

Stable Service Patterns in Scheduled Transport Competition

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Abstract

In a horizontal product differentiation model, it is shown that a stable service pattern in scheduled transport competition only exists if consumers are sufficiently sensitive to the quality of non-scheduling service characteristics. Since this sensitivity is related to travel distance, the modelling results can explain why competition in local public transport results in unstable service patterns while competition in longer-distance air transport does not have the same problem.

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1.0 Introduction

In Western Europe, most public and private sector companies in local and regional public transport have long-lasting exclusive rights that are increasingly tendered for (for example, Sweden and France). Bus companies in Great Britain are the exception. In 1985, the British government removed quantity regulation from the provision of local bus services, allowing bus operators to compete freely. In addition, services that are not produced in the market are tendered (such as late night and Sunday services). Consequently, British operators, including those in metropolitan areas (except London), have been subject to on-the-road competition since 1986. This experience with on-the-road competition in British urban areas led policy makers to believe that competition eventuates in unstable service patterns and low levels of integration of services, so that competitive tendering of exclusive rights has become the leading concept for opening the market in public transport.¹

This approach to competition in urban public transport is also reminiscent of some ancient practices of bus operators/drivers. Competition between privately-operated buses had led to unsafe situations due not only to the roadworthiness of vehicles, but also to the on-the-road behaviour of bus drivers and the scheduling strategies of operators trying to outsmart each other to get customers on board. These consumers generally boarded the first bus that arrived and bought a ticket on the bus. These practices led to the regulation of public transport in Western Europe around 1930. Foster (1985) and Foster and Golay (1986) have given an overview of practices before regulation. These practices can be distinguished on the basis of whether they relate to scheduling or to behaviour on the road.

A familiar practice with regard to scheduling behaviour is ‘head running’, when an operator schedules a bus just before a rival’s. Other strategies operators use for gaining market share and/or forcing the competitor out of business that are closely related to ‘head running’ are ‘schedule-matching’ — putting a bus at the same published time as a rival’s, ‘nursing’ — scheduling a bus that accompanies a rival’s bus and getting as much traffic away from it as possible, and ‘blanketing’ — nursing with two buses (one before and one after the rival bus). These strategies clearly lead

¹Proposal from the European Commission of 26 July 2000 for a Regulation of the European Parliament and of the Council on action by Member States concerning public service requirements and the award of public service contracts in passenger transport by rail, road and inland waterway COM(2000)7 provisional, pp. 5 and 9.

to unstable service patterns. In reaction to such practices by a competitor, an operator can always improve its position by choosing another time of departure.

'Head running' also creates counterpart behaviour on the road. 'Hanging back', 'hanging the road' or 'crawling' means that buses go slowly so as to pick up as much traffic as possible that would otherwise go with the bus behind. A variant is 'waiting' at a bus stop until the bus has a fuller load or until the next bus comes over the horizon. 'Passing' or 'overtaking' means that a bus that is behind tries to get in front of the competitor. Related to this are 'tailing', namely keeping just behind another bus to overtake it or 'cut in', and 'chasing', with the same objective of overtaking a bus in front or 'cutting in'. These practices are likely to result in 'leapfrogging' or 'jockeying', a situation in which buses continually pass and re-pass each other. Alternatively, a bus driver can decide to go 'missing' at a bus stop or 'racing', that is, missing several bus stops. Quite similar is 'turning' a bus before the end of the route and driving it back in the opposite direction. These last strategies are not only relevant with regard to scheduling competition but may also occur if demand at certain places along the line is expected to be low.

There is persistent anecdotal evidence that these practices recurred in the early years of British deregulation. However, a microanalysis of these practices has never been carried out. The effects of deregulation have only been studied at a sector (meso) level (for example, Mackie *et al.*, 1995; Tyson, 1992) and it has become increasingly difficult to carry out a microanalysis. Operators learned quite fast that merger and acquisition were superior strategies to competition, resulting in market concentration. There are currently five bus groups dominating the UK market, with territorial monopolies throughout the country, allowing locally based small operators to operate some fringe services. The market can be considered to be settled. Hence, it appears that competition on the road was not sustainable. Sector studies have given some indication of instability. An important indication is the large number of changes in services that operators register. Tyson (1992) indicates that in the seven metropolitan areas a total of 9,628 registration changes were made in 1990 (and 9,566 in 1989).

This instability of service provision does not appear to be a problem in air transport, also a scheduled transport mode. Air transport in the United States was deregulated in 1978 followed by intra-European (European Union) air transport in 1997. In both cases, unstable scheduling did not surface. For interregional bus services in Great Britain there is a mixed pattern. The initial positive experience with the deregulation of interregional bus services in 1980 gave rise to the deregulation of local and

regional bus services. The British government argued that fares had been reduced, new services had been provided, bus usage had increased and better quality services had been introduced, and as a consequence proposed to extend deregulation to all bus services (Banister, 1985). However, analysis of the results over a longer period of time showed that competition did not always lead to stable service patterns, particularly for shorter distances (Douglas, 1987). Comparison of these experiences with competition in different scheduled transport modes gives the impression that scheduling stability depends on the length of the journey.

Consumers can be assumed to select an operator on the basis of three factors: scheduling, price, and quality. It is not yet clear how operators trade-off these factors in competition, whether they copy each others' strategies, or whether they successfully attempt to differentiate on the basis of one or more factors. There are a number of intuitions with regard to scheduling, pricing, and quality strategies that need to be explained in relation to each other.

Scheduling seems to provide little opportunity for operators to distinguish themselves. An operator can easily copy departure times, frequency, and origin–destination pairs from another operator. Operating the same types of vehicles between the same nodes and on the same routes do not allow for many elements of sustainable differentiation between operators. These outputs only become distinctive if the size of operations differs. Providing a larger network increases the chance that a service is available that corresponds to the specific basic transport needs of a consumer. Cancian *et al.* (1995) showed that competition between television news programmes on the basis of choosing broadcasting times alone is insufficient to create a stable equilibrium (Nash, in pure strategies). Their model assumes a circular market with a uniform distribution of consumers along the circle, consumers who can only view news programmes at times that are later than their preferred time (that is, after coming home from work, study, and so on), and two stations that both have to choose a broadcasting time. This model shows that each station wants to broadcast just before its competitor's in order to catch the largest audience possible for their news programme. A simple parallel can be made with scheduled transport. The model would then result in operators who want to schedule their departures just in front of their competitors.

In scheduled transport, it has been argued that operators do not compete in prices. Mackie *et al.* (1995, p. 241) state for local bus transport that '... fares are not a particularly potent marketing weapon. On a typical 2 mile journey with a full cash fare of 50 pence, a ride time of 15 minutes and walk and wait time of 15 minutes and a value of time of 2 pence per minute of in-vehicle time, fare is less than one third of generalized costs.

The impulse to board the first bus is strong; correspondingly the first marketing imperative is on service rather than on fare.' This hypothesis is based on the observation that fares did not decrease after deregulation, as was expected.

Another hypothesis given for the same observation is that (ibid.) '...operators recognize that fare competition is mutually destructive. Reaction periods are short; the serious entrant knows that price initiatives will be instantly matched by the incumbent', indicating that fare competition is too strong a marketing weapon in the sense that even a few pence price difference would become a highly visible and tangible sales advantage that no operator can concede to its competitor. This hypothesis has also been expressed as an argument against the deregulation of US domestic air transport. Although price competition was one of the main arguments for deregulation, Brenner (1975) argued that an operator usually finds it less costly to follow a fare reduction and remain competitive rather than to lose market share.

Intuitively, the first intuition seems to be relevant for short-distance travel in which a few per cent difference in fares is insignificant in absolute terms, while waiting time at the bus stop is a significant component of total travel time and considered rather unpleasant. The second intuition seems to be relevant for long-distance travel in which a few per cent difference in fares is a significant amount in absolute terms, while waiting time constitutes only a small share of the total travel time.

With regard to quality elements, one of the greatest challenges for any operator is to establish such product differentiation in a meaningful fashion and by doing so to create consumer loyalty that determines consumers' travel behaviour. Many of the elements in which differences exist are, by their nature, subjective. Examples are friendliness and courtesy of personnel, vehicle type, interior design, seats, meals/snacks, lounges, advertising image, and so on. Differentiation gives consumers different preferences for the services of one operator over others. Long-distance trips differ from short-distance trips in this respect. Consumers are more sensitive to quality aspects in long-distance trips than in short-distance trips, so operators in long-distance passenger transport are better able to differentiate themselves.

These intuitions will be clarified by putting them in their context. It will be demonstrated that the sensitivity to quality, without excluding other factors that may also play a role, can explain the difference in scheduling stability between long-distance and short-distance scheduled transport. This will be done by analysing a circular horizontal differentiation model in which two scheduled transport operators compete for consumers. In contrast to a similar circular horizontal differentiation model used by Cancian

et al. (1995), operators are not only able to choose times of departure, but also fares and quality of service offered. Horizontal product differentiation implies that there is no univocal ranking of products among consumers. For example, different consumers have different preferences for the colour design of vehicles. By contrast, in the case of vertical differentiation all consumers have the same preference ranking over alternatives. For example one can safely assume that all consumers prefer more seating space. Basically, the differentiation in quality that is considered in this paper has both horizontal and vertical aspects. In order not to complicate the analysis too much, we focus on horizontal differentiation only. Probably the most dominant factor differentiating operators are consumer loyalty programmes and they create horizontal differentiation between operators. On the basis of Neven and Thisse (1990), Ansari *et al.* (1998), and Irmen and Thisse (1998), it can be expected that focusing on vertical differentiation does not qualitatively affect the results we obtain concerning scheduling decisions.

The rest of this paper proceeds as follows. Section 2 outlines the specification of the model. Section 3 analyses operators' scheduling behaviour. Section 4 gives an interpretation of the modelling results. Finally, Section 5 draws some conclusions. Technical extensions of the model and details of the proofs can be found in Appendices A and B, respectively.

2.0 The Model

Stability in competition is analysed through a horizontal product differentiation model. This model describes a situation in which two operators compete for consumers on a fixed origin–destination pair. Both operators are assumed to use similar production technology. Consumers' preferences for these services are uniformly distributed over a two-dimensional location space. Their preferences are indicated by (x_i, y_i) . The x -axis takes the form of a circumference and indicates a consumer's specific preferred time of departure. This circumference realistically presents any repetitive time segment (such as an hour or a day) of the combined schedule of both operators. The circumference is $x \in [0, 1]$. For simplicity, both operators operate only one service or departure per time period $x \in [0, 1]$. The y -axis, the width of the circumference, indicates specific consumer tastes with regard to other aspects of product differentiation. The width of the circumference is assumed to be $y \in [0, 1]$. This gives an upper limit to the differentiation that is considered. Both operators determine their products by choosing departure time x and their y .

Consumers incur costs for a deviation from their preferred departure time (x -axis). The non-availability of a departure at the preferred time creates a disutility. This waiting of a consumer for a departure is subject to a directional constraint in both short-distance and long-distance transport. In local public transport consumers often do not know the timetable, but they have a notion with regard to the level of frequency. Given a high frequency of departures, they simply go to a stop at their preferred travel time and wait for a vehicle to arrive. Upon arrival at the stop, a consumer waits for subsequent vehicles to arrive, those that have already passed the stop are gone. Consumers take departures following their arrival at the place of departure, which is a backward constraint on consumers' waiting. This travel behaviour is often implicitly or explicitly assumed in public transport models (Evans, 1987). In long-distance transport (such as air travel) when frequency is not that high, consumers know the timetable and plan their trip on the basis of the required time of arrival at the destination. This situation has also a directional constraint on consumers' travel behaviour. A consumer can only leave home or work earlier in order to take departures before the one that arrives at the preferred time (for example to make an appointment). Later departures are too late. A consumer can only take departures that arrive before the consumer's required arrival time (see Cancian *et al.*, 1995), which implies a forward constraint to consumers' waiting. Our model analyses the case of a backward constraint. Using a forward constraint would give rise to mirror image results for the location strategies of operators on the circumference.

Consumers also incur costs for a deviation from their preferred quality (y -axis). Deviations from preferred quality are assumed to imply quadratic costs. Perceived discomfort/irritation can be considered to increase non-linearly (that is, convex) as the deviation from preferred quality increases. In Appendix A it is shown that there is no subgame perfect Nash equilibrium in pure strategies if these costs are taken to be linear, creating an unstable service pattern.

By contrast, linear waiting costs are assumed for the non-availability of a departure at the time desired. The value of waiting time is based on the value of alternative uses of time (opportunity costs). The value of alternative uses of time can often be related to salary, or in the case of goods transport, the interest cost of capital. These costs are linear in time.

The prices charged, the deviation from the preferred departure time (x -axis), and the deviation from preferred quality, can be traded off against each other. If the departure time of a service deviates from a consumer's preferred time, the consumer incurs costs of t per time period x . Similarly, if the service deviates from a consumer's preferred variety, the consumer

incurs costs of ϕ per unit y . Consequently, the utility U for a consumer i located at (x_i, y_i) and travelling with operator 1 or 2 respectively are:

$$U_i(1) = V - p_1 - t(1 - x_i) - \phi(y_1 - y_i)^2$$

$$U_i(2) = \begin{cases} V - p_2 - t(x_2 - x_i) - \phi(y_2 - y_i)^2 & \text{if } 0 < x_i \leq x_2 \\ V - p_2 - t(1 + x_2 - x_i) - \phi(y_2 - y_i)^2 & \text{if } x_2 < x_i \leq 1 \end{cases},$$

where V is the maximum willingness to pay, p_1 and p_2 are the prices of operators 1 and 2 respectively, x_2 is the time difference between both operators' departures, and y_1 and y_2 are the respective locations of operator 1 and 2 with respect to the quality they offer. The maximum willingness to pay reflects the existence of competition from other modes and competition between alternative ways of spending time (not all requiring transport). For simplicity, it is assumed that V is sufficiently large. The x -axis locations of both operators' departures are normalised around the location of departure of operator 1. This implies that the location of operator 1, x_1 , is 0 by assumption and that the location of operator 2, x_2 , is reinterpreted as the time between both operators' departures. This time is measured as the clockwise distance of operator 2's location of departure from operator 1's location of departure. With regard to the y -axis locations, it is assumed that $y_2 \geq y_1$. This assumption does not influence the results in a qualitative way because quality differentiation is horizontal.

Since both operators provide a single departure and share identical production technologies, the production costs of both operators are assumed to be C . These production costs can be left out of consideration because they do not qualitatively affect the strategies of operators. Consequently, profits are given by $\pi_1 = p_1 D_1$ and $\pi_2 = p_2 D_2$ where D_1 and D_2 are the demands of operator 1 and operator 2 respectively. The decisions of operators are modelled in a dynamic two-stage game. In the first stage both operators simultaneously determine their products by choosing locations x_2, y_1 , and y_2 . In the second stage operators choose prices. This order reflects the fact that prices are easily changed at very short notice. Scheduling decisions and quality are more difficult and require more time to implement.

The two stages of the game are subgames of the overall game. Service patterns will only be stable if, in the overall game, there exists a subgame perfect Nash equilibrium in pure strategies. Dynamic games are solved by backward induction. At stage 1, operators anticipate their optimal strategies at stage 2, and establish their optimal strategies for stage 1 accordingly. Consequently, the two stages of the overall game need to be analysed in reverse order. This generates two questions. First, at stage 2, given any chosen locations of departures and qualities, what are the

equilibrium prices p_1 and p_2 ? Second, at stage 1, what is the optimal choice of locations of departures and qualities for both operators taking the effect on prices into account?

Operators' demands can be established by equating the utility functions for consumers with a preferred departure time of $0 < x_i \leq x_2$ and $x_2 < x_i \leq 1$ respectively. This generates the indifferent consumers who get the same utility from using either the service of operator 1 or that of operator 2, that is

$$\underline{y} = \frac{p_2 - p_1 - t(1 - x_2) + \varphi(y_2^2 - y_1^2)}{2\varphi(y_2 - y_1)} \quad \text{for } 0 < x_i \leq x_2,$$

and

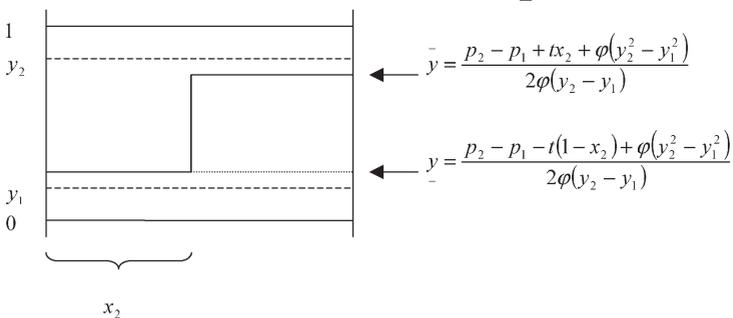
$$\bar{y} = \frac{p_2 - p_1 + tx_2 + \varphi(y_2^2 - y_1^2)}{2\varphi(y_2 - y_1)} \quad \text{for } x_2 < x_i \leq 1.$$

It is noted that $\underline{y} < \bar{y}$.

A graphical representation in case the indifferent consumers \underline{y}, \bar{y} belong to the interval $[0,1]$ is given in Figure 1, where the area above the line segments that position the indifferent consumer represents the demand for operator 2, and the area under these line segments represent the demand for operator 1.

This figure shows that of the consumers with a preferred departure time of $0 < x_i \leq x_2$, a total of $x_2(1 - \underline{y})$ will choose operator 2 because it offers the closest departure. The other consumers with a preferred departure time $0 < x_i \leq x_2$, a total of $x_2\underline{y}$, prefer to pass operator 2's departure and wait for the departure of operator 1. For these consumers the better match of the service of operator 1 with their quality preferences compensates for the longer waiting time. Similarly, of the consumers who have a preferred departure time of $x_2 < x_i \leq 1$ a total of $(1 - x_2)\bar{y}$ prefer

Figure 1
Representation of indifferent consumer if $\underline{y}, \bar{y} \in [0, 1]$



operator 1, which offers the closest departures, and a total of $(1 - x_2)(1 - \bar{y})$ prefer to pass the departure of operator 1 and wait for the departure of operator 2.

3.0 Analysis

The main question is whether there exists a stable service pattern and, if so, under what conditions. This boils down to looking for a subgame perfect Nash equilibrium in pure strategies.

Proposition 1. *If consumers are sufficiently sensitive to quality, that is $\varphi \geq t$, a subgame perfect Nash equilibrium in pure strategies exists, where for any location x_2 , that is, the distance between the departure times of operator 1 and 2, equilibrium locations in quality and fares are given by $y_1^* = 0$, $y_2^* = 1$, and $p_1^* = p_2^* = \varphi$. In such an equilibrium the location x_2 is undetermined. In contrast, if consumers are insufficiently sensitive to quality, there is no subgame perfect Nash equilibrium in pure strategies.*

Proof. As shown above the respective demands if $\underline{y}, \bar{y} \in [0, 1]$ will be

$$D_1 = x_2 \left(\frac{p_2 - p_1 - t(1 - x_2) + \varphi(y_2^2 - y_1^2)}{2\varphi(y_2 - y_1)} \right) + (1 - x_2) \left(\frac{p_2 - p_1 + tx_2 + \varphi(y_2^2 - y_1^2)}{2\varphi(y_2 - y_1)} \right)$$

and

$$D_2 = x_2 \left(1 - \frac{p_2 - p_1 - t(1 - x_2) + \varphi(y_2^2 - y_1^2)}{2\varphi(y_2 - y_1)} \right) + (1 - x_2) \left(1 - \frac{p_2 - p_1 + tx_2 + \varphi(y_2^2 - y_1^2)}{2\varphi(y_2 - y_1)} \right).$$

These demands for operator 1 and operator 2 can be rewritten as

$$D_1 = \frac{p_2 - p_1 + \varphi(y_2^2 - y_1^2)}{2\varphi(y_2 - y_1)} \quad \text{and} \quad D_2 = 1 + \frac{p_1 - p_2 - \varphi(y_2^2 - y_1^2)}{2\varphi(y_2 - y_1)}.$$

It should be noted that these demands do not depend on x_2 . Consequently, both operators are indifferent with regard to the distance between their departures. This implies that an operator has no incentive to change a location in reaction to the location of its competitor.

On the basis of these demands, profits for operator 1 and operator 2 will be

$$\pi_1 = p_1 \left(\frac{p_2 - p_1 + \varphi(y_2^2 - y_1^2)}{2\varphi(y_2 - y_1)} \right) \quad \text{and} \quad \pi_2 = p_2 \left(1 + \frac{p_1 - p_2 - \varphi(y_2^2 - y_1^2)}{2\varphi(y_2 - y_1)} \right).$$

Maximising these profits with respect to fares gives the following reaction curves

$$p_1 = \frac{p_2 + \varphi(y_2^2 - y_1^2)}{2} \quad \text{and} \quad p_2 = \frac{p_1 + 2\varphi(y_2 - y_1) - \varphi(y_2^2 - y_1^2)}{2}.$$

Substituting these reaction curves into each other gives the subgame perfect prices

$$p_1^* = \frac{2\varphi(y_2 - y_1) + \varphi(y_2^2 - y_1^2)}{3} \quad \text{and} \quad p_2^* = \frac{4\varphi(y_2 - y_1) - \varphi(y_2^2 - y_1^2)}{3}.$$

Substituting the subgame perfect prices into the expression for demand and multiplying these by their own prices generates the following equilibrium profits

$$\pi_1 = \frac{(2\varphi(y_2 - y_1) + \varphi(y_2^2 - y_1^2))^2}{18\varphi(y_2 - y_1)} \quad \text{and} \quad \pi_2 = \frac{(4\varphi(y_2 - y_1) - \varphi(y_2^2 - y_1^2))^2}{18\varphi(y_2 - y_1)}.$$

Maximising the profits with respect to the location of quality gives

$$\frac{\partial \pi_1}{\partial y_1} = \frac{[2\varphi(y_2 - y_1) + \varphi(y_2^2 - y_1^2)][\varphi(y_2^2 - y_1^2) - 4y_1\varphi(y_2 - y_1) - 2\varphi(y_2 - y_1)]}{18\varphi(y_2 - y_1)^2},$$

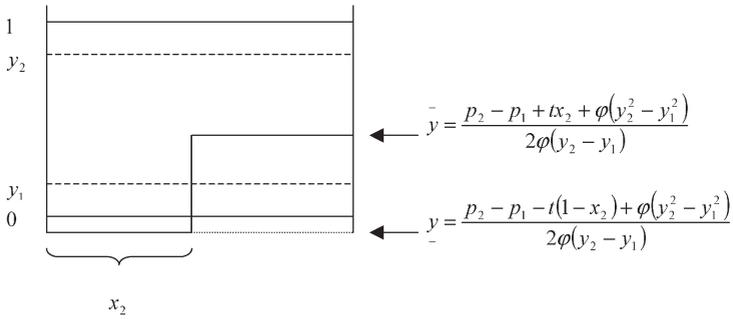
and

$$\frac{\partial \pi_2}{\partial y_2} = \frac{[4\varphi(y_2 - y_1) - \varphi(y_2^2 - y_1^2)][\varphi(y_2^2 - y_1^2) - 4y_2\varphi(y_2 - y_1) + 4\varphi(y_2 - y_1)]}{18\varphi(y_2 - y_1)^2}.$$

The expressions show that $\partial \pi_1 / \partial y_1 < 0$ and $\partial \pi_2 / \partial y_2 > 0$. Hence, operator 1 will choose to offer quality $y_1^* = 0$ and operator 2 will offer $y_2^* = 1$. Substituting these locations into the subgame perfect prices and equilibrium profits gives $p_1^* = p_2^* = \varphi$ and $\pi_1^* = \pi_2^* = \varphi/2$, respectively. An operator cannot generate higher profits by deviating from this differentiation in quality and associated fares if the conditions for this division of demand between both operators are satisfied, that is, $y, \bar{y} \in [0, 1]$. Substituting the equilibrium qualities $y_1^* = 0$, $y_2^* = 1$ and prices $p_1^* = p_2^* = \varphi$ into

$$\underline{y} = \frac{p_2 - p_1 - t(1 - x_2) + \varphi(y_2^2 - y_1^2)}{2\varphi(y_2 - y_1)}$$

Figure 2
Representation of indifferent consumer if ($\underline{y} < 0, \bar{y} \in [0, 1]$)



shows that $\underline{y} \in [0, 1]$ if $\phi \geq t(1 - x_2)$. Doing the same for

$$\bar{y} = \frac{p_2 - p_1 + tx_2 + \phi(y_2^2 - y_1^2)}{2\phi(y_2 - y_1)}$$

shows that $\bar{y} \in [0, 1]$ if $\phi \geq tx_2$. Consequently,

$$\pi_1^*, \pi_2^* = \phi/2 \text{ if } \begin{cases} \phi \geq t(1 - x_2) & \text{for } 0 \leq x_2 < 1/2 \\ \phi \geq tx_2 & \text{for } 1/2 \leq x_2 \leq 1 \end{cases}$$

These conditions will be fulfilled for any $x_2 \in [0, 1]$ if $\phi \geq t$ so that a necessary condition for equilibrium is that the sensitivity to quality is sufficiently large in relation to the perceived inconvenience of ‘waiting’.

It has to be checked whether the operators cannot realise a higher profit than π_1^*, π_2^* by choosing qualities and prices so that the indifferent consumers \underline{y} and/or the indifferent consumers \bar{y} have a preferred quality outside the area defined by $\underline{y}, \bar{y} \in [0, 1]$. Figure 2 shows the situation in which one of the operators deviates from his equilibrium strategy by choosing quality or price so that $\underline{y} < 0$ and $\bar{y} \in [0, 1]$.

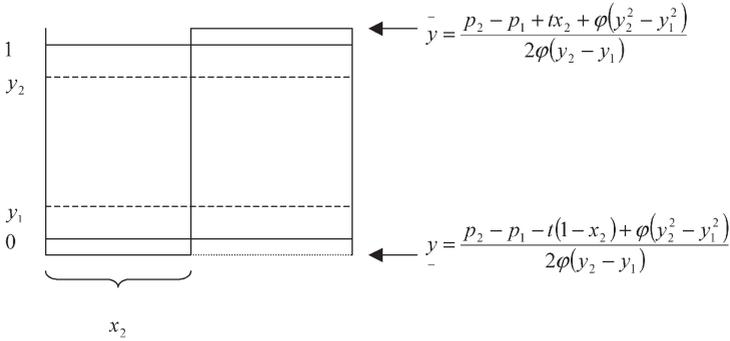
In Appendix A it has been shown that both operators cannot realise higher profits by deviating from their equilibrium strategy by choosing locations and prices so that either ($\underline{y} < 0, \bar{y} \in [0, 1]$) or ($\underline{y} \in [0, 1], \bar{y} > 1$).

If the sensitivity to quality is sufficiently small in relation to the perceived inconvenience of waiting, that is, if

$$\phi < \frac{t}{2(y_2 - y_1)},$$

then $\bar{y} - \underline{y} > 1$ and it must be that ($\underline{y} < 0, \bar{y} \in [0, 1]$), ($\underline{y} \in [0, 1], \bar{y} > 1$), or ($\underline{y} < 0, \bar{y} > 1$). In Appendix A it has also been shown that no Nash

Figure 3
Representation of indifferent consumer if $(\underline{y} < 0, \bar{y} > 1)$



equilibrium in pure strategies exists if $(\underline{y} < 0, \bar{y} \in [0, 1])$ or $(\underline{y} \in [0, 1], \bar{y} > 1)$. The division of demand if $(\underline{y} < 0, \bar{y} > 1)$ is shown in Figure 3.

In this division of demand the respective demands for operator 1 and operator 2 are $D_1 = 1 - x_2$ and $D_2 = x_2$ so that the profits will be $\pi_1 = p_1(1 - x_2)$ and $\pi_2 = p_2x_2$. It can be easily seen from these profits that there are no subgame perfect equilibrium prices. And even if there were, then operators would not prefer the same distance x_2 between departures. Operator 1 would like to have a zero distance, that is, $x_2 = 0$, between both operators' locations, and operator 2 would prefer maximum distance, that is, $x_2 = 1$, between departures. Consequently no Nash equilibrium in pure strategies exists in the division of demand if $(\underline{y} < 0, \bar{y} > 1)$.

Consequently, there is a subgame perfect Nash equilibrium in pure strategies if the sensitivity to quality is sufficiently large, that is, if $\phi \geq t$, but no Nash equilibrium in pure strategies exists if this sensitivity to quality is sufficiently small.

4.0 Interpretation of Results

The difference in consumers' sensitivity to quality could explain why competition in local bus transport results in unstable service patterns while air transport does not seem to have the same problem.

4.1 Short-distance transport

Given the short-distances travelled, operators in local bus transport have very limited scope for meaningful product differentiation that could

make consumers loyal. A consumer will generally take the first bus that arrives at the stop. This occurs in the division of demand of Figure 3. In that case each operator prefers a different distance between departure times and will continuously try to attain his preferred distance. The preferred distance between departures for both operators is conflicting and no subgame perfect equilibrium in pure strategies with regard to the timing of departures exists.

This result corresponds to the generally-shared view on competition in urban public transport in continental Europe. The European Commission, in its proposal for new regulation of the award of public service contracts in public transport by Member States, articulates this view, stating that without exclusive rights 'service patterns are unstable'.² This perception is based on the experiences in Great Britain of the deregulation of local public transport.

Evans (1990) and Mackie *et al.* (1995) observed that after deregulation of local public transport the number of travellers did not increase despite the doubling of service levels (frequencies) and consequent reductions in waiting times. This 'puzzling result' could not be explained by the decline in bus usage due to higher real incomes and lower real motoring costs. One explanation that has been put forward was that frequency increases are compensated by offsetting effects, for example uncertainty caused to passengers by route instability such as irregular intervals, frequently changing timetables, and so on (Evans, 1990; Mackie *et al.*, 1995).

The model for low consumer sensitivity to quality predicts this instability of service patterns. In the division of demand of Figure 3 the preferred location of operator 1 is just before the departure of operator 2 ($x_2 = 0$), while operator 2 prefers a larger distance between departures ($x_2 > 0$). Given $x_2 = 0$, operator 2 will choose a later departure, but in reaction operator 1 will follow in order to reduce the distance to $x_2 = 0$ again, and so on. Basically, every operator would like to be at a bus stop just before its competitor in order to take the whole market. This implies that operators are 'head-running' each other.

With regard to pricing, Mackie *et al.* (1995) stated that pricing is not a particularly potent marketing weapon. The first priority is on service rather than fares as travellers tend to board the first bus. This behaviour of consumers is confirmed by the model for the situation in which an operator's

²Proposal from the European Commission of 26 July 2000 for a Regulation of the European Parliament and of the Council on action by Member States concerning public service requirements and the award of public service contracts in passenger transport by rail, road and inland waterway COM(2000)7 provisional, p. 5.

demand is independent of prices and $D_1 = 1 - x_2$ and $D_2 = x_2$. In this case the model shows that $\partial\pi_1/\partial p_1 \geq 0$ and $\partial\pi_2/\partial p_2 \geq 0$. Hence, operators are competing over the timing of departures rather than pricing. Evans states that (1990, p. 266) ‘The operators match fares, and tacitly collude to maintain the pre-existing fare structure. The operators also tacitly collude to increase fares simultaneously at a rate at least as fast as inflation. Fares remain the same on high-demand as on low-demand routes, and on actively competitive as on non-competitive routes’. The model shows that the upward pressure on prices will only be limited by the maximum willingness to pay. Consequently, the pricing discipline comes from external factors, for example the existence of competition from another mode (such as private car, cycling, walking, and so on) or competition between activities that consumers can undertake (and that may or may not require transport). This clarifies why fares move at a similar rate to inflation and are not related to the level of demand on a route or the level of competition on a route.

4.2 Long-distance transport

In long-distance transport, such as air transport, consumers are more sensitive to quality. Travel times are much longer and, consequently, consumers are more appreciative of comfort. This allows for much greater quality differentiation than in short-distance travel. Hence, consumers will more easily take an alternative flight. Non-price and non-scheduling factors increase in importance for longer distances, as can be seen for air transport in the Table 1.

The factors, as distinguished in the Avmark Aviation Economist, are ‘schedule’, ‘fare’, ‘airline’, ‘airplane’, and ‘other’. ‘Schedule’ and ‘fare’ are equivalent to the endogenous variables ‘schedule’ and ‘price’. ‘Airline’,

Table 1
Most Influential Factors in Choosing a Flight

<i>Factors as distinguished by the model</i>	<i>Factors as distinguished by Source</i>	<i>Length of flight</i>		
		<i>Under 2 hours (%)</i>	<i>2–5 hours (%)</i>	<i>Over 5 hours (%)</i>
Schedule	Schedule	70.1	43.0	19.3
Price	Fare	11.4	13.7	16.5
Quality	Airline	15.5	36.2	53.0
	Airplane	2.5	6.6	10.2
	Other	0.5	0.6	1.0

Source: Avmark Aviation Economist/IAPA in Gialloreto, 1988.

'airplane', and 'other' can be clustered into the variable 'quality'. Pressured by competition, airlines have become increasingly creative and effective in differentiating their services in terms of quality, including the creation of consumer loyalty.

This situation is described by a sufficient sensitivity to quality, that is, $\phi \geq t$, as depicted in Figure 1. Given this division of demand, operators maximally differentiate in quality, that is, $y_1 = 0$ and $y_2 = 1$. This quality differentiation relaxes competition over scheduling. The model shows that an operator cannot improve his position by choosing another time of departure given the time of departure of his competitor. Stronger consumer loyalty eliminates operators' incentive to 'head-run' each other, which creates a subgame perfect equilibrium in timing departures. Intuitively, one would say that an operator who moves to a later point of time increases his market, but sufficiently strong consumers' sensitivity to quality balances this tendency. Consumers tend to wait for the departure of their preferred operator.

With regard to price, the intuition cannot be confirmed that operators do not seem to compete on price because pricing is too strong a marketing weapon. According to the model, operators do compete in price. Further analysis is needed in order to ascertain whether (and if so, why) operators do not seem to compete in price in long-distance transport. One observation (Brenner, 1975) has been that scheduled transport operators tend to rival each other on frequency instead of price. Such competition increases costs, putting upward pressure on prices, which could offset any reduction in price that is normally associated with the introduction of competition.

4.3 Short-distance vs. long-distance

The impact of travel distance on stability can also be seen in the deregulation of 'long-distance' bus services in Great Britain. In 1980, the British government abolished a system of quantity control in the express coach market. The deregulation allowed for free entry and competition on routes served by nationalised carriers. Douglas (1987) made a thorough assessment of the effects of this deregulation on market structure, fares, service quality and costs, using data provided by express coach operators. He showed that, on short-distance connections, independent operators entering the market were unable to develop a significant market share since the two national carriers simply matched or undercut their fares and increased or manipulated departure times (Douglas, 1997). Only 39 per cent of such services to and from London survived during the first few years of deregulation. The national carriers maintained some competitive advantages after deregulation such as sole access to major and centrally

located bus stations, a considerable network of services and an extensive sales network. This made them better able to survive than the new independent operators on the market.

On long-distance connections, however, several independents captured a permanent place in the market. Specialisation, by offering a premier quality service, allowed the independents to build up immunity to the purely fare and scheduling responses of nationalised carriers and thus capture a share of the market (Douglas, 1997). In this way, the independents were especially successful on the Anglo-Scottish routes. The longer travel distance offered operators greater scope for product differentiation. Consequently, more operators were able to provide services even if they departed at the same time. In the first few years of deregulation, 67 per cent of new long-distance services to or from London survived. Hence, product differentiation on long-distance connections made entrants resistant to the pricing and scheduling responses of incumbent operators by creating consumer loyalty, stabilising scheduling competition.

5.0 Conclusion

For operators in scheduled passenger transport, it is a great marketing challenge to create an image that is distinct from their competitors. Operating the same or similar types of vehicles on the same routes does not have many elements of visible, tangible, differentiation vis-à-vis competitors. Many of the elements in which differences do exist are, by their nature, subjective and difficult to prove to the sceptical consumer. Examples are friendliness and courtesy of personnel, interior design, seats, meals/snacks, lounges, advertising image, and so on. Long-distance operators are better able than short-distance operators to realise such product differentiation in a meaningful fashion. Over longer distances, consumers are generally more flexible with regard to departure times and are more appreciative of their preferred quality of service.

In a two-dimensional horizontal product differentiation model with linear waiting costs for departures and non-linear sensitivity to deviations from preferred quality of service, it is shown that a stable service pattern requires consumers to be sufficiently sensitive to differences in quality between operators. In reaction, operators will differentiate their products in terms of quality. This differentiation prevents consumers from simply taking the vehicle that is closest to their preferred departure time. Consumers will be inclined to take an earlier or later departure if they feel more comfortable with that operator or if there are additional benefits provided

by this operator. In contrast, if consumers are not sensitive to quality differences, operators would prefer to schedule their vehicles just before their competitor in order to 'steal' its customers.

On the basis of Neven and Thisse (1990), Ansari *et al.* (1998), and Irmen and Thisse (1998), it can be expected that this result will not change if the dimension of quality differentiation is modelled as vertical differentiation rather than horizontal differentiation. It is clear that the quality differentiation considered has not only horizontal but also vertical aspects. One example is seat width and pitch. For the same price, every consumer would appreciate more space. The most dominant factor differentiating operators is probably an operator's consumer loyalty programme and that creates horizontal differentiation. A formal analysis for the case of vertical differentiation is beyond the scope of this paper.

The sensitivity to quality differences is a plausible factor that could clarify the difference in existence of a stable equilibrium between different modes of scheduled transport. A modal spectrum can be created on the basis of the relevance of quality differentiation for the selection of an operator. The low-relevance end of this spectrum includes examples as container freight operations (such as liner shipping) and local public transport. The high-relevance end would probably include inter-continental passenger air transport. For the low-relevance modes, experience shows that competition results in unstable service patterns. For the high-relevance mode, stability has never appeared to be a problem.

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Appendix A

A Linear Preference for Quality Model

The utility functions change if the costs that consumers incur for a deviation from their preferred quality (y -axis) are linear instead of quadratic. If discomfort increases linearly for larger deviations from preferred quality the utility of a consumer for using the service from operator 1 and operator 2 respectively are

$$U_i(1) = V - p_1 - t(1 - x_i) - \phi|y_1 - y_i|,$$

and

$$U_i(2) = \begin{cases} V - p_2 - t(x_2 - x_i) - \phi|y_2 - y_i| & \text{if } 0 < x_i \leq x_2 \\ V - p_2 - t(1 + x_2 - x_i) - \phi|y_2 - y_i| & \text{if } x_2 < x_i \leq 1 \end{cases}.$$

Proposition 2. *There is no subgame perfect Nash equilibrium in pure strategies if the costs are linear in deviation from the preferred quality.*

Proof. Equating the generalised costs for the intervals $0 < x_i \leq x_2$ and $x_2 < x_i \leq 1$ allows us to establish the demand functions for both operators. A consumer i with quality preference y_i and preferred departure time x_i has the following generalised costs if he uses the service from operator 1 or

2 respectively:

If $0 < x_i \leq x_2$

$$\left\{ \begin{array}{ll} \left\{ \begin{array}{l} U_i(1) = V - p_1 - t(1 - x_i) - \varphi(y_1 - y_i) \\ U_i(2) = V - p_2 - t(x_2 - x_i) - \varphi(y_2 - y_i) \end{array} \right. & \text{if } y_i \leq y_1 \leq y_2 \\ \left\{ \begin{array}{l} U_i(1) = V - p_1 - t(1 - x_i) - \varphi(y_i - y_1) \\ U_i(2) = V - p_2 - t(x_2 - x_i) - \varphi(y_2 - y_i) \end{array} \right. & \text{if } y_1 < y_i < y_2 \\ \left\{ \begin{array}{l} U_i(1) = V - p_1 - t(1 - x_i) - \varphi(y_i - y_1) \\ U_i(2) = V - p_2 - t(x_2 - x_i) - \varphi(y_i - y_2) \end{array} \right. & \text{if } y_1 \leq y_2 \leq y_i. \end{array} \right.$$

If $x_2 < x_i \leq 1$

$$\left\{ \begin{array}{ll} \left\{ \begin{array}{l} U_i(1) = V - p_1 - t(1 - x_i) - \varphi(y_1 - y_i) \\ U_i(2) = V - p_2 - t(1 + x_2 - x_i) - \varphi(y_2 - y_i) \end{array} \right. & \text{if } y_i \leq y_1 \leq y_2 \\ \left\{ \begin{array}{l} U_i(1) = V - p_1 - t(1 - x_i) - \varphi(y_i - y_1) \\ U_i(2) = V - p_2 - t(1 + x_2 - x_i) - \varphi(y_2 - y_i) \end{array} \right. & \text{if } y_1 < y_i < y_2 \\ \left\{ \begin{array}{l} U_i(1) = V - p_1 - t(1 - x_i) - \varphi(y_i - y_1) \\ U_i(2) = V - p_2 - t(1 + x_2 - x_i) - \varphi(y_i - y_2) \end{array} \right. & \text{if } y_1 \leq y_2 \leq y_i. \end{array} \right.$$

Equating these generalised costs generates the consumers who have no preference to using the service of operator 1 or operator 2. This gives the following expressions:

$$\text{If } 0 < x \leq x_2 \left\{ \begin{array}{ll} p_2 - p_1 = t(1 - x_2) - \varphi(y_2 - y_1) & \text{if } y \leq y_1 \leq y_2 \\ \frac{p_2 - p_1}{2} - \frac{t(1 - x_2) - \varphi(y_1 + y_2)}{2} = y & \text{if } y_1 < y < y_2 \\ p_2 - p_1 = t(1 - x_2) + \varphi(y_2 - y_1) & \text{if } y_1 \leq y_2 \leq y. \end{array} \right.$$

$$\text{If } x_2 < x \leq 1 \left\{ \begin{array}{ll} p_2 - p_1 = -tx_2 - \varphi(y_2 - y_1) & \text{if } y \leq y_1 \leq y_2 \\ \frac{p_2 - p_1}{2} + \frac{tx_2 + \varphi(y_1 + y_2)}{2} = y & \text{if } y_1 < y < y_2 \\ p_2 - p_1 = -tx_2 + \varphi(y_2 - y_1) & \text{if } y_1 \leq y_2 \leq y. \end{array} \right.$$

A consumer with quality preference y_i will prefer operator 1 to operator 2 if ‘=’ is changed into ‘>’ and will prefer operator 2 if ‘=’ is changed into ‘<’. The expressions show that only when $y_1 < y < y_2$ is there a division of consumers within an interval ($0 < x \leq x_2$ or $x_2 < x \leq 1$), some of whom prefer operator 1 while others prefer operator 2 and a few are indifferent. In the case of $y \leq y_1 \leq y_2$ and $y_1 \leq y_2 \leq y$ there is no indifferent consumer. For example, if $y \leq y_1 \leq y_2$ all consumers in interval $0 < x \leq x_2$ (x_2) prefer operator 2 if $p_2 - p_1 < t(1 - x_2) - \varphi(y_2 - y_1)$. However, if $y \leq y_1 \leq y_2$ not all x_2 consumers will prefer operator 1 if $p_2 - p_1 > t(1 - x_2) - \varphi(y_2 - y_1)$,

unless $y_1 = y_2$. In this case, if $y_1 < y_2$, the indifferent consumer is actually between y_1 and y_2 , violating the condition $y \leq y_1 \leq y_2$. Hence, if $y \leq y_1 \leq y_2$ or $y_1 \leq y_2 \leq y$, $y_1 = y_2$ is a necessary condition for possible divisions of demand.

On the basis of these expressions for the indifferent consumer, it is now possible to define the following possible divisions of demand between both operators:

If $y \leq y_1 \leq y_2$

$$\left\{ \begin{array}{l} \left\{ \begin{array}{l} D_1 = x_2 \\ D_2 = 1 - x_2 \end{array} \right. \quad \text{if } y_1 = y_2, p_2 - p_1 > t(1 - x_2), p_2 - p_1 < -tx_2 \\ \left\{ \begin{array}{l} D_1 = 1 - x_2 \\ D_2 = x_2 \end{array} \right. \quad \text{if } y_1 = y_2, p_2 - p_1 < t(1 - x_2), p_2 - p_1 > -tx_2. \end{array} \right.$$

If $y_1 < y < y_2$

$$\left\{ \begin{array}{l} D_1 = \left(\frac{p_2 - p_1 - t(1 - x_2) - \varphi(y_1 + y_2)}{2} \right) x_2 \\ \quad + \left(\frac{p_2 - p_1 + tx_2 + \varphi(y_1 + y_2)}{2} \right) (1 - x_2) \\ D_2 = \left(1 - \frac{p_2 - p_1 - t(1 - x_2) - \varphi(y_1 + y_2)}{2} \right) x_2 \\ \quad + \left(1 - \frac{p_2 - p_1 + tx_2 + \varphi(y_1 + y_2)}{2} \right) (1 - x_2). \end{array} \right.$$

If $y_1 \leq y_2 \leq y$

$$\left\{ \begin{array}{l} \left\{ \begin{array}{l} D_1 = x_2 \\ D_2 = 1 - x_2 \end{array} \right. \quad \text{if } y_1 = y_2, p_2 - p_1 > t(1 - x_2), p_2 - p_1 < -tx_2 \\ \left\{ \begin{array}{l} D_1 = 1 - x_2 \\ D_2 = x_2 \end{array} \right. \quad \text{if } y_1 = y_2, p_2 - p_1 < t(1 - x_2), p_2 - p_1 > -tx_2. \end{array} \right.$$

For $y \leq y_1 \leq y_2$ and $y_1 \leq y_2 \leq y$ the possible divisions of demand and associated conditions are identical. Analysing $D_1 = x_2$, $D_2 = 1 - x_2$, it becomes immediately clear that the conditions $p_2 - p_1 > t(1 - x_2)$ and $p_2 - p_1 < -tx_2$ cannot be fulfilled at the same time. The division $D_1 = 1 - x_2$, $D_2 = x_2$ faces a similar problem. Its associated conditions can be rewritten as $p_2 < p_1 + t(1 - x_2)$ and $p_1 < p_2 + tx_2$. These pricing strategies of operator 2 and operator 1 respectively clearly do not constitute equilibrium in the pricing subgame, so prices will not be stable.

This leaves us with the division of demand if $y_1 < y < y_2$. Operators' respective demands can be simplified into

$$D_1 = \frac{p_2 - p_1 + \varphi(y_1 + y_2)}{2} \quad \text{and} \quad D_2 = 1 + \frac{p_1 - p_2 - \varphi(y_1 + y_2)}{2}.$$

Profits are then

$$\pi_1 = p_1 D_1 = p_1 \left(\frac{p_2 - p_1 + \varphi(y_1 + y_2)}{2} \right)$$

and

$$\pi_2 = p_2 D_2 = p_2 \left(1 + \frac{p_1 - p_2 - \varphi(y_1 + y_2)}{2} \right).$$

Maximising these profits with respect to fares give the operators' reaction curves

$$p_1 = \frac{p_2 + \varphi(y_1 + y_2)}{2} \quad \text{and} \quad p_2 = 1 + \frac{p_1 - \varphi(y_1 + y_2)}{2}.$$

Substituting these reaction curves into each other gives the subgame perfect prices

$$p_1 = \frac{2 + \varphi(y_1 + y_2)}{3} \quad \text{and} \quad p_2 = \frac{4 - \varphi(y_1 + y_2)}{3}.$$

Substituting these prices into the expressions for profits gives

$$\pi_1 = \frac{[2 + \varphi(y_1 + y_2)]^2}{18} \quad \text{and} \quad \pi_2 = \frac{[4 - \varphi(y_1 + y_2)]^2}{18}.$$

These profits show that π_1 increases in y_1 and that π_2 decreases in y_2 . Consequently, the equilibrium locations of quality are $y_1 = y_2$. However, these equilibrium locations of quality violate the condition $y_1 < y < y_2$ that defines demand. If $y_1 = y = y_2$ the division of demand collapses into

$$\left\{ \begin{array}{l} \left\{ \begin{array}{l} D_1 = x_2 \\ D_2 = 1 - x_2 \end{array} \right. \quad \text{if } p_2 - p_1 > t(1 - x_2), p_2 - p_1 < -tx_2 \\ \left\{ \begin{array}{l} D_1 = 1 - x_2 \\ D_2 = x_2 \end{array} \right. \quad \text{if } p_2 - p_1 < t(1 - x_2), p_2 - p_1 > -tx_2 \end{array} \right.$$

It has been shown for $y \leq y_1 \leq y_2$ and $y_1 \leq y_2 \leq y$ that the associated conditions for this division of demand cannot be fulfilled at the same time so no equilibrium exists. This outcome suffers from the same instability as Hotelling's principle of minimum differentiation (1929) as demonstrated by d'Aspremont *et al.* (1979). Hence, none of the three divisions of demand generates a subgame perfect Nash equilibrium in pure strategies.

Appendix B

Analysis of Figure 2

Operators can choose prices, locations of departures and qualities so that either the indifferent consumers \underline{y} or the indifferent consumers \bar{y} have a preferred quality outside the area defined by $\underline{y}, \bar{y} \in [0, 1]$. If $(\underline{y} < 0, \bar{y} \in [0, 1])$, the division of demand between operator 1 and operator 2 will be

$$D_1 = (1 - x_2) \left(\frac{p_2 - p_1 + tx_2 + \varphi(y_2^2 - y_1^2)}{2\varphi(y_2 - y_1)} \right)$$

and

$$D_2 = x_2 + (1 - x_2) \left(1 - \frac{p_2 - p_1 + tx_2 + \varphi(y_2^2 - y_1^2)}{2\varphi(y_2 - y_1)} \right).$$

Alternatively, if $(\underline{y} \in [0, 1], \bar{y} > 1)$ the division of demand will be

$$D_1 = x_2 \left(\frac{p_2 - p_1 - t(1 - x_2) + \varphi(y_2^2 - y_1^2)}{2\varphi(y_2 - y_1)} \right) + (1 - x_2)$$

and

$$D_2 = x_2 \left(1 - \frac{p_2 - p_1 - t(1 - x_2) + \varphi(y_2^2 - y_1^2)}{2\varphi(y_2 - y_1)} \right).$$

Proposition 3. *In equilibrium, operators cannot realise higher profits by deviating so that $(\underline{y} < 0, \bar{y} \in [0, 1])$ or $(\underline{y} \in [0, 1], \bar{y} > 1)$.*

It has to be shown that operators cannot increase their profits by deviating from their equilibrium qualities and prices with the result that some indifferent consumers will prefer quality outside the area defined by $\underline{y}, \bar{y} \in [0, 1]$, that is, either $(\underline{y} < 0, \bar{y} \in [0, 1])$ or $(\underline{y} \in [0, 1], \bar{y} > 1)$. In the division of demand if $(\underline{y} < 0, \bar{y} \in [0, 1])$ profits will be

$$\pi_1 = p_1 \left[(1 - x_2) \left(\frac{p_2 - p_1 + tx_2 + \varphi(y_2^2 - y_1^2)}{2\varphi(y_2 - y_1)} \right) \right]$$

and

$$\pi_2 = p_2 \left[x_2 + (1 - x_2) \left(1 - \frac{p_2 - p_1 + tx_2 + \varphi(y_2^2 - y_1^2)}{2\varphi(y_2 - y_1)} \right) \right].$$

Maximising these profits with respect to fares gives the following reaction curves

$$p_1 = \frac{p_2}{2} + \frac{tx_2 + \varphi(y_2^2 - y_1^2)}{2}$$

and

$$p_2 = \frac{p_1}{2} - \frac{tx_2 + \varphi(y_2^2 - y_1^2)}{2} + \frac{\varphi(y_2 - y_1)}{1 - x_2}.$$

Substituting these reaction curves into each other gives the subgame perfect prices

$$p_1^* = \frac{tx_2 + \varphi(y_2^2 - y_1^2)}{3} + \frac{2\varphi(y_2 - y_1)}{3(1 - x_2)}$$

and

$$p_2^* = -\frac{tx_2 + \varphi(y_2^2 - y_1^2)}{3} + \frac{4\varphi(y_2 - y_1)}{3(1 - x_2)}.$$

Using the envelope theorem, the optimal locations for operator 1 can be found by calculating (Tirole 1988, p. 281):

$$\frac{d\pi_1}{dx_2} = p_1^* \left(\frac{\partial D_1}{\partial x_2} + \frac{\partial D_1}{\partial p_2} \frac{dp_2^*}{dx_2} \right) = 0,$$

where

$$\left(\frac{\partial D_1}{\partial x_2} + \frac{\partial D_1}{\partial p_2} \frac{dp_2^*}{dx_2} \right) = \frac{t(2 - 3x_2)}{6\varphi(y_2 - y_1)} - \frac{y_1 + y_2}{6} + \frac{1}{3(1 - x_2)}$$

and

$$\frac{d\pi_1}{dy_1} = p_1^* \left(\frac{\partial D_1}{\partial y_1} + \frac{\partial D_1}{\partial p_2} \frac{dp_2^*}{dy_1} \right) = 0,$$

where

$$\left(\frac{\partial D_1}{\partial y_1} + \frac{\partial D_1}{\partial p_2} \frac{dp_2^*}{dy_1} \right) = \frac{tx_2(1 - x_2)}{6\varphi(y_2 - y_1)^2} + \frac{1 - x_2}{2} - \frac{1 + (1 - x_2)y_2}{3(y_2 - y_1)}.$$

Similarly for operator 2

$$\frac{d\pi_2}{dx_2} = p_2^* \left(\frac{\partial D_2}{\partial x_2} + \frac{\partial D_2}{\partial p_1} \frac{dp_1^*}{dx_2} \right) = 0,$$

where

$$\left(\frac{\partial D_2}{\partial x_2} + \frac{\partial D_2}{\partial p_1} \frac{dp_1^*}{dx_2} \right) = -\frac{t(2 - 3x_2)}{6\varphi(y_2 - y_1)} + \frac{y_1 + y_2}{6} + \frac{2}{3(1 - x_2)}$$

and

$$\frac{d\pi_2}{dy_2} = p_2^* \left(\frac{\partial D_2}{\partial y_2} + \frac{\partial D_2}{\partial p_1} \frac{dp_1^*}{dy_2} \right) = 0,$$

where

$$\left(\frac{\partial D_2}{\partial y_2} + \frac{\partial D_2}{\partial p_1} \frac{dp_1^*}{dy_2} \right) = \frac{tx_2(1-x_2)}{6\varphi(y_2-y_1)^2} - \frac{1-x_2}{2} + \frac{2-(1-x_2)y_1}{3(y_2-y_1)}.$$

It can be easily checked that

$$\frac{tx_2(1-x_2)}{6\varphi(y_2-y_1)^2} + \frac{1-x_2}{2} - \frac{1+(1-x_2)y_2}{3(y_2-y_1)} < 0$$

so that

$$\frac{d\pi_1}{dy_1} < 0$$

and that

$$\frac{tx_2(1-x_2)}{6\varphi(y_2-y_1)^2} - \frac{1-x_2}{2} + \frac{2-(1-x_2)y_1}{3(y_2-y_1)} > 0$$

so that

$$\frac{d\pi_2}{dy_2} > 0.$$

Consequently, operator 1 and operator 2 do not have incentive to deviate from $y_1^* = 0$ and $y_2^* = 1$ respectively.

Substituting these equilibrium qualities into the profit functions gives

$$\pi_1 = p_1 \left[(1-x_2) \left(\frac{p_2 - p_1 + tx_2 + \varphi}{2\varphi} \right) \right]$$

and

$$\pi_2 = p_2 \left[x_2 + (1-x_2) \left(1 - \frac{p_2 - p_1 + tx_2 + \varphi}{2\varphi} \right) \right].$$

These profits can be compared with the equilibrium profits

$$\pi_1^*, \pi_2^* = \varphi/2 \quad \text{where} \quad \begin{cases} \varphi \geq t(1-x_2) & \text{if } 0 \leq x_2 < 1/2 \\ \varphi \geq tx_2 & \text{if } 1/2 \leq x_2 \leq 1 \end{cases}$$

for a change in price of operator 1 or operator 2 with the result that $\underline{y} < 0$. Given $p_2^* = \varphi$ operator 1 will need to charge $p_1 > 2\varphi - t(1-x_2)$ so that $\underline{y} < 0$. It can easily be checked that the resulting for operator 1 is smaller than its equilibrium profit, that is, $\pi_1 < \pi_1^*$. Similarly, given $p_1^* = \varphi$ operator 2 will need to charge $p_2 < t(1-x_2)$ so that $\underline{y} < 0$. Also operator 2 does not increase its profit by doing so as $\pi_2 < \pi_2^*$.

It should be noted that the division of demand if ($\underline{y} < 0, \bar{y} \in [0, 1]$) and the division of demand if ($\underline{y} \in [0, 1], \bar{y} > 1$) are simply mirror images in

which the roles of operator 1 and operator 2 are switched. Consequently, both operators cannot realise higher profits by deviating from their equilibrium qualities and prices.

Proposition 4. *There is no subgame perfect Nash equilibrium in pure strategies in the divisions of demand if ($\underline{y} < 0, \bar{y} \in [0, 1]$) or ($\underline{y} \in [0, 1], \bar{y} > 1$).*

From the expression for $d\pi_1/dx_2$ and $d\pi_2/dx_2$ it can be easily checked that $d\pi_1/dx_2 = 0$ is incompatible with $d\pi_2/dx_2 = 0$:

$$\frac{d\pi_1}{dx_2} = 0 \rightarrow \frac{t(2 - 3x_2)}{6\phi(y_2 - y_1)} = \frac{y_2 + y_1}{6} - \frac{1}{3(1 - x_2)}$$

and

$$\frac{d\pi_2}{dx_2} = 0 \rightarrow \frac{t(2 - 3x_2)}{6\phi(y_2 - y_1)} = \frac{y_2 + y_1}{6} + \frac{2}{3(1 - x_2)},$$

which can only be true if

$$\frac{1}{1 - x_2} = 0.$$

Consequently, there will not be an equilibrium location x_2 that is not also a corner-solution. The existence of a corner solution requires that $x_2 = 0$, $d\pi_1/dx_2 \leq 0$ and $d\pi_2/dx_2 \leq 0$, or $x_2 = 1$, $d\pi_1/dx_2 \geq 0$ and $d\pi_2/dx_2 \geq 0$.

Substituting $x_2 = 0$ into the total derivatives shows that $d\pi_1/dx_2 > 0$. Consequently, there cannot be a corner solution in which $x_2 = 0$. Doing the same for $x_2 = 1$ shows that $d\pi_1/dx_2 > 0$ and $d\pi_2/dx_2 > 0$. However, substituting the subgame perfect prices into the conditions

$$\frac{p_2 - p_1 - t(1 - x_2) + \phi(y_2^2 - y_1^2)}{2\phi(y_2 - y_1)} \leq 0$$

and

$$\frac{p_2 - p_1 + tx_2 + \phi(y_2^2 - y_1^2)}{2\phi(y_2 - y_1)} < 1,$$

and substituting for $x_2 = 1$ shows that these two conditions cannot be fulfilled for $y_1, y_2 \in [0, 1]$ because $\bar{y}, \underline{y} > 1$. Consequently, $x_2 = 1$ cannot be a corner solution either. In this division of demand if ($\underline{y} < 0, \bar{y} \in [0, 1]$), no Nash equilibrium in pure strategies exists. Also no Nash equilibrium in pure strategies exists in the division of demand if ($\underline{y} \in [0, 1], \bar{y} > 1$) because the analysis is similar to the division of demand if ($\underline{y} < 0, \bar{y} \in [0, 1]$).