Consumption, Production, Welfare B:
General Equilibrium: exchange

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Exchange economies

- Individual $i$ possesses $(\bar{x}^i_1, \bar{x}^i_2)$ of two goods.
- If there are two individuals, we can make an Edgeworth box of size $(\bar{x}^1_1 + \bar{x}^2_1, \bar{x}^1_2 + \bar{x}^2_2)$
Are there gains from trade?

• People have their initial endowments
• Arrows indicate higher utility levels for respective individuals (red for 1; blue for 2)
• Initial endowments are not Pareto-efficient allocation
• Gains from trade (for one or even both)
Contract curve

- Set of allocations such that all gains from trade are exploited (set of Pareto-efficient allocations)
- No claim about fair allocation
- Can be defined as solution to $\max U_i(x_1^i, x_2^i)$ such that $U_j(x_1^j, x_2^j) \geq \bar{U}_j$ for any $\bar{U}_j$
- It has to be the case that the slope of indifference curves should be the same
- If there are no cost of bargaining, then you may expect individuals to agree on a point on the contract curve
Price ratio and Contract curve

- For any (yellow) point on the contract curve that is agreed upon you can define an implicit price ratio (green line) that agents trade for.
- If they trade $x$ apples for $y$ oranges, then one apple costs in terms of oranges $\frac{y}{x}$.
- Note that prices themselves are not defined, only price ratio.
Competitive exchange

• In a competitive exchange economy, individuals take prices as given and maximize utility given these prices and their endowment.
• In an equilibrium of such an economy, price ratio has to be such that demand has to be equal to supply for both commodities (or, in other words, individual desires have to be realized).
• Can the green price ratio line indicate competitive equilibrium prices (ratio)?
• No, demand for commodity 2 (1) is larger (smaller) than supply.
Does competitive exchange equilibrium exist? I

- At price ratio 1, excess demand for good 2
- At price ratio 2, excess supply of good 2
- If demand is a continuous function of price ratio, there must be a price ratio where demand and supply for good 2 are in equilibrium
Does competitive exchange equilibrium exist? II

- What about equilibrium for good 1?
- Budget line for individual 1 can be represented as $p_1 x_1^1 + p_2 x_2^1 = p_1 \bar{x}_1^1 + p_2 \bar{x}_2^1$
- And for ind. 2 by $p_1 x_1^2 + p_2 x_2^2 = p_1 \bar{x}_1^2 + p_2 \bar{x}_2^2$
- Equilibrium for commodity 2 implies $x_2^1 + x_2^2 = \bar{x}_2^1 + \bar{x}_2^2$
- But then it follows that $p_1 (x_1^1 + x_1^2) = p_1 (\bar{x}_1^1 + \bar{x}_1^2)$, or equilibrium for good 1.
- Walras’ Law: summing budget constraints implies that the values of excess market demands must sum to zero. Thus, if there is equilibrium in (n-1) markets, then from the budget constraints it follows that there is equilibrium in the last market.
Welfare economics I

• Where should this competitive equilibrium be located? On or off the contract curve?

• **First welfare theorem**: a competitive equilibrium is Pareto efficient (on contract curve)

• „Proof“: for all individuals and all goods $j$ and $k$ utility maximization given prices implies $-\frac{\partial U_i}{\partial x_k} = -\frac{p_k}{p_j}$. As prices are the same for everyone, this implies slopes of indifference curves of all individuals must be equal. But that is definition of contract curve.

• Outcome can be „unfair“, depending on the initial endowments
Welfare economics II

• Can any point on the contract curve be a competitive equilibrium?
  – (points off the contract curve cannot be outcome of competitive equilibria)
• **Second welfare theorem**: if you can redistribute the initial endowments, then any point on the contract curve can be the outcome of a competitive market.
  – (competitive markets not necessarily in conflict with whatever notion of fairness one has)
  – Choose price ratio such that you „separate“ indifference curves of individuals
  – Choose initial endowment point on this separating line
Core Allocations

• The core is defined as the set of allocations such that no coalition can do better and rejecting the allocation and just trade among themselves

• Core is a concept from cooperative game theory
  – If a group of individuals can make binding, costless agreements (negotiations), then they would choose a point in the core
  – Example glove game: suppose there are three individuals, to with a left hand glove only, one with a right hand glove only. A pair, and only a pair, is worth 1, -. How should the 1 € be divided for the allocation to be in the core?
  – What about the glove game with $n$ left hand gloves and $m$ right hand glove individuals, and 1 € for each pair.

• What is the core in the exchange economy?
  – Core has to be part of contract curve
Orange segment is core of our earlier economy
Core and larger Economies I

• Suppose we multiply the economy and have two of each type of individuals.

• Will the set of core allocations (what each individual will get) be larger, smaller?
  – Certainly not larger: individuals will never agree to get less than the utility level of their initial endowment, and two individuals will always get on their contract curve
  – Smaller? Consider the following thought experiment
Core and larger Economies II

- Blue is endowment point, green is on contract curve (in core) with two individuals. Why is it not in the core with four individuals?
Core and larger Economies III: argument

- Consider a coalition of two individuals 1 and one individual 2. These three individuals can make themselves better off when trading among themselves. How?
  - Leave second individual 2 at her endowment point
  - Give first individual 2 the allocation at green point
  - Share the burden of giving this one individual 2 the green point with both individuals 1.
  - Thus, compared to the green point, both individuals 1 are strictly better off, one individual 2 is not worse off. These three can block duplicating the green point – you can actually make all three better off.
Infinite duplication of the economy

- You can copy the same economy many times. What will be the core?
- Previous argument suggests that if we draw a straight line from the contract curve through the endowment point, it should not be possible to make one type of individuals better off on this straight line than on the point on the contract curve.
  - Otherwise, you should be able to replicate the economy and find a group that can become better off
- Core of a very large economy is the competitive equilibrium (and vice versa)