

Competition, Disclosure and Signaling.

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Abstract

Market competition creates strategic incentives for firms to communicate private information about their own product quality through signaling rather than voluntary disclosure. This provides a new explanation of infrequent voluntary disclosure of quality attributes in many markets. In a duopoly where firms may disclose quality before setting prices that may also signal quality, we show that for a large section of the parameter space, non-disclosure by *all* firms is the unique symmetric outcome *even if disclosure cost vanishes*. A high quality firm may not disclose even if it has strong competitive advantage over a low quality rival. In the non-disclosure outcome, signaling conveys all private information to consumers; however, distortions in prices and consumption necessary to meet the incentive constraints for signaling may sometimes provide a rationale for mandatory disclosure regulation.

Keywords: Voluntary Disclosure, Signaling, Private Information, Quality, Oligopoly, Price Competition.

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1 Introduction

In a large number of markets ranging from educational and health services to consumer goods and financial assets, sellers have important information about quality attributes¹ of their own products that are not publicly observable by potential buyers or their competitors. In many of these markets, firms have the option of voluntarily disclosing this information in a credible and verifiable manner through a variety of means such as independent third party certification², labeling, rating by industry associations (or, government agencies) and through informative advertising.

It would appear, as some economists have argued,³ that a firm whose product is actually better than the average ought to have a positive incentive to voluntarily disclose the quality of its product to buyers. This should then induce every firm whose quality is above the average *undisclosed* quality to also disclose, and through an unraveling argument, all private information about quality should be revealed through voluntary disclosure. If disclosure is costly, this argument suggests that firms whose product qualities are above a certain threshold ought to disclose.

In practice, however, voluntary disclosure often does not occur even when credible low cost mechanisms for facilitating such disclosure exist. Various studies⁴ report evidence that in the absence of statutory disclosure requirement, hospitals did not disclose risk adjusted mortality, schools did not report standardized test scores, and about half of Health Maintenance Organizations (HMOs) did not disclose quality through the National Committee on Quality Assurance. Even for a relatively simple product category like salad dressings, some producers did not label the fat content of their products even when their fat content was lower than that of many others. Evidence of relatively low voluntary disclosure activity is also found in the striking lack of product information in many advertisements even though the threat of penalty for false advertising (such as that under current FTC regulation) could make such information credible.⁵

The absence of affordable mechanisms to credibly disclose quality (such as third party certifi-

¹Quality attributes include satisfaction from consuming the product, durability, safety, potential health hazards and environmental damage that buyers care about.

²In some cases, certification or quality rating is provided or sold by an independent third party in response to demand for information by consumers. In many of these situations, the quality rating agency needs to obtain data directly from sellers (such as the Leapfrog Group's hospital quality ratings) and therefore, requires seller participation, which effectively requires the seller to voluntarily disclose quality.

³See, Viscusi (1978), Grossman (1981), Milgrom (1981), Jovanovic (1982).

⁴See, the excellent recent survey by Dranove and Jin (2010). See also, Jin (2005) and Mathios (2000).

⁵See, for instance, Abernethy and Butler, 1992 and Abernethy and Franke, 1996.

cation) in many markets may be partially attributed to inadequate demand for such mechanisms and therefore, to the unwillingness of firms to disclose voluntarily. Indirect evidence of inadequate voluntary disclosure is also provided by the fact that public authorities feel the need to impose mandatory disclosure laws, and that market outcomes do change significantly after imposition of mandatory disclosure laws.⁶

Existing explanations of observed non-disclosure are *all* based on the presumption that voluntary disclosure is the *only* channel available to a firm for communication of private information about its product quality; it is assumed that when disclosure does not occur, there is no way for consumers to derive any information about product quality (except possibly for information revealed by the fact that a firm does not disclose). While such an assumption may be relevant in certain contexts, in a large class of markets consumers can *infer* the private information of firms from their observable market behavior such as pricing, advertising, warranties etc. In other words, firms may *signal* their product quality to consumers. In such markets, non-disclosure is not necessarily equivalent to non-revelation, and whether or not a firm discloses voluntarily should be seen as a *choice between alternative channels of communication*.

In this paper, we provide a new approach to understanding observed non-disclosure that is based on the availability of signaling as an alternative means of communicating private information. We suggest that a firm may often choose to not disclose its product quality, not because it wishes to prevent consumers from knowing its true product quality, but because it prefers to use signaling as the channel of communication of private information to consumers. More specifically, our explanation of non-disclosure is based on the possibility of signaling *and* market competition. We argue that competition is the important reason why firms prefer to use signaling rather than disclosure as the channel of communication.

We consider a duopoly where firms engage in price competition. Prior to price setting, firms decide whether or not to voluntarily disclose private information about their own quality. Disclosure perfectly communicates true product quality to *all* buyers. When a firm does not disclose, its price may signal product quality to consumers. We show that for a large subset of the parameter space, firms do not disclose their product quality even if their product is of the highest quality and *even*

⁶See, Zarkin et al (1993), Mathios (2000), Jin and Leslie (2003) and Dranove and Jin (2010).

if disclosure is not costly; instead, they signal quality through prices.

Daughety and Reinganum (2008b) were the first to systematically study the choice between voluntary disclosure and signaling of private information about product quality.⁷ Their analysis is in the framework of a monopoly where the marginal cost of production is increasing in quality. They show that signaling involves distortion of full information profits and that this distortion is higher for better quality; therefore, the seller ought to disclose voluntarily if his product quality is above a threshold (as long as the cost of disclosure is not too high);⁸ almost all quality types disclose as the disclosure cost vanishes so that non-disclosure can only be explained through disclosure frictions. Our paper shows that market competition changes the above outcome very significantly.

Specifically, we analyze a simple model where the products of the two firms differ only in quality; there is no other form of product differentiation. Product quality may be one of two types: *high* or *low*. Consumers are identical and have unit demand with valuations depending on quality. Firms have *pure* private information about their product quality i.e., a firm's product quality is unknown to its rival as well as all buyers. Note that firms may not be aware of the true product quality of other firms because of protection of trade secrets, zealous safeguarding of results of product quality tests, confidential settlement of consumer complaints and many other factors. We view quality disclosure as a relatively long term decision that takes place prior to (short run) price competition. Firms, knowing their own product quality, simultaneously decide whether or not to disclose. After observing the outcome of voluntary disclosure, firms simultaneously choose prices. If quality is not disclosed, consumers may make inferences about the product quality on the basis of the price they observe.

Non-disclosure implies that the rival firm remains uncertain about the true quality of the firm at the pricing stage, and this modifies the nature of price competition. In particular, when a firm's quality is not revealed through disclosure, it must think about how pricing affects consumers' beliefs about its product quality. Reducing price to undercut a rival may induce consumers to believe that

⁷Bernhardt and Leblanc (1995) also touch on this tradeoff between disclosure and signaling in a somewhat different context. Fishman and Hagerty (2003) incorporate both disclosure and signaling but do not model them as substitutes; signaling occurs "along with" rather than "instead of" disclosure.

⁸Daughety and Reinganum (2008c) study a version of the model with two quality types where high quality may be supplied at *lower* unit cost (because of lower expected liability). While results differ, the primary incentive to disclose is also based on the signaling distortion of monopoly profit.

its product is of lower quality, and therefore not attract more buyers.⁹ These consumer beliefs discipline price competition and, in an indirect sense, non-disclosure is a way for the high quality firm to credibly precommit to not undercut. This is particularly profitable for a firm whose rival has not disclosed. In that case, non-disclosure leads to a pricing game of incomplete information where the market outcome is one where firms signal quality through prices and low quality types earn sufficient rent so as to not imitate high quality firms. In fact, the signaling outcome can often be one where both low and high quality firms charge high prices; this, in turn, creates strategic incentives for non-disclosure. While in the monopoly case, signaling distortion reduces market profits of firms relative to the full information (or full disclosure) case,¹⁰ under market competition it increases profits by softening price competition.

When the quality premium that buyers are willing to pay is low (relative to the cost difference between low and high quality), i.e., the low quality product generates higher social surplus than the high quality product, the unique symmetric equilibrium is one where neither firm discloses its product quality. In particular, a high quality firm has little to gain from disclosure as it has a competitive disadvantage in the market when its rival has low quality product, while disclosing can only intensify competition when the rival produces high quality.

Surprisingly, even when the high quality firm generates more surplus and has a competitive advantage over any low quality rival, full non-disclosure may occur for a large section of the parameter space. Indeed, under certain conditions, this happens even when disclosure cost is close to zero, and even if the high quality product is supplied at lower marginal cost (for instance, due to lower expected liability damage payments) than the low quality product – a situation of very strong competitive advantage for the high quality firm. On one hand, disclosure allows the high quality firm to exploit its competitive advantage in the market by pricing more aggressively without triggering adverse beliefs about its product quality among buyers. On the other hand, disclosing high quality makes the rival more aggressive in pricing, and eliminate the strategic gain of softer competition when firms are bound by the beliefs of buyers. The latter (strategic) effect may dominate, particularly when the rival does not disclose, and lead to non-disclosure by both quality

⁹We show that these beliefs are not arbitrary and, in fact, meet the D1 criterion, a rather strong refinement criterion (that essentially requires that beliefs should be based on consideration of which type of a firm has higher incentive to charge any particular out of equilibrium price).

¹⁰See, for instance, Bagwell and Riordan (1991, 1992).

types. For low values of the disclosure cost, an equilibrium with full disclosure may coexist with full non-disclosure; even then, the non-disclosure equilibrium may be Pareto dominant in terms of expected profits of both types.

The fact that we obtain *full* non-disclosure even when there is no friction in the disclosure process is significant for three reasons. First, it explains why no disclosure may be observed in an industry even in the presence of credible, effective and fairly low cost ways of disclosing product information (such as labeling, increasing the product information content of existing advertisements that are often uninformative, certification by industry associations etc.). Second, it explains why even the highest quality firms may not disclose. Finally, our explanation indicates that improvements in the disclosure mechanism that reduce the cost of voluntary disclosure, or reduce other imperfections in the communication process following disclosure (such as better dissemination of information to consumers), need not necessarily increase the likelihood of actual disclosure by firms. This is of relevance to the design of public policy.

We also examine the effect of mandatory disclosure regulation. We show that mandatory disclosure regulation may improve welfare for two reasons. First, mandatory disclosure leads to fiercer competition compared to the nondisclosure equilibrium and may create higher sales. A second reason pertains to the case when the quality premium is relatively high and the market outcome is one with non-disclosure. In that case, non-disclosure leads to a signaling outcome where (in order to generate sufficient rent to dissuade the low quality firm from imitating the high quality price) the low quality firm enjoys a high market share even when the other firm is of high quality and generates higher social surplus. Mandatory disclosure corrects for this misallocation and increases the market share of high quality firms. This, in turn, provides a different explanation of why consumers in some industries increase their purchase from high quality firms (vertical sorting) after the imposition of mandatory disclosure regulation.¹¹ Note that here mandatory disclosure regulation is beneficial not because it increases the information available to consumers (all private information is still revealed through signaling in the absence of such regulation); rather, the benefits are related to the change in the price structure of the market. We also show that mandatory disclosure may reduce profits of both low and high quality types and this provides an explanation

¹¹See, for instance, Ippolito and Mathios (1990).

of why many industries lobby *against* mandatory disclosure regulation.¹²

Related Literature. The fact that signaling of private quality information under incomplete information leads to softer competition and higher profits than under full information (or equivalently, full disclosure) has been noted in the literature (Daughety and Reinganum, 2007, 2008a, Janssen and Roy, 2010). In particular, Daughety and Reinganum (2007) show that if competing firms can precommit to disclose or not disclose *before knowing their own product qualities* (i.e., before acquiring private information), they may choose to commit to non-disclosure. In this paper, we assume that firms cannot credibly pre-commit to disclosure or non-disclosure before knowing their own product qualities; this is a more natural assumption for many markets.

As mentioned earlier, existing explanations of non-disclosure in the literature on voluntary disclosure ignore the role of signaling as an alternative to disclosure. In much of this literature, non-disclosure is generated through imperfections or frictions in the voluntary disclosure mechanism; as these frictions disappear, these models generate full revelation through disclosure. Examples of such frictions include high cost of disclosure (Grossman and Hart 1980, Jovanovic 1982), insufficient ability of consumers to understand the information disclosed and to make sophisticated inference (Fishman and Hagerty, 2003), and unawareness of consumers about disclosures made by firms (Dye and Sridhar, 1995). Levin, Peck and Ye (2009) argue that market competition may reinforce the role of disclosure cost in generating non-disclosure by reducing the ability of a firm to appropriate the potential value added by disclosure; however, as disclosure cost becomes negligible, their model too generates full disclosure¹³.

A smaller, more recent literature has shown the possibility of *partial* non-disclosure even where there is no disclosure cost or other friction in the disclosure process. In a duopoly where *firms know each other's product qualities* (but these are unknown to consumers), Board (2009) shows that the higher quality firm always discloses, but its rival may not disclose in order to be perceived as having a product of "average" quality (conditional on being below the disclosed quality of its rival) so as to increase the extent of vertical product differentiation. Hotz and Xiao (2011) generate a similar result in a variation of this model that allows for horizontal product differentiation. In

¹²See, for instance, references cited in Hotz and Xiao (2011).

¹³In a similar vein, Cheong and Kim (2004) argue that if the number of firms is large, competition makes the incentive for costly disclosure small.

a monopoly where the seller has private information about both horizontal and vertical attributes and is assumed to either disclose both attributes or none at all, Sun (2011) shows that seller types with unfavorable horizontal attributes may not want to disclose quality even if it is high (see also, Celik 2011).

In contrast to this literature, we view the true opportunity cost of disclosure as being endogenously generated through the signaling outcome. Further, we demonstrate the possibility of *full* non-disclosure even when the (direct) cost of disclosure vanishes and there are no other imperfections in the disclosure mechanism. Note that full non-disclosure implies that no firm discloses, not even those with the highest quality product.

Caldieraro, Shin and Stivers (2008) study signaling and disclosure in a duopoly where it is common knowledge that one firm's product is of low and the other's product of high quality. Firms know each others' product qualities (no private information) but consumers do not know which of the two products is of high quality. Their model generates full disclosure unless the cost of doing so is too high. In contrast, our model deals with a situation where firms have (pure) private information, a firm can only disclose its own quality, and our results are closer to the evidence on non-disclosure.

Our paper is also related to a recent theoretical literature on the informational content of advertisements (see, among others, Anderson and Renault 2006, Mayzlin and Shin 2010). Revealing product information through advertising is akin to voluntary disclosure, while the value of non-informative advertising is likely to be related to signaling. Our results indicate that there may be an alternative explanation of the lack of informational content in advertisements.

The rest of the paper is organized as follows. The next section presents the model. Sections 3 - 5 present the main results on the equilibrium disclosure outcome for various cases that depend on whether or not low quality generates more surplus than high quality, and whether or not low quality is supplied at lower cost. Section 6 contains a discussion of the welfare implications of mandatory disclosure regulation. Section 7 concludes.

2 The Model

There are two firms, $i = 1, 2$, in the market. Each firm's product may be of either high (H) or low (L) quality. The true product quality is known only to the firm that supplies the product; it is not known to the rival firm or to the consumers. It is, however, common knowledge that the *ex ante* probability that a firm's product is of high quality is $\alpha \in (0, 1)$. The products of the firms are not differentiated in any dimension other than quality. Firms produce at constant unit cost and the unit cost $c_s \geq 0$ of a firm depends only on its true quality s , $s = H, L$. The unit cost subsumes both current production cost (including the cost of compliance with any form of prevalent regulation) as well as the expected future costs related to current sale of product (such as those arising through liability, damages, legal and other costs associated with settlement of disputes and complaints).

There is a unit mass of identical consumers in the market; consumers have unit demand and each consumer's valuation of a unit of quality s is given by V_s , $s = H, L$, where

$$V_H > V_L, V_s > c_s, s = L, H.$$

We will consider two kinds of cost regimes. In the *regular cost configuration*, higher quality is produced at higher unit cost, *i.e.*,

$$c_L < c_H. \tag{1}$$

In contrast, under *cost reversal*, the effective cost of supplying the low quality product is higher than that of supplying the high quality product, *i.e.*,

$$c_L \geq c_H. \tag{2}$$

Cost reversal may, in particular, arise when the expected liability damage payment associated with low quality is large enough.

Within the regular cost configuration (where (1) holds), we have two scenarios. First, the scenario where the quality premium $V_H - V_L$ that buyers are willing to pay for the high quality

product is lower than the cost difference:

$$V_H - V_L \leq c_H - c_L. \quad (3)$$

We will refer to this as the *low quality premium* scenario. In this case, $V_H - c_H \leq V_L - c_L$ so that the production and consumption of the low quality good creates more social surplus than that of the high quality good. If the inequality in (3) holds strictly, then in price competition under complete information, the low quality producer has a competitive advantage over a high quality rival, reducing the latter's market share to zero. The second scenario is one where the quality premium $V_H - V_L$ that buyers are willing to pay for the high quality product exceeds the cost difference:

$$V_H - V_L > c_H - c_L. \quad (4)$$

We will refer to this as the *high quality premium* scenario. In this case, the high quality good creates more social surplus than the low quality good; in price competition under complete information, the high quality producer has a competitive advantage over a low quality rival, reducing the latter's market share to zero.

Formally, the game proceeds in four stages. First, nature independently draws the type (or quality) τ_i of each firm i from a distribution that assigns probabilities α and $1 - \alpha$ to H and L , respectively; the realization of τ_i is observed only by firm i . Next, both firms (having observed their own types), simultaneously decide whether or not to *voluntarily disclose* their type publicly by incurring a cost $d > 0$. Disclosure is assumed to be credible and verifiable. Apart from disclosure cost, there is no other imperfection in the disclosure mechanism and the information conveyed is communicated perfectly to all consumers in the market. After observing the information disclosed in the second stage, in the third stage, firms choose their prices simultaneously. Finally, (after observing the information disclosed voluntarily and the prices) consumers decide whether to buy and if so, from which firm. The payoff of each firm is its expected profit net of disclosure cost (if any). The payoff of each consumer is her expected net surplus.

The solution concept used is that of perfect Bayesian equilibrium (PBE) where the out-of-

equilibrium beliefs satisfy the D1 criterion (Cho and Kreps, 1987) in every subgame.¹⁴

Before moving on to analysis of the model, it is useful to briefly discuss the incentive to disclose product quality in the *monopoly version* of our model. It is easy to show that if the firm does not disclose, then the high quality monopolist's profit is always (strictly) lower than his optimal profit under full information¹⁵. Therefore, as long as the disclosure cost d is below a threshold, the firm chooses to voluntarily disclose when its product is of high quality and all information is revealed through disclosure. The monopoly benchmark of our model confirms the results obtained by Daughety and Reinganum (2008b) and indicates that the non-disclosure outcome in our model arises as a consequence of both market competition *and* the availability of signaling as an alternative means of communicating quality.

3 Low Quality Premium

In this section, we analyze the incentives of firms to voluntarily disclose information about their product quality when the quality premium that buyers are willing to pay is relatively low:

$$V_H - V_L \leq c_H - c_L.$$

Observe that in this case $c_H > c_L$ (regular cost configuration holds), and that

$$V_H - c_H \leq V_L - c_L. \tag{5}$$

¹⁴This implies that following every possible disclosure outcome in the second stage, we consider the D1 equilibrium of the pricing subgame. The D1 refinement essentially requires that after observing an out of equilibrium price set by a particular firm, buyers should speculate which type of the firm has greater incentive to deviate to this price (given the equilibrium strategy of the rival firm). To make this comparison of incentives, one can think about the the smallest level of (expected) quantity that must be sold at that price to make the deviation gainful for one type, and compare it that to that for the other type.

¹⁵The argument is as follows. The full information profit of the high quality monopolist is $(V_H - c_H)$. To obtain the same level of expected profit under incomplete information about quality, the high quality monopolist must charge the price V_H with probability one *and* sell to all consumers at that price. The latter requires that buyers' equilibrium beliefs associate the price V_H with only high quality and therefore, the low quality type must not be charging the price V_H . Thus, the low quality seller's price must fully reveal his quality and therefore, in order to be able to sell, the low quality seller's price $\leq V_L < V_H$ and its profit is less than $(V_H - c_L)$. The low quality seller would then always gain by deviating from such an equilibrium by charging V_H and selling to all consumers.

i.e., the low quality product creates more surplus and therefore has a competitive advantage over the high quality product. We argue that in this scenario, the only symmetric equilibrium is one where neither firm discloses its product quality voluntarily (whatever its true product quality). Further, this holds no matter how small the cost of voluntary disclosure. As no information is communicated through disclosure, firms compete in prices under incomplete information about each other's product qualities, and signal their product qualities to consumers through prices.

We begin with the observation that *if a high quality firm discloses its product quality, then it earns negative net profit independent of the disclosure policy of its rival, and no matter how small the disclosure cost.* This reflects the competitive disadvantage of the high quality firm; if it discloses its quality, it is fully expropriated by the rival firm when the latter is of the low quality type. As a result, a disclosing high quality firm can only make money in the state where the rival is of high quality type. This, however, leads to intense price competition in the state where both firms are of high quality, thus eliminating all profits. This is easy to see if the rival high quality firm has also revealed its type fully through disclosure, but it can also be shown to hold if the rival's type is not fully revealed. Thus, net of disclosure cost, the disclosing high quality firm earns negative expected profit. The formal argument is contained in the appendix (see, proof of Proposition 1). It follows that *in equilibrium, a high quality firm will never choose to disclose with positive probability.*¹⁶

Next, we consider the possibility of a symmetric equilibrium where each firm discloses when its product is of low quality, but not when its product is of high quality. In such an equilibrium, the types of both firms are fully revealed after the disclosure stage, so that firms engage in Bertrand-like price competition under complete information. Using (5), it is easy to check that in this equilibrium, a high quality firm obtains zero profit independent of the type of its rival, while a low quality firm earns strictly positive profit only in the state where its rival has high quality product and its expected net profit is

$$\alpha(c_H - (V_H - V_L) - c_L) - d. \tag{6}$$

Now, suppose that the low quality type of firm deviates from this proposed equilibrium, and does

¹⁶If $d = 0$, there is always a symmetric equilibrium where a firm discloses if, and only if, its product quality is high. The disclosing high quality firm earns zero profit in equilibrium. By deviating to non-disclosure, it still earns zero profit (as the rival firm fully reveals its type at the disclosure stage). This equilibrium outcome is clearly not robust to positive disclosure cost.

not disclose. Then, firm 2, when it sets its price, will believe that firm 1 is of high quality (as only high quality firms do not disclose in this equilibrium). Firm 2, which reveals its type fully through disclosure in this equilibrium, will therefore set a price equal to c_H if it is of high quality, and a price equal to $c_H - (V_H - V_L)$ if it is of low quality. Firm 1, whose true product quality is low, can now set a price just below $c_H - (V_H - V_L)$ and sell to the entire market whatever be the type of the rival, earning an expected profit close to $(c_H - (V_H - V_L) - c_L)$, which is clearly higher than its equilibrium payoff (6). Therefore, the deviation is gainful, and there cannot be a symmetric equilibrium where low quality firms disclose for sure.

The essential argument here is that by not disclosing its quality, the low-quality firm is able to change the rival firm's perceived likelihood of his being a high quality type. As the high quality type is less competitive, this makes the rival less aggressive in price competition. This, in turn, allows the non-disclosing low quality firm to increase its profit by aggressively undercutting the rival at the price competition stage. Note that as the low quality firm is not interested in communicating its type to buyers, the only advantage of disclosure for a low quality firm would have to come from the way in which it modifies price competition; however, as the low quality type is associated with lower marginal cost and is more competitive, disclosure of its type can only make its rival more aggressive in price competition. Of course, non-disclosure also saves the firm its disclosure cost. This general reasoning also rules out a symmetric equilibrium where low quality firms disclose with positive probability; the details of this argument are contained in the appendix (see, proof of Proposition 1).

The above discussion can be summarized as follows: *there does not exist a symmetric equilibrium where a firm discloses with positive probability and, therefore, the only possible symmetric equilibrium outcome is full non-disclosure.* We now show that a full non-disclosure outcome is indeed an equilibrium.

We begin by describing this equilibrium. In this equilibrium, both low and high quality types choose to not disclose their product quality; as a result, no additional information about the type of any firm is revealed in the disclosure stage of the game. On the equilibrium path, the price setting game is an incomplete information game where the prior beliefs assign probabilities α and $1 - \alpha$ to high and low quality types for each firm. This is exactly the pricing game analyzed in Janssen

and Roy (2010). They show that there is a unique symmetric D1 equilibrium in the pricing game of the following kind: each firm charges a certain price $p_H \in [c_H, V_H]$ when its product is of high quality, and randomizes with a continuous probability distribution over an interval $[\underline{p}_L, \bar{p}_L]$ when its product is of low quality, where

$$c_L < \underline{p}_L < \bar{p}_L < p_H.$$

Consumers are indifferent between buying from the low quality firm at price \bar{p}_L and from a high quality firm at price p_H i.e.,

$$\bar{p}_L = p_H - (V_H - V_L).$$

In this equilibrium, consumers always buy from a low quality firm unless both firms are of high quality. At price \bar{p}_L , a low quality firm can sell only in the state where its rival is of high quality (it is undercut for sure when its rival is of low quality) and therefore, the low quality firm's expected equilibrium profit is given by

$$\alpha(p_H - (V_H - V_L) - c_L). \tag{7}$$

The mixed strategy followed by the low quality firm balances its market power when the rival is of high quality type (and charges price p_H), and its incentive to undercut the rival's price when the latter is of low quality.

In this equilibrium of the pricing game, prices signal quality perfectly. The high quality type of a firm charges a higher price, but loses business to the rival firm with higher probability than the low quality type. The latter plays an important role in deterring imitation of the high quality price by the low quality firm (which has lower marginal cost). The low quality firm earns strictly positive expected profit - in fact, just enough profit so as to not have any incentive to imitate the high quality type. It is easy to see from (7) that generating sufficient rent for the low quality firm requires that the rival firm charge a high enough price when its product is of high quality. As a result not only the low quality firm, but also the high quality firm may choose price above marginal cost (sometimes as high as the full information monopoly price) and earn considerable rent. More generally, signaling softens price competition to sustain rents that are important to

deter imitation of higher price by low quality firms. The reason why these rents are not dissipated through price competition has a lot to do with the beliefs of buyers. Reducing price (even slightly) to steal business from the rival may not be worthwhile for a high quality firm, if this leads buyers to adversely revise their beliefs about the quality of its product.

We now argue that neither type of a firm has any incentive to deviate from the proposed equilibrium at the disclosure stage. Earlier in this section, we have argued that a high quality firm that discloses earns negative expected profit whether or not its rival discloses; thus, a high quality firm cannot gain by deviating and disclosing its type. Suppose the low quality type of a particular firm, say firm 1, deviates and discloses its type. In this case we are in a subgame where the product quality of firm 1 is commonly known to be low quality, but firm 2's quality remains private information. The equilibrium of this price setting game with one-sided incomplete information is characterized formally in the appendix (see, proof of Proposition 1), but the key arguments are as follows. As one would expect, the price set by the high quality type of firm 2 (which has a competitive disadvantage vis-a-vis firm 1) would be bid down to its marginal cost c_H . As firm 1 faces Bertrand competition from the low quality type of firm 2, its market power arises solely from the fact that it can charge a price $c_H - (V_H - V_L)$ and steal all business in the state where the rival firm has high quality. In equilibrium, the disclosing low quality firm 1 earns profit only in the state where its rival is of the high quality type, and that profit is exactly equal to $[c_H - (V_H - V_L) - c_L]$; its net expected profit is therefore $\alpha [c_H - (V_H - V_L) - c_L] - d$. It is clear that this can never be larger than the equilibrium profit (7) of the low quality firm, as $p_H \geq c_H$. As a result, deviation by a low quality type from the proposed equilibrium is not gainful. Thus, full non-disclosure is an equilibrium outcome and therefore, must be the unique symmetric equilibrium outcome.¹⁷ The next proposition summarizes our discussion in this section, and is one of the key results of this paper:

Proposition 1 *Suppose that (5) holds i.e., quality premium is relatively low. Then, for all $d > 0$ i.e., no matter how small the cost of disclosure, the unique symmetric equilibrium outcome is one where, independent of true product quality, neither firm discloses voluntarily; firms communicate private quality information only by signaling through prices.*

¹⁷Non-disclosure remains an equilibrium if $d = 0$, but the uniqueness result does not hold.

4 High Quality Premium

In this section, we consider the scenario where the quality premium that buyers are willing to pay is relatively high i.e., $V_H - V_L > c_H - c_L$ so that

$$V_H - c_H > V_L - c_L, \tag{8}$$

which implies that the high quality product generates higher surplus, and has a competitive advantage over the low quality product. Further, in this section, we will assume that the regular cost configuration obtains i.e.,

$$c_H > c_L. \tag{9}$$

The case of cost reversal is analyzed in the next section.

The nondisclosure result in the previous section is significantly driven by the competitive disadvantage of the high quality firm which makes it impossible for the firm to make positive profit under full disclosure. Under (8), this is no longer the case; the high quality type has a competitive advantage. By disclosing its type, a high quality firm can reduce its price without taking into account how such pricing may affect the beliefs of consumers about its product quality. This increases its ability to compete aggressively and steal business from a low quality competitor. Interestingly enough, even under these circumstances, full non-disclosure may be an equilibrium.

To see this, suppose that both firms of both types choose not to disclose. The continuation pricing game is one of two-sided incomplete information where each firm's product is of high quality with probability α . The unique symmetric D1 equilibrium of this game has been broadly described in the previous section. In this equilibrium, a high quality firm charges a high price and loses the entire market to its rival when the latter is of the low quality type; both of these enable the rival low quality firm to earn sufficient rent so as not to imitate the higher price charged by its own high quality type. However, when (8) holds, for the low quality firm to earn rent it must be able to attract buyers at a price above its marginal cost, and thus the high quality firm must charge a price sufficiently higher than c_H . Recall that in this signaling equilibrium, a high quality firm sells only in the state where its rival is also of the high quality type. Despite this, high quality

firms can sustain high prices because of the out-of-equilibrium beliefs of buyers; buyers correctly perceive that a low quality type has greater incentive (than a high quality type) to charge a (out of equilibrium) price between the high quality price and the low quality prices. Therefore, using the D1 criterion, a high quality firm that undercuts is perceived to be a low quality firm and loses all market to its rival.¹⁸

Now, consider the incentive of a high quality firm to unilaterally deviate from the full non-disclosure equilibrium and disclose in the first stage. The gain from such a deviation is that in the pricing game, the disclosing high quality firm is no longer constrained by the beliefs of buyers and can reduce its price freely; this implies that it no longer loses business to its rival in the state where the latter is of the low quality type. Indeed, it can now make money in that state of the world.¹⁹ However, there is also a downside to this deviation: the disclosing high quality firm no longer precommits to not undercutting the rival firm when the latter is of the high quality type; this makes price competition between the high quality types more intense. By disclosing, the high quality firm reduces its profit in the state where its rival is also of high quality. If α , the probability that the rival is of the high quality type, is large enough, then the gain from unilateral disclosure is likely to be outweighed by the loss, and the deviation is not likely to be gainful. Further, if the competitive advantage of the high quality firm over the low quality firm given by $[(V_H - c_H) - (V_L - c_L)]$ is small, then the disclosing high quality firm will have to charge a rather low price in order to avoid being undercut by its low quality rival and therefore, the gain from disclosing (which arises only in the state where the rival is of low quality type) is likely to be small i.e., the deviation from full non-disclosure is not likely to be gainful. Under such conditions, full non-disclosure is an equilibrium.

The next proposition formalizes this result. The proof of this proposition is contained in the appendix.

Proposition 2 *Suppose that (8) and (9) hold i.e., the buyers' quality premium is relatively high and the marginal cost of supplying the high quality product is more than that of the low quality*

¹⁸This argument is spelt out in details in Janssen and Roy, 2010.

¹⁹Even if the low quality firm charges c_L , the high quality firm can make profit as $c_L + (V_H - V_L) > c_H$.

product. Let \hat{d} be defined by

$$\begin{aligned}\hat{d} &= (1 - \alpha)[(V_H - c_H) - (V_L - c_L)] - \frac{\alpha}{2}(c_H - c_L), \text{ if } \frac{V_L - c_L}{V_H - c_L} \geq \frac{1}{2} \\ &= [V_H - c_H) - (V_L - c_L)] - \alpha \frac{(V_H - c_H)(V_L - c_L)}{V_H - c_L}, \text{ if } \frac{V_L - c_L}{V_H - c_L} < \frac{1}{2}.\end{aligned}$$

There exists a symmetric equilibrium with full non-disclosure if, and only if, $d \geq \max\{0, \hat{d}\}$. In such an equilibrium neither firm discloses voluntarily whatever their true product qualities, and firms signal quality to consumers through prices.

Observe that a lower value of \hat{d} implies that the full non-disclosure outcome is more likely. Also, note that \hat{d} is decreasing in α , the probability that a firm is of the high quality type, and increasing in $[V_H - c_H) - (V_L - c_L)]$, the competitive advantage of the high quality firm. This reflects the fact that (as discussed above) an increase in α and a decrease in the competitive advantage of a high quality product reduce the incentive of a high quality firm to unilaterally deviate from a non-disclosure outcome and disclose its type. More particularly, observe that if α is close enough to 1 i.e., the prior likelihood that a firm's product is of high quality is large enough, then $\hat{d} < 0$ so that full non-disclosure is an equilibrium for all $d \geq 0$ i.e., no matter how small the cost of disclosure (in fact, even if disclosure cost is exactly zero). The same is true if the competitive advantage of the high quality firm is small enough. To summarize:

Corollary 1 *Suppose that (8) and (9) hold. There exists a critical $\bar{\alpha} \in (0, 1)$ such that if the ex ante probability of high quality type $\alpha > \bar{\alpha}$, then there exists a symmetric equilibrium with full non-disclosure no matter how small the disclosure cost. The same conclusion holds if $[V_H - c_H) - (V_L - c_L)]$, the competitive advantage of the high quality firm is small enough.*

Thus, even if the premium that buyers are willing to pay for high quality is large enough so that the high quality producer has a competitive advantage over a low quality rival, it may not wish to exercise that advantage fully by disclosing its type; further, this may occur even if there are no frictions in the disclosure process.

The arguments underlying the above non-disclosure outcome do not rule out the fact that a high quality firm may have an incentive to disclose its quality if it knows that its rival firm will

also choose to disclose when it has a high quality product. In other words, there may exist an equilibrium where high quality firms disclose their product quality, but not low quality firms. Note that in this equilibrium by not disclosing its quality, the low quality firm implicitly reveals its information and therefore does not have an incentive to pay a cost to disclose. The next result shows that if the disclosure costs are not too high, there is indeed an equilibrium where high quality firms disclose information.

The argument is as follows. As the equilibrium is fully revealing, in the price competition game, firms make no profit when both firms are of the same type. In case the firms are of different types, the high quality firm wins the competition and sets a price equal to $c_L + (V_H - V_L)$, which is larger than c_H under (8). The ex ante equilibrium profit of a high type firm in this case is therefore equal to

$$(1 - \alpha)(c_L + (V_H - V_L) - c_H) - d, \quad (10)$$

whereas the low quality firms earn zero profits. A necessary condition for this to be an equilibrium is that the net profit of the high quality firm given by (10) should be non-negative i.e.,

$$d \leq (1 - \alpha)(c_L + (V_H - V_L) - c_L). \quad (11)$$

If this satisfied, the only reason why this would not be an equilibrium outcome is if the high quality firm has an incentive not to disclose. Suppose that the high quality type of firm 1 deviates and does not disclose. Given that we consider an equilibrium where (only) the low quality does not disclose, firm 2 now believes that firm 1 is of low quality type, and therefore sets a price equal to c_L if it is itself of low quality type, and $c_L + (V_H - V_L)$ if it is of high quality type. To determine the optimal pricing strategy of firm 1 in the pricing game, the out-of-equilibrium beliefs of consumers at price $p > c_H$ are important. As the equilibrium profit of a low quality firm is zero, this type has a greater incentive to deviate to any such price (deviation is gainful for this type as long as it can expect to sell *any* strictly positive amount, but that is not true for the high quality type). Therefore, the D1 criterion implies that consumers should believe that any $p > c_H$ charged by firm 1 is actually set by a low quality type, and no consumer should buy at such price. Thus, the high quality firm cannot gain by deviating and not disclosing its quality. Thus, (11) is necessary and

sufficient for existence of this equilibrium.

Proposition 3 *Suppose that (8) and (9) hold i.e., the buyers' quality premium is relatively high and the marginal cost of supplying the high quality product is more than that of the low quality product. Let $\tilde{d} > 0$ be defined by*

$$\tilde{d} = (1 - \alpha)[(V_H - V_L) - (c_H - c_L)].$$

There exists a symmetric equilibrium where high quality firms disclose their product quality with probability one and low quality firms do not disclose if, and only if, $d \leq \tilde{d}$.

Observe that a lower value of \tilde{d} implies that the disclosure equilibrium is less likely. Note that \tilde{d} is decreasing in α , the probability that a firm is of the high quality type. We have noted earlier that higher values of α make it more likely that we have an equilibrium with full non-disclosure; we can see now that it also makes disclosure less likely. Likewise, a decrease in the competitive advantage of the high quality firm given by $[(V_H - V_L) - (c_H - c_L)]$ makes full non-disclosure more likely and disclosure less likely.

It is interesting to see whether or not both disclosure and non-disclosure can coexist as symmetric equilibrium outcomes. The following result combines the two propositions above to determine the range of values of d for which the equilibria coexist. For some parameter constellation there are values of the disclosure cost d such that none of the pure strategy equilibria discussed so far exist, and only a mixed strategy equilibrium exist where high quality discloses with a certain probability i.e., partial disclosure may occur.

Proposition 4 *Suppose that (8) and (9) hold. If either,*

$$\frac{V_L - c_L}{V_H - c_L} \geq \frac{1}{2} \tag{12}$$

or,

$$\frac{1}{V_H - c_L} + \frac{1}{V_H - c_H} < \frac{1}{V_L - c_L}, \tag{13}$$

then $\hat{d} < \tilde{d}$, and both the full nondisclosure and the disclosure equilibrium (with only high quality

firms disclosing) coexist if (and only if) $\hat{d} < d \leq \tilde{d}$. If neither (12) nor (13) holds, then $\tilde{d} \leq \hat{d}$ and the two kinds of equilibrium do not coexist for any value of the disclosure cost d ; further, for $d \in [\tilde{d}, \hat{d}]$ there is no symmetric equilibrium where both quality types choose pure strategies at the disclosure stage, but there is a symmetric equilibrium with partial disclosure where each high quality firm discloses with probability $q \in (0, 1)$ and low quality firms do not disclose.

In case both the full non-disclosure and the disclosure equilibrium coexist, one may wonder whether one can rank them according to which equilibrium is preferred by the firms. A few observations are in place. First, low quality firms will always prefer the non-disclosure equilibrium as they make positive profit in any such equilibrium, whereas their profits are equal to zero in the disclosure equilibrium (they lose all market to their rival when the latter has high quality and dissipate all profit through Bertrand competition when the rival's type is low quality). Second, high quality firms may also prefer the non-disclosure equilibrium. In the disclosure equilibrium, a high quality earns profit only in case the other firm has low quality product, and if this probability is small i.e., α is large, then high quality firms make low profits in a disclosure equilibrium. On the other hand, the profits of high quality firms are increasing in α in the no disclosure equilibrium. The next proposition shows that there is a reasonably large set of parameter values such that *both* type of firms are better off in the non-disclosure equilibrium.

Proposition 5 *Suppose that (8) and (9) hold and further, both full nondisclosure and full disclosure by high quality types coexist as equilibrium outcomes. Then, there exist a critical value $\alpha^* < 1$, such that for all $\alpha > \alpha^*$ the nondisclosure equilibrium yields higher pay-off to both low and high quality firms. In particular, if (12) holds, then $\alpha^* < 1/2$.*

There are two implications of the above proposition. First, when both disclosure and non-disclosure are equilibrium outcomes, under certain conditions, the latter may be preferred by all players (of all types) so that they are likely to focus on the non-disclosure equilibrium and the latter is more likely to obtain. Second, the fact that the non-disclosure equilibrium provides more payoff to both low and high quality firms implies that both high and low quality firms may be worse off under a mandatory disclosure regulation and therefore, the industry may resist such regulation unitedly. To fully understand when firms are likely to oppose mandatory disclosure regulation,

we would have to consider the entire region of the parameter space where full non-disclosure is an equilibrium and see when the payoff of all types of firms are reduced through mandatory disclosure. But that detracts us from the main purpose of this paper which is to examine the possibility of *voluntary* disclosure. We will return to some comments on the welfare effects of mandatory disclosure in a later section.

5 Cost Reversal

In this section, we analyze the situation where the effective marginal cost of supplying the low quality product exceeds that of supplying high quality:

$$c_L \geq c_H. \tag{14}$$

This is particularly likely when the low quality product may lead to health, environmental or other hazards that make the firm potentially liable for payment of compensatory and punitive damages in the future; the expected liability and therefore, the effective *expected* marginal cost, may then be higher for the low quality product. Note that (14) implies that

$$V_H - c_H > V_L - c_L \tag{15}$$

so that, as in the previous section, the high quality firm creates greater surplus and has a competitive advantage in this market. Indeed, the relative disadvantage of the low quality firm is now much more pronounced than in the previous section - it produces a worse product at higher cost than the high quality firm. Surprisingly, there may be strong strategic incentives for the high quality firm to not disclose its quality even in this setting.

We begin with the observation that in this scenario, a low quality firm can never earn positive profit whether or not its quality is revealed at the disclosure stage. This is fairly obvious when firms reveal their quality at the disclosure stage; the low quality firm loses all market to its rival when the latter is of the high quality type, and Bertrand competition with a rival low quality firm drives its profit to zero. When firms do not reveal their qualities at the disclosure stage, our discussion in

the previous section for the case where $c_H > c_L$ indicated that despite its competitive disadvantage, the low quality firm earns rent in a signaling equilibrium. This is no longer the case when $c_L \geq c_H$. Even if no information is revealed at the disclosure stage, signaling no longer generates rent for the low quality type. This is because in any signaling equilibrium following full non-disclosure, *the low quality type charges a higher price than the high quality type*; as it has higher marginal cost, the low quality type has greater incentives to charge higher price. As a result, the low quality firm cannot sell in the state where the rival is of the high quality type (nobody buys a lower quality product unless the price is lower), and Bertrand competition with a rival low quality type then drives its price to marginal cost c_L .

The fact that low quality firms cannot make money in the market has two implications. First, as disclosure cost is positive, low quality firms will never choose to disclose quality. If there is a symmetric equilibrium with disclosure, it must be one where high quality firms disclose. Second, if a high quality firm does not disclose, the low quality type will gain by imitating any price above c_L at which the high quality type sells. As a result, if the high quality type does not disclose, it is constrained to price below c_L which restricts its profitability. This, in turn, ought to create strong incentive for disclosure by the high quality firm.

So, consider for a moment the possibility of a symmetric equilibrium where each firm discloses if, and only if, it is of high quality. In any such equilibrium, price competition takes place under full information and in particular, the high quality firm earns zero profit if the rival produces high quality and if the rival has low quality, it sells to all consumers at price $c_L + (V_H - V_L)$ so that the expected net profit of the high quality firm in this equilibrium is

$$(1 - \alpha)[c_L + (V_H - V_L) - c_H] - d. \quad (16)$$

Now, suppose firm i of high quality deviates from this equilibrium and does not disclose its type. Given the equilibrium strategies, its rival would believe that it is in a continuation game where firm i 's product is of low quality for sure. Expecting firm i to charge c_L , the rival firm would charge c_L if it is of low quality type and $c_L + (V_H - V_L)$ if it is of high quality type. As $c_L > c_H$ firm i whose true product quality is high could now charge a price slightly below c_L . Upon observing

this out-of-equilibrium price set by firm i (that is below the marginal cost of low quality type), consumers must infer that such a price could only be set by a high quality firm, and they will all prefer to buy from firm i with probability 1. After this deviation, the expected net profit of a high quality firm is therefore approximately $c_L - c_H$, and this exceeds its equilibrium payoff (16) as long as

$$(1 - \alpha)(V_H - V_L) - \alpha(c_L - c_H) < d. \quad (17)$$

(When $c_L = c_H$, the above inequality implies that the high quality firm's profit (16) in the proposed equilibrium is negative). Thus, under (17), there is no symmetric equilibrium where firms reveal their product quality through disclosure. It can be shown that the deviation from an equilibrium with disclosure outlined above is actually the optimal deviation for a high quality firm and therefore, if the above condition does not hold, there is always a symmetric equilibrium where high quality firms disclose. Finally, (17) is satisfied for all $d > 0$ if

$$\alpha > \frac{V_H - V_L}{(V_H - V_L) + (c_L - c_H)},$$

in which case there is no symmetric equilibrium with disclosure no matter how small the cost of disclosure (and in fact, even if the disclosure cost is equal to zero). Thus, we have:

Proposition 6 *Assume (14). There is no equilibrium where the product qualities of both firms are fully revealed through voluntary disclosure if*

$$d > (1 - \alpha)(V_H - V_L) - \alpha(c_L - c_H). \quad (18)$$

Further, this holds for all $d > 0$ i.e., no matter how small the cost of disclosure, if

$$\alpha > \frac{V_H - V_L}{(V_H - V_L) + (c_L - c_H)} \quad (19)$$

If, on the other hand,

$$(1 - \alpha)(V_H - V_L) - \alpha(c_L - c_H) \geq d, \quad (20)$$

then there is a symmetric equilibrium where each firm voluntarily discloses quality if, and only if,

it is of high quality type.

A complete proof of this proposition is contained in the appendix.

Proposition 6 provides a necessary and sufficient condition (condition (18)) under which there is no symmetric equilibrium with disclosure. Observe that (18) does not require the disclosure cost to be prohibitive i.e., larger than the profit that a disclosing high quality firm makes in the market. Indeed, under restriction (19), the disclosure cost may be arbitrarily small, even zero. The non-disclosure that we uncover is once again driven by the strategic incentives of firms to soften price competition. In particular, by deviating and not disclosing its type, the high quality firm induces a change in the posterior belief of the rival firm by making the latter believe that it is a low quality type; as the low quality type has higher marginal cost and generates significantly smaller surplus, this makes the rival less aggressive in its pricing (particularly, when the latter is also of the high quality type). The deviating firm which is actually of high quality type can now take advantage of the increase in rival's price and still reveal its true type to consumers through its own price. The strong competitive advantage of the high quality firm is a double-edged sword; it apparently creates strong incentive for disclosure in order to reveal type to consumers without being subject to signaling constraints, but also creates strong incentive to hide its true type from the rival in order to make the latter price less aggressively. When the latter incentive dominates, as it does under (19), disclosure does not occur even if there is no cost of disclosure.

As the primary reason for the high quality firm to hide its type is to soften price competition with the high quality type of the rival firm, the incentive to deviate from disclosure increases with α , the probability that the rival is of the high quality type; if this probability exceeds a threshold indicated in (19), disclosure does not occur no matter how small the disclosure cost. Similarly, an increase in $(c_L - c_H)$, the cost advantage of the high quality producer, leads to an increase in the profit that the high quality firm can make by hiding its type while still revealing its type to consumers (by pricing just below c_L); an increase in $(c_L - c_H)$ reduces the possibility of disclosure. Finally, if $(V_H - V_L)$, the buyers' quality premium is large, then the incentive for disclosure is high because the disclosing high quality firm can charge a high enough price; non-disclosure is more likely if $(V_H - V_L)$ is small.

When condition (18) holds, a symmetric equilibrium can be of only two possible types: neither

type of any firm discloses (and we have a pure signaling outcome in the market), or high quality firms randomize between disclosure and non-disclosure. Our last proposition provides necessary and sufficient conditions for these outcomes.

Proposition 7 *Assume (14).(a) There is a symmetric equilibrium with full non-disclosure where neither firm of any type discloses its product quality voluntarily if*

$$d \geq (1 - \alpha)(V_H - V_L). \quad (21)$$

(b) *If*

$$\max\{0, (1 - \alpha)(V_H - V_L) - \alpha(c_L - c_H)\} < d < (1 - \alpha)(V_H - V_L), \quad (22)$$

there is a symmetric equilibrium with partial non-disclosure where each high quality firm randomizes between disclosure and non-disclosure while low quality firms do not disclose.

The proof of this proposition is contained in the appendix. Observe that conditions (20), (21) and (22) are mutually exclusive and exhaust the parameter space.

It is easy to check from (21) that, in line with arguments outlined above, an increase in α , the *ex ante* probability of being high quality type, and a decrease in the quality premium, $(V_H - V_L)$, make the full non-disclosure outcome more likely. Observe that the randomization in part (b) of the above proposition (as well as Proposition 4 in the previous section) indicates that competition does not necessarily induce similar firms with similar product qualities to replicate each others' disclosure acts; this may explain why some firms appear to disclose voluntarily and others not, even when they have similar product attributes.

6 Discussion: Effect of Mandatory Disclosure

In this section, we informally discuss the welfare effects of *mandatory* disclosure regulation. This is of considerable interest in the context of our model where, in the absence of such regulation, we have shown that *voluntary* disclosure does not often occur.

In much of the literature on mandatory disclosure, it is generally presumed that the purpose of mandatory disclosure regulation is to provide more information to consumers (in markets where

firms do not disclose voluntarily), and welfare gains result from buyers being better informed while making decisions. However, when firms can signal their product quality, the absence of voluntary disclosure need not imply that consumers do not acquire the relevant information before purchase. Indeed, in our model, even if firms do not disclose their product quality voluntarily, they always communicate their product quality to consumers through price signals and consumers always make their purchase decisions under complete information. Therefore, mandatory disclosure regulation cannot lead to more informed decision making by consumers even when firms do not disclose voluntarily. This raises the question about whether there is any *other rationale* for mandatory disclosure rules (that appear to be prevalent in increasing number of industries). Our framework allows us to shed some light on this.

We argue that mandatory disclosure changes the information structure of *firms*, and their need to signal product quality through prices. This, in turn, affects the strategic interaction of firms in the market, the resulting prices and quantities sold, as well as the composition of goods sold in the market. In particular, as goods of different qualities differ in the social surplus they create, changes in the quality composition of goods bought by consumers may increase total social surplus. Of course, one must also take into account the disclosure cost incurred as a consequence of the regulation. We comment on this trade-off by looking at specific cases.

We first consider the scenario analyzed in Section 3 where (5) holds, and the low quality good generates higher social surplus. The unique symmetric equilibrium is one where no firm discloses. In the pricing game that follows full non-disclosure, firms signal quality through prices and, in particular, the low quality firms charge lower prices so as to steal business from high quality rivals. All consumers buy the low quality product, if it is available. In other words, the quality that generates higher surplus is consumed whenever it is available, and no disclosure cost is incurred. It follows that if all consumers buy with probability one in the signaling equilibrium i.e., there is no quantity distortion, total surplus is maximized and mandatory disclosure cannot increase total surplus (in fact, it reduces total surplus because of the disclosure costs that firms incur). However, if $V_H - c_L > 2(V_H - c_L)$, then in order to deter low quality firms from imitating the price charged by high quality firms, the signaling equilibrium involves distortion in the total quantity sold in the state of the world where both firms are of high quality; in particular, high quality firms charge

$p_H = V_H$ and consumers buy with probability $\eta = \frac{2(V_H - c_L)}{V_H - c_L} < 1$ in that state of the world. The (expected) total surplus generated without mandatory disclosure then equals

$$(1 - \alpha)^2(V_L - c_L) + \alpha^2 \frac{2(V_H - c_L)}{V_H - c_L} (V_H - c_H).$$

Under mandatory disclosure, expected total surplus equals

$$(1 - \alpha)^2(V_L - c_L) + \alpha^2(V_H - c_H) - 2d$$

so that mandatory disclosure increases welfare if the disclosure cost is not too high:

$$2d < \alpha^2 \left(1 - \frac{2(V_H - c_L)}{V_H - c_L} \right) (V_H - c_H).$$

Thus, in this case where the low quality good generates higher social surplus, the only possible argument for mandatory disclosure comes from the fact that by forcing firms to set more competitive prices, it removes the distortion in total quantity sold that may arise in a signaling equilibrium. Note that the quantity distortion effect of high prices that can arise in our model with a unit demand structure captures qualitatively the usual deadweight loss effect of high market power with downward sloping demand curve.²⁰

Next, we consider the case analyzed in Section 4 where (8) and (9) hold. Here, the high quality good creates higher social surplus. Our analysis in Section 4 indicates that depending on parameter values, the symmetric equilibrium outcome may be one with voluntary disclosure or one with full non-disclosure, or both. It is easy to see that imposition of mandatory disclosure cannot lead to any improvement over the voluntary disclosure equilibrium outcome. So, consider the full non-disclosure equilibrium, where prices signal product quality. We will argue that even if there is no total quantity distortion in the signaling equilibrium, mandatory disclosure may improve welfare for a very different reason. In order to focus on the latter, assume that $V_H - c_L < 2(V_H - c_L)$ so that all consumers buy with probability one in the signaling equilibrium. In the signaling equilibrium, a

²⁰If we consider consumer surplus as a welfare measure, then there is a positive effect of mandatory disclosure whenever the price charged by the high quality firm in the signaling outcome (that obtains in the absence of mandatory disclosure regulation) is strictly greater than c_H , as then the expected price paid by the consumer is lower and the consumer is strictly better off under mandatory disclosure.

low quality firm sells to all buyers when its rival is of high quality type. The expected total surplus generated in the signaling outcome following full non-disclosure is

$$\alpha^2(V_H - c_H) + (1 - \alpha^2)(V_L - c_L). \quad (23)$$

Now, consider the effect of mandatory disclosure regulation. At the pricing stage, firms engage in Bertrand competition under complete information; the high quality sellers, if any, serve the market, and all consumers buy the high quality product if it is available. Thus, mandatory disclosure regulation corrects the signaling distortion in prices that makes consumers buy the low quality good even when the high quality product is available. The expected total surplus generated under mandatory disclosure

$$\alpha(2 - \alpha)(V_H - c_H) + (1 - \alpha)^2(V_L - c_L) - 2d. \quad (24)$$

Comparing (23) and (24), it is obvious that mandatory disclosure improves total surplus over a non-disclosure equilibrium if, and only if,

$$d < 2\alpha(1 - \alpha) [(V_H - c_H) - (V_L - c_L)].$$

Thus, when the buyers' quality premium is high so that the high quality good generates more surplus, mandatory disclosure may positively affect welfare by ensuring that consumers buy the socially efficient type of good whenever it is available. This positive effect on welfare makes mandatory disclosure optimal if disclosure cost is small, if there is considerable uncertainty about product quality ($\alpha(1 - \alpha)$ is high), and if the additional surplus generated by the high quality good (over the low quality good) is large.²¹

Finally, it is worth noting that the sharpening of competition between firms after imposition of mandatory disclosure regulation may lead either high or low quality firms (depends on which type generates smaller surplus) to exit the market in the event that the market profit of such a firm does not cover the disclosure cost. This may lead to higher market power and concentration in the

²¹Note that if α is high enough (for instance, close to 1), or if the surplus difference between low and high quality goods is small, then (as indicated in Section 4), the non-disclosure outcome is more likely; however, in those situations, mandatory disclosure regulation is unlikely to be welfare improving.

future. A proper analysis of the dynamic benefits and costs of mandatory disclosure rules should be an interesting topic for future research.

7 Conclusion

We have analyzed a simple model of strategic voluntary disclosure in a market where firms have private information about their own product quality, and can signal their product quality through prices when they do not disclose voluntarily. Voluntary disclosure of product quality is modeled as a (relatively long term) decision that is made prior to (short run) price competition. We show that competition between firms and the possibility of signaling may together create strong incentives to not disclose product quality and instead, signal quality through prices. Non-disclosure inhibits sharp price undercutting by high quality firms as they take into account how pricing affects the beliefs of buyers about product quality, and leads to a signaling outcome characterized by sufficient market power and rent to dissuade imitation of high quality pricing by low quality types. A seller whose product quality type has a competitive advantage (creates more surplus) may want to disclose quality in order to be able to price aggressively and steal business from the rival firm when the latter is of the disadvantaged quality type (without bothering about the beliefs of buyers); but such a seller may also wish to hide its quality type from the rival firm in order to make the latter less aggressive in price competition (particularly in the state where the rival is also of the same type).

We show that no matter how small the disclosure cost, non-disclosure by all firms is the unique symmetric equilibrium outcome when the quality premium that buyers are willing pay is relatively low so that the low quality product generates more social surplus. When the quality premium is high (high quality generates more surplus), non-disclosure by all firms remains an equilibrium outcome for a large section of the parameter space; non-disclosure is more likely if the likelihood of being a high quality producer and the competitive disadvantage of the low quality seller are higher; if the latter are sufficiently high, non-disclosure is an equilibrium even if disclosure cost vanishes. However, with high quality premium, if the disclosure cost is small, there also exists an equilibrium where high quality firms disclose their product quality. When both kinds of equilibrium coexist, there is a large set of parameter values for which the nondisclosure equilibrium generates higher

profits for firms of all quality. Finally, we show that non-disclosure may also be likely when the low quality product has higher effective marginal cost than the high quality product - a situation where a high quality firm has tremendous competitive advantage over a low quality rival and therefore, all the more reason to disclose its quality.

Our results provide a new explanation of the observed reluctance of firms to disclose quality attributes; this explanation is based on the availability of an alternative channel of communicating quality viz, signaling, and the existence of sufficiently strong market competition. Further, we can explain full non-disclosure by *all* firms - even those with better products, and even when there are no disclosure frictions. Our analysis indicates that the social cost of non-disclosure is not the consumers' inability to make informed decisions; indeed, in our framework, consumers can infer private information of firms from prices. Rather it is related to the distortions in prices and market shares of producers of different qualities when they signal their quality through prices. In some cases, this may create some limited role for mandatory disclosure regulation.

The limitations of our analysis arise from the simplicity of our model. Our model ignores imperfections in both signaling and voluntary disclosure as modes of communicating private information. In a more general model, signaling may be imperfect and noisy, and (as some of the literature on voluntary disclosure has emphasized) disclosure may not communicate information to all buyers. Some of these imperfections may soften competition and others may have the reverse effect. It is difficult to say which imperfections are more important and ought to be incorporated. Therefore, as a first enquiry, we have abstained from treating imperfections in the transmission mechanisms. We have also restricted the analysis to the case where consumers are homogeneous in their preferences for the products and the case of only two types of quality. These assumptions allow us to bring out the implications of competition in its starkest form, and to ensure a clean characterization of the signaling outcome.

Finally, we have not allowed for repeated price competition. In a dynamic setting, if qualities remain unchanged over time, future price competition depends on the information revealed in the current period, but not on the channel through which it is revealed i.e., whether it is revealed through signaling or disclosure. While a full analysis of this kind of dynamic game is complex, it appears that the trade-off between signaling and disclosure may remain qualitatively unaffected.

A proper analysis of such a dynamic model, however, should also take into account the possibility that product quality may change and firms may acquire new private information over time; the latter is likely to reinforce the effects on the choice between disclosure and signaling uncovered by our current analysis. This will be a useful direction to pursue in future research.

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APPENDIX

Proof of Proposition 1(remaining details from Section 3). Much of the proof has been outlined in the main text of Section 3. Three parts of the proof were left out; these are as follows: (i) showing that there is no equilibrium where a high quality firm discloses with positive probability, (ii) showing that there is no symmetric equilibrium where low quality firms randomize between disclosure and non-disclosure (and high quality firms do not disclose); and (iii) showing that the low quality firm has no incentive to deviate from the equilibrium with full non-disclosure.

We begin with (i). Suppose there is an equilibrium where firm 1 of H type discloses with strictly positive probability. Consider the price competition that follows disclosure of high quality by this firm. It is obvious that firm 1 can never earn strictly positive profit in the state of the world where firm 2 is of L type; this is because (using(5)) the latter will always undercut any price above c_H charged by firm 1 by a sufficient amount so as to steal all business. This occurs independent of whether or not firm 2's type is revealed at the disclosure stage. To earn strictly positive expected profit, firm 1 must sell at a price above c_H when firm 2 is of H type. So, consider the situation where firm 2 is also of H type. If firm 2's type is fully revealed at the end of the disclosure stage, then it is obvious that the two firms will engage in Bertrand price competition leading to marginal cost pricing. What if firm 2's type is not fully revealed at the end of the disclosure stage? As firm 1 has revealed its product quality to be high, it will be uninhibited in undercutting firm 2. As a result, firm 2 must earn zero expected profit in equilibrium; if it charges price above c_H , it is undercut by firm 1 with probability one at the upper bound of its (possibly mixed) price distribution. As firm 2 earns positive profit when it is of L type, H type of firm 2 has greater incentive to charge any out-of-equilibrium price that undercuts a price above marginal cost charged by firm 1. The D1 criterion suggests that following any such price undercutting by firm 2, buyers should believe that firm 2's product is of high quality. Therefore, firm 2 will be uninhibited in undercutting firm 1. In particular, if firm 1 charges price above marginal cost, it is undercut by firm 2 with probability one at the upper bound of its (possibly mixed) price distribution, leading to zero expected profit for firm 1. Thus following disclosure, firm 1 of H type earns negative net profit (taking into account disclosure cost $d > 0$).

Next, we prove (iii). Consider the pricing subgame after the low quality type of firm 1 deviates

and discloses. In equilibrium, firm 1's expected profit (ignoring any disclosure cost) is $\geq \alpha(c_H - (V_H - V_L) - c_L)$ as it can set a price just below $c_H - (V_H - V_L)$ and sell in case firm 2 is of H type. Suppose that (5) holds with strict inequality. In that case, $c_H - (V_H - V_L) - c_L > 0$ which means that firm 1's expected profit > 0 . Let us denote by \bar{p}_1 the upper bound of the support of the possibly mixed price strategy firm 1 chooses. Then, price $\bar{p}_1 > c_L$ and at this price, firm 1 is undercut by firm 2 with probability one if the latter is of L type. So, at price \bar{p}_1 , firm 1 sells only in the state where firm 2 is of H type. If $\bar{p}_1 > c_H - (V_H - V_L)$, then at \bar{p}_1 , firm 1 is undercut with probability one by a margin of more than $(V_H - V_L)$ in the state where firm 2 is of type H so that it sells zero at price \bar{p}_1 . Therefore, $\bar{p}_1 \leq c_H - (V_H - V_L)$ so that the only way for firm 1 to earn at least $\alpha(c_H - (V_H - V_L) - c_L)$ as profit in the equilibrium of this subgame is that it sets $\bar{p}_1 = c_H - (V_H - V_L)$, obtains equilibrium profit equal to $\alpha(c_H - (V_H - V_L) - c_L)$ while firm 2 of H type earns zero profit; all consumers buy from firm 1 when firm 2 is of H type. In particular, firm 2 of H type charges a price equal to c_H in equilibrium. The other details of the mixed strategy equilibrium can be easily derived. Firm 1's profit at any price $p \leq \bar{p}_1$

$$[\alpha + (1 - \alpha)(1 - F_L^2(p))] (p - c_L)$$

where $F_L^2(p)$ is the mixed strategy chosen by the L type of firm 2. Setting this expression equal to $\alpha[c_H - c_L - (V_H - V_L)]$, the equilibrium payoff of firm 1 gives us a continuous mixed strategy distribution of firm 2 of type L on $[\alpha(c_H - (V_H - V_L)) + (1 - \alpha)c_L, c_H - (V_H - V_L)]$. A similar consideration for the low type of firm 2 makes clear that firm 1 should choose a similar mixed strategy, but with a mass point at price $\bar{p}_1 = c_H - (V_H - V_L)$ of probability α . Consumers buy at the lowest price when both firms are of L type. Next, suppose that (5) holds with equality. In that case, neither firm has any market power. The unique equilibrium outcome is that firm 1 charges its marginal cost c_L and each type of firm 2 charges its marginal cost; all firms are zero profit. Thus, we have that if the low quality type of firm 1 deviates and discloses, its net profit in the D1 equilibrium of the continuation pricing game is $\alpha(c_H - (V_H - V_L) - c_L) - d \leq (p_H - (V_H - V_L) - c_L)$, the payoff of the low quality firm if it does not deviate (as indicated in (7)), where $p_H \geq c_H$.

Finally, we return to the proof of (ii). Suppose there is a symmetric equilibrium where each

firm discloses with positive probability $x \in (0, 1)$ if it is of L type and discloses with probability 0 when it is of H type. We will show that given the proposed equilibrium strategy of firm 1, firm 2 of L type must earn strictly higher net expected profit from non-disclosure than from disclosure so that it would never randomize between them in equilibrium. There are two possibilities: (a) firm 1 discloses and (b) firm 1 does not disclose. Given its equilibrium strategy, (a) occurs with probability $x(1 - \alpha)$ and (b) with probability $1 - x(1 - \alpha)$. We will show that non-disclosure is better for firm 2 of L type in both situations (a) and (b). Suppose (a) occurs; in that case firm 1 is known to be of L type. If firm 2 of L type discloses in the first stage, Bertrand competition leads to a net profit of $-d$. As the guarantee itself at least zero net net expected profit under non-disclosure, disclosure leads to lower payoff in the event that (a) occurs. So, suppose that (b) occurs. In this case, the pricing game is one where firm 1's type is H with (updated) probability $\hat{\alpha} = \frac{\alpha}{1-x(1-\alpha)}$ and L with probability $1 - \hat{\alpha}$. If firm 2 of L type has not disclosed, it is a symmetric pricing game of incomplete information virtually identical to the one (characterized in Janssen and Roy (2010) and described in Section 3) when both firms do not disclose for sure in the first stage, except that we replace α , the probability of being H type, by $\hat{\alpha}$. As indicated in the text in Section 3, the payoff of firm 2 (which is of L type) is (using (7)):

$$\hat{\alpha}(p_H - (V_H - V_L) - c_L)$$

On the other hand, if firm 2 (which is of L type) discloses the pricing game is a game of one sided incomplete information where firm 1 is of type H with probability $\hat{\alpha} \in (0, 1)$ and firm 2 is of L type for sure. Using the characterization of the equilibrium of this pricing game in the proof of (iii) above, we have that the payoff to firm 2 from disclosure is

$$\hat{\alpha}(c_H - (V_H - V_L) - c_L) - d$$

As $p_H \geq c_H$ and $d > 0$, disclosure yields strictly higher net expected profit for firm 2. The proof is complete.

Proof of Proposition 2 In the full non-disclosure outcome, the pricing strategies in the unique symmetric D1 equilibrium of the pricing game characterized by Janssen and Roy (2010) depend

on whether or not the condition $\frac{V_L - c_L}{V_H - c_L} \geq \frac{1}{2}$ holds. If $\frac{V_L - c_L}{V_H - c_L} \leq \frac{1}{2}$, then the high quality price satisfies $p_H = \max\{c_H, c_L + 2(V_H - V_L)\}$ and the equilibrium is one where all consumers buy with probability one. If $\frac{V_L - c_L}{V_H - c_L} > \frac{1}{2}$, then $p_H = V_H$ and the equilibrium is one where some consumers do not buy in the state where both firms are of high quality.²²

First consider the case where $\frac{V_L - c_L}{V_H - c_L} \geq \frac{1}{2}$. When both firms do not disclose information with probability one, the unique D1 equilibrium of the pricing game is one where the high quality firm sets a price $p_H = c_L + 2(V_H - V_L)$, the low quality firms choose a mixed pricing strategy over the interval $[c_L + \alpha(V_H - V_L), c_L + (V_H - V_L)]$ and the consumers buy at the lowest low quality price if at least one of the firms is of low quality and otherwise randomly buys at one of the two shops. The equilibrium profits of the high quality firm are given by $\frac{\alpha}{2} [2(V_H - V_L) - (c_H - c_L)]$.

We will now show that under the conditions specified in the Proposition, no type has an incentive to deviate. It is easy to see that if the L type of any firm deviates and discloses then it must lose all market to the rival in the state where the latter is of type H . As the disclosing low quality firm only makes profits in the state where the rival is also of type L , this leads to Bertrand competition driving the price charged by firm 1 of type L to c_L . The deviating firm makes a negative net profit of $-d$ by disclosing and such a deviation is therefore not profitable.

Next, consider the case where the H type of a firm, say firm 1, deviates and discloses its type. This leads to a subgame where firm 1 is known to be of H type and firm 2's type remains private information. In this subgame, it is easy to see that if firm 1 charges any price above $c_L + (V_H - V_L)$, it is undercut with probability one by its rival of both types. Therefore, the highest price firm 1 of type H after disclosure will not be larger than $c_L + (V_H - V_L)$. Moreover, it is clear that the low type of firm 2 has to set a price equal to c_L . As the low type of firm 2 cannot make any profit, the high type of firm 2 can also not make any profit in equilibrium as otherwise the low type of firm 2 would have an incentive to deviate and imitate its high type's price. The equilibrium then is one where both firm 1 (who is of high type) and the high type of firm 2 set a price equal to $c_L + (V_H - V_L)$ and consumers buy at firm 1 (who has disclosed its information). The high type of firm 2 cannot profitably undercut as consumers believe him to be of low type and therefore will not

²²The fact that some consumers do not buy can be re-interpreted a la Bagwell and Riordan (1992) in the sense that the high quality firm rationally chooses such a high price, that consumers respond with a low demand in order to prevent the low quality firm from imitating its pricing behavior.

buy at a deviating price. These consumers' beliefs are consistent with the D1 requirement. (Note that this is the only subgame where a firm benefits from disclosing its information).

Therefore, the net profit of firm 1 of type H after disclosure are equal to $[c_L + (V_H - V_L) - c_H] - d$. Comparing this to the profit of a H type firm in the candidate equilibrium shows that the deviation is not gainful if

$$\frac{\alpha}{2} [2(V_H - V_L) - (c_H - c_L)] \geq [c_L + (V_H - V_L) - c_H] - d,$$

or

$$d \geq (1 - \alpha)(V_H - V_L) - (1 - \alpha/2)(c_H - c_L) = \hat{d}.$$

Now, consider the case where $\frac{V_L - c_L}{V_H - c_L} < \frac{1}{2}$. In this case Janssen and Roy (2010) show that in case firms never disclose information, there exists a unique D1 equilibrium where the high quality firm sets a price $p_H = V_H$,²³ the low quality firms choose a mixed pricing strategy over the interval $[c_L + \alpha(V_L - c_L), V_L]$ and the consumers buy at the lowest low quality price if at least one of the firms is of low quality and otherwise buys with a probability $\eta = 2\frac{V_L - c_L}{V_H - c_L}$ and if she buys, she randomly does so at one of the two shops. The equilibrium profits of the high quality firm in this case are given by $\alpha \left[\frac{(V_H - c_H)(V_L - c_L)}{V_H - c_L} \right]$. As in the previous case, a deviation of a low quality type is not profitable. Consider therefore the case where the high quality type of firm 1 deviates and discloses information. Comparing the profits in the candidate equilibrium with the best possible deviation pay-off of $[c_L + (V_H - V_L) - c_H] - d$ gives

$$\alpha \left[\frac{(V_H - c_H)(V_L - c_L)}{V_H - c_L} \right] \geq [c_L + (V_H - V_L) - c_H] - d.$$

This can be rewritten as

$$d \geq (V_H - V_L) - (c_H - c_L) - \alpha \frac{(V_H - c_H)(V_L - c_L)}{V_H - c_L} = \hat{d}.$$

Proof of Proposition 4 It is straightforward to see that $\hat{d} < \tilde{d}$ if $\frac{V_L - c_L}{V_H - c_L} \geq \frac{1}{2}$. In case, $\frac{V_L - c_L}{V_H - c_L} < \frac{1}{2}$ straightforward algebra shows that $\hat{d} < \tilde{d}$, if and only if, $\frac{1}{V_H - c_L} + \frac{1}{V_H - c_H} \geq \frac{1}{V_L - c_L}$. In these cases the statements of the Proposition on the necessary and sufficient conditions of the coexistence

²³Note that if $\frac{V_L - c_L}{V_H - c_L} = \frac{1}{2}$, $c_L + 2(V_H - V_L) = V_H$.

of a disclosure and nondisclosure equilibrium immediately follow. On the other hand, in case, $\frac{V_L - c_L}{V_H - c_L} < \frac{1}{2}$, it may be that $\hat{d} < \tilde{d}$ and then there is a region of disclosure cost d such that no pure strategy equilibrium exists. The proof concludes by showing that in this case there is a mixed strategy equilibrium where the high quality firm randomizes between disclosing and not disclosing and the low quality type chooses not to disclose.

Proof. So, suppose that $\hat{d} < d < \tilde{d}$ and that a high quality firm chooses to disclose with probability q . In this case three possible pricing subgames can arise in equilibrium. First, if both firms disclose they are of high quality, there will be Bertrand competition resulting in no profits for either firm. Second, if one firm discloses it is of high quality and the other firm does not disclose, we are in the pricing equilibrium analyzed in Proposition 2 so that the high quality disclosing firm always sells and makes a profit equal to $c_L + (V_H - V_L) - c_H$. Third, if no firm discloses we are in the pricing game analyzed in Janssen and Roy (2010) with the exception that now the firms believe their rival is of low quality with probability $\alpha(1 - q)/[1 - \alpha q]$ because of Bayesian updating. The expected profit of a high quality firm in this case is therefore

$$\pi_H^* = \frac{\alpha(1 - q)}{1 - \alpha q} \left[\frac{(V_H - c_H)(V_L - c_L)}{V_H - c_L} \right].$$

For a high quality type to be indifferent between disclosing and not-disclosing it therefore has to be the case that

$$\alpha q \cdot 0 + (1 - \alpha q) [c_L + (V_H - V_L) - c_H] - d = \alpha q \cdot 0 + \alpha(1 - q) \left[\frac{(V_H - c_H)(V_L - c_L)}{V_H - c_L} \right],$$

where the two terms on both sides of the expression reflect the pay-off of disclosing (respectively not disclosing) in case the other firm discloses and does not disclose. This can be rewritten as

$$(1 - \alpha q) [(V_H - c_H) - (V_L - c_L)] - \alpha(1 - q) \left[\frac{(V_H - c_H)(V_L - c_L)}{V_H - c_L} \right] - d = 0.$$

It is easy to see that the LHS of this equation is decreasing in q , it equals $\hat{d} - d > 0$ if $q = 0$ and it equals $\tilde{d} - d < 0$ if $q = 1$. Thus, there must be a unique value of q with $0 < q < 1$ such that the indifference equation holds. As the argument showing that the low quality type does not have

an incentive to deviate is identical as before, we conclude that in case $\frac{V_L - c_L}{V_H - c_L} < \frac{1}{2}$ and $d_3 < d < d_2$ there exists a mixed strategy equilibrium where the high quality firm randomizes between disclosing and not disclosing. ■

Proof of Proposition 5. We know that in the disclosure equilibrium the L type makes zero profits, while the L type makes positive profits in the nondisclosure equilibrium. We therefore only need to check the profits of the H type firms in the different equilibria. In the disclosure equilibrium, the H type makes an ex ante profit of $(1 - \alpha)(c_L + V_H - V_L - c_H) - d$. As indicated above, if $(V_L - c_L)/(V_H - c_L) \geq 1/2$, in the nondisclosure equilibrium where the H type charges a price $p_H < V_H$, and it makes a net profit of $\alpha(V_H - V_L) - \frac{\alpha}{2}(c_H - c_L)$. Straightforward calculations show that the latter expression is larger than the former if

$$\alpha > \frac{(V_H - V_L) - (c_H - c_L) - d}{2(V_H - V_L) - \frac{3}{2}(c_H - c_L)}.$$

Denote the RHS of this inequality by α^* if $(V_L - c_L)/(V_H - c_L) \geq 1/2$. It is easy to see that

$$\alpha^* < \frac{(V_H - V_L) - (c_H - c_L)}{2(V_H - V_L) - \frac{3}{2}(c_H - c_L)} < \frac{1}{2}.$$

If $(V_L - c_L)/(V_H - c_L) < 1/2$, the nondisclosure equilibrium is one where H type firms charge a price $p_H = V_H$, and as indicated above, each H type firm makes a profit of $\alpha \left[\frac{(V_H - c_H)(V_L - c_L)}{V_H - c_L} \right]$. Straightforward calculations show that this profit expression is larger than its net profit $(1 - \alpha)(c_L + V_H - V_L - c_H) - d$ in the disclosure equilibrium if

$$\alpha > \frac{(V_H - V_L) - (c_H - c_L) - d}{\frac{(V_H - c_H)(V_L - c_L)}{V_H - c_L} + (V_H - V_L) - (c_H - c_L)}.$$

Denote the RHS of this inequality by α^* in case $(V_L - c_L)/(V_H - c_L) \leq 1/2$. As long as both disclosure and full non-disclosure equilibrium co-exist, it is clear that there exists a critical value $\alpha^* < 1$ such that for all $\alpha > \alpha^*$ both types of firms make more profit in the no disclosure equilibrium.

In the proofs of Propositions 6 and 7 we use the following Lemma, determining equilibrium pricing strategies of the different pricing subgames when $c_L > c_H$.

Lemma 1 *Assume (14). Consider the game of price setting that follows the voluntary disclosure stage.*

(i) Suppose that the types of both firms are fully revealed at the disclosure stage. Then, both firms charge price equal to marginal cost earning zero (gross) profit when they have identical types, and if their types differ, the L type firm charges its marginal cost selling zero output while the H type firm sells to the entire market charging price equal to $V_H - (V_L - c_L)$ and earning (gross) profit equal to $[(V_H - c_H) - (V_L - c_L)]$.

(ii) Suppose that the type of only one firm (say, firm 1) is fully revealed at the end of the disclosure stage. Let $x \in (0, 1)$ denote the probability that the other firm (firm 2) is of H type assigned by the updated posterior belief after the disclosure stage. (ii.a) Suppose the revealed type of firm 1 is H. Suppose further that

$$\frac{V_H - V_L}{c_L - c_H} \geq \frac{x}{1 - x}. \quad (25)$$

Then, firm 1 charges $[c_L + (V_H - V_L)]$, sells only in the state where rival is of type L and earns gross expected profit $(1 - x)(c_L + V_H - V_L - c_H)$. Firm 2 of type L sells zero with probability one and follows a mixed strategy; firm 2 of type H charges c_L , sells to all consumers and earns gross profit equal to $(c_L - c_H)$. Next, suppose that (25) does not hold. Then, firm 1 follows a mixed strategy that has a mass point at $(c_L + V_H - V_L)$ and a continuous distribution on an interval $[\underline{p}, c_L]$ where $\underline{p} < c_L$ while firm 2 of type H follows a mixed strategy that has a mass point at c_L and whose support is the interval $[\underline{p}, c_L]$; the equilibrium (gross) profits of both firms are equal to $(1 - x)(c_L + V_H - V_L - c_H)$. Firm 2 of type L follows a mixed strategy and sells zero, earning zero gross profit. (ii.b) Suppose the revealed type of firm 1 is L. Then, firm 1 as well as both types of firm 2 charge a common price c_L and all consumers buy from firm 2 with probability one; firm 1 as well as firm 2 of L type earn zero gross profit while firm 2 of H type earns gross profit equal to $(c_L - c_H)$.

(iv) Suppose that neither firm's type is revealed fully at the end of the disclosure stage. In particular, consider the symmetric situation where the updated posterior belief assigns identical probability $x \in (0, 1)$ to the event that either firm is of H type. The unique symmetric equilibrium is one where both firms of type L charge price c_L earning zero (gross) profit while each firm of type H follows a mixed strategy with continuous distribution on support $[(1 - x)c_L + xc_H, c_L]$ earning (gross) expected profit equal to $(1 - x)(c_L - c_H)$.

Proof of Lemma. The proof of part (i) is obvious. Consider (ii.a). In the event that firm 2

is of type L , firm 1 can sell to the entire market at price $c_L + (V_H - V_L) > c_H$, and therefore its equilibrium expected (gross) profit $\geq (1 - x)(c_L + V_H - V_L - c_H) > 0$. If firm 1 sells only in the state where rival is of H type, price competition would drive its profit to zero. Therefore, it must sell in the state where rival is of L type, and thus firm 2 of type L must earn zero gross profit in equilibrium. If firm 2 of type H sells at any price above c_L with positive probability, it will be imitated by firm 2 of type L . Therefore, in equilibrium, the price charged by firm 2 of type H does not exceed c_L . The decision problem for firm 1 (which is of type H) is then whether to forsake the market in the state where firm 2 is of type H and sell only in the state where the latter is of type L charging a deterministic price $c_L + V_H - V_L$, or to compete for the market even when its rival is of type H . First, suppose (25) holds. It is optimal for firm 1 to forsakes the market when rival is of type H and therefore, charge $c_L + (V_H - V_L)$ with probability one. Firm 2 of type L sells zero with probability one and follows a mixed strategy whose distribution function ϕ is continuous with support is $[c_L, \infty)$ where

$$\phi(p) = 1 - \frac{c_L + V_H - V_L - c_H}{p + V_H - V_L - c_H}, p > c_L.$$

This distribution function makes firm 1 of type H indifferent between charging $c_L + (V_H - V_L)$ and any price above that. Firm 2 of type H charges c_L with probability one. Next, suppose that (25) does not hold i.e., $\frac{V_H - V_L}{c_L - c_H} < \frac{x}{1 - x}$. In this case, firm 1 follows a mixed strategy that has a mass point at $(c_L + V_H - V_L)$ and a continuous distribution on the interval $[\underline{p}, c_L]$ where \underline{p} is given by

$$\underline{p} - c_H = (c_L + V_H - V_L - c_H)(1 - x).$$

Note that $\underline{p} < c_L$. In particular, the distribution function $F^H(p)$ followed by firm 1 is given by:

$$\begin{aligned} F^H(p) &= 0, p \leq \underline{p} \\ &= 1 - (1 - x) \frac{c_L + V_H - V_L - c_H}{p - c_H}, p \in [\underline{p}, c_L] \\ &= 1 - (1 - x) \frac{c_L + V_H - V_L - c_H}{c_L - c_H}, p \in [c_L, c_L + V_H - V_L) \\ &= 1, p \geq c_L + V_H - V_L. \end{aligned}$$

Firm 2 of type H follows the distribution function $\Gamma^H(p)$ where

$$\begin{aligned}\Gamma^H(p) &= 0, p \leq \underline{p} \\ &= 1 - \left(\frac{1-x}{x}\right) \left[\frac{c_L + V_H - V_L - c_H}{p - c_H} - 1\right], p \in [\underline{p}, c_L) \\ &= 1, p \geq c_L.\end{aligned}$$

Note that firm 2 puts probability mass $\left(\frac{1-x}{x}\right)\left(\frac{V_H-V_L}{c_L-c_H}\right) \in (0, 1)$ on price equal to c_L . Finally, firm 2 of type L follows a mixed strategy with the distribution function ϕ as outlined in above which makes firm 1 of type H indifferent between charging $c_L + (V_H - V_L)$ and any price above that. It is easy to check that given the strategy of firm 2, firm 1 of type H is indifferent between charging $c_L + V_H - V_L$ and any price in $[\underline{p}, c_L]$ and is strictly worse off charging a price in $(c_L, c_L + V_H - V_L)$. On the other hand, given the equilibrium strategy of firm 1, firm 2 of type L can never make strictly positive profit and therefore, has no incentive to deviate from its prescribed strategy; firm 2 of type H is indifferent between all prices in the interval $[\underline{p}, c_L]$ and strictly prefers to not set a price below \underline{p} .

Proof. Next, consider *(ii.b)*. As firm 1 is known to be of type L , it can never sell in the state where the rival firm is of type H which leads to severe competition between L type firms and an outcome where both L types charge their marginal cost while firm 2 of type H sells to all consumers though the latter cannot charge a price above of c_H without being imitated by its own L type; therefore both types of firm 2 charge price equal to c_L . All consumers (strictly prefer to) buy from firm 2. The out-of-equilibrium beliefs assign probability one to the event that firm 2 is of type L if it charges a price above c_L .

Finally, consider *(iv)*. If a firm is of H type, it can always charge a price just below c_L and guarantee itself profit arbitrarily close to $(c_L - c_H)$ in the state where the rival firm is of L . Therefore, the equilibrium profit of the H type firm must be strictly positive which also implies that in any symmetric equilibrium, a firm of H type must sell in the state where the rival is of L type (if H type firms sell only in the state where both firms are of H type, Bertrand price competition will lead to zero profit). This, in turn, implies that L type firms must sell zero in the state where rival is H type and therefore Bertrand competition between L type firms leads to marginal cost pricing

for those firms. Even though consumers would prefer to buy high quality at price slightly above c_H rather than buy low quality at price c_L , a type H firm cannot charge a price above of c_H without being imitated by its own L type (that earns zero profit in equilibrium). The unique symmetric (D1) equilibrium of this game is one where both firms of type L charge price c_L while each firm of type H follows a mixed strategy with distribution function Ψ and support $[(1-x)c_L + xc_H, c_L]$ where

$$\Psi(p) = 1 - \left(\frac{1-x}{x}\right) \left[\frac{c_L - c_H}{p - c_H} - 1\right], p \in [(1-x)c_L + xc_H, c_L].$$

The out-of-equilibrium beliefs assign probability one to the event that firm 2 is of type L if it charges a price above c_L . ■

Proof of Proposition 6. Suppose that (18) holds and that, contrary to the proposition, there exists $d > 0$ and an equilibrium where the type of both firms are fully revealed with probability one at the voluntary disclosure stage. In any such equilibrium, the market outcome is identical to the full information outcome (except for the disclosure cost being incurred) and therefore the equilibrium profit (gross of any disclosure cost) of each firm of L -type is 0 and that of each firm of H -type is at most $(1-\alpha)(c_L + V_H - V_L - c_H)$ as such a firm can make money only if its rival is of L -type. The equilibrium strategy of firm 1 at the voluntary disclosure stage can be one of three kinds: (1) Disclose if, and only if, realized type is H ; (2) Disclose if, and only if, realized type is L , (3) Disclose independent of realized type. Consider case (1). Suppose firm 2 of type H deviates and does not disclose its type. Given the equilibrium strategy of firm 2, firm 1 must then infer that firm 2 is of type L with probability one. It would then be rational for firm 1 to believe that firm 2 would never charge a price lower than c_L which means that independent of firm 2's type, it would never charge a price strictly less than c_L . The deviation strategy of firm 2 of type H would then be to charge a price $c_L - \epsilon$ for $\epsilon > 0$ arbitrarily small. Upon observing this out-of-equilibrium price set by firm 2, consumers must infer that firm 2 is of type H with probability 1 (under D1 criterion as only H type firm could gain by charging price below c_L). All consumers would therefore buy from firm 2 yielding firm 2 of type H a deviation profit $c_L - c_H - \epsilon$ and this exceeds its expected equilibrium payoff for some $\epsilon > 0$ as long as $(1-\alpha)(c_L + V_H - V_L - c_H) - d < c_L - c_H$ which follows from (18); thus, the deviation is gainful. Next, consider cases (2) and (3). Here, firm 1 of

type L earns negative payoff after disclosure (it makes zero profit under full information) while it can certainly ensure zero payoff by not disclosing (and charging c_L in the continuation game).

Next, we show that if (18) does not hold i.e., (20) holds, there is a symmetric equilibrium where each firm voluntarily discloses quality when it is of H type, but not if it is of L type. On this equilibrium path, the price competition game is one of complete information and the outcome is as indicated in part (i) of Lemma 1 and equilibrium payoff of each firm of L -type is 0 and that of each firm of H -type is $(1 - \alpha)(c_L + V_H - V_L - c_H) - d$. The out-of-equilibrium beliefs (after the price setting stage) for a firm does not disclose type is as follows: if it charges price $> c_L$, is is of type L with probability one, and if it charges price $< c_L$, is is of type H with probability one. Note that these satisfy the D1 criterion; the equilibrium profit of L type firm is zero and therefore it has a higher incentive (than H type) to deviate and charge a price above c_L while only a type H firm would find it profitable to charge price below c_L . Suppose that firm 2 of H -type deviates and does not disclose its type. Then, firm 1 believes that it is in a complete information pricing game (in part (i) of Lemma 1) where the type of firm 2 is L for sure. If firm 1 has revealed its type to be H , it will charge price $c_L + (V_H - V_L)$ and expect to sell to all consumers. If firm 1 has revealed its type to be L , it will charge price c_L . If firm 2 of type H charges price above c_L , out-of-equilibrium beliefs of consumers assign probability one to the event that firm 2 is of type L and therefore, strictly prefer to buy from firm 1. So this deviation cannot be gainful. If firm 2 of type H charges a price below c_L , its maximum expected profit is $c_L - c_H$, and this does not exceed its equilibrium payoff $(1 - \alpha)(c_L + V_H - V_L - c_H) - d$ if (20) holds. This completes the proof.

Proof of Proposition 7. (a) In an equilibrium where both firms disclose with probability zero, the equilibrium path at the pricing stage is one where firms signal quality through prices, as described in part (iii) of Lemma 1. In particular, a firm charges c_L for sure if it is of L type and earns zero profit. If a firm is of H -type it follows a mixed strategy with no mass point whose support is an interval $[\underline{p}_H, c_L]$ where $c_H < \underline{p}_H$ and earns equilibrium payoff $(c_L - c_H)(1 - \alpha)$. The out-of-equilibrium consumer beliefs are that a firm charging price $> c_L$ is of low quality with probability one. Clearly, a low quality firm has no incentive to deviate and disclose quality. Suppose a firm, say firm 2, of type H deviates and discloses its true quality. The continuation pricing game is as described in part (ii.a) of Lemma 1, firm 2 charges $c_L + (V_H - V_L)$ with positive probability and at

that price, sells to all consumers only when it's rival is of type L and the deviation profit of firm 2 is

$$(1 - \alpha)(c_L + V_H - V_L - c_H) - d \leq (c_L - c_H)(1 - \alpha), \text{ using (21).}$$

(b) We begin by defining the equilibrium strategies. At the disclosure stage, each firm of type L chooses not to disclose while each firm of type H discloses with probability $p \in (0, 1)$. We will show later how this probability is determined. At the pricing stage, the strategies of firms on the equilibrium path are as follows. If both firms actually disclose their types, then they are both of type H , so that both firms charge price equal to c_H and earn net profit (net of disclosure cost 0). If firm 2 discloses, but not firm 1, then the firms' pricing strategies are as indicated in part (ii) of Lemma 1 where the posterior belief assigns probability $x = \frac{\alpha(1-p)}{1-\alpha p}$, $1 - x = \frac{1-\alpha}{1-\alpha p}$ to H and L types for firm 2. The outcome is symmetric when firm 2 discloses but not firm 1. If neither firm discloses, the firms' pricing strategies are as indicated in part (iii) of Lemma 1 where each firm is of type H, L with probabilities

$$x = \frac{\alpha(1-p)}{1-\alpha p}, 1-x = \frac{1-\alpha}{1-\alpha p}. \quad (26)$$

In this equilibrium, the expected net profit of a firm of L type is 0 and no such firm can unilaterally deviate and gain strictly positive net profit. All we need to show now is that there is a value of $p \in (0, 1)$ so that each firm of type H is indifferent between disclosure and non-disclosure if its rival plays according to the prescribed strategies. Given the strategy followed by firm 1, the reduced form expected profit of firm 2 of type H when it discloses is 0 if its rival also discloses (and this occurs with probability αp) and $(1-x)(c_L + V_H - V_L - c_H) - d$ if its rival does not disclose (which occurs with probability $1 - \alpha p$), so that the expected net profit from disclosure is

$$\begin{aligned} \pi_2^H(D) &= (1-x)(c_L + V_H - V_L - c_H)(1 - \alpha p) - d \\ &= (1-\alpha)(c_L + V_H - V_L - c_H) - d. \end{aligned}$$

On the other hand, the reduced form expected profit of firm 2 of type H when it does not disclose, but its rival does disclose (this occurs with probability αp) is $(c_L - c_H)$, if (25) holds, and $(1-x)(c_L + V_H - V_L - c_H)$, otherwise; if the rival firm does not disclose (which occurs with probability

$1 - \alpha p$), firm 2's expected profit is $(1 - x)(c_L - c_H)$. The expected net profit of firm 2 of type H from nondisclosure can be written as

$$\begin{aligned}\pi_2^H(ND) &= [1 - (1 - p)\alpha](c_L - c_H), \text{ if } p \geq 1 - \left\{ \frac{1 - \alpha}{\alpha} \frac{V_H - V_L}{c_L - c_H} \right\} \\ &= \frac{1 - \alpha}{1 - \alpha p} [p\alpha(V_H - V_L) + (c_L - c_H)], \text{ if } p < 1 - \left\{ \frac{1 - \alpha}{\alpha} \frac{V_H - V_L}{c_L - c_H} \right\}\end{aligned}$$

where the second line is valid only if $\frac{V_H - V_L}{c_L - c_H} < \frac{\alpha}{1 - \alpha}$. It is easy to check that

$$\frac{1 - \alpha}{1 - \alpha p} [p\alpha(V_H - V_L) + (c_L - c_H)] - [1 - (1 - p)\alpha](c_L - c_H) \rightarrow 0$$

as $p \rightarrow 1 - \left\{ \frac{1 - \alpha}{\alpha} \frac{V_H - V_L}{c_L - c_H} \right\}$. Thus, $\pi_1^H(ND)$ is continuous in p on $[0, 1]$. As $p \rightarrow 0$, $\pi_1^H(ND) \rightarrow [1 - \alpha](c_L - c_H)$ so that, using (22), $\pi_1^H(D) - \pi_1^H(ND) \rightarrow (1 - \alpha)(V_H - V_L) - d > 0$. On the other hand, $\lim_{p \rightarrow 1} \pi_1^H(ND) \geq (c_L - c_H)$ so that, using (22),

$$\lim_{p \rightarrow 1} [\pi_1^H(D) - \pi_1^H(ND)] \leq (1 - \alpha)(V_H - V_L) - \alpha(c_L - c_H) - d < 0.$$

From the intermediate value theorem, there exists $\hat{p} \in (0, 1)$ such that $\pi_1^H(D) = \pi_1^H(ND)$ for $p = \hat{p}$. This completes the construction of the equilibrium.