers may actually not observe each others’ cost). What is easy to see, however, is that the upstream firm does not have an incentive to charge one of the retailers such a high price that it effectively forecloses this firm from actively selling in the market. The reason is that given that the equilibrium upstream price is larger than the vertically integrated monopoly price, the upstream firm benefits from downstream competition, and even though it is imperfect because of consumer search, it is better than not having any downstream competition.

In the rest of this subsection, we show that the upstream cannot locally increase profits by price discriminating between the two retailers who know each others’ cost. To this end, let us assume that the upstream firm deviates and charges \(w_1\) to firm 1 and \(w_2 (< w_1)\) to firm 2 and that \(p^m(w_i) > \rho\) \((i = 1, 2)\). The proof proceeds by showing that the upstream is always better off by charging both firms \(w_1\) instead of charging one retailer a lower price. Given that any asymmetric pricing strategy of the upstream firm is dominated by a symmetric one, it follows that charging both firms the \(w^*\) derived in the previous sections has to be locally optimal.

If \(w_1 > w_2\) and both retailers know each others’ cost, both downstream firms will randomize continuously over an interval \([b, \rho]\) and firm 1 will have a mass point at \(\rho\). For any price \(p \in [b, \rho]\) that firm 1 charges, the pricing strategy of firm 2, denoted by \(F_2\), should satisfy

\[
(p - w_1)D(p)(1 - \lambda + 2\lambda(1 - F_2(p))) = (1 - \lambda)(\rho - w_1)D(\rho)
\]

\(^{16}\)This is only one set of beliefs that work. There are many other beliefs the retailers may hold, and depending on these beliefs the upstream firm either has or does not have an incentive to deviate.
in order to make firm 1 indifferent between any of its prices in the support of the mixed strategy distribution. This gives

\[ F_2(p) = 1 - \left( \frac{1 - \lambda}{2\lambda} \left( \rho - w_1 \right) D(\bar{p}) - \left( p - w_1 \right) D(p) \right). \]

From (5), the lower limit \( b \) is implicitly defined by

\[ (1 + \lambda)(b - w_1) D(b) = (1 - \lambda)(\rho - w_1) D(\rho). \]  

(6)

Note that for a given \( \rho \) this implies that the lower bound of the price distribution in this asymmetric case where \( w_1 > w_2 \) is equal to the lower bound of the price distribution in case both firms are charged a wholesale price of \( w_1 \). Moreover, observe that \( F_2(p) \) coincides with the distribution that would have been used if both downstream firms were charged \( w_1 \).

For any price that firm 2 charges in the interval \([b, \rho)\), \( F_1 \) should satisfy

\[ (p - w_2) D(p) (1 - \lambda + 2\lambda(1 - F_1(p))) = (1 + \lambda)(b - w_2) D(b) = (1 - \lambda + 2\lambda \gamma_1)(\rho - w_2) D(\rho), \]

in order to make firm 2 indifferent between any of its prices in the support of the mixed strategy distribution. This yields

\[ F_1(p) = 1 - \frac{(1 - \lambda + 2\lambda \gamma_1)(\rho - w_2) D(\rho) - (1 - \lambda)(p - w_2) D(p)}{2\lambda(p - w_2) D(p)}. \]  

(7)

From the above, firm 1’s mass point at \( \rho \) is equal to

\[ \gamma_1 = 1 - F_1(\rho) = \frac{(1 - \lambda)(w_1 - w_2)(\rho - b)}{2\lambda(b - w_1)(\rho - w_2)}. \]  

(8)

Observe that

\[ F_1(p) < F_2(p) \iff (1 - \lambda + 2\lambda \gamma_1)(\rho - w_2)(p - w_1) > (1 - \lambda)(\rho - w_1)(p - w_2) \iff 2\lambda \gamma_1(\rho - w_2)(p - w_1) > (1 - \lambda)(\rho - p)(w_1 - w_2), \]

which, given the expression for \( \gamma_1 \), is certainly the case

\[ (1 - \lambda)(w_1 - w_2) \frac{\rho - b}{b - w_1}(p - w_1) > (1 - \lambda)(\rho - p)(w_1 - w_2) \iff (\rho - b)(p - w_1) > (\rho - p)(b - w_1) \iff (\rho - w_1)(p - b) > 0. \]
This means that $F_1$ first-order stochastically dominates $F_2$.

The above implies that the upstream firm always earns larger profit when charging both retailers $w_1$ than when charging one retailer $w_1$ and the other one a lower $w_2$. This is true because in the asymmetric case retail prices are higher (firm 2 behaves in the same way in both cases and firm 1 charges higher prices), and hence upstream demand is lower. Moreover, one of the firms (firm 2) gets, on average, a larger proportion of this smaller demand and pays less to the upstream firm. All these factors reduce the upstream firm’s profit.

So for the upstream firm any asymmetric pricing is dominated by some form of symmetric pricing and we have shown before that of all symmetric prices, charging $w^*$ to both retailers gives the highest profit. Thus, for the upstream firm, there is no profitable local deviation from $w_1 = w_2 = w^*$.

The above analysis remains true as long as $p^m(w_2) > \rho$. However, when the upstream firm sets $w_2$ such that $p^m(w_2) < \rho$, retailer 2 will not charge prices up to $\rho$ and the price distributions will be truncated at $p^m(w_2)$ with firm 2 having a mass point at $p^m(w_2)$ and firm 1 having a mass point at $\rho$. Thus, the above argument that the upstream firm’s demand is decreasing if one decreases $w_2$ from an initial level of $w_1$ is a local argument, and does not continue to hold if $w_2$ is decreased sufficiently. The fact that the retailers choose mixed strategies and that we do not have clean expressions for how upstream profits depend on $w$ (let alone how they depend on asymmetric choices of $w_1$ and $w_2$) makes it difficult to come up with a global argument. Given that it is not clear that this case of retailers knowing each others’ cost level is more interesting than the case where they do not, we have decided not to develop this point further.

### 5.3 Retail Oligopoly

Finally, we show that the qualitative properties of the equilibria under retail duopoly extend to the case where there are more than two retail firms. The effects shown by the numerical analyses we present here are easily interpreted. Roughly speaking, there are two effects. First, the region of parameters where the knowledge consumers have on the wholesale price makes a difference becomes smaller. This in quite intuitive if we recall the result by Stahl (1989) that the reservation price is increasing in the number of firms. In the context of our model, this implies that when the number of firms is larger the upper bound of the retail price distribution is given by the retail monopoly price for smaller values of $s$. As this is the region where the two models coincide, the region where there is a difference between the two information scenarios becomes smaller. Second, when the search cost is small, the triple marginalization effect becomes stronger. That is, the larger the number of
firms, the stronger is the effect that the equilibrium wholesale price is decreasing in $s$. The reason is that with a larger number of firms, the more probability mass the retail price distribution gives to higher prices and the smaller the effect of an increase in the wholesale price on the demand for the upstream firm.

![Figure 8: Upstream and weighted average downstream prices for the two models for $N = 3$.](image)

We show these effects in two different ways. For $N = 3$, we show that Figure 8 depicting the relationship between wholesale and expected retail price as a function of $s$ is similar to the corresponding figure for $N = 2$ (apart from the two differences noted above). Next, we also show the relationship between wholesale price and the number of firms when the search cost is small. As explained above, Figure 9 shows that this relationship is increasing.

### 6 Discussion and Conclusion

This paper has looked at the implications of vertical industry structure in markets where consumer search is important. In particular, we have looked at an industry structure where one upstream manufacturer sells its product to a downstream retail oligopoly. The retailers have homogeneous products and compete in prices. A fraction of consumers does not know the prices and has to incur a search cost to discover the next price. In this industry structure we have looked at the importance of consumers not observing the wholesale price. In the standard search literature following Stahl (1989) retailers’ cost is assumed to be exogenously given and known to consumers. There is also a literature, following Benabou and Gertner (1993) and Dana (1994), where retailers’ cost is exogenous, but unknown to consumers. In this
paper we focus on the difference between markets where consumers do and do not observe the wholesale price in the case where this price is endogenously determined by an upstream manufacturer.

We find that whether or not the wholesale price is observed by consumers has both important qualitative and quantitative effects on market outcomes. The most important qualitative difference concerns the implications of changes in the search costs. In the case where the wholesale price is observed expected retail prices are increasing in search cost, and upstream prices are non-monotonic. In contrast, in the case where the wholesale price is unobserved expected retail prices and upstream prices are decreasing in search cost. The main reason for this latter result is that the upstream firm internalizes the fact that retailers can charge higher margins in case search costs are higher, and this causes the upstream firm to reduce its price.

From a quantitative perspective we find that welfare can be around 20% larger in markets where the wholesale price is observed compared to when it is not observed. When this price is not observed, consumers’ reservation price is not affected by the actual choice of the wholesale price. This makes the demand, from the perspective of the upstream firm less elastic, so that it has an incentive to increase its price, squeezing the profits of the downstream firms. In equilibrium, the actual choice of the wholesale price has to be equal to the price that is expected by consumers, but this can only arise at very high levels of the wholesale price, much higher than the price a vertically integrated monopolist would set.
Overall, this paper draws attention to the importance of taking into account the incentives of upstream firms for the study of retail markets where consumer search is important. Given the findings in this paper, we expect that also other findings in the consumer search literature may get exacerbated once the upstream perspective is taken into account. In addition the paper opens up the question whether other topics in the study of vertical market structures (such as exclusive dealing, retail price maintenance, etc.) should be reconsidered for markets where consumer search is important. Finally, the paper redirects the theoretical literature on consumer search to consider non-reservation price equilibria.

References


Appendix: Proofs

Lemma 2. For every \( \lambda \in (0, 1) \), there exists a critical value \( \hat{s}^*(\lambda) \) such that for all \( s < \hat{s}^*(\lambda) \) \( \bar{p}(w) = \rho(w) \) and for all \( s > \hat{s}^*(\lambda) \) \( \bar{p}(w) = p^m(w) \).

Proof. It is clear that \( p^m(w) \) is independent of \( s \) and larger than \( w \). Stahl (1989, Proposition 7) shows that \( \rho(w) \) is increasing in \( s \). Also, it is clear that for every \( \lambda \in (0, 1) \) there is an \( s^m(\lambda) > 0 \) such that \( ECS(p^m) = s^m(\lambda) \). It then follows that \( \hat{s}^*(\lambda) = s^m(\lambda) \). \( \square \)

Proposition 3. Suppose \( D(p) = 1 - p \). For all \( \lambda \), if \( s > \hat{s}^*(\lambda) \) so that \( \bar{p}(w) = p^m(w) \), \( w^* = 0.5 \). Moreover, when \( s \) approaches 0, then \( w^* \) approaches 0.5.

Proof. To prove the first part, note that as

\[
1 - F(p|c) = -\frac{1 - \lambda}{2\lambda} + \frac{1 - \lambda (\bar{p}(w) - c) D(\bar{p}(w))}{(p - w) D(p)}
\]

upstream profits can be rewritten as

\[
\pi_u(w) = \left( 1 - \lambda \right) \int_{b(w)}^{\bar{p}(w)} f(p) \frac{(\bar{p}(w) - c) D(\bar{p}(w))}{(p - w)} dp \bigg) c
\]

\[
= \left( \frac{1 - \lambda}{32} \frac{(1 - w)^4}{(p - w)^3} \right) \int_{b(w)}^{\bar{p}(w)} \frac{1 + c - 2p}{(p - w)^3(1 - p)^2} dp \bigg) c
\]

\[
= \left( \frac{1 - \lambda}{32} \frac{(1 - w)^4}{(p - w)^3} \right) \left[ - \frac{(1 - w)^2}{2 (p - w)^2} + 2 \frac{(1 - w)}{p - c} - \frac{1 - w}{1 - p} - \ln(p - w)(1 - p) \right] b(w) \bigg) c.
\]

Now, suppose that \( \bar{p}(w) = p^m(w) = \frac{1 + c}{2} \) so that \( 1 - \bar{p}(w) = \bar{p}(w) - c = \frac{1 - w}{2} \). Moreover, in this case we can write

\[
b(w) = \frac{1 + c - (1 - w) \sqrt{2\lambda}}{2}
\]

so that \( 1 - b(w) = b(w) - w = \frac{1 - w}{2} \left( 1 - \sqrt{\frac{2\lambda}{1 + \lambda}} \right) \). We will argue that the term in square brackets is therefore only a function of \( \lambda \) and not of \( c \). To see this, note that for \( k = 1, 2 \)

\[
\frac{(1 - \frac{w}{2})^k}{(\bar{p}(w) - w)^k} - \frac{(1 - \frac{w}{2})^k}{(b(w) - w)^k} = \frac{(1 - \frac{w}{2})^k}{(1 - \bar{p}(w))^k} - \frac{(1 - \frac{w}{2})^k}{(1 - b(w))^k} = \frac{1 - \sqrt{\frac{2\lambda}{1 + \lambda}}}{2\lambda} - 1.
\]
Moreover,
\[
\frac{(\bar{p}(w) - w)(1 - \bar{p}(w))}{(b(w) - w)(1 - b(w))} = \left(\frac{1}{1 - \sqrt{\frac{2}{1+\lambda}}}\right)^2.
\]

Thus, all four terms in square brackets evaluated at \( p = \bar{p}(w) \) and \( p = b(w) \) as the upper and lower bound respectively are functions of \( \lambda \) only and can therefore be expressed as some complicated function \( g(\lambda) \). Upstream profits can thus be written as

\[
\pi_u(w) = \left(\frac{(1 - \lambda)^2(1 - w)}{32\lambda} g(\lambda)\right) w,
\]

for some function \( g(\lambda) \). It is easy to see that this expression has a maximum at \( w^* = 0.5 \).

As we know that \( \hat{s}^o(\lambda) \equiv \hat{s}^o(\lambda; 0.5) \), it follows that at \( s = \hat{s}^o(\lambda) \rho(0.5) = p^m(0.5) = 0.75 \) and from lemma (2.2) we know that for all \( s > \hat{s}^o(\lambda) \), \( \rho(0.5) > p^m(0.5) \) so that \( \bar{p}(w) = p^m(w) \). Above we have shown that if \( \bar{p}(w) = p^m(w) \), the optimal \( w^* = 0.5 \). Together this implies that \( w^* = 0.5 \) for all \( s > \hat{s}^o(\lambda) \).

Let us then consider the limit when \( s \) goes to 0. In this case \( \bar{p}(w) = \rho(w) \) and both \( \rho(w) \) and \( b(w) \) go to \( w \) as \( s \) goes to 0. Thus, the demand for both shoppers and non-shoppers equals \( 1 - w \) and the upstream firm maximizes \( w(1 - w) \), which is maximized at \( w^* = 0.5 \).

\[ \square \]

**Proposition 6.** In the case where consumers do not observe the upstream price \( w \) and demand is given by \( D(p) = 1 - p \), it is the case that for all \( \lambda \) there exists a \( \hat{s}^{no}(\lambda) \equiv \hat{s}^{no}(\lambda; w = 0.5) \) such that for all \( s > \hat{s}^{no}(\lambda) \) \( w^* = 0.5 \).

**Proof.** To prove the first part note that when \( \bar{p}(w) = p^m(w) \), the downstream market behaves in exactly the same way as in the case where consumers observe \( w \) and therefore we can use the proof of Proposition 3 to argue that in that case \( w^* = 0.5 \). As we know that \( \hat{s}^{no}(\lambda) \equiv \hat{s}^{no}(\lambda; 0.5) \), it follows that at \( s = \hat{s}^{no}(\lambda) \rho^* = p^m(0.5) = 0.75 \) and from Proposition 4 we know that for all \( s > \hat{s}^{no}(\lambda) \), \( \rho^* > p^m(0.5) \) so that \( \bar{p}(w) = p^m(w) \). We know that if \( \bar{p}(w) = p^m(w) \), the optimal \( w^* = 0.5 \). Together this implies that \( w^* = 0.5 \) for all \( s > \hat{s}^{no}(\lambda) \).

\[ \square \]

**Proposition 7.** In the case where consumers do not observe the upstream price \( w \) and demand is given by \( D(p) = 1 - p \), and a reservation price equilibrium exists when \( s \) approaches 0, then \( w^* \) approaches \( 1/(1 + \lambda) > 0.5 \).

**Proof.** Consider the limiting behavior of \( w^* \) when \( s \) approaches 0. As \( \hat{s}(\lambda) > 0 \), it will be the case that the upper bound \( \bar{p}(w) = \rho \) and thus independent of \( c \). The
search cost $s$ does not directly enter the profit function $\pi_u(w)$. For a fixed $w$, $b$ is a function of $w$ and $\rho$, and thus $\lim_{s \to 0} \rho = w$, we have

$$\lim_{s \to 0} \pi_u(w) = \lim_{\rho \to c} \pi_u(w).$$

A necessary condition for the optimal choice of $w^*$ is that it satisfies the first-order condition. Before deriving the first-order condition, we rewrite profits as

$$w(1 - \rho)^2(1 - \lambda)^2 \left[ \left( \frac{\rho - w}{b - w} \right)^2 - 1 + \frac{(\rho - w)^2}{(1 - \rho)^2(1 - \lambda)^2} \left( \frac{1 + \lambda}{1 - \lambda} \frac{1 - b}{1 - \rho} \right)^2 - 1 \right] - \frac{w(1 + \lambda)^2(1 - b)b'(w)}{\lambda(1 - w)}.$$

Using $(1 + \lambda)(1 - b)(b - w) = (1 - \lambda)(1 - \rho)(\rho - w)$ and opening the big bracket we get

$$\frac{w(1 - \rho)^2(1 - \lambda)^2}{2\lambda(1 - w)^2} \left( \left( \frac{1 + \lambda}{1 - \lambda} \frac{1 - b}{1 - \rho} \right)^2 - 1 \right) - \frac{w(1 + \lambda)^2(1 - b)b'(w)}{\lambda(1 - w)}.$$

Here recall that $\rho$ is not a function of $w$ when $w$ is unobserved. The derivative of the second term is zero in the limit because all of its parts include multiples of $(\rho - c)$.

This leaves the first term, the derivative of which is

$$\frac{(1 - \lambda)^2(1 - \rho)^2}{2\lambda(1 - w)^2} \left( \left( \frac{1 + \lambda}{1 - \lambda} \frac{1 - b}{1 - \rho} \right)^2 - 1 \right) - \frac{w(1 + \lambda)^2(1 - b)b'(w)}{\lambda(1 - w)}.$$

From $(1 + \lambda)(1 - b)(b - w) = (1 - \lambda)(1 - \rho)(\rho - w)$, we can derive $b'(w)$ by implicit differentiation:

$$(1 + \lambda)(1 + w - 2b)b'(w) - (1 + \lambda)(1 - b) = (1 - \lambda)(1 - \rho). \quad (10)$$

Using the fact that when $s$ approaches 0, $\rho$ and $b$ approach $w$ we have that $\lim_{\rho \to w} b'(w) = \frac{2\lambda}{1 + \lambda}$. Thus, the limit of the derivative of profits can be written as

$$\lim_{\rho \to w} \frac{\partial \pi_u(w)}{\partial w} = \lim_{\rho \to w} \left[ \frac{(1 - \lambda)^2(1 - \rho)^2}{2\lambda(1 - w)^2} \left( \left( \frac{1 + \lambda}{1 - \lambda} \frac{1 - b}{1 - \rho} \right)^2 - 1 \right) - \frac{w(1 + \lambda)^2(1 - b)b'(w)}{\lambda(1 - w)} \right]$$

$$= \frac{(1 - \lambda)^2}{2\lambda} \left( \left( \frac{1 + \lambda}{1 - \lambda} \right)^2 - 1 \right) - 2(1 + \lambda)w$$

$$= 2 - 2(1 + \lambda)w.$$
Equating the derivative to zero gives the equilibrium upstream price in the limit

\[ w^* = \frac{1}{1 + \lambda}. \]

\( \square \)

**Proposition 8.** There exists a critical value of \( \lambda \), denoted by \( \lambda^* \), such that for all \( \lambda < \lambda^* \) there exists a \( \tilde{s}(\lambda) \) such that for all \( s < \tilde{s}(\lambda) \) a reservation price equilibrium does not exist, where \( \lambda^* \approx 0.47 \) solves

\[
\frac{2(1 - \lambda(1 + \lambda))}{\lambda} - \frac{(1 - \lambda)^2}{\lambda^2} \log \left( \frac{1 + \lambda}{1 - \lambda} \right) = 0.
\]

**Proof.** The limit of the second-order condition with respect to \( w \) can be found in a similar way as the limit of the first-order condition of the proof of Proposition 6. Using expression (9), taking the second derivative and eliminating all the terms with zero limit we are left with

\[
\frac{2w(1 - \lambda)^2(1 - \rho)^2}{\lambda(1 - w)^2} \left[ \frac{1}{1 - b} - \frac{1}{1 - \rho} - \frac{\log \left( \frac{(1 - b)(1 + \lambda)}{(1 - \lambda)(1 - \rho)} \right)}{1 - w} \right] + \frac{(1 - \lambda)^2(1 - \rho)^2}{\lambda} \left( \frac{(1 + \lambda - b)(1 - \rho)^2}{1 - \lambda(1 - \rho)} - 1 \right) - \frac{2(1 + \lambda)^2(1 - b)b'(w)}{\lambda(1 - w)^2} + \frac{w(1 + \lambda)^2}{2\lambda(1 - w)} \left( 2b'(w)^2 - 2(1 - b)b''(w) \right)
\]

Using implicit differentiation on (10) we have

\[
(1 + w - 2b)b''(w) + 2b'(w) - 2(b'(w))^2 = 0.
\]

Using the fact that in the limit \( w^* = \frac{1}{1 + \lambda} \) and \( b'(w) = \frac{2\lambda}{1 + \lambda} \) one arrives at \( b''(w) = -4 \left( 1 - \frac{2}{1 + \lambda} \right) \). Thus, the limit of the second derivative of profits evaluated at the equilibrium upstream price gives

\[
-\frac{2(1 - \lambda)^2}{\lambda^2} \log \left( \frac{1 + \lambda}{1 - \lambda} \right) + \frac{(1 - \lambda)^2}{\lambda} \frac{1 + \lambda}{(1 - \lambda)^2} - \frac{4(1 + \lambda)^2}{\lambda} + 4 + \frac{4(1 - \lambda)}{\lambda}.
\]

So,

\[
\lim_{\rho \to c} \frac{\partial^2 \pi_u(w)}{\partial w^2} \bigg|_{c=c^*} = 2 \left( \frac{2(1 - \lambda(1 + \lambda))}{\lambda} - \frac{(1 - \lambda)^2}{\lambda^2} \log \frac{1 + \lambda}{1 - \lambda} \right).
\]

By inquiring when

\[
\frac{2\lambda(1 - \lambda - \lambda^2)}{(1 - \lambda)^2} > \log \frac{1 + \lambda}{1 - \lambda}
\]

it can be verified when the limit of the second order condition is negative. As (i) at \( \lambda = 0 \), the LHS = RHS, as (ii) the derivative of the LHS is first (for small values
of \( \lambda \) larger than the derivative of the RHS and then smaller and (iii) at \( \lambda = 1 \), the LHS < RHS, there exist a critical value of \( \lambda \), denoted by \( \lambda^* \) such that for \( \lambda < \lambda^* \) the limit of the second-order condition is positive, while it is negative for \( \lambda > \lambda^* \). Thus, for small values of \( \lambda \) there exists values of \( s \) small enough such that a necessary condition for an equilibrium to exist (namely that the upstream firm chooses an optimal price and that the SOC is fulfilled) is not satisfied. Numerically, one can verify that \( \lambda^* \approx 0.475 \). 

Lemma 9. The critical boundaries \( \hat{s}^o(\lambda) \) and \( \hat{s}^{no}(\lambda) \) are equal: \( \hat{s}^o(\lambda) = \hat{s}^{no}(\lambda) \).

Proof. Fix \( s \) and \( \lambda \). For any \( w \) it should be the case that either \( \rho(w) > p^{m}(w) \), or \( \rho(w) < p^{m}(w) \), or \( \rho(w) = p^{m}(w) \). Moreover, in any equilibrium when consumers do not observe \( w \), the consumers anticipate \( w \) correctly. Therefore, if one of these three cases occurs whenever consumers do not observe \( w \), the same case should occur when consumer observe \( c \). Thus, \( \hat{s}^o(\lambda) = \hat{s}^{no}(\lambda) \). □

Before we prove Proposition 9, it is useful to prove the following two Lemmas.

Lemma A1. If for all \( p \in (0, p^{m}) \), \( \pi''_r(p) \pi_r(p) - (\pi'_r(p))^2 < 0 \), then \( \rho'(w) > 0 \) in the case where consumers observe \( w \).

Proof. Note that as \( ECS(\rho(w),c) = s \) so that

\[
\frac{\partial ECS(\rho, w)}{\partial \rho} \frac{d\rho}{dw} + \frac{\partial ECS(\rho, w)}{\partial w} = 0.
\]

As expected consumer surplus decreases with \( w \) when \( \rho \) is held constant, \( \rho'(w) > 0 \) immediately follows if we can show that \( \frac{\partial ECS(\rho, w)}{\partial \rho} > 0 \). As

\[
\frac{\partial F(p)}{\partial \rho} = 1 - \frac{\lambda}{2\lambda} \left( \frac{D(\rho) + (\rho - w)D'(\rho)}{(p - w)D(p)} \right) = 1 - \frac{\lambda \pi'(\rho)}{2\lambda \pi(p)}
\]

and

\[
\frac{\partial F(p)}{\partial p} = 1 - \frac{\lambda}{2\lambda} \left( \frac{(\rho - c)D(\rho)}{(p - w)^2D^2(p)} (D(p) + (p - w)D'(p)) \right) = 1 - \frac{\lambda \pi(\rho)\pi'(\rho)}{2\lambda \pi^2(p)}
\]

we can write

\[
\frac{\partial ECS(\rho)}{\partial \rho} = \int_{b}^{\rho} D(p) \frac{F(p)dp}{\partial \rho} = D(\rho)F(\rho) - D(b)F(b)\frac{\partial b}{\partial \rho} - \int_{b}^{\rho} D(p)\frac{\partial F(p)}{\partial \rho}dp
\]

\[= D(\rho) - \int_{b}^{\rho} D(p)\frac{\pi'(\rho)/\pi(\rho)}{\pi'(p)/\pi(p)} \frac{\partial F(p)}{\partial p}dp.\]
Given that $\rho < p^m$ and the assumption that $\frac{\pi'(p)/\pi(p)}{\pi'(\rho)/\pi(\rho)} < 1$ for all $p < p^m$ it follows that
\[
\frac{\partial E\text{CS}(\rho)}{\partial \rho} > D(\rho) - \left[1 - \int_b^\rho \frac{\partial F(p)}{\partial p} dp\right] = 0.
\]

Lemma A2. For a given $w$, the upstream profit is decreasing in $\rho$ for $p^m(w) > \rho \geq w$.

Proof. Recall
\[
F(p|c) = 1 - \left(\frac{1 - \lambda (\rho - w)D(\rho) - (p - w)D(p)}{2\lambda (p - w)D(p)}\right),
\]
and take the derivative of $F$ with respect to $\rho$. This gives
\[
\frac{(1 - \lambda)(D(\rho) + (\rho - w)D'(\rho))}{2(p - w)\lambda D(p)}.
\]
This expression is negative because given $p^m(w) > \rho$, $D(\rho) + (\rho - w)D'(\rho) > 0$. So for a larger $\rho$ the distribution of retail prices first order stochastically dominates the one for a smaller $\rho$. Given that the downstream demand is downward-sloping, upstream profit decreases in $\rho$.

Proposition 10. If (i) $s$ and $\lambda$ are such that a reservation price equilibrium exists in the case where retailers’ cost is unobserved, (ii) the upper bound of the price distribution in both the observed and the unobserved case is given by the reservation price and (iii) for all $p \in (0,p^m)$, $\pi''_r(p)\pi_r(p) - (\pi'_r(p))^2 < 0$, where $\pi_r(p) = (p - w)D(p)$, then the optimal upstream price in the case where retailers’ cost is unobserved is larger than the optimal upstream price in the case where retailers’ cost is observed.

Proof. Denote the optimal price of the upstream firm in case retailers’ cost is observed by $w^o$ and in case retailers’ cost is not observed by $w^{no}$. As in any equilibrium of the unobserved case $w^{no}$ is correctly anticipated by the consumers, the equilibrium reservation price $\rho = \rho(w^{no})$. Importantly, this reservation price does not depend on $w^{no}$ out of equilibrium.

For a given $w$ and a corresponding reservation price $\rho$, the derivative of upstream profits is smaller in absolute value in the unobserved case than in the observed case. This follows from Lemmas A.1 and A.2 because the former states that when $w$ increases $\rho(w)$ also increases, while the latter states that when $\rho$ increases the upstream profit falls. For the same increase in $w$, in the observed case profit will fall more because also $\rho(w)$ increases, whereas in the unobserved case $\rho$ is held constant at $\rho(w)$. 

37
The above means that \( w^{no} = w^o \) cannot be the case because the derivative of expected profit with respect to \( w \) is larger for the unobserved case than for the observed case, so if the latter is zero the former is positive. Thus if \( w^o \) maximizes upstream profits, \( w^{no} \) cannot maximize profits for a the reservation price fixed at \( \rho(w^{no}) \).

Now let us consider \( w^{no} < w^o \). From \( w^{no} < w^o \) we have \( \rho(w^{no}) < \rho(w^o) \) by Lemma A.1, and because the upstream profit is decreasing in \( \rho \) by Lemma A.2, we get

\[
\pi_u(w^o, \rho(w^o)) < \pi_u(w^o, \rho(w^{no})).
\]

(The last expression holds even if \( w^o > \rho(w^{no}) \). In such case the downstream prices are deterministic and equal to \( w^o \). So \( \pi_u(w^o, \rho(w^{no})) = w^oD(w^o) \) which is strictly larger than \( \pi_u(w^o, \rho(w^o)) \) because \( \rho(w^o) > w^o \).

By definition of the observed equilibrium \( \pi_u(w^o, \rho(w^o)) > \pi_u(w^{no}, \rho(w^{no})) \), so with \( \pi_u(w^o, \rho(w^o)) < \pi_u(w^o, \rho(w^{no})) \) we get

\[
\pi_u(w^{no}, \rho(w^{no})) < \pi_u(w^o, \rho(w^{no})).
\]

The latter implies that for unobserved \( w \), the upstream firm has higher profit when charging \( w^o \) than when charging \( w^{no} \), thus \( w^{no} < w^o \) cannot hold.