Going Where the Ad Leads You: On High Advertised Prices and Searching Where to Buy*

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Abstract

An important role of informative advertising is to inform consumers of the simple fact that the shop that advertises sells a particular product. This information may help consumers to save on their search activities: instead of wandering around, a consumer can simply visit the shop that has advertised, knowing that there he can find the commodity he is looking for. The implications of this simple fact have not been studied before. Using game theoretic reasoning in a model that combines consumer search and firms' advertising we show that firms may find it optimal to advertise prices that are higher than non-advertised prices. The important mechanism underlying this result is that advertising lowers the expected search cost for consumers. Through this analysis we provide a new insight into the role of informative advertising.

Keywords: consumer search, informative advertising, pricing strategy

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Abstract

An important role of informative advertising is to inform consumers of the simple fact that the shop that advertises sells a particular product. This information may help consumers to save on their search activities: instead of wandering around, a consumer can simply visit the shop that has advertised, knowing that there he can find the commodity he is looking for. The implications of this simple fact have not been studied before. Using game theoretic reasoning in a model that combines consumer search and firms' advertising we show that firms may find it optimal to advertise prices that are higher than non-advertised prices. The important mechanism underlying this result is that advertising lowers the expected search cost for consumers. Through this analysis we provide a new insight into the role of informative advertising.

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1 Introduction

Imagine yourself attending a conference in a foreign country. You sit with your laptop in a hotel room and realize that when your battery expires you cannot charge it because of the different electricity outlet system. You decide to go out in town to search for an electricity converter and enter a first electronics shop where the charming sales representative tells you that they unfortunately do not carry such an item in their store. The same story repeats in a number of stores, after which you disappointingly go back to the hotel. In a last desperate attempt you ask at the hotel lobby whether they, by any chance, would know a shop where they carry the item you are looking for. Triumphantly, the clerk at the desk tells you that a firm has left an advertisement behind informing people that they carry all different types of electronic converters one may ever wish to use (possibly with the prices at which they sell). You are very happy for this piece of information, immediately go to the shop and are prepared to buy at any (somewhat reasonable) price.

This story contains an element that we believe is important in many markets, not just when hanging around in far away destinations: namely that part of the search activities of people is not about "searching for firms with the lowest price", but rather about "searching for firms that sell the product". This distinction has not been made before. The typical search model only considers situations where all firms in the market carry the product and the only reason for consumers to search (further) is to look for a price-quality combination that better fits the individual's preferences. That is, the literature on consumer search is not about the "real" search activity of consumers when they are uncertain about which firms carry the product.

Another important aspect of the story above is that a potentially important role of advertising is simply to inform consumers about the fact that the advertising firm carries the product, thereby helping the consumer to save on expected search cost. If the uncertainty about which firm carries the product is very large, then the reduction in "real" search cost may be quite significant. This, so the present paper argues, may lead to advertised prices being higher than non-advertised prices. This is contrary to conventional wisdom expressed in the literature on informative advertising, according to which informative price advertising leads to better informed consumers and therefore to more competition and lower prices (see, e.g., Farris and Albion (1980) and Tirole (1998, Section 7.3)). Thus, the paper contributes to the strategic literature on advertising by arguing that in the presence of uncertainty about which firms sell the product, informative price advertising may lead to higher prices compared to the non-advertised prices. This insight is also important in empirical work on advertising. It shows that one should be careful to conclude from an observed positive correlation between advertising and prices that advertising is persuasive, see e.g. Boulding et al. (1994) and Clark (2005).

The mechanism we uncover is potentially important in understanding emerging markets that use multimedia technologies. A first issue in such markets concerns discussions one can find in the popular press and in online discussion groups on personalized advertising. [1] Google's CEO Eric Schmidt has claimed that over the next few years he wants to be able to send personalized ads on GPS-based in-car communication systems to consumers to help them find the shops they are looking for. One interpretation of this would be that personalized advertising helps consumers to get products that are really close to their tastes, i.e., in a world where product heterogeneity is important. Another interpretation of personalized advertising is one where these ads help consumers to economize on their search costs by directing their search activities on the firm(s) from which they received an ad. This second interpretation comes very close to what we analyze in this paper.

A second aspect of these markets relates to the commercial advertisements one sees when using websites, like http://www.allbookstores.com/, where book prices of different online bookshops are quoted. These websites offer the option to search for the lowest price or to buy directly at one particular online bookseller. Apparently, this bookseller is willing to pay for this link suggesting that a significant fraction of consumers does not continue the search, but simply buys immediately at the firm where they know they can buy the book. Something similar happens at well-known search engines as Google and Yahoo, where firms pay to get their firm's website listed, knowing that (a fraction of) consumer are likely to click on one of the first-listed sites first. Firms may exploit this search behavior by charging higher prices to compensate for the advertising expenditures.[2]

This paper studies "searching for the product" and "high prices through informative advertising" in relation. To this end, we develop a three-stage model. In the first stage, firms decide whether or not they want to allocate shelve-space to a particular type of product. Doing so has an opportunity cost of not using that space for having some other commodity on display. We call all firms who decide to carry the product "active firms". In the second stage, active firms decide on their price and on whether they advertise the fact that they carry the product (and the price at which they sell it) by

[1] See, e.g., http://blogs.zdnet.com/micro-markets/index.php?p=131.

[2] The main difference between this example and our paper is that prices are typically not mentioned in these advertisements. Without price advertisement it is, however, easier to see that advertising firms may set higher prices (see Section 6 for more details on this) and the present paper basically makes the additional point that even if prices were advertised, they still can be higher and attract consumers.

sending an advertisement to consumers. In the third stage, after potentially having received some ads, consumers decide on whether or not to search for a firm that carries the product, with a potentially lower price than the firms that advertised. However, if the firm has not advertised, the consumer does not know whether or not the firm carries the product in the first place.

The simplest search model we can imagine that makes the point that advertised prices can be higher than non-advertised prices, even under informative price advertisement, has two types of consumers: low-valuation consumers having low search costs c_L and high-valuation consumers having higher search costs c_H. For simplicity, we normalize c_L to be equal to 0. One may think of the high demand consumers as having high income from demanding jobs to justify the positive correlation between the size of consumers' search cost and their willingness to pay. In previous literature this positive relation between a consumers' willingness to pay and her search costs has been used by e.g. Iyer (1998) and Coughlan and Soberman (2005).

In such a model, there may be many different types of equilibria, depending on the parameter configurations. We focus on equilibria where advertised prices are higher than non-advertised prices. The simplest such equilibrium has the following structure. Low-valuation consumers search all firms and then buy at the lowest-priced firm. As soon as high-valuation consumers receive an advertisement, they buy at the advertising firm. High-demand consumers who get no advertisements do not search for an active firm as they find the probability that these firms do not carry the product too high compared to the search costs they have to make. Non-advertising firms therefore completely concentrate on the low-valuation consumers, whereas advertising firms concentrate on high-valuation consumers. The probability that firms are active and the intensity with which active firms advertise are determined endogenously in such a way that firms are indifferent between being inactive, advertising high prices and not-advertising and setting low prices. We show for which parameter values such an equilibrium exists and in addition we show for which subset of these parameter values the equilibrium is unique. The main result is thus best interpreted as a screening result: high-valuation consumers buy at the advertising firms (if any) at high prices and the low-valuation consumers buy at the non-advertising firms (if any) at low prices.

The paper is, of course, related to the large literatures on consumer search and advertising (see, e.g., the seminal papers by Stigler (1961), Diamond (1971), Stahl (1989, 1994) and Butters (1977)). The main difference with the consumer search literature is, as we mentioned, the uncertainty consumers face of not finding the product at a shop they visit. As Diamond (1971) has shown, the price uncertainty in search models can lead to monopoly prices. Rao and Syam (2001) obtain the same type of result in an advertising model. Here uncertain prices in the sense of unadvertised prices are above advertised prices. Our results are exactly the reverse: known, advertised prices are higher than unknown, unadvertised prices. One of the reasons why the Diamond effect does not play a role in our model is that there is a fraction of consumers with zero search costs.

In the advertising literature the main distinction is between persuasive and informative advertising. As indicated above, the main role of advertising in our model is to inform consumers of the availability of the product at the firm. In this way, advertising creates value to consumers and this value is evaluated differently by different consumers. The value that is created is different from that studied in the literature on persuasive advertising (see, e.g., Dixit and Norman (1978)), however. This persuasive advertising literature basically argues that advertising changes the preferences of individuals and the demand effect that emerges is mainly dependent on psychological factors (exogenous to the model) determining how much people's preferences have been affected. In our paper, preferences of individuals are unaffected and the only effect of advertising, namely the reduction in expected search costs, is endogenously determined.

In the informative advertising literature (see, Meurer and Stahl (1994) and Soberman (2004)), price differentiation sometimes arises from horizontal product differentiation. Informative advertising plays two roles in this context. First, it creates awareness of products so that consumers know what best fits their tastes. This strengthens the product differentiation aspect, giving firms an incentive to raise prices. On the other hand, it also leads to more consumers with full information giving firms an incentive to reduce prices. When product differentiation is important enough, the first aspect dominates the second so that advertising can lead to price increases. In our model, however, all firms are *ex ante* identical and any form of differentiation is thus endogenously created.

The literature that combines consumer search and advertising is much more limited.[3] Robert and Stahl (1993) is the first paper where consumers' ignorance about prices can be resolved by consumers searching for prices or by firms informing consumers about the prices they charge through advertising. Following on their work, Stahl (2000) and Janssen and Non (forthcoming)

[3]Apart from the literature mentioned here, there is also a recent paper by Stivers and Tremblay (2005). Their model is however very different from the standard search models as they model search costs as the wedge between producer prices and consumer prices, very much like the analysis in traditional tax studies. Moreover, they assume that advertising lowers the search costs of consumers. In such a world, they show that it is possible that advertising raises the price the firms ask, while at the same time decreasing the price (including search costs) that consumers have to pay.

check the robustness of the model by investigating the properties of different modeling assumptions. Janssen and Non show that in some equilibria advertised prices may be higher than non-advertised prices. This result in their model is driven by the assumption that less-informed consumers can buy at firms that advertise without incurring search cost, giving advertising firms an advantage above non-advertising firms that have to be searched for. In the present paper we formally explain how this difference in search cost to buy from an advertising and a non-advertising firm can emerge out of the uncertainty consumers face when they visit a shop that did not advertise.

The rest of this paper is organized as follows. Section 2 presents the model. In Section 3 we analyze a simplified version of the model where we show that the result of high advertised prices does not depend on price uncertainty but instead is caused by uncertainty about whether the firm carries the product. Section 4 formally characterizes one equilibrium where advertised prices are higher than non-advertised prices. In Section 5 we will elaborate on the existence of other equilibria and we will show that the equilibrium with high advertised prices is unique in some parameter region. Section 6 concludes.

2 The model

Consider a homogeneous goods market where (at most) two firms produce without incurring production costs.[4] The only cost relevant for our analysis is the opportunity cost S firms face for shelving the product. The decisions firms take are modeled as a three-stage game. In the first stage, firms decide on whether or not to carry the product. We will denote the probability of a firm being active by β. If a firm decides to be inactive it makes no profits or losses.

In the second stage of the model, firms decide simultaneously on their advertising strategy and price. Firms do not know the outcome of the first stage (the decision to be active) and therefore, it is just as if firms play these two stages simultaneously. An active firm can decide to advertise that it sells the product and at which price. Note here that advertising is purely informative: an advertisement informs about existence and price. Advertising is an 'all-or-nothing' decision, that is, a firm either advertises to the complete market or does not advertise at all. The cost of advertising is A. We will denote the probability with which a firm advertises by α. The pricing strategy depends on whether a shop advertises or not. We will therefore specify a pricing strategy conditional on advertising and a pricing strategy conditional on not advertising. Denote by $F_1(p)$ the price distribution conditional

[4]Extending the model to more than two firms does not change the results but only complicates the analysis. Production costs can be easily introduced, but they are simply normalized to be equal to 0.

on advertising and let $F_0(p)$ denote the price distribution conditional on not advertising. We use $\overline{p}_1$ to indicate the highest price and $\underline{p}_1$ to indicate the lowest price in the support of $F_1(p)$. Similarly, $\overline{p}_0$ and $\underline{p}_0$ denote the highest and lowest price in the support of $F_0(p)$.[5]

The 'all-or-nothing' advertising technology used in the model may seem somewhat unrealistic at first sight. Many other advertising models (e.g., Butters (1977), Stahl (1994), Robert and Stahl (1993)) assume that firms choose an advertising reach, indicating the fraction of consumers who are informed by an advertisement. This advertising intensity generally depends on the price chosen, and so a firm's strategy in this context can be denoted by a price distribution $F(p)$ and an advertising function $\kappa(p)$, indicating the advertising reach conditional on a price p. When the advertising costs are linear and given by $A\kappa(p)$ it can be shown[6] that the two formulations are equivalent, meaning that if in the 'all-or-nothing' model α, $F_0(p)$ and $F_1(p)$ are part of an equilibrium then $F(p) = (1-\alpha)F_0(p) + \alpha F_1(p)$ and $\kappa(p) = \frac{\alpha f_1(p)}{(1-\alpha)f_0(p)+\alpha f_1(p)}$ form an equilibrium in the 'advertising reach' model. On the other hand, when $F(p)$ and $\kappa(p)$ are part of an equilibrium in the 'advertising reach' model then $\alpha = \int_{\underline{p}}^{\overline{p}} \kappa(p)dF(p)$, $F_0(p) = \frac{\int_{\underline{p}}^{p} 1-\kappa(p)dF(p)}{1-\alpha}$ and $F_1(p) = \frac{\int_{\underline{p}}^{p} \kappa(p)dF(p)}{\alpha}$ form an equilibrium in the 'all-or-nothing' model. Since the two formulations are equivalent and the 'all-or-nothing' model is easier to analyze, we will use this formulation throughout the paper.

In the third stage of the game, consumers receive the advertisements that are sent and decide on their search strategy. There is a unit mass of consumers with unit demand. Consumers come into two types. A fraction γ has a low valuation θ_L for the product and zero search costs.[7] A fraction $1-\gamma$ has a high valuation θ_H for the product and strictly positive search costs c. We assume $\theta_H - c > \theta_L$. Consumers search sequentially, conditional on the advertisements that are sent. Sequential search means that the consumers first look at the advertisements they have received. They then decide on

[5] Note that the price distributions $F_0(p)$ and $F_1(p)$ can describe randomized price strategies as well as pure strategies. For instance, if advertising firms use the pure strategy of always setting price p^* $F_1(p) = 0$ for $p < p^*$ and $F_1(p) = 1$ for $p \geq p^*$. It is however easy to see that in equilibrium advertising firms always use a randomized price strategy. For an advertising firm there is a strictly positive probability $\alpha\beta$ that the competitor advertises as well and so a standard undercutting argument can be used to show there are no atoms in $F_1(p)$.

[6] Details are available on request.

[7] Note that because of the zero search costs these consumers know all active firms and prices. Firms compete for these consumers and therefore an undercutting argument shows there are no atoms in $F_0(p)$ for $p \leq \theta_L$. In an earlier version, we analyzed a model where the low-valuation consumers had a strictly positive search cost. The equilibrium with high advertised prices is also an equilibrium in this modified model, but we have not been able to prove uniqueness.

whether to visit one additional firm, to buy immediately from the cheapest advertising firm, or not to buy at all. After searching one firm they again decide on whether to visit a second firm, to buy from the cheapest known firm or not to buy at all. Note that consumers only search non-advertising firms, since they already know that advertising firms are active and the ad also tells at which price the active firm sells. We assume that for the high-valuation consumers every first visit to a firm costs c.[8] Visiting an advertising firm, a consumer has to bear the search costs, but is sure to find the product. Visiting a non-advertising firm, a consumer incurs the same search costs, but in this case is not sure to find the product. This implies that the 'real' search cost for searching a non-advertising firm is higher than the search cost for searching an advertising firm.

We will solve this model for a symmetric perfect Bayesian equilibria, the standard equilibrium notion for games with asymmetric information. Symmetry implies that all *ex ante* identical players play the same strategy. Bayesian updating plays a role when consumers form expectations about the probability a firm is active given that it did not advertise.

3 Equilibrium under price certainty

For didactical purposes, and to highlight that not price uncertainty, but uncertainty about product availability, is the key element driving the result mentioned in the introduction, we first consider a version of the model where firms do not freely choose prices, but instead choose between an exogenously given high price p_H and low price p_L, with $\theta_H - c \geq p_H > \theta_L > p_L$. The question we address is whether there exists an equilibrium with the same features as the equilibrium mentioned in the introduction: the high price is advertised, the low price is not advertised and high-valuation consumers do not search. Such an equilibrium has price certainty for searching consumers as all firms that do not advertise set the same price. We concentrate on a symmetric equilibrium and in this case that means that both firms are indifferent between choosing the high price p_H and advertising it and choosing the low price and not advertising it. It is clear that for a symmetric equilibrium to exist, it must be the case that the probability with which a firm is active is strictly smaller than 1 as otherwise a consumer who did not receive any advertisements or received only one advertisement will infer that the non-advertising firm(s) is (are) active and charges a low price, making it profitable to search. Thus, in such an equilibrium a firm has to be indifferent between being active and not being active implying that the equilibrium pay-off should equal 0.

[8]For simplicity, we follow the search literature in assuming "free recall", i.e., consumers do not bear any costs for return visits to firms they already visited once.

We pose that there exists a symmetric equilibrium where advertising firms charge price p_H and non-advertising firms charge a price p_L, where high-valuation consumers buy at an advertising firm without searching and where high-valuation consumers who get no advertisements do not search at all. Low-valuation consumers observe all prices and only buy if a firm sells at p_L.

We now have to check that this is indeed an equilibrium, i.e., that none of the players has an incentive to deviate. We first look at consumer behavior and note that if it is not optimal to search after not receiving any advertisements then it certainly is not optimal to search after receiving one advertisement as the utility of following the ad is positive. We therefore concentrate on the case where high-valuation consumers did not receive any advertisements. If their first search leads to an active firm they will stop searching since a lower price can not be found. If they search once and do not find an active firm the utility of a second search is $-c + \frac{(1-\alpha)\beta}{1-\alpha\beta}(\theta_H - p_L)$.[9] If this expression is below zero, the utility of not searching, consumers will not search a second time and the utility of the first search also is $-c + \frac{(1-\alpha)\beta}{1-\alpha\beta}(\theta_H - p_L) < 0$. Therefore if this inequality holds consumers will not search at all.

Given the strategy of the high-valuation consumers, the pay-offs to a firm choosing p_H and advertising are given by

$$\pi_H = p_H(1-\gamma)(1-\frac{1}{2}\beta\alpha) - A - S$$

and the pay-offs to a firm choosing p_L and not advertising are given by

$$\pi_L = p_L\gamma(1-\frac{1}{2}\beta(1-\alpha)) - S.$$

As $\pi_H = \pi_L = 0$ in the equilibrium we are looking for, we can determine α and β endogenously. It follows that $\beta = 4 - 2\frac{A+S}{(1-\gamma)p_H} - 2\frac{S}{\gamma p_L}$ and $\alpha = \frac{2}{\beta} - \frac{2}{\beta}\frac{A+S}{(1-\gamma)p_H}$. As $0 < \alpha < 1$ and $0 < \beta < 1$ this gives restrictions

$$\frac{A+S}{p_H(1-\gamma)} < 1,$$

$$\frac{S}{p_L\gamma} < 1$$

and

$$3 - 2\frac{A+S}{p_H(1-\gamma)} - 2\frac{S}{p_L\gamma} < 0.$$

[9] Note that consumers only search non-advertising firms and therefore the probability of finding an active firm is given by $\frac{(1-\alpha)\beta}{1-\beta\alpha}$.

Firms should also not have an incentive to deviate. If a firm would set p_H and not advertise, its profits are $-S$ while if it sets a price p_L and advertises this price, its profits are given by

$$p_L(1-\gamma) + p_L\gamma(1-\frac{1}{2}\beta(1-\alpha)) - A - S = p_L(1-\gamma) - A.$$

For a deviation not to be profitable it should be that $p_L(1-\gamma) < A$.

It is easy to find parameter values such that all restrictions are satisfied. For example, take $\theta_L = 1, p_L = 0.9, \gamma = 0.5, A = 0.75, S = 0.375$ and $p_H = 2.5$. Then if we choose θ_H and the search cost parameter c such that $2.5 + c < \theta_H < 0.9 + 2.4c$, all conditions are satisfied.

4 Existence of an equilibrium with high advertised prices

The previous section has shown that the result of high advertised prices is not driven by price uncertainty but instead by uncertainty about the availability of the product. In this section we will construct a symmetric equilibrium of the full model specified in Section 2 where firms are free to choose any price. In this equilibrium again advertising firms set higher prices than non-advertising firms and high-valuation consumers do not search. Intuitively, when high-valuation consumers do not search the non-advertising firms compete for the low-valuation consumers. Advertising firms however also sell to the high-valuation consumers and therefore have an incentive to ask higher prices. Even in such a case high-valuation consumers do not want to search for the lower non-advertised prices as long as the probability of a firm being active, β, is low enough.

Proposition 4.1 *If the following four conditions are satisfied*

1. $(1-\gamma)\theta_L < A < (1-\gamma)(\theta_H - c) - S$
2. $S < \gamma\theta_L$
3. $1 - \frac{S}{\gamma\theta_L} - \frac{A+S}{(1-\gamma)(\theta_H - c)} < 0$
4. $\theta_H(1 - \frac{S}{\gamma\theta_L}) - \frac{S}{\gamma}\ln\frac{\gamma\theta_L}{S} < \frac{A+S}{(1-\gamma)(\theta_H - c)}c,$

then there exists a symmetric equilibrium. In this equilibrium consumer search behavior is characterized by

(i) *Low-valuation consumers buy at the cheapest active firm out of all firms, provided the price is not above θ_L.*

(ii) *High-valuation consumers who receive at least one advertisement buy immediately at the the advertising firm with the lowest price, provided the price is not above* $\theta_H - c$.

(iii) *High-valuation consumers who do not receive an ad do not search.*

Firm behavior is characterized by

(i) *Firms are active with probability* $\beta = 2 - \frac{S}{\theta_L \gamma} - \frac{A+S}{(1-\gamma)(\theta_H - c)}$.

(ii) *Active firms advertise with probability* $\alpha = \frac{1 - \frac{A+S}{(1-\gamma)(\theta_H - c)}}{2 - \frac{S}{\theta_L \gamma} - \frac{A+S}{(1-\gamma)(\theta_H - c)}}$.

(iii) *Non-advertising firms choose prices according to distribution*

$$F_0(p) = 1 - \frac{(\theta_L - p)(1 - \beta(1-\alpha))}{p\beta(1-\alpha)}$$

with $\overline{p}_0 = \theta_L$ *and* $\underline{p}_0 = \theta_L(1 - \beta(1-\alpha))$.

(iv) *Advertising firms choose prices according to distribution*

$$F_1(p) = 1 - \frac{(\theta_H - c - p)(1 - \beta\alpha)}{p\beta\alpha}.$$

with $\overline{p}_1 = \theta_H - c$ *and* $\underline{p}_1 = (\theta_H - c)(1 - \beta\alpha)$.

Proof

We start with some general observations. First, it is easy to see that α and β are between 0 and 1. The restriction $\beta < 1$ is ensured by condition 3. The second part of condition 1 together with condition 2 ensures that $\beta > 0$. The second part of condition 1 also gives that $\alpha > 0$ and $\alpha < 1$ follows from condition 2.

A second observation is that both $F_0(p)$ and $F_1(p)$ are increasing in p, $F_0(\underline{p}_0)$ and $F_1(\underline{p}_1)$ equal 0 and $F_0(\overline{p}_0)$ and $F_1(\overline{p}_1)$ are equal to 1. Therefore $F_0(p)$ and $F_1(p)$ are proper cdf's. We finally note that for $\underline{p}_1 > \overline{p}_0 = \theta_L$ to hold $A + S$ should be larger than $(1-\gamma)\theta_L$. The first part of condition 1 ensures that this is indeed the case.

We next show that under the conditions specified, none of the players has an incentive to deviate. We first consider the search behavior of consumers. It is easily seen that since low-valuation consumers have no search costs they will search all firms and so know all active firms and their prices. For them it is optimal to buy at the cheapest of these firms, provided the price is not above the valuation for the product. High-valuation consumers who get two advertisements know that both firms are active and also know both prices. Again it is optimal to buy at the cheapest of the two firms.

Next, consider the case where a high-valuation consumer did not receive any advertisement. Suppose such a consumer has searched already once and found an inactive firm so that the consumer has to decide whether to search once more. He will not search for a second time when the utility from searching is smaller than the utility from not searching, which gives

$$-c + \frac{\beta(1-\alpha)}{1-\alpha\beta} \int_{\underline{p}_0}^{\theta_H} (\theta_H - p) f_0(p) dp < 0.$$

Integrating by parts and rearranging terms gives that searching a second time is not profitable when $\int_{\underline{p}_0}^{\theta_H} F_0(p)dp < \frac{1-\alpha\beta}{\beta(1-\alpha)}c$. This is the case if

$$\theta_H(1 - \frac{S}{\theta_L \gamma}) - \frac{S}{\gamma} \ln \theta_L \frac{\gamma}{S} < \frac{A+S}{(1-\gamma)(\theta_H - c)} c,$$

which is condition 4.

If it is not optimal to search after having found an inactive firm then it clearly is not optimal to continue searching when the consumer would have found an active firm asking a price p_0 in the first search. So, if a consumer searches one time, he will (under condition 4) certainly not search a second time. However, this implies that the consideration of whether or not to search the first time is exactly identical to the consideration of searching a second time after having found an inactive firm. Therefore, under condition 4 it is indeed not optimal to search at all if no advertisement was received. Note that if it is not optimal to search after not having received any advertisements then it clearly is not optimal to search after having received one advertisement as well.

We next consider the behavior of firms. Let $\pi_0(p)$ denote the profits from not advertising and setting a price p and let $\pi_1(p)$ denote the profits from advertising a price p. Given the consumer behavior specified in the proposition we have for $p \leq \theta_L$

$$\pi_0(p) = p\gamma(1 - \beta(1-\alpha)F_0(p)) - S.$$

Substituting α, β and $F_0(p)$ gives that $\pi_0(p) = 0$ for all $\underline{p}_0 \leq p \leq \theta_L$. For $p < \underline{p}_0$ we have that $\pi_0(p) = p\gamma - S < \underline{p}_0\gamma - S = 0$ and for $p > \theta_L$ the firm does not sell anything so that $\pi_0(p) = -S < 0$. This shows that for a non-advertising firm it is indeed optimal to choose a price between $\underline{p}_0$ and $\overline{p}_0$.

An advertising firm setting a price between θ_L and $\theta_H - c$ makes a profit of

$$\pi_1(p) = p(1-\gamma)(1-\beta\alpha F_1(p)) - S - A.$$

For prices above $\theta_H - c$, it does not generate any sales so that

$$\pi_1(p) = -S - A$$

and for prices below θ_L we have that

$$\pi_1(p) = p\gamma(1-\beta(1-\alpha)F_0(p)) + p(1-\gamma) - A - S.$$

When we substitute the values for α, β, $F_0(p)$ and $F_1(p)$ we see that for $\underline{p}_1 \le p \le \overline{p}_1$ $\pi_1(p) = 0$. Moreover, $\pi_1(p) < 0$ for $p > \overline{p}_1$ and for $p < \underline{p}_1$. So for an advertising firm deviating from $F_1(p)$ is not profitable. We note that in equilibrium both advertising and non-advertising firms make zero profits. Therefore firms are indifferent between advertising and not advertising and between being active and being inactive, making $0 < \alpha < 1$ and $0 < \beta < 1$ part of the optimal strategy. □

The interpretation of the four restrictions used in Proposition 4.1 is fairly straightforward. The first three restrictions guarantee that α and β are between 0 and 1 and that deviating is not profitable. The first part of the first restriction is necessary to guarantee that it is not optimal to advertise a low price of θ_L. The second part of the first restriction ensures that advertising a price $\theta_H - c$ is attractive. The second restriction guarantees that a firm can make nonnegative profit when not advertising. The third restriction basically says that the shelving and advertising costs should not be too small. This is needed to guarantee that firms do not always want to be active, which in turn is needed as otherwise high-valuation consumers want to continue searching after receiving one advertisement with a relatively high price. Finally, the last restriction basically gives a lower bound on the search costs c such that consumers do not want to search.

The first restriction implies that $\theta_H - c > \theta_L$, which can be interpreted as that the difference in valuations between the two types of consumers should be high enough. This restriction is necessary for any equilibrium where advertised prices are higher. To make this clear, suppose to the contrary that $\theta_H - c < \theta_L$. The maximum price that an advertising firm will then ask is still $\theta_H - c$. To see this, note that if an advertising firm would set a price between $\theta_H - c$ and θ_L it would only sell to low-valuation consumers and so there would be no reason to advertise. For non-advertising firms it could however be profitable to set a price above $\theta_H - c$. There are two reasons for this. First, low-valuation consumers are willing to pay θ_L and so a non-advertising firm setting a price θ_L has a probability of $1-\beta$ to be the only firm in the market and to sell to the low-valuation consumers. Second, searching high-valuation consumers are willing to pay θ_H when they find an active non-advertising firm. This difference in willingness to pay between an advertising firm and a non-advertising firm comes from the search costs being sunk costs when visiting a non-advertising firm. It can be shown that when $\theta_H - c < \theta_L$ it is indeed always profitable for a non-advertising firm to ask a higher price than an advertising firm and so our result can not hold anymore.

We now want to check whether all four restrictions can jointly hold, that is, whether there really exists a parameter region where the above equilibrium exists. To do so we set, without loss of generality, $\theta_L = 1$.[10] It is easily checked that the four restrictions can jointly hold when for instance $\gamma = 0.1$, $S = 0.099$, $A = 1$, $c = 0.04$ and $1.3 < \theta_H < 2.5$. Moreover, we would like to establish the following result, arguing that the existence conditions become more easy to satisfy when both θ_H and c are relatively large.

Proposition 4.2 *If for some values A^*, S^*, θ_H^*, c^* and γ^* the equilibrium of Proposition 4.1 exists, then the equilibrium also exist for values A^*, S^*, $\theta_H^* + x$, $c^* + x$ and γ^*, where x is an arbitrary positive number.*

The proof can be found in appendix A. Below we provide an intuitive explanation for the result. The expressions for the strategy of the firms $(F_0(p), F_1(p), \alpha, \beta)$ given in Proposition 4.1 do not depend on θ_H or c in isolation, but instead only depend on $\theta_H - c$, since this is the maximum price that advertising firms can ask. As the first three restrictions are imposed to guarantee that α and β are in between 0 and 1, these restrictions remain unaffected as long as $\theta_H - c$ is unaffected.

Restriction four ensures that consumers do not want to search. Consumers who do not receive any advertisement have to make a tradeoff between incurring search costs c and having a probability of finding an active firm, which would give an uncertain payoff of $\theta_H - p$, where p is the uncertain price being found. Suppose that for some parameters θ_H^* and c^* it is optimal not to search, so the search costs are above the expected payoff. Then if both c^* and θ_H^* increase by x the expected payoff, given by $\frac{\beta(1-\alpha)}{1-\alpha\beta}(\theta_H - p)$, with $\frac{\beta(1-\alpha)}{1-\alpha\beta} < 1$, also increases but with less than x. Therefore, if both c^* and θ_H^* increase by x it is still not optimal to search and restriction four is still satisfied.

[10] Looking at the equilibrium shows that if the parameters θ_L, θ_H, A, S and c are multiplied by a scalar x the same equilibrium emerges, except that the prices are multiplied by x. To ease the analysis we decided to normalize the parameters such that $\theta_L = 1$.

5 Other equilibria

The model we specified in Section 2 has many possible equilibria. It is natural to inquire whether for some parameter values the equilibrium we have identified in the previous section is the unique symmetric perfect Bayesian equilibrium. If the equilibrium is unique, we can legitimately argue that there is a parameter region where advertised prices are higher than non-advertised prices. If such a parameter region does not exist, then there is a possibility that advertised prices are higher, but this cannot be guaranteed. In this section, we do three things. First, we provide some observations on how the reservation price characterizing consumer search behavior is determined. This reservation price is needed in the remainder of this section. Next, we illustrate by means of an example that there are parameter values where the equilibrium described in the previous section exists, but it is not unique. Finally, we prove that when θ_H and c are relatively large, the equilibrium described in the previous section is unique.

As in any consumer search model, we can define a reservation price r by $\int_{\underline{p}_0}^{r+c} F_0(p)dp = \frac{1-\alpha\beta}{(1-\alpha)\beta}c$, where $\int_{\underline{p}_0}^{r+c} F_0(p)dp$ is the benefit of an additional search provided the firm found is active and did not advertise; $\frac{(1-\alpha)\beta}{1-\alpha\beta}$ is the probability a firm is active conditional on that it does not advertise. When high-valuation consumers receive an advertisement they buy immediately when the lowest advertised price is at or below r, while for higher prices they search until they find an active non-advertising firm asking a price at or below $r + c$ or until all firms have been searched. When high-valuation consumers have not received any advertisement they search when $r+c < \theta_H$, they do not search when $r + c > \theta_H$ and they are indifferent between searching and not searching if $r + c = \theta_H$. When they search, they continue to search until they find an active firm asking a price at or below $r + c$ or until all firms have been searched. The derivation of this result follows the usual lines, except that we need to take into account that consumers have to pay search costs when visiting any firm, independent of whether it advertises or not. Moreover, there is a difference between advertising and non-advertising firms in the sense that when a consumer observes the price set by a non-advertising firm, its search costs are already sunk, while it still has to make the search costs just after having observed the advertised price.

Example (Pure Consumer Search). In the previous section we noted that the equilibrium with high advertised prices exists for $\gamma = 0.1$, $S = 0.099$, $A = 1$, $c = 0.04$ and $1.3 < \theta_H < 2.5$. We now show that for these very same parameter values a 'pure search equilibrium' exists where firms do not advertise ($\alpha = 0$) and where all firms are active ($\beta = 1$). In this example we look at the case where $r + c < \theta_L < \theta_H - c$, implying that all high-valuation consumers search. A traditional undercutting argument shows that when

$p > r + c$ there are no atoms and so $\pi_0(\overline{p}_0) = -S$ for any $\overline{p}_0 > r + c$. On the other hand, $\overline{p}_0$ cannot be smaller than $r + c$ either since deviation to $r + c$ would be profitable. For any price $p \leq \overline{p}_0 = r + c$ profits are given by

$$\pi_0(p) = p\gamma(1 - F_0(p)) + p(1 - \gamma)\frac{1}{2} - S.$$

Equating this to $\pi_0(r + c) = \frac{1}{2}(1 - \gamma)(r + c)$ gives

$$F_0(p) = 1 - \frac{(1 - \gamma)(r + c - p)}{2p\gamma},$$

and $\underline{p}_0 = \frac{\frac{1}{2}(1-\gamma)}{\gamma+\frac{1}{2}(1-\gamma)}(r + c)$. Furthermore, $\int_{\underline{p}_0}^{r+c} F_0(p)dp = c$ gives $r = c\frac{1-\kappa}{\kappa}$, with $\kappa = 1 - \frac{\frac{1}{2}(1-\gamma)}{\gamma} \ln \frac{\frac{1}{2}(1-\gamma)+\gamma}{\frac{1}{2}(1-\gamma)}$. This equilibrium holds whenever $\pi_0 > 0$, $\pi_1 < \pi_0$ and $r + c < \theta_L = 1$. Note that the first restriction holds if, and only if, $\pi_0 = \frac{1}{2}(r + c)(1 - \gamma) - S > 0$, while the second holds if $\frac{1}{2}(1 - \gamma)r < A$. Substituting the parameter values that are given above shows that these three restrictions do hold for these values. $\square$

The intuition behind the co-existence of multiple equilibria result is as follows. For the given parameter values, S is very close to γ. In the equilibrium with high advertised prices described in Section 4 the maximum profits from not advertising, realized when the competitor advertises its high price or is not active at all, are $\gamma - S$. When γ is close to S these maximum profits are low and to make not advertising attractive the probability of obtaining these profits should be high. This implies that β has to be low. A low value of β also means that high-valuation consumers have no incentives to search, even when the search costs c are low. The equilibrium with high advertised prices can therefore also exist for low values of c. For such low values of c it is, however, also possible to have equilibria where consumers search (as in the above example). If consumers continue to search, it is not profit maximizing to advertise and a standard consumer search equilibrium emerges as discussed in the example.

In the remainder of this section we will show, however, that this co-existence argument crucially depends on the assumed small value of c. The next proposition shows that when θ_H and c are relatively large, the equilibrium of Proposition 4.1 is unique.

Proposition 5.1 *Fix parameter values $\theta_H^*, c^*, S^*, \gamma^*$ and A^* such that the equilibrium described in Proposition 4.1 exists. Then there exists a positive real number $\widehat{x}$ such that for all $x > \widehat{x}$ the equilibrium described in Proposition*

4.1 exists and is unique for all parameter values $\theta_H^* + x, c^* + x, S^*, \gamma^*$ and A^* .

The existence part of this proposition has already been shown in Section 4. The proof of the uniqueness part can be found in appendix A. Here we will provide an intuitive explanation. The equilibrium we defined in Proposition 4.1 has high-valuation consumers not searching in case they did not receive an advertisement. Other equilibria where consumers do not search are shown not to overlap with the equilibrium with high advertised prices. This is because in these equilibria the consumers behave in the same way and therefore there is enough continuity in the firms' pricing and advertising decision problem to prevent overlap in the equilibrium regions. Overlap can occur between equilibria with different consumer behavior. For example, as we showed above, the high advertised price equilibrium with no consumer search (partly) overlaps with an equilibrium where high-valuation consumers do search. Therefore it is necessary to rule out the co-existence of equilibria where consumers (partly) search, i.e., equilibria with $\beta > 0$ and $\alpha < 1$.[11] We show in appendix A that these equilibria, except for one equilibrium, do not exist for x high enough. The equilibrium that does exist even when x gets infinitely large is shown not to overlap for x high enough. The two main arguments used to obtain this result are as follows. First, as argued before, consumers who did not receive any advertisements need to compare the search costs c with the possible gains from searching, $\frac{\beta(1-\alpha)}{1-\alpha\beta}(\theta_H - p)$. When a constant x is added to both the search costs and θ_H the search costs generally increase more than the gains from searching. Therefore, the higher the constant x the more difficult it is to have an equilibrium where consumers search. Second, when high-valuation consumers search, non-advertising firms may decide to concentrate on high-valuation consumers and since the costs of searching a non-advertising firm are sunk at the moment the consumer has arrived at the shop these firms can ask a price equal to the maximum advertised price plus the search costs c without loosing any customers. When the search costs increase this option of not advertising and setting a high price becomes increasingly more attractive. This pricing strategy is, however, detrimental for consumer search since consumers who expect a high price will not search. So, this second argument exploits the fact that firms that advertise are committed to the price they offer before consumers search, whereas non-advertising firms are not committed. The two arguments together rule out other equilibria for the case where θ_H and c are relatively large.

[11] When $\beta = 0$ no firm is active and searching is not profitable.

6 Conclusion and extensions

The core of the argument developed in this paper centers around the uncertainty consumers face concerning the shops that carry the product they are looking for: some firms do have the product, others do not. This uncertainty is important in explaining consumer search behavior, but so far this type of uncertainty has not been considered in the large literature on consumer search. An important role of advertising in such a situation is to inform consumers that the advertising firm indeed sells the product. Advertising therefore can lower consumers' expected search costs. Since visiting an advertising firm comes with lower expected search costs than finding the product in a non-advertising firm, advertising firms have an advantage above non-advertising firms. In this paper we show that advertising firms can use this advantage to set higher prices. We have argued that the mechanism we uncover may be important in understanding recent developments in emerging markets using multi-media technologies.

We analyzed the case where advertisements contain both information on the availability of the product and on the price the advertising firm asks. As we noted in the introduction, however, in many cases where our argument applies advertisements may not contain price information. It therefore would be interesting to analyze a model where advertisements only contain information on product availability. The analysis of such a model is not straightforward because of complications arising in consumer search behavior. Without price advertisement, consumers not only have to decide on whether they will search, but also on where to search: an advertising firm or a non-advertising firm. Another complicating factor is that when prices are not advertised, advertising firms may exploit the fact that search cost are sunk at the time the price is revealed to consumers. Still, the argument used in this paper suggests that an equilibrium exists with high-valuation consumers visiting and buying at an advertising firm and not-searching in case they did not receive an advertisement. Such an equilibrium with advertising firms setting high prices could hold as long as the probability of finding an active firm is low enough.

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Appendix A

Proof of Proposition 4.2

Assume that for A^*, S^*, θ_H^*, c^* and γ^* the equilibrium exists, and so all four restrictions hold. The first three restrictions do not depend on θ_H or c in isolation but instead only depend on $\theta_H - c$. Since this value does not change when x is added to both θ_H^* and c^* these three restrictions still hold. The fourth restriction depends on θ_H and c in isolation. When x is added to both θ_H^* and c^* this restriction changes to

$$\theta_H^*(1-\frac{S^*}{\gamma^*})+x(1-\frac{S^*}{\gamma^*})-\frac{S^*}{\gamma^*}\ln\frac{\gamma^*}{S^*} < \frac{A^*+S^*}{(1-\gamma^*)(\theta_H^*-c^*)}c^*+\frac{A^*+S^*}{(1-\gamma^*)(\theta_H^*-c^*)}x. \tag{1}$$

Since (restriction 4 of Proposition 4.1)

$$\theta_H^*(1-\frac{S^*}{\gamma^*})-\frac{S^*}{\gamma^*}\ln\frac{\gamma^*}{S^*} < \frac{A^*+S^*}{(1-\gamma^*)(\theta_H^*-c^*)}c^*$$

and (due to restriction 3)

$$x(1-\frac{S^*}{\gamma^*}) < \frac{A^*+S^*}{(1-\gamma^*)(\theta_H^*-c^*)}x$$

inequality (1) holds and so when x is added to θ_H^* and c^* all restrictions are still satisfied and the equilibrium exists.

Proof of Proposition 5.1.

To prove Proposition 5.1 we need to check all possible equilibria of the model and show that neither of these overlaps with the equilibrium of Proposition 4.1 for x large enough. To this end, we classify the possible equilibria by the probabilities α and β. In the equilibrium of Proposition 4.1 these probabilities are strictly between 0 and 1. We first concentrate on equilibria for which it is easy to show that they do not overlap with the equilibrium of Proposition 4.1 at all.

(i) An equilibrium where $\beta = 0$ implies that profits of being active should be below zero. If a firm would deviate and choose not to advertise and to set a price $\theta_L = 1$, its profits are $\gamma - S$. Hence, an equilibrium where $\beta = 0$ only holds for $\gamma < S$, while condition (2) of Proposition 4.1 stipulates that $\gamma > S$.

(ii) In an equilibrium with $\alpha = 1$ and $0 < \beta < 1$ we should have that $\pi_1 = 0$. First suppose $\underline{p}_1 > \theta_L$. If a firm deviates to not advertising a price $\theta_L = 1$ profits are at least equal to $\gamma - S$. As deviating should not be profitable, this equilibrium only holds for $\gamma < S$. Now suppose $\underline{p}_1 \leq \theta_L$. It is easy to show that in this case θ_L is in the support of $F_1(p)$ and so $\pi_1(\theta_L) = 1-\beta+\beta(1-F_1(\theta_L))-A-S = 0$, or $1-\beta+\beta(1-F_1(\theta_L)) = A+S$. Deviating to not advertising a price θ_L gives a profit that is at least as large as $\pi_0(\theta_L) = \gamma(1-\beta+\beta(1-F_1(\theta_L))) - S = \gamma(A+S) - S$. As deviating should not be profitable it follows that $\gamma(A+S) - S$ has to be smaller than 0. This restriction leads to a parameter region that does not overlap with a region defined by $S < \gamma$ and $A > 1 - \gamma$.

(iii) $\alpha = 1$ and $\beta = 1$. In this case an equilibrium does not exist. When every firm is active and advertises, we get Bertrand competition and equilibrium prices equalling 0. This leads to negative profits $-A - S$.

Note that the equilibria that are left to analyze are equilibria with $\alpha < 1$ and $\beta > 0$. For these equilibria we cannot universally show that they do not overlap with our equilibrium and in fact some of these equilibria partially overlap. As we will show these equilibria however only overlap for relatively small values of θ_H and c. Apart from classifying equilibria by α and β we also need to distinguish three different cases, depending on the way consumers search. In the remainder of the proof we use the optimal consumer behavior as specified Section 5, so that the reservation price r is given by $\int_{\underline{p}_0}^{r+c} F_0(p)dp = \frac{1-\alpha\beta}{(1-\alpha)\beta}c$.

(a) $r > \theta_H - c$**.**

In this case high-valuation consumers do not search at all. First we show that for $\beta = 1$ an equilibrium does not exist. In this case the definition of r gives $\int_{\underline{p}_0}^{\theta_H} F_0(p)dp < c$. As high-valuation consumers do not search profits from not advertising a price above θ_L are $-S$ and so non-advertised prices are always at or below θ_L. Therefore, $\int_{\underline{p}_0}^{\theta_H} F_0(p)dp = \int_{\underline{p}_0}^{\theta_L} F_0(p)dp + \int_{\theta_L}^{\theta_H} 1dp = \int_{\underline{p}_0}^{\theta_L} F_0(p)dp + \theta_H - \theta_L > c$ (where the inequality follows from the model assumption $\theta_H - c > \theta_L$). This contradicts the definition of r.
Two cases with $0 < \beta < 1$ are left to be analyzed: $\alpha = 0$ and $0 < \alpha < 1$. The equilibrium with $\alpha = 0$ does not overlap the equilibrium of Proposition 4.1. There can only be an equilibrium with $0 < \beta < 1$ and $\alpha = 0$ if a firm that deviates and advertise a price $\theta_H - c$ makes a nonpositive profit. However, $\pi_1(\theta_H - c) < 0$ yields $(1-\gamma)(\theta_H - c) - A - S < 0$, which contradicts the first restriction of Proposition 4.1.
An equilibrium with $0 < \alpha < 1$ has two possibilities. The first is that $\underline{p}_1 > \theta_L$. In this case the only equilibrium is the one described in Proposition 4.1. The other possibility is that $\underline{p}_1 \leq \theta_L$. In this case we have for $p \leq \theta_L$

$$\pi_0(p) = p\gamma(1 - \beta\alpha F_1(p) - \beta(1-\alpha)F_0(p)) - S$$

and

$$\pi_1(p) = p\gamma(1 - \beta\alpha F_1(p) - \beta(1-\alpha)F_0(p)) + p(1-\gamma)(1 - \beta\alpha F_1(p)) - A - S.$$

Standard arguments show that for $p \leq \theta_L$ $F_0(p)$ and $F_1(p)$ have no atoms and therefore are continuous. This gives that there exists a price p^* that is in the support of both $F_0(p)$ and $F_1(p)$. To see this, suppose that such a price does not exist. Then there are prices p_0^* and p_1^* with $p_0^* < p_1^*$ (the reverse case is similar) such that a price between p_0^* and p_1^* is never asked. But then $\pi_0(p_0^*) < \pi_0(p_1^*)$, contradicting this equilibrium. For the shared price $p^* \leq \theta_L = 1$ we have $\pi_0(p^*) = \pi_1(p^*)$ which gives $A = p^*(1-\gamma)(1-\beta\alpha F_1(p^*)) \leq 1-\gamma$, contradicting the first restriction of Proposition 4.1.

(b) $r < \theta_H - c$.

In this case consumers who get no advertisements search for a non-advertising firm. We need to distinguish two different cases: $\alpha = 0$ and $0 < \alpha < 1$. We start with $0 < \alpha < 1$. Define $z = 1 - \beta + \beta\alpha(1 - F_1(r)) + \beta(1-\alpha)(\frac{1}{2} + \frac{1}{2}(1 - F_0(r+c)))$. It is easy to prove that $\overline{p}_1 \leq r$ since $\pi_0(p) > \pi_1(p)$ for $p > r$. In case $\underline{p}_1 \leq \theta_L$ we can use the same arguments as in case (a) with $0 < \beta < 1$ and $0 < \alpha < 1$ to show that a necessary condition for such an equilibrium to exist is that $A \leq 1-\gamma$, contradicting the first condition of Proposition 4.1. In case $\underline{p}_1 > \theta_L$, we have $\pi_1(p) = p(1-\gamma)(1-\beta\alpha F_1(p)) - A - S$ and so $\overline{p}_1 = r$. Profits π are therefore given by $r(1-\gamma)(1-\alpha\beta) - A - S$, which gives $1 - \alpha\beta = \frac{\pi+A+S}{r(1-\gamma)}$. We also know that for any $\theta_L < p \leq r + c$ profits from not advertising are given by $\pi_0(p) = p(1-\gamma)z - S$. As this is increasing in p, it follows that a non-advertising firm never asks a price between θ_L and $r+c$, but instead will ask $r+c$. It then also follows that $\underline{p}_0 \leq \theta_L$ as otherwise the reservation price is not well-defined. For an equilibrium to hold it should be that $\pi_0(r+c) \leq \pi_0(\underline{p}_0) \leq 1 - S$. We therefore get as a first restriction

$$(r+c)(1-\gamma)z \leq 1.$$

On the other hand, we know that $z = 1 - \beta\alpha - \frac{1}{2}(\beta - \beta\alpha)F_0(r+c)$. Using $1 - \alpha\beta = \frac{\pi+A+S}{r(1-\gamma)}$ and $\beta - \alpha\beta < \frac{\pi+A+S}{r(1-\gamma)}$ we get $z \geq \frac{1}{2}\frac{\pi+A+S}{r(1-\gamma)} > 0$ or

$$(r+c)(1-\gamma)z \geq \frac{1}{2}(\pi + A + S)\frac{r+c}{r}.$$

As r is bounded above by $\theta_H - c$ and $\pi \geq 0$, it follows that $\frac{1}{2}(\pi + A + S)\frac{r+c}{r} > 1$ for c and θ_H high enough. This means that the restriction $(r+c)(1-\gamma)z \leq 1$

cannot hold for x high enough and the equilibrium cannot exist for high values of θ_H and c.
In case $\alpha = 0$ we have $z = 1 - \beta \frac{1}{2} F_0(r + c)$. This gives $z \geq \frac{1}{2}$ and the same argument as before can be used to show that this equilibrium cannot hold for high values of θ_H and c.

(c) $r = \theta_H - c$**.**

In this case consumers who do not receive any advertisements search with probability μ for a non-advertising firm. We need to distinguish four different subcases. The first subcase where $\alpha = 0$ and $0 < \beta < 1$ can be ruled out using the same argument as used in case (a).

The second subcase has $\alpha = 0$ and $\beta = 1$. For $p \leq \theta_L$ profits are

$$\pi_0(p) = p\gamma(1 - F_0(p)) + p(1 - \gamma)\frac{1}{2}\mu - S$$

while for $p > \theta_L$ profits are given by

$$\pi_0(p) = p(1 - \gamma)\frac{1}{2}\mu - S.$$

As the latter expression is increasing in p, it follows that it is not profitable to set prices strictly in between θ_L and θ_H. As $r = \theta_H - c$, it follows that non-advertising firms should charge prices equal to θ_H as well as prices below θ_L. Thus, for $p \leq \theta_L$ we have that

$$F_0(p) = 1 - \frac{(\theta_H - p)(1 - \gamma)\frac{1}{2}\mu}{p\gamma}.$$

Moreover, $F_0(p) = F_0(\theta_L)$ for all $\theta_L < p < \theta_H$ and θ_H is charged with strictly positive probability. One of the restrictions for this equilibrium to hold is that $\pi_0 = \frac{1}{2}\mu(1 - \gamma)\theta_H - S > 0$ (or, rewritten, $\frac{1}{2}\mu > \frac{S}{(1-\gamma)\theta_H}$). As the definition of the reservation price in this case gives $\int_{\underline{p}_0}^{\theta_H} F_0(p)dp = c$ we have that

$$\begin{aligned}
c &= \int_{\underline{p}_0}^{\theta_L} F_0(p)dp + (\theta_H - \theta_L)(1 - \frac{(\theta_H - \theta_L)(1 - \gamma)\frac{1}{2}\mu}{\gamma}) \\
&< \theta_L + \theta_H - \theta_L - (\theta_H - \theta_L)\frac{(\theta_H - \theta_L)(1 - \gamma)\frac{1}{2}\mu}{\gamma} < \theta_H - (\theta_H - \theta_L)\frac{(\theta_H - \theta_L)S}{\gamma\theta_H}.
\end{aligned}$$

Note that this can be rewritten as

$$\theta_H - c - (\theta_H - \theta_L)\frac{(\theta_H - \theta_L)S}{\gamma\theta_H} > 0.$$

It is easy to see that the LHS of this inequality is negative if θ_H and c become large in the way indicated in the proposition. Therefore, this subcase cannot arise for θ_H and c large enough.

The third subcase has $0 < \alpha < 1$ and $\beta = 1$. As in the previous subcase, we can show that non-advertising firms do not ask prices between θ_L and θ_H, that $\underline{p}_0 \leq \theta_L$ and that there will be an atom at θ_H. Moreover, if $\underline{p}_1 \leq \theta_L$, we can use the argument used in case (a) above that there exists a price p^* that is in the supports of both $F_0(p)$ and $F_1(p)$ and this again yields $A \leq 1 - \gamma$, a contradiction with the first restriction of Proposition 4.1.
We therefore concentrate on the case that $\underline{p}_1 > \theta_L$. Using standard arguments one can show that for $p \leq \theta_L$ the price distribution $F_0(p)$ is given by

$$F_0(p) = \frac{1}{1-\alpha}\left[1 - \frac{(\theta_H - p)(1-\gamma)(1-\alpha)\frac{1}{2}\mu}{p\gamma}\right].$$

As $\overline{p}_1 = \theta_H - c$, we need to have that in equilibrium $\pi_1(\theta_H - c) = \pi_0(\theta_H)$ yielding $(\theta_H - c)(1-\gamma)(1-\alpha) - A - S = \frac{1}{2}\mu(1-\gamma)(1-\alpha)\theta_H - S$, or $1 - \alpha = \frac{A}{(1-\gamma)(\theta_H - c - \frac{1}{2}\mu\theta_H)}$. The definition of r gives $\int_{\underline{p}_0}^{\theta_H} F_0(p)dp = c$ and $\pi_0(\theta_H) > 0$ gives $\frac{1}{2}\mu > \frac{S(\theta_H - c)}{\theta_H(A+S)}$ and $1 - \alpha > \frac{A+S}{(1-\gamma)(\theta_H - c)}$. This gives

$$c = \int_{\underline{p}_0}^{\theta_L} F_0(p)dp + (\theta_H - \theta_L)\frac{1}{1-\alpha}(1 - \frac{(\theta_H - \theta_L)(1-\gamma)(1-\alpha)\frac{1}{2}\mu}{\gamma})$$

$$< \frac{\theta_H}{1-\alpha}(1 - \frac{(\theta_H - \theta_L)(1-\gamma)(1-\alpha)\frac{1}{2}\mu}{\gamma}) = \theta_H + \theta_H\frac{\alpha}{1-\alpha} - \theta_H\frac{(\theta_H - \theta_L)(1-\gamma)\frac{1}{2}\mu}{\gamma}$$

$$< \theta_H + \theta_L\frac{S}{\gamma}\frac{(1-\gamma)(\theta_H - c)}{A+S} + \theta_H\frac{(\theta_H - c)(1-\gamma)}{A+S}\left[1 - \frac{S}{\gamma} - \frac{A+S}{(1-\gamma)(\theta_H - c)}\right]$$

Note that this can be rewritten as

$$\theta_H - c + \theta_L\frac{S}{\gamma}\frac{(1-\gamma)(\theta_H - c)}{A+S} + \theta_H\frac{(\theta_H - c)(1-\gamma)}{A+S}\left[1 - \frac{S}{\gamma} - \frac{A+S}{(1-\gamma)(\theta_H - c)}\right] > 0.$$

If θ_H and c become large in the way indicated in the proposition this restriction can hold if, and only if, $1 - \frac{S}{\gamma} - \frac{A+S}{(1-\gamma)(\theta_H - c)} \geq 0$, contradicting restriction 3 of Proposition 4.1.

The fourth subcase has $0 < \alpha < 1$ and $0 < \beta < 1$. As before, non-advertising firms do not ask prices between θ_L and θ_H, the support of non-advertised prices should include an interval at and below θ_L and we can exclude the case where $\underline{p}_1 \leq \theta_L$ by alluding to the same shared argument of a common price in the support of two distributions as before. So, we concentrate on the case where $\underline{p}_1 > \theta_L$ and distinguish two cases: (I) $\overline{p}_0 = \theta_L$ and (II) $\overline{p}_0 = \theta_H$.

(I) From the definition of r we have that $\int_{\underline{p}_0}^{\theta_L} F_0(p)dp + \theta_H - \theta_L = \frac{1-\alpha\beta}{\beta(1-\alpha)}c$, which can be rewritten as $\int_{\underline{p}_0}^{\theta_L} F_0(p)dp + (\theta_H - c) - \theta_L = \frac{1-\beta}{\beta(1-\alpha)}c$. From $\overline{p}_1 = \theta_H - c$ it follows that $\pi_1(\theta_H - c) = (\theta_H - c)(1-\gamma)(1-\alpha\beta) - A - S = 0$, which implicitly defines $\alpha\beta$. Furthermore, the equilibrium profits are given by $\pi_0(\theta_L) = \gamma(1-\beta+\beta\alpha) + (1-\gamma)z - S = 0$, with $z = (1-\beta)(\frac{1}{2}\mu + \frac{1}{2}\mu^2) + \beta(1-\alpha)\frac{1}{2}\mu$. As deviating to a price of θ_H should not be profitable, we have that $\pi_0(\theta_H) = \theta_H(1-\gamma)z - S < 0$, or $z < \frac{S}{\theta_H(1-\gamma)}$. Using this and the implicit expression for $\alpha\beta$ we arrive at $S = \gamma(1-\beta+\beta\alpha) + (1-\gamma)z < \gamma(1-\beta+1-\frac{A+S}{(\theta_H-c)(1-\gamma)}) + \frac{S}{\theta_H}$, which can be rewritten as $1-\beta > \frac{S(\theta_H-1)}{\theta_H\gamma} - 1 + \frac{A+S}{(\theta_H-c)(1-\gamma)}$. Therefore, the definition of r gives

$$\beta(1-\alpha)\left[\int_{\underline{p}_0}^{\theta_L} F_0(p)dp + (\theta_H - c) - \theta_L\right] > c\left[\frac{S}{\gamma} - 1 + \frac{A+S}{(\theta_H-c)(1-\gamma)}\right] - \frac{c}{\theta_H}\frac{S}{\gamma}.$$

Note that the LHS of this inequality is smaller than $\theta_H - c$. If $\frac{S}{\gamma} - 1 + \frac{A+S}{(\theta_H-c)(1-\gamma)} > 0$ and θ_H and c become large in the way indicated in the proposition the RHS of the inequality grows unboundedly. Therefore, the only way for the inequality to continue to hold when c and θ_H grow larger is if $\frac{S}{\gamma} - 1 + \frac{A+S}{(\theta_H-c)(1-\gamma)} \leq 0$, contradicting the third restriction of Proposition 4.1.

(II) We then consider the case $\overline{p}_0 = \theta_H$. Again, $\overline{p}_1 = \theta_H - c$ and so $\pi_1(\theta_H - c) = 0$ defines $1-\alpha\beta = \frac{A+S}{(\theta_H-c)(1-\gamma)}$. Using the same definition of z as before we derive $\pi_0(\theta_H) = \theta_H(1-\gamma)z - S = 0$ (or $z = \frac{S}{\theta_H(1-\gamma)}$) and for $p \leq \theta_L$

$$F_0(p) = \frac{1}{\beta(1-\alpha)}(1 - \frac{S - p(1-\gamma)z}{p\gamma}).$$

The definition of r gives $\int_{\underline{p}_0}^{\theta_H} F_0(p)dp = \frac{1-\alpha\beta}{(1-\alpha)\beta}c$. Substituting the expressions for $F_0(p)$, z and $1-\alpha\beta$ gives

$$\theta_H - \frac{S}{\gamma}(\theta_H - 1) - \frac{S}{\gamma}\ln\frac{\gamma\theta_H + S}{S\theta_H} = \frac{A+S}{(\theta_H-c)(1-\gamma)}c.$$

Note that this is an expression in the model parameters A, S, γ, θ_H and c and that if it holds for one particular set of parameter values, it certainly does not hold anymore when we increase θ_H and c.