Abstract

We study vertical relations in markets with consumer and retailer search. We obtain three important new results. First, we provide a novel explanation for price dispersion that does not depend on some form of heterogeneity among consumers. Price dispersion takes on the form of a bimodal distribution. Second, under competitive conditions (many retailers or small consumer search cost), social welfare is significantly smaller than in the double marginalization outcome. Manufacturers’ regular price is significantly above the monopoly price, squeezing retailers’ markups and providing an alternative explanation for incomplete cost pass-through. Third, firms’ prices are decreasing in consumer search cost.

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1 Introduction

For markets to be truly competitive, consumers must be able to compare price offers of different firms (Stigler, 1961). In markets with consumer search frictions, prices may be considerably higher than their competitive levels. In this paper, we argue that in vertical industries comprising of manufacturers and retailers, the ability of retailers to compare wholesale prices is also vital in establishing competitive outcomes. Even a small retailer search cost, combined with a small consumer search cost, may bring about large welfare losses. In addition, the paper shows that (bimodal) price dispersion arises naturally even if all buyers and sellers are identical.\(^1\) Finally, we show that to explain pricing patterns at the retail level it is important to simultaneously consider pricing behaviour at the wholesale level.

Our baseline model has a simple vertical structure. Manufacturers sell their product to retailers and retailers resell the product to final consumers. The key feature of the model is that both final consumers and retailers engage in search and have to pay a search cost to acquire new information. Final consumers search among retailers, while retailers search among manufacturers. We consider homogeneous goods markets so that in the absence of search costs, competition would force prices to be equal to marginal cost. In the baseline model, all retailers have identical retailer search cost and all consumers have identical consumer search cost and identical demand functions.

Retailer and consumer search are important in many real-world markets. While there is a large theoretical and empirical literature documenting the importance of search costs in retail markets, this is the first paper combining consumer search with retailer search in wholesale markets. Even though one may argue that retailers are professional traders with substantial knowledge of market conditions, search frictions in the wholesale market may be substantial if the relevant prices of alternative suppliers are hard to identify.\(^2\) On the other hand, it may be argued that retailers often have long-term contracts with their current suppliers and it may be (almost) costless for them to continue their relations with these suppliers.\(^3\) This is perfectly consistent with

\(^1\)Typically, to explain price dispersion, either consumers have different search costs (as, for example, in Varian (1980) and Stahl (1989)) or sellers have different costs (as in Reinganum (1979)).

\(^2\)Retailers often sign complex contracts with manufacturers, specifying delivery conditions, packages, buy-back policies, branding, and product positioning so that the relevant price may be hard to know ex-ante.

\(^3\)Explicitly considering long-term contracts raises a whole series of new considerations. For example, (i) can retailers and/or manufacturers get out of the contract at any point in time (presumably at a cost) or will they only have the possibility to switch after the term of the contract has expired, or
our paper. In search theoretic terms, this is equivalent to assuming that the retailer’s first search is free. It is our aim to highlight the connection between the contractual arrangements between retailers and their suppliers and the cost that retailers would need to incur in order to deal with alternative suppliers. Both search and switching costs are relevant in this regard and we will discuss the similarities and differences between the effects of these two different types of costs.\footnote{At a fundamental level, the difference between search and switching costs is that the first relates to an informational friction, while the second does not (see Wilson, 2012).}

Since our baseline model is a simple Diamond-type setup with a vertical industry structure, one may expect the interaction between manufacturers and retailers to result in the Diamond paradox arising at both levels of the product chain.\footnote{Diamond (1971) showed that, as long as all consumers have a positive, possibly (arbitrary) small, search cost, firms will choose monopoly pricing in equilibrium. This is what is often called the Diamond paradox. The main idea underlying the Diamond paradox is that for any price that is smaller than the monopoly price, the following is true: if this price is expected to be charged, then consumers who observe a slightly larger price have no incentive to continue to search for other prices and, therefore, firms have an incentive to set such a slightly larger price.} Thus, the classic double marginalization outcome would materialize, where retailers charge the retail monopoly price given the wholesale price that is set by manufacturers, and given this retail behaviour, manufacturers set their optimal monopoly price as well. Since there is no price dispersion, neither consumers nor retailers have an incentive to search. Indeed, we show that double marginalization constitutes an equilibrium for some parameter values, but - more importantly- it is not an equilibrium for other parameter values.

To understand the logic behind this result, consider the following very stylized environment where a single manufacturer sells its product to a retailer who then re-sells it to a final consumer. Assume this consumer has access to an alternative retailer who charges a price $p_o$. The situation is depicted in Figure 1. The outside option for the consumer acts as a limit price for the retailer. Whenever this option is sufficiently attractive ($p_o$ low enough), the manufacturer can increase his profit by charging $p_o$ to the retailer and using the limit price to squeeze the retail margin, appropriating all the surplus. In this case, double marginalization is not an equilibrium. If this option is rather unattractive ($p_o$ large enough), the reduction in demand brought about by a higher price is larger than the share of surplus that the manufacturer gives up if he charges the double marginalization wholesale price ($w^m$) and, hence, double marginalization constitutes an equilibrium.
In a consumer search model, search costs and the price distribution jointly determine the relative attractiveness of the outside option endogenously. Under competitive conditions (a relatively large number of retailers and small consumer search cost), consumers have a good alternative to buy at alternative retailers, giving manufacturers an incentive to deviate from the double marginalization wholesale price in order to appropriate the retailers’ margin.

We show that if double marginalization does not constitute an equilibrium, manufacturers randomize over two prices: the double marginalization price and a higher price where retailers are squeezed. Because retailers incur a search cost to visit the next manufacturer, they are also willing to buy at the highest of these two prices. In a competitive market where retail profits are low, a small retailer search cost is sufficient to deter retailers from continuing to search. In markets where retailers are relatively large, manufacturers may partially squeeze larger retailers if retailers have a search cost above a critical level. Retail price strategies are deterministic and increasing: retailers set a larger price if they face a larger wholesale price. As a consequence, a binary retail price distribution results, and is induced by the manufacturers. This result obtains for small, but strictly positive, consumer search cost where it is not profitable for retailers to pass on increases in wholesale prices as their optimal price responses are restricted by consumers’ reservation price. The fact that manufacturers are able to secure a sizeable
surplus is consistent with a recent study by Noton and Elberg (2015) who find that suppliers of supermarkets typically extract substantially more than half of the surplus even if the suppliers are very small relative to the supermarket.

Thus, even though the model is set up such that all the conditions are satisfied for Diamond-type results to emerge at both levels of the product chain, consumer and retailer search cost do matter. For any given value of the retailer search cost, lower consumer search costs make it more profitable for manufacturers to squeeze retailers, and (a)symmetrically, for any given value of the consumer search cost (if it is below a critical value) lower retailer search cost deters manufacturers from squeezing them.

Janssen and Shelegia (2015a) present a search model with a vertical industry structure where a monopoly manufacturer sells to multiple retailers. Janssen and Shelegia (2015a) focus on the implications of the wholesale contract between the manufacturer and retailers not being observed by consumers. The possibility for the manufacturer to squeeze retailers also plays an important role in that paper, and it may be so strong that an equilibrium fails to exist. Our paper presents the first search model with upstream competition where retailers may search among alternative suppliers. In our paper with multiple manufacturers, non-existence of a reservation price equilibrium does not arise as each manufacturer has an incentive that their retailers are successful in selling to final consumers, whereas with a monopoly manufacturer retailers and consumers cannot be supplied by alternative manufacturers. In addition, if retailer search costs are small enough, manufacturers face another constraint not to increase their wholesale price, which is not relevant in the monopoly manufacturer model.

The empirically observed distribution of retail prices is double-peaked, with two modal prices accounting for a large share of price realizations (see Hosken and Reiffen (2004), Pesendorfer (2002) and more recently Hendel and Nevo (2013)) and the typical time-series is \textit{V-shaped} with a high price for a number of periods followed by a discrete, short-lived drop which we identify as sales. The bold curve in Figure 2 provides an example of such a pattern. In the context of dynamic infratemporal price discrimination, some models have explained the binary nature of retail prices. Pesendorfer (2002)

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6 Other vertical models with consumer search include Janssen and Shelegia (2015b) and Lubensky (2013). Janssen and Shelegia (2015b) consider a monopolistic manufacturer in a model where product differentiation between retailers plays a role. Lubensky (2013) considers a single manufacturer dealing exclusively with a large number of retailers who face a competitive fringe of independent sellers.

7 Conlisk et al. (1984) is an early example where a monopolist faces two types of consumers who enter the market over time. The monopolist regularly serves the high demand consumers only, but when sufficiently many low valuation consumers have accumulated, the monopolist drops prices to serve the whole market.
presents a model of durable good consumption whereby consumers differ in their willingness to pay and firms optimally vary prices over time in order to price discriminate among consumers (see also Sobel, 1984). Recently, Hendel and Nevo (2013) and Hendel et al. (2014) provide a dynamic model where consumers and firms can stock the product and firms discriminate between price sensitive and price-insensitive consumers.

Dynamic pricing and storage holdings are certainly important in explaining the V-shaped retail pricing patterns in many industries. Our simple model focuses attention, however, on another possible explanation, namely the interaction between manufacturers and retailers in markets where consumer and retailer search are important. The key to the bimodal price distribution in our model is that the manufacturer’s profit function has two peaks, one arising from the incentives to squeeze retailers. Equilibrium price dispersion that is generated in the theoretical search literature is typically continuous (see, e.g., Stahl, 1989) and our model is the first in this literature that can explain binary price dispersion through the interaction between consumer and retailer search. Even though our baseline model abstracts from many features of the relationship between manufacturers and retailers, an important result of our model is that sales at the retail level are linked to lower wholesale prices. In addition, when wholesale prices are high, retail margins are low, explaining incomplete cost-pass through independent of the shape of demand. While this pattern has so far not been documented in the literature, it is consistent with the price series in the well-known Dominick’s database. A typical example is presented by the grey curve in Figure 2, where it is clear that retail sales coincide with (or are induced by) contemporaneous reductions in wholesale prices and that retail markups are decreasing in wholesale prices. In our model, price dispersion

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8Heidhues and Köszegi (2014) survey the literature on bimodal price dispersion and note that there may be some evidence that there is more variability at the sales price, while there exists one regular price. This can be explained by an atom in the upper bound of the price distribution and a continuous distribution of sales prices (see, e.g., Robert and Stahl (1993) who combine advertising and search, or Heidhues and Köszegi (2014) who introduce loss averse consumers). In an extension, we show that we can generate a pattern close to this by introducing shoppers as in Stahl (1989). Below we discuss that in the extension, manufacturers randomize over two disjoint intervals where the highest interval is relatively narrow and has prices concentrated at levels just below the consumer reservation price.

9Varian (1980) and Stahl (1989), and a large subsequent literature, analyze static models where consumers differ in their information about other firms’ prices and show that random prices can arise in equilibrium.

10See the Online Appendix for a more systematic statistical analysis.

11Many industries are characterized by incomplete cost pass-through (Weyl and Fabinger, 2013), i.e. higher wholesale prices induce lower margins at the retail level. For instance, using real exchange rates as a source of wholesale price variation, Campa and Goldberg (2005) find that the typical product has a cost pass-through elasticity smaller than one. Research on incomplete cost-pass through identifies demand functions for which firms do not fully pass on cost to consumers. Our model endogenously
Figure 2: Retail and Wholesale Prices Reported in Dominick’s Database for one of the most Popular Miller Beer Products in Store 121.

is associated with substantial welfare losses. As the lowest price in the wholesale price distribution is the double marginalization wholesale price, these equilibria are welfare-inferior to the Double Marginalization Equilibrium. To highlight the idea that search frictions at both levels of the retail chain significantly strengthen the Diamond paradox, we refer to these equilibria as Double Diamond Equilibria.\footnote{One could argue that both types of equilibria suffer from a Double Diamond paradox as also in the Double Marginalization there exists a Diamond paradox at both levels. However, once the Diamond paradox is understood, there is no interesting, new paradox that yields Double Marginalization. Moreover, we already have a well-known name in the literature for the first equilibrium, namely Double Marginalization. Thus, we reserve the name Double Diamond equilibrium for the second type of equilibrium only.}

Interestingly, when the consumer search cost approaches zero, Double Diamond Equilibria exist for almost all values of the other parameters and they converge to the manufacturer choosing the double marginalization wholesale price with arbitrary small probability and setting the consumer reservation price with probability close to one. Expected wholesale and retail prices are increasing as the search cost is decreasing.\footnote{Janssen and Shelegia (2015a) have a similar result in a monopoly manufacturer model in which a fraction of consumers have zero search cost. The reason why lower consumer search cost leads to higher (expected) prices is somewhat different in the two models, however. In Janssen and Shelegia (2015a) this result holds true for linear demand and it is the manufacturer that charges a deterministic price that is decreasing in search cost. In the DDE in our paper, the manufacturers randomize, but the two prices over which they randomize are independent of search cost (as they follow from the manufacturers’ indifference condition). It is the probability distribution over these two prices that is generates incomplete cost pass-through, regardless of the shape of demand.} Thus, what is typically
considered essential for competition, namely the combination of many retailers and small search costs, is disastrous for social welfare in vertically related industries where both retailers and consumers have a small search cost. Numerically, we show that the difference in outcomes between Double Diamond Equilibria and the Double Marginalization Equilibrium can be quantitatively substantial. For example, for linear demand, we show that the total surplus generated in these equilibria can be in the order of 40% lower than the total surplus generated in the Double Marginalization Equilibrium (which is already considered to be low), whereas consumer surplus is more than 60% lower! These results suggest that vertical relations in search markets have important implications that have so far been unexplored.

A natural question is to what extent these results depend on the Diamond paradox where, in equilibrium, no consumer compares prices. The literature has reacted to the Diamond paradox by constructing search models that do not have this feature. Wolinsky (1986) introduced product heterogeneity to accommodate active search, while Stahl (1989) introduced shoppers that have zero search cost. In an extension, we enrich our model by introducing search cost heterogeneity à la Stahl (1989), where some consumers (called shoppers) have zero search cost and, therefore, buy always from the lowest price retailer. The remaining consumers (non-shoppers) face a positive search cost. In the absence of a vertical structure, retailers facing the same marginal cost randomize their prices over a continuous support such that the highest price is (at most) the monopoly price. Once we introduce manufacturers and retailer search, many results of our baseline model generalize. First, retailers optimally choose a deterministic markup as a function of the wholesale price and manufacturers randomize over different prices so that sales are manufacturer induced. Second, for a range of consumer search costs, the distribution of wholesale prices has two disjoint intervals, with one interval having the manufacturer monopoly price as the upper bound and the other interval having the non-shoppers’ reservation price (which lies above the double marginalization price) as the upper bound. Finally, the size of the upper interval can be very small so that retail prices are concentrated just below the consumer reservation price, mimicking a mass point at the upper bound. If the consumer search cost of non-shoppers is sufficiently small, however, this equilibrium can no longer be sustained and more competitive results apply. As this model is the competitive manufacturer version of Janssen and Shelegia (2015a), we show that their results do not naturally carry over to markets changing with search cost (to make the consumers indifferent). Moreover, our result is true independent of the specific shape of the demand function.
with upstream competition.\footnote{In the online Appendix, we also explore (i) search cost heterogeneity and (ii) consumers using strategies that do not satisfy the reservation price property. The results presented there show that the bimodal price distribution is robust to such extensions.}

Our paper also has interesting implications for empirical studies assessing the classical question of the effect of retail concentration on prices (see, e.g., Berger and Hannan (1989) and Bikker and Haaf (2002) for the banking industry, or Cotterill (1986) for the food industry). In our model, retail margins are lower in Double Diamond Equilibria and these equilibria typically exist when there are many retailers in the market. However, this does not imply that competitive retail markets create higher welfare. Manufacturers find it easier to squeeze retailers when retail markets are competitive, resulting in much higher wholesale prices and higher retail prices than in concentrated retail markets.

The rest of the paper is organized as follows. The next section presents the baseline model. Section 3 presents some general characterization results that all equilibria have to satisfy. In particular, we show that the double marginalization equilibrium has the lowest wholesale and retail price in any possible equilibrium. Section 4 shows the conditions that together are necessary and sufficient for a Double Marginalization Equilibrium to exist. Section 5 focuses on Double Diamond Equilibria. It characterizes these equilibria and determines necessary and sufficient conditions for these equilibria to exist. This Section also discusses the welfare implications. Section 6 deals with an extension where manufacturers choose to set individualized prices to retailers, rather than post prices that are equal for all retailers that buy from them. Section 7 introduces shoppers into this model with individualized prices. Section 8 concludes.

\section{The Basic Model}

To focus on search frictions in a vertical industry structure, consider a standard market for a homogenous product, supplied by $m \geq 2$ manufacturers. Our arguments do not depend on the manufacturers’ cost structure, so we normalize their cost to be equal to 0. Manufacturer $i$ (she) sells the product for a price $w_i$ per unit to $n \geq 2$ retailers, and each retailer $j$ (he) resells the product to consumers at a retail price $p_j$ at no additional cost. There is a unit mass of consumers (often referred to as they) and each individual consumer has a demand function $D(p)$ so that they derive a surplus...
of $S(\tilde{p}) = \int_{\tilde{p}}^{\infty} D(p)dp$ when buying at price $\tilde{p}$. The easiest way to think about this setup is one where manufacturers post prices and each manufacturer sets the same price to all retailers. Alternatively, we may think of markets where manufacturer $i$ sets an individualized take-it-or-leave it price $w_{ij}$ to each retailer $j$ that visits her. When presenting the results, we first focus on posted prices and in Section 6 we show that qualitatively our results continue to hold in this alternative setting. Moreover, when discussing the model with shoppers in Section 7, we explicitly use that manufacturers set individualized prices.

The novel feature of our model is that we have multiple manufacturers and retailers so that both retailers and consumers engage in costly sequential search to discover the relevant prices. Retailers (consumers) have to pay a positive search cost $s_R$ (respectively $s_C$) for any additional price quotation (either $w$ for retailers or $p$ for consumers) they observe. Since our goal is to understand the different roles played by retailers’ and consumers’ search costs, we allow $s_R \neq s_C$. For simplicity, we follow most of the search literature and assume that the first price quotation is free, but this is not essential in our model since both retailers and consumers have positive expected surplus in any equilibrium. When retailers decide to stop their search and pay a wholesale price $w'$, their marginal cost is $w'$ for each unit sold. When consumers stop searching, they decide to buy at the lowest price $p$ they have observed and buy $D(p)$ units.

A retailer’s search strategy is characterized by a reservation price $\rho_R$: at any price $w \leq \rho_R$, the retailer buys, otherwise he continues to search. For the first search, each retailer either visits a manufacturer at random or, in the case of long-term relationships, he visits his current supplier. Reservation price strategies are optimal for the retailer since he does not update his beliefs about the distribution of wholesale prices, following any history of observed prices (see, e.g., Kohn and Shavell, 1974). Consumers face a slightly different problem, since they may learn about the distribution of manufacturer prices from the observed history of retail prices. In this paper, however, we consider equilibria in which consumers’ optimal search behaviour is also characterized by a reservation price strategy. We denote this reservation price by $\rho_C$: at any price

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15 We assume throughout a vertical structure of independent firms signing linear pricing contracts. We believe that this is the natural starting point for any analysis of retailer search. For a discussion of this assumption, see Section 8.

16 For the case of search cost heterogeneity, see the online Appendix.

17 See Rothschild (1974) for the observation on the (non)optimality of the reservation price rule when the search environment is (not) stable.

18 Other equilibria, where consumers choose non-reservation price strategies, are presented in the online Appendix. The market outcomes are qualitatively similar in both cases.
consumers buy, otherwise they continue to search. For the first search, an equal share of consumers visits each retailer. The determination of $\rho_R$ and $\rho_C$ depends on the equilibrium features, and we discuss the details in different sections.

The timing of the interaction is as follows. When manufacturers post prices, they first set their wholesale prices. Retailers then search among manufacturers according to an optimal search strategy. When all retailers have stopped searching, taking their wholesale price as given, they simultaneously set retail prices without knowing the wholesale prices of others. Consumers then engage in sequential search among the given retail prices.

Given some wholesale price $w$, we define the profit-maximizing price for a monopolist retailer as

$$p^m(w) = \arg\max_{p \geq 0} (p - w)D(p).$$

(1)

We refer to $p^m(w)$ as the retail monopoly price. Given the retailer’s optimal price $p^m(w)$, a single manufacturer would choose the wholesale monopoly price defined by

$$w^m = \arg\max_{w \geq 0} wD(p^m(w)).$$

(2)

We shall assume throughout that both maximization problems are well-defined. In particular, we assume that manufacturers’ and retailers’ (monopolistic) profit functions are single-peaked.\textsuperscript{19} We refer to $w^m$ as the wholesale monopoly price and $p^m(w^m)$ as the double marginalization (DM) retail price. Together they induce the DM outcome.

Following the vertical contracting literature (see, e.g., McAfee and Schwartz, 1994), we consider symmetric Perfect Bayesian equilibria satisfying a natural extension of passive beliefs to our environment. Namely, we assume that whenever a given player observes an off-the-equilibrium path action, she puts positive probability on minimal deviation paths only. For each information set, the minimal deviation path specifies a set of actions for every other player that induces that information set such that the number of players whose action does not belong to the support of their equilibrium strategy profile is minimal. Notice that passive beliefs satisfy the minimal deviation property.

**Definition 1.** A symmetric perfect Bayesian reservation price equilibrium (RPE) is a wholesale price $w^*$, a retailer search and pricing strategy $(\rho_R^*, p^*(w))$ and a consumer

\textsuperscript{19}The problem of the retailer is quasi-concave if the demand function is not too convex (log-concavity suffices). In such a case, the manufacturer’s profit function is also quasi-concave. If See Weyl and Fabinger (2013) for details.
search strategy $\rho^*_C$ such that in every information set, a player’s strategy is optimal given the strategies of the other players and beliefs that put zero probability on action profiles that do not belong to a minimal deviation path.\textsuperscript{20}

If consumers visit a retailer who has posted an unexpected retail price, they are not sure which firm has actually deviated from the equilibrium strategy: the retailer they have visited or the manufacturer from which the retailer they have visited had procured the product. Out-of-equilibrium beliefs are therefore of some importance to determine the optimal search behaviour of the consumer. First, they may believe that the retailer they visited has deviated to another retail price, while all manufacturers followed their equilibrium price strategies. In such a case, they do not need to update their beliefs about the other retailers when deciding whether to continue their search. This is akin to passive beliefs. Second, consumers may believe that a manufacturer has deviated to set a different wholesale price, inducing the retailer to charge a different price (consistent with her equilibrium strategy). In this case, they will take into account that other retailers may have visited the same deviating manufacturer, bought at the same wholesale price, and sell at the same (off-the-equilibrium-path) retail price. Only these two belief systems are consistent with minimum deviation paths, and, therefore, we shall restrict our analysis to them.

3 General Characterization Results

We first provide a general characterization of some of the properties that all equilibria in this model have to satisfy. This allows us to reduce the number of possible strategy profiles to consider. Our first result shows that in any symmetric equilibrium the firms’ prices are not smaller than the DM prices.

\textbf{Proposition 1.} In any symmetric equilibrium, wholesale and retail prices set by manufacturers and retailers are such that $w^* \geq w^m$ and $p^*(w^*) \geq p^m(w^m)$.

This is an important result.\textsuperscript{21} It shows that when both consumers and retailers have positive search costs, we cannot expect competition in the market to result in

\textsuperscript{20}A definition like this is often implicitly used in the consumer search literature. Formally, if one does not require reservation prices to be optimal off the equilibrium path, marginal cost pricing may be considered equilibrium behaviour if consumers’ reservation prices are equal to marginal cost. By requiring retail and consumer strategies to be optimal for all beliefs that are consistent with a minimal deviation path, the above definition rules out situations like this.

\textsuperscript{21}With minor changes in the proof, we can extend Proposition 1 to hold for any (potentially non-symmetric) equilibrium.
social welfare levels that are better than those resulting from a market served by a monopolist at both the wholesale and the retail level. Thus, it constitutes a relatively straightforward extension of the Diamond paradox.

In addition, we show that in any equilibrium, the wholesale monopoly price is always set with some positive probability and if other prices are also set with positive probability, there has to be a non-negligible gap between the wholesale monopoly price and the rest of the support.

**Proposition 2.** In any symmetric equilibrium, manufacturers set $w^m$ with some strictly positive probability. In addition, there exists some $\varepsilon > 0$ such that the open set of prices $(w^m, w^m + \varepsilon)$ does not belong to the support of the wholesale price distribution.

The first part of the Proposition is the combination of two basic observations. First, in any equilibrium, the retail price is increasing in the wholesale price. It follows that if a manufacturer chooses the lowest price in the support of her distribution, she guarantees that both retailers and consumers buy without further search. Second, of all prices that can be on the equilibrium path according to Proposition 1, the wholesale monopoly price gives the manufacturer the highest profits if retailers and consumers buy immediately without continuing to search. Thus, if the wholesale monopoly price was not set with positive probability on the equilibrium path, it would be optimal to deviate to it.

For the second part, notice that for any wholesale price sufficiently close to the wholesale monopoly price, retailers are able to charge the retailer monopoly price and, given positive search costs, still sell to all incoming consumers. Since, by definition, this yields lower profits to the manufacturer, such prices are never part of the equilibrium price distribution.

## 4 Double Marginalization Equilibrium

In our model, Double Marginalization (DM) is the natural counterpart to the monopoly outcome that arises in Diamond’s seminal paper. In the strategy profile that leads to the DM outcome, (i) each manufacturer sets the wholesale monopoly price, (ii) each retailer visits one single manufacturer, buys at that wholesale price, and then sets the retail monopoly price, and (iii) consumers visit a single retailer and buy at that retail price. In this section, we show under which conditions DM is and is not an equilibrium outcome.
Figure 3: Manufacturer Profits from DM and Squeezing as a Function of the Retailer’s Reservation Price
The intuition behind the results presented here can be understood by considering Figure 3, where we depict the standard DM outcome, which yields a profit of \( w^m D(p^m) \) (corresponding to the darker areas). If consumers expect this to be the equilibrium outcome, then \( \rho_C \) represents the consumer reservation price for a small search cost \( s_C \). If retailers’ search cost is sufficiently high, i.e., \( \rho_R > \rho_C \), the manufacturer will be able to extract all surplus by charging \( \rho_C \), thereby obtaining higher profits (the sum of the four gray areas is larger than the black area). Therefore, DM will not be an equilibrium for low enough \( s_C \) if \( s_R \) is sufficiently high. Figure 3 includes two more relevant cases. First, if the retailers’ search cost is at an intermediate level, the retailers’ reservation price will be around \( \bar{\rho}_R \). In that case, the manufacturer cannot fully expropriate the retailer surplus since he will decide to search for a better deal with another manufacturer if she charges \( w > \bar{\rho}_R \). However, the manufacturer may still profitably deviate and charge \( w = \bar{\rho}_R \), so that the equilibrium does not correspond to the DM outcome. Finally, if the retailers’ search cost is even lower, the reservation price will drop to \( \underline{\rho}_R \), which is so low that the manufacturer will not find it attractive to deviate. Thus, the DM outcome constitutes an equilibrium if retailers’ search cost is low enough.

More formally, to have a DME, each manufacturer should choose \( w^m \), retailers’ behaviour as a function of the wholesale price \( w \) is given by

\[
\sigma^*_R(w) = \begin{cases} 
\text{buy and set } p^m(w) & \text{if } w \leq \rho_R \text{ and } p^m(w) \leq \rho_C, \\
\text{buy and set } \rho_C & \text{if } w \leq \rho_R \text{ and } p^m(w) > \rho_C, \\
\text{buy and set } w & \text{if } \rho_C < w < \rho_R, \\
\text{continue to search} & \text{if } w > \rho_R,
\end{cases}
\] (3)

while consumers’ optimal search rule \( \sigma^*_C(p) \) as a function of the retail price \( p \) is given by

\[
\sigma^*_C(p) = \begin{cases} 
\text{buy at } p & \text{if } p \leq \rho_C, \\
\text{continue to search} & \text{if } p > \rho_C.
\end{cases}
\] (4)

If this strategy profile constitutes an equilibrium, there is no active search and no price dispersion. Retailers are effective monopolists in their incoming demand and, therefore, optimize by setting the monopoly retail price \( p^m(w) \).

**Definition 2.** A symmetric (pure) strategy profile \( \sigma^{DM} \) is a Double Marginalization (DM) strategy profile if \( \sigma^{DM} = (w^*, \sigma^*_R(w), \sigma^*_C(p)) \) is such that each manufacturer sets \( w^* = w^m \) as in (2), each retailer follows the strategy \( \sigma^*_R(w) \) as in (3), and consumers
follow the search rule $\sigma_C(p)$ as defined by (4). If $\sigma^{DM}$ is an equilibrium, then we call it a Double Marginalization Equilibrium (DME).

Given the construction of these strategies, it is clear that the only profitable deviation may come from a manufacturer who charges a higher wholesale price at the expense of a lower retailer margin. To derive necessary and sufficient conditions for this deviation not to be profitable, we still have to specify the reservation prices $\rho_R$ and $\rho_C$. As we mentioned in Section 2, the retailers’ reservation price is relatively easy to characterize. We can compute the wholesale price $\rho_R > w_m$ such that retailers are indifferent between buying the product at that price and continuing to search for a lower (equilibrium) price $w_m$ that they expect from other manufacturers. Given that, after observing $\rho_R$ a retailer would set a retail price equal to $\min\{\rho_C, p^m(\rho_R)\}$. The retailer reservation price $\rho_R$ is implicitly determined by

$$\left(\min\{\rho_C, p^m(\rho_R)\} - \rho_R\right) \frac{D(\min\{\rho_C, p^m(\rho_R)\})}{n} = (p^m(w_m) - w_m) \frac{D(p^m(w_m))}{n} - s_R,$$

where because of the first random search, consumer demand is evenly split among all retailers. The above equation is rewritten as

$$(p^m(w_m) - w_m) D(p^m(w_m)) - (\min\{\rho_C, p^m(\rho_R)\} - \rho_R)D(\min\{\rho_C, p^m(\rho_R)\}) = n s_R.$$  

The characterization of the consumers’ reservation price is more complicated because $\rho_C > p^m(w_m)$. If consumers observe the reservation price (whatever its precise value), then they know that either this retailer has deviated or the manufacturer has deviated that has sold the product to the retailer he has visited. Notice that both interpretations of the deviation are consistent with the minimal deviation property on beliefs, as discussed in Section 2.

Assume for now that consumers blame the manufacturer for a possible deviation from the equilibrium retail price.\footnote{Note that if $s_R$ is relatively large, the RHS may be negative and $\rho_R > \rho_C$ in that case. Also, if we were to consider retailer switching cost instead of search cost, the same equation would apply as there is no uncertainty for retailers as regards what price to expect on the next search in the DM strategy profile.} In this case, if a consumer continues to search, there is a chance of $\frac{1}{m}$ that the next retailer has bought the product from the same

\footnote{In an online Appendix, we provide the analysis for the case in which consumers blame retailers for a deviation and show that the equilibrium prices are exactly the same; only the equilibrium existence conditions differ.}
manufacturer. The consumer reservation price $\rho_C$ is then given by

$$\int_{\rho_C}^{\infty} D(p)dp = \frac{1}{m} \int_{\rho_C}^{\infty} D(p)dp + \frac{m-1}{m} \int_{p^m(w^m)}^{\infty} D(p)dp - s_C,$$

that is,

$$\frac{m-1}{m} \int_{p^m(w^m)}^{\rho_C} D(p)dp = s_C. \tag{5}$$

We have two reasons to focus on the case in which consumers blame a manufacturer for a possible deviation. First, our main results suggest that retailer search brings about equilibrium outcomes that may be significantly worse than the DM outcome. We do not want this argument to depend on specific choices of out-of-equilibrium beliefs. Later in this section, we show that the most critical deviation to consider is the manufacturer choosing a wholesale price $w$ equal to the consumer reservation price $\rho_C$, thereby fully squeezing the retail margins. This deviation is more profitable if the reservation price is low, which happens if consumers blame retailers (see the online Appendix). In other words, if the double marginalization equilibrium is not an equilibrium if consumers blame the manufacturer, it is certainly not an equilibrium if consumers blame the retailer. Second, by assuming consumers blame the manufacturer, the next section provides a clear characterization of all reservation price equilibria.

We are now in a position to characterize the conditions under which the DM strategy profile is an equilibrium. As argued above, it suffices to focus on potential deviations by manufacturers. Moreover, it suffices to verify whether manufacturers can profitably deviate to a wholesale price $w$ equal to the consumer reservation price $\rho_C$, thereby fully squeezing the retail margins. This deviation is more profitable if the reservation price is low, which happens if consumers blame retailers (see the online Appendix). In other words, if the double marginalization equilibrium is not an equilibrium if consumers blame the manufacturer, it is certainly not an equilibrium if consumers blame the retailer. Second, by assuming consumers blame the manufacturer, the next section provides a clear characterization of all reservation price equilibria.

We are now in a position to characterize the conditions under which the DM strategy profile is an equilibrium. As argued above, it suffices to focus on potential deviations by manufacturers. Moreover, it suffices to verify whether manufacturers can profitably deviate to a wholesale price that induces retailers to buy and charge the consumers’ reservation price (whenever retailers react to a deviation by setting the retail monopoly price, the manufacturer’s profit is always lower than in the candidate equilibrium, by definition of the wholesale monopoly price). There are two cases to consider. First, suppose $s_C$ is so large that $\rho_C D(\rho_C) \leq w^m D(p^m(w^m))$. It is clear that in this case no deviation is profitable. Whether or not retailers buy from the manufacturer that deviated, the profit the deviating manufacturer makes is smaller than the equilibrium profit $w^m D(p^m(w^m))$. Formally, we can define $\bar{w}$ as the smallest wholesale price above $p^m(w^m)$ such that if retailers buy and react by selling themselves at $\bar{w}$ and consumers react by buying as well, the manufacturer is indifferent between setting this price and

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[^24]: Note that $\rho_C$ is increasing in $s_C$ and that because of the single peakedness assumption, $\rho_C D(\rho_C)$ is decreasing in $s_C$ for all $\rho_C > p^m(w^m)$. 

the wholesale monopoly price, i.e.,

\[ \bar{w} D(\bar{w}) = w^m D(p^m(w^m)). \] (6)

Whenever consumers observe the retail price \( \bar{w} \), their pay-off of buying (resp. continuing search) is given by \( \int_{\bar{w}}^{\infty} D(p)dp \) (resp. \( \frac{1}{m} \int_{\bar{w}}^{\infty} D(p)dp +(1-\frac{1}{m}) \int_{p^m(w^m)}^{\infty} D(p)dp - s_C \)). Thus, we can define a threshold \( \bar{s}_C(m) \) by

\[ \bar{s}_C(m) = \frac{m-1}{m} \int_{p^m(w^m)}^{\bar{w}} D(p)dp \] (7)

at which the consumer is indifferent between continuing to search after observing a price \( \bar{w} \) and buying. Thus, for \( s_C \geq \bar{s}_C(m) \), the manufacturer’s deviation is not profitable, even if retailers and consumers react by buying at this price. In that case the DME exists, while for smaller search costs, we have to inquire into the retailers’ decision problem.

Second, suppose then \( \rho_C D(\rho_C) > w^m D(p^m(w^m)) \), or equivalently that \( s_C < \bar{s}_C(m) \). In this case if the retailer still buys at \( \rho_C \) (even if fully squeezed), which requires that the retailer search cost is larger than the profit she would make if she continues to search, i.e., \( s_R > \frac{1}{n}(p^m(w^m) - w^m)D(p^m(w^m)) \), it is clearly optimal for the manufacturer to deviate and the DME does not exist. But even if \( s_R < \frac{1}{n}(p^m(w^m) - w^m)D(p^m(w^m)) \), it is still profitable to deviate if, and only if, \( \rho_R D(\rho_C) > w^m D(p^m(w^m)) \), where \( \rho_R \) is defined by

\[ (\rho_C - \rho_R) \frac{D(\rho_C)}{n} = (p^m(w^m) - w^m) \frac{D(p^m(w^m))}{n} - s_R. \] (8)

This equation can be understood as follows. Following a wholesale price \( \rho_R \), the retailer is indifferent between accepting it and charging \( \rho_C \) to consumers or searching further at the cost of \( s_R \).\(^{25}\) Since the retailer has passive beliefs, he expects the next manufacturer to offer him a price \( w^m \) so that he expects to obtain a profit \( (p^m(w^m) - w^m)D(p^m(w^m)) \). From (8) it follows that

\[ \rho_R D(\rho_C) = \rho_C D(\rho_C) - (p^m(w^m) - w^m)D(p^m(w^m)) + ns_R, \]

\(^{25}\)It must be that \( \rho_C < p^*(\rho_R) \) for otherwise \( \rho_R \) would be dominated by \( w^m \) and the manufacturer would not find it optimal to charge it.
and so a manufacturer’s gain from deviating is positive if

\[ \rho_R D(\rho_C) - w^m D(p^m(w^m)) = \rho_C D(\rho_C) - p^m(w^m) D(p^m(w^m)) + n s_R > 0. \]  

(9)

Notice that the RHS of (9) is strictly increasing in \( s_R \). Define \( \bar{s}_R(n) \) to be the highest search cost such that a retailer may consider searching to get to buy at the wholesale monopoly price, i.e., \( n \bar{s}_R(n) = (p^m(w^m) - w^m) D(p^m(w^m)) \). Since \( \rho_C < \infty \), the RHS of (9) is positive at \( \bar{s}_R(n) \). As a result, there exists some threshold retailer search cost level \( 0 < s^*_R(m, n, s_C) < \bar{s}_R(n) \) defined by

\[ s^*_R(m, n, s_C) = \frac{1}{n} \left( p^m(w^m) D(p^m(w^m)) - \rho_C D(\rho_C) \right), \]  

(10)

such that for any \( s_R \in (s^*_R(m, n, s_C), \bar{s}_R(n)) \) partial squeezing of retailers is optimal and a double marginalization equilibrium does not exist. If, however, \( s_R < s^*_R(m, n, s_C) \), the deviation is not profitable and the DM strategy profile constitutes an equilibrium.

From the above, we can characterize the existence of the DME as follows. The fact that it is not optimal for the manufacturer to deviate to prices above the consumer reservation price is a consequence of single-peakedness and proved in the Appendix.

**Proposition 3.** For any given \( m \) and \( n \), the Double Marginalization strategy profile is an equilibrium if and only if \( s_C \) and \( s_R \) are such that (i) \( s_C > \bar{s}_C(m) \) as defined in (7) or (ii) \( s_C < \bar{s}_C(m) \) and \( s_R < s^*_R(m, n, s_C) \) as defined in (10).

The critical threshold value \( s^*_R(m, n, s_C) \) is decreasing in \( n \) and increasing in \( s_C \) (via \( \rho_C \)). Importantly, the threshold value approaches 0 when \( n \) becomes very large or when \( s_C \) is close to 0. In this case, it is profitable for the manufacturer to deviate for almost all retailer search cost levels. Accordingly, retailer search cost does not need to be significantly large not to have DM. In addition, \( \bar{s}_C(m) \) is increasing in \( m \). Thus, in markets that are thought of as being competitive because the search frictions are small and there are many retailers and manufacturers, DM is not an equilibrium.

Figure 4 illustrates Proposition 3. The region to the right of the thick black line indicates the parameter region where the DM strategy profile is an equilibrium. Above the \( s^*_R(m, n, s_C) \) curve and to the left of \( \bar{s}_C(m) \), the strategy profile is not an equilibrium because manufacturers are better off by partially or fully squeezing retailers.

To provide some quantitative illustration of our findings so far, we briefly consider the special case of a linear demand function \( D(p) = 1 - p \). As is well-known, the
wholesale and retail monopoly prices are equal to \( w^m = \frac{1}{2} \) and \( p^m(w^m) = \frac{1+w^m}{2} = \frac{3}{4} \), respectively. Accordingly, the profit of manufacturers (resp. retailers) is given by \( \frac{1}{8} \) (resp. \( \frac{1}{16} \)). Social welfare and consumer surplus are given by \( \frac{7}{32} \) and \( \frac{1}{32} \). To see for which parameter values this is an equilibrium, we need to compute the consumer and retailer reservation prices, \( \rho_C \) and \( \rho_R \), given by (5) and (8), and calculate \( s_C(m) \) and \( s^*_R(m,n,s_C) \). By (5), we derive the consumer reservation price to be \( \rho_C(s_C) = 1 - \frac{1}{4} \sqrt{\frac{m-1-32ms_C}{m-1}}. \) It follows that \( \overline{s}_C(m) = \frac{4\sqrt{2-5}}{32} m^{-\frac{1}{2}}. \) Proposition 3 shows that the DME exists, if and only if, \( s_R \leq s^*_R(m,n,s_C) \) or \( s_C \geq \overline{s}_C(m) \). In Figure 4, we show this region for \( m = n = 2 \). In this case \( \overline{s}_C(2) = \frac{4\sqrt{2-5}}{64} \), which is approximately 33% of the consumer surplus generated in the DME. Thus, the DM strategy profile is not an equilibrium for a large set of relevant parameter values.

5 Double Diamond Equilibria

We now show that manufacturers may partially or fully squeeze retailers along the equilibrium path by setting a wholesale price that is larger than the wholesale monopoly price with positive probability. This will result in retail prices that are higher than the DM price and in welfare that is smaller than the welfare generated in the DME. \[s^*_R(m,n,s_C) = \frac{1}{4} \left( 1 - \frac{8ms_C}{m-1} - \frac{1}{4} \sqrt{1 - \frac{32ms_C}{m-1}} \right). \] It can be easily shown that \( s^*_R(m,n,s_C) \) is strictly increasing and convex in \( s_C \) and strictly decreasing in both \( m \) and \( n \).

\[\text{Figure 4: The Region where the DME does (not) Exist for a Given Pair } (m,n)\text{.}\]
Interestingly, these equilibria exist whenever the DME does not, i.e., when the consumer search cost is small and the retailer search cost is not too small (depending on how many retailers there are in the market). We call these equilibria Double Diamond Equilibria (DDE) because the combination of search costs for consumers and retailers renders market outcomes that are significantly worse than those in the Diamond model. Moreover, the discontinuity as search costs become negligible is even more severe here than in the original Diamond model. Since manufacturers randomize in equilibrium between two prices, the consumers’ reservation price no longer depends on off-the-equilibrium path beliefs, but it is directly pinned down by equilibrium strategies.

The simplest equilibrium generating market outcomes that are worse than the DM outcome has manufacturers randomizing over two wholesale prices: the wholesale monopoly price $w^m$ defined by (2) is set with probability $\gamma$ and a higher wholesale price $w^{dd} > w^m$ is set with probability $1 - \gamma$. So, manufacturers adopt a mixed strategy given by

$$\sigma_M = \begin{cases} w^m & \text{w.p. } \gamma \\ w^{dd} & \text{w.p. } 1 - \gamma. \end{cases} \quad (11)$$

We know that retailers will react to $w^m$ by setting the retail monopoly price $p^m$. We also know that in any equilibrium where consumers choose reservation price strategies, it must be the case that retailers choose $p(w) = \min\{p^m(w), \rho_C\}$ (if $w \leq \min\{\rho_R, \rho_C\}$). As the manufacturer has to be indifferent between choosing $w^m$ and $w^{dd}$ (and $w^m$ is the unique maximizer of $wD(p^m(w))$), it therefore has to be that in a DDE, retailers react to $w^{dd}$ by choosing $\rho_C$. Thus, we may have two types of DDE: one where with positive probability manufacturers fully squeeze retailers and set $w^{dd} = \rho_C$ and one where they partially squeeze retailers and set $w^{dd} = \rho_R < \rho_C$ with positive probability.

Formally, we define a Double Diamond strategy profile as follows.

**Definition 3.** A symmetric (pure) strategy profile $\sigma^{DD}$ is a Double Diamond (DD) strategy profile if $\sigma^{DD} = (\sigma_M, \sigma^R_R(w), \sigma^C_C(p))$ is such that each manufacturer sets a strategy as in (11), each retailer follows the strategy $\sigma^R_R(w)$ as in (3), and consumers follow the search rule $\sigma^C_C(p)$ as in (4). If $\sigma^{DD}$ is an equilibrium, then we call it a Double Diamond Equilibrium (DDE).

We first inquire into the conditions for the existence of a DDE whereby retailers are fully squeezed. In this case, as manufacturers have to be indifferent between charging
Given that manufacturers now explicitly randomize, $\rho_C$ is defined as follows. Consumers who visit a retailer charging a retail price $\rho_C$ are indifferent between buying and continuing search if, and only if,

$$\int_{\rho_C}^{\infty} D(p)dp = \frac{1}{m} \int_{\rho_C}^{\infty} D(p)dp + \left(1 - \frac{1}{m}\right) \left(\gamma \int_{p^m(w^m)}^{\infty} D(p)dp + \left(1 - \gamma\right) \int_{\rho_C}^{\infty} D(p)dp\right) - s_C.$$ 

To understand the RHS of this expression, note that if a consumer continues to search for a lower retail price, there are two cases to consider: (i) with probability $\frac{1}{m}$, another retailer has visited the same manufacturer setting the high wholesale price $\rho_C$ (with the retail price $\rho_C$ as the best response), and (ii) when another retailer visits one of the other manufacturers, because of manufacturers’ randomized pricing, the retailer sets $p^m(w^m)$ with probability $\gamma$ and $\rho_C$ with probability $1 - \gamma$. Thus, after observing the retail price $\rho_C$, consumers are indifferent if, and only if,

$$\frac{m - 1}{m} \int_{p^m(w^m)}^{\rho_C} D(p)dp = \frac{s_C}{\gamma}.$$  

(12)

In an equilibrium where retailers are fully squeezed, it must be that $\rho_C = w^{dd}$. Thus, the consumer indifference condition (12) does not define $\rho_C$ as is usually the case, but instead determines $\gamma$, the probability that manufacturers choose the wholesale monopoly price, i.e.,

$$\gamma = \frac{s_C}{\frac{m - 1}{m} \int_{p^m(w^m)}^{w^{dd}} D(p)dp}.$$ 

Thus, for every $w^{dd}$ that is determined by the manufacturer’s indifference condition, consumers are indifferent between continuing to search and buying at $w^{dd}$ for every consumer search cost $s_C$, because the probability that manufacturers choose the lower price $w^m$ is adjusted. Note that this expression is linear in $s_C$ and, as will be important later, that $\gamma$ approaches 0 when $s_C$ approaches 0. Note that if $s_C < s_C(m)$, as defined in the previous section, this equation will always define $\gamma$ in such a way that it is smaller than 1.

We next consider the retailer strategy. If a retailer visits a manufacturer with a
wholesale price of $\rho_C$, the best he can do is to buy and sell at $\rho_C$ if

$$\gamma(p^m(w^m) - w^m)D(p^m(w^m)) \leq s_R.$$  \(27\)

This equation is easily understood: the chance of observing a wholesale price of $w^m$ on the next search equals $\gamma$ and the expected pay-off that is obtained should be smaller than the search cost.

Thus, a DDE with full squeezing of retailers exists if $s_C < \bar{s}_C(m)$ and

$$\frac{s_C(p^m(w^m) - w^m)D(p^m(w^m))}{\frac{(m-1)n}{m} \int_{p^m(w^m)}^\infty D(p)dp} \leq s_R. \quad (13)$$

We have the following result.

**Proposition 4.** For any given $m$ and $n$, there exists a DDE where retailers are fully squeezed with strictly positive probability if, and only if, $s_C < \bar{s}_C(m)$ and $s_R$ is such that (13) holds.

Note that the depth of a sale, denoted by $w^d - w^m$, is independent of the search costs $s_C$ and $s_R$ and that the probability of a sale $\gamma$ is increasing in $s_C$. The lower bound on $s_R$ such that retailers do not want to continue to search goes to 0 if $s_C$ approaches 0 or if the number of retailers $n$ is large. The reason why retailers do not want to continue to search after observing $\rho_C$ if $s_C$ is close to 0 (and $s_R$ is also small, but larger than the threshold value defined by the LHS of (13)) is that the probability with which manufacturers charge the monopoly wholesale price $w^m$ is also close to 0 so that the chance of getting a lower wholesale price and making a profit on the next search can be made arbitrarily small.

Note that in a full squeezing DDE, $\gamma$ ranges between 0 and 1, depending on whether $s_C$ is close to 0 or close to $\frac{m-1}{m} \int_{p^m(w^m)}^\infty D(p)dp$. For small $s_C$ values in this range, we have an equilibrium with a regular high price and an irregular lower sale price that is induced by the manufacturer, very similar to what we have observed in Figure 1. At the sales price, retailers’ margins are positive (and are in fact equal to the monopoly level given the wholesale price), while they are 0 at the regular price. For larger $s_C$ values

\[\text{If the retailer cost is a switching cost rather than a search cost, all retailers are assigned to a manufacturer before manufacturers decide on prices and retailers know all manufacturer prices but have to pay a cost } s_R \text{ to switch to alternative suppliers, this condition needs to be modified to } \frac{(p^m(w^m) - w^m)D(p^m(w^m))}{n} \leq s_R.\]

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in this range, we have that the low price is charged most of the time and becomes the regular price.

We next consider the possibility that the manufacturers partially squeeze retailers in a DDE. In this case, it is clear that the following conditions should be satisfied. First, manufacturers should be indifferent between charging $w^m$ and $\rho_R$:

$$\rho_R D(\rho_C) = w^m D(p^m(w^m)).$$  \hfill (14)

As the RHS is constant, it follows from the single-peakedness of $pD(p)$ and the fact that the manufacturer has to be indifferent between choosing $w^m$ and $\rho_R < \rho_C$ that consumers’ reservation price under partial squeezing has to be smaller than under full squeezing.

Second, after observing $\rho_C$, consumers should be indifferent between buying and continuing to search:

$$\frac{m - 1}{m} \int_{\rho^m(w^m)}^{\rho_C} D(p)dp = \frac{s_C}{\gamma}. \hfill (15)$$

This condition is similar to what we saw before in the full squeezing DDE, the only difference being that $\rho_C$ is now determined differently. Finally, at $\rho_R$ retailers should be indifferent between buying and continuing to search for the lower wholesale price $w^m$:

$$(\rho_C - \rho_R) \frac{D(\rho_C)}{n} = \frac{\gamma}{n} (p^m(w^m) - w^m) D(p^m(w^m)) + \frac{1 - \gamma}{n} (\rho_C - \rho_R) D(\rho_C) - s_R,$$

which reduces to

$$p^m(w^m) D(p^m(w^m)) -\rho_C D(\rho_C) = \frac{ns_R}{\gamma}. \hfill (15)$$

To investigate when such a partial squeezing DDE exists, consider for any given $m$ and $n$ all pairs $(s_C, s_R)$ such that (10) is satisfied. In the previous section, we have argued that these $(s_C, s_R)$ pairs define the boundary of the region where the DME exists. These $(s_C, s_R)$ pairs satisfying (10) can, however, also be re-interpreted as partial squeezing DDE where $\gamma = 1$. From equation (14), (15) and (12) it is then clear, however, that if $(\rho_R, \rho_C, 1)$ is a solution for the parameters $(m, n, s_C, s_R)$, then for any $\gamma \in (0, 1)$ it should be the case that $(\rho_R, \rho_C, \gamma)$ is a solution for the parameters $(m, n, s_C', s_R')$, where $s_C' = \gamma s_C$ and $s_R' = \gamma s_R$. This fact is used in the proof of the next proposition to argue that for small values of $s_C$ and $s_R$, the partial squeezing DDE exists in the region in between the DME and the full squeezing DDE. Thus, at least one of these types of
equilibria always exists.

**Proposition 5.** For any given set of parameter values \(m, n, s_C\) and \(s_R\), there exists a unique RPE where consumers blame manufacturers for an out-of-equilibrium price observation. The equilibrium is either of the DM or the DD type.

In the online Appendix we show that if consumers blame retailers for observing an out-of-equilibrium price, the equilibrium prices remain the same and it is only the region where a DME exists that shrinks. Therefore, equilibrium outcomes are not determined by the particular choice of out-of-equilibrium beliefs, as long as they fulfill the minimum deviation path property.

**Welfare**

DDE yield very inefficient outcomes, and this inefficiency is largest in competitive conditions. Indeed, DDE exist when \(s_C\) is small and \(n\) is relatively large (or \(s_R\) is not too small). Further, as consumer search cost decreases, by (12), the equilibrium value of \(\gamma\) also decreases. In particular, when \(s_C\) goes to zero, \(\gamma\) converges to 0 and so the randomized market prices converge to \(w = p = \rho_C\). Competitive forces, far from improving equilibrium outcomes, lead to equilibria with lower social welfare in markets with search frictions in both layers of the product chain. In particular, the social welfare loss is greatest when the consumer search cost approaches zero.

How much social welfare in the DDE is lower than that in the DME depends on the shape of the demand function, the number of firms, and the level of search costs. We now illustrate that the effects can be large by considering linear demand \(D(p) = 1 - p\).

From the above, it follows that a full squeezing DDE exists if \(s_C < \bar{s}_C(m) = \frac{4\sqrt{2} - 5}{32} \frac{m-1}{m}\) and \(s_R > \frac{2m}{(4\sqrt{2} - 5)(m-1)n}s_C\). From (6) it follows that in this equilibrium \(\rho_C = \frac{2 + \sqrt{2}}{4}\) and \(\gamma = \frac{32}{4\sqrt{2} - 5} \frac{m-1}{m}s_C\). The region where this equilibrium exists is also depicted in Figure 5. Total surplus and consumer surplus are increasing in \(s_C\). When \(s_C\) approaches 0, total and consumer surplus are at their lowest level and equal to approximately 0.135 and 0.01, respectively, which is around 38% and 66% lower than the corresponding figures for the DM outcome, which is already known to generate low welfare levels. The remaining area in Figure 5 is where the partial squeezing DDE exists.

Figure 6 shows how total surplus and consumer surplus vary with changes in \(s_C\) and \(s_R\) for a given level of the other search cost. In the left panel, we show the dependence on \(s_C\) for \(s_R = 0.016\) that is such that the equilibrium gradually changes from a DDE
with full squeezing to one with partial squeezing and finally, for large enough values of $s_C$, to the DME. The right panel of Figure 6 shows a similar picture for the impact of $s_R$ for a value of $s_C$ equal to 0.005. Notice that the impact of search cost on welfare is large and that the two search costs have opposite effects: welfare is lowest for large values of $s_R$ and small values of $s_C$.

The panels clearly show the discontinuity of total surplus and consumer surplus at a search cost equal to 0 and how the interaction between retail and consumer search.

Figure 5: Equilibrium for Different Search Costs.

Figure 6: Welfare and Consumer Surplus as a Function of Search Costs.
severely affects this discontinuity. If both retailer and consumer search costs are equal to 0, we have marginal cost pricing and total surplus and consumer surplus being both equal to 0.5. With retailer search costs being equal to 0 and consumer search costs approaching 0, we have the Diamond paradox with monopoly pricing, resulting in total surplus and consumer surplus being equal to 0.375 and 0.125, respectively. With both search costs approaching 0 at a ratio such that the full squeezing DDE prevails, we have a limiting total surplus and consumer surplus of 0.135 and 0.01.28

6 Individualized Wholesale Prices

So far we have considered the case where the manufacturers post a single price to every incoming retailer. In this section we will re-do our analysis for the case where manufacturers are free to choose individualized wholesale prices to different retailers. In order to keep the analysis as close as possible to the benchmark model, we restrict manufacturers to make independent and identically distributed offers to different retailers. We leave the case of correlated offers for future research.

Let us first consider the conditions under which Double Marginalization is an equilibrium outcome. While the retailers’ reservation price is unchanged, consumer’s reservation price changes. Regardless of whether they blame the manufacturer or the retailer, there is no reason for consumers to update their beliefs regarding the distribution of prices in another retailer after they have observed an off-the-equilibrium retail price. Hence, the consumer reservation price $\rho_C$ is now implicitly determined by

$$\int_{\rho_{m(w^m)}}^{\rho_C} D(p)dp = s_C,$$

which is the same definition as when consumers blame retailers.29 Since the rest of the analysis is unchanged, a DM equilibrium exists if $s_C > \frac{m}{m-1}5_C(m)$ or if $s_C > \frac{m}{m-1}5_C(m)$ but $s_R < s_R^*(m, n, \frac{m-1}{m}s_C)$ as defined in (10). As a result, the area in which DME exists shrinks if manufacturers have the ability to choose different prices for different retailers.

We now turn our attention to the characterization of the DDE in which retailers are fully squeezed. The probability $\gamma$ that manufacturers choose the monopoly wholesale

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28 The case where consumer search cost equals 0 while retailer search costs are positive is somewhat different and dealt with in Honda (2015).
29 See the online Appendix. The difference between these two cases is that if consumers blame retailers from a deviation by the manufacturer, deviations become more profitable and the region in which an equilibrium exists shrinks.
price is now defined by
\[ \int_{p^m(wm)}^{p_C} D(p) dp = \frac{s_C}{\gamma}, \] (16)
since each manufacturer uses an independent randomization to determine her price for every retailer that buys from her.\(^{30}\) Also, the description of the optimal retail strategy does not change, and thus even if retailers are fully squeezed, they will not want to continue to search if their search cost is high enough, which (given the slight adaptation in the definition of \(\gamma\)) is now the case if
\[ \frac{s_C(p^m(w^m) - w^m) D(p^m(w^m))}{n \int_{p^m(wm)}^{p_C} D(p) dp} \leq s_R. \]

Thus, defining the LHS of this inequality by \(\underline{s}_R\) it is clear that for all \((s_C, s_R)\) with \(s_C < \frac{m-1}{m} s_C(m)\) and \(s_R > \underline{s}_R\) a Double Diamond Equilibrium exists where retailers are fully squeezed as in this case we can always find a \(\gamma\), with \(0 < \gamma < 1\), such that (16) is satisfied.

Finally, we can use the same argument as in the case of posted prices, to argue that if \(s_C < \frac{m-1}{m} s_C(m)\) and \(s_R^I(m, n, m^{-1} s_C) < s_R < \underline{s}_R\) there exists a Double Diamond Equilibrium where retailers are partially squeezed.

7 Shoppers

It is known that the Diamond paradox is a consequence of all consumers not searching beyond the first firm. Introducing a fraction \(\lambda\) of consumers (shoppers) with zero search cost as in Stahl (1989) removes the paradox. One may think that our Double Diamond paradox is similar in that it does not hold in the presence of shoppers who actively search for the best price. In this section, we show that this is not the case and that the main features of the DDE survive if \(\lambda\) is not too large: (i) manufacturers randomize, while retailers react with a pure strategy to this randomization (so that sales are induced by manufacturers), (ii) with positive probability, prices are above the double marginalization retail price and (iii) the size of the upper interval can be very small so that “regular” retail prices are concentrated in an interval just below the

\(^{30}\) We do not allow a manufacturer to condition her randomization device to one retailer on the outcome of the randomization to another retailer. If that were possible, independent randomization would not be optimal. To do so would require, however, that the manufacturer decisions are sequential. Our analysis applies when a manufacturer simultaneously randomizes to all retailers that visit her.
consumer reservation price. In this way we may explain the observation that sales prices tend to be somewhat more variable than the regular price (cf., Heidhues and Köszegi (2014)).

We continue assuming that all retailers have a positive search cost. While one may think that some consumers enjoy shopping and really like to compare many prices before buying, retailers are professionals with a clear opportunity cost of their time. We focus on the case where \( s_R \) is large enough so that the retailers’ reservation price is not binding. In this section, we explicitly revert to the case where manufacturers set individualized wholesale prices to retailers. We continue to assume that manufacturers make independent offers to different retailers, which significantly simplifies the analysis, as we argue below. Finally, for analytic tractability, we focus on a duopolistic retail market (i.e. \( n = 2 \)).

We denote by \( G(w) \) the distribution of wholesale prices. As we focus on equilibria where some wholesale prices are smaller than \( w^m \), while others are larger, we denote by \( G_1(w) \) the distribution of wholesale prices conditional on \( w \leq w^m \) and by \( G_2(w) \) the distribution of wholesale prices conditional on \( w^m < w \leq \rho_C \). Let \( \gamma \) be the probability that \( G_1(w) \) is charged, while \( 1 - \gamma \) is the probability that \( G_2(w) \) is charged. Clearly, no wholesale price larger than \( \rho_C \) will ever be part of an equilibrium. We denote by \( F(p \mid w) \) the distribution of retail prices following a wholesale price \( w \). As manufacturers charge individualized prices to retailers, they randomly sample from \( G(w) \) whenever they are visited by a retailer. In this environment, if \( G(w) \) is atom-less, then the equilibrium retail price is a pure strategy \( p(w) \). This follows as for any atom-less \( G(w) \), the distribution of retail prices yielding positive markups will also be atom-less and, given our assumptions on demand, this implies that there is a unique maximizer. With posted wholesale prices, the situation would be more complicated as in that case, after observing a particular \( w \), retailers know that with positive probability other retailers visit the same manufacturer, so that there is a positive probability they all have the same cost. In that case, retailers would like to undercut each other to attract the shoppers and randomize their pricing decisions.

We proceed as follows. We construct an equilibrium with features that are similar to the DDE and show which conditions should be satisfied. In the Proposition we then formally show for which values of \( \lambda \) and \( s_C \) all required conditions hold. We can write the retailer’s optimization problem after observing wholesale price \( w \) in the support of \( G(w) \) as one of choosing which cost \( \tilde{w} \) he would like to pretend to have and price
accordingly. Thus, the retail price $p(w)$ is the maximizer of

$$(p(\tilde{w}) - w) \left[ \frac{1 - \lambda}{2} + \lambda (1 - \gamma G_1(\tilde{w})) \right] D(p(\tilde{w})), \quad \text{if } \tilde{w} \leq w^m$$

$$(p(\tilde{w}) - w) \left[ \frac{1 - \lambda}{2} + \lambda (1 - \gamma)(1 - G_2(\tilde{w})) \right] D(p(\tilde{w})), \quad \text{if } \tilde{w} > w^m$$

which we rewrite as $(p(\tilde{w}) - w)Q(\tilde{w})$. In equilibrium, the retailer would like to price as if his cost were $w$ so that

$$p'(w)Q(w) + (p(w) - w)Q'(w) = 0. \quad (17)$$

This expression implicitly determines $p(w)$ as a function of $Q(w)$. Notice that with individualized prices, a manufacturer who chooses a wholesale price $w$ will face a demand of exactly $Q(w)/m$. A manufacturer will find it optimal to randomize, if she is indifferent charging any $w$ in the support of $G(w)$. So, we should have

$$wQ'(w) + Q(w) = 0.$$

Substituting this condition into (17) we obtain a differential equation with solution

$$p(w) = w(c_i - \ln(w)),$$

where $c_i$, $i = 1, 2$, are constants that depends on the lower and upper interval of prices, respectively. We have two different boundary conditions, depending on whether or not $w \leq w^m$. For the lower interval of prices we should have $p(w^m) = p^m(w^m),^{31}$ yielding $c_1 = \frac{p^m(w^m)}{w^m} + \ln(w^m)$, whereas for the upper interval, we should have $p(\rho_C) = \rho_C$, yielding $c_1 = 1 + \ln \rho_C$. Thus, we can write the equilibrium strategy of the retailer as

$$p^*_1(w) = w\left(\frac{p^m(w^m)}{w^m} + \ln \frac{w^m}{w}\right) \text{ if } w \leq w^m,$$

$$p^*_2(w) = w\left(1 + \ln \frac{\rho_C}{w}\right) \text{ if } w^m < w \leq \rho_C.$$
Writing $\pi_1(w) = wD(p_1^*(w))$ and $\pi_2(w) = wD(p_2^*(w))$, the wholesale price distributions can be derived in the usual way:

$$G_1(w) = \frac{1}{\gamma} \left( 1 - \frac{[(1 - \lambda) + 2\lambda(1 - \gamma)] \pi_1(w^m) - \pi_1(w)}{2\lambda} \right)$$ \tag{19}

and

$$G_2(w) = 1 - \frac{1}{1 - \gamma} \left( \frac{[(1 - \lambda) + 2\lambda(1 - \gamma)] \pi_1(w^m) - \pi_2(w)}{2\lambda} \right).$$ \tag{20}

To fully characterize the equilibrium, we still need to determine $\gamma$. As in the DDE, $\gamma$ follows from the non-shoppers’ indifference condition between searching and buying after observing $\rho_C$. So, we should have

$$\gamma \int_{w_1}^{w^m} \int_{p_1^*(w)}^{\rho_C} D(p)dpdG_1(w) + (1 - \gamma) \int_{w_2}^{\rho_C} \int_{p_2^*(w)}^{\rho_C} D(p)dpdG_2(w) = s_C.$$ \tag{21}

Let $\pi_1^R(w) = [(1 - \lambda)/2 + \lambda(1 - \gamma G_1(w))](p_1^*(w) - w)D_1(p_1^*(w))$ and $\pi_2^R(w) = [(1 - \lambda)/2 + \lambda(1 - \gamma - (1 - \gamma)G_2(w))](p_2^*(w) - w)D_2(p_2^*(w))$. Then we denote the lower bound on the retailer search cost such that they are indeed not willing to search by

$$\gamma \int_{w}^{w^m} \pi_1^R(w)dG_1(w) + (1 - \gamma) \int_{w_2}^{\rho_C} \pi_2^R(w)dG_2(w) = \bar{s}_R(\lambda, s_C).$$ \tag{22}

We are now able to state the following proposition, where we define when the equilibrium we have characterized above exists.

**Proposition 6.** If $\lambda$ is small enough, then there exists critical values $s_C(\lambda), \bar{s}_C(\lambda)$ and $s_R(\lambda, s_C)$ such that a unique reservation price equilibrium exists where manufacturers set individualized prices to retailers for all $(s_C, s_R)$ such that $s_C(\lambda) < s_C < \bar{s}_C(\lambda)$ and $s_R \geq s_R(\lambda, s_C)$. In this equilibrium, the wholesale price distribution is atom-less and given by (19) and (20), retailers buy and set prices $p_1^*(w)$ and $p_2^*(w)$ as defined in (18) and $\gamma$ is defined by (21). Moreover, $s_C(\lambda)$ goes to 0 as $\lambda$ approaches 0, while $\gamma$ is increasing in $s_C$.

Thus, in the presence of shoppers some, but not all, important features of the DDE continue to hold. For a range of search cost parameters, manufacturers randomize over two intervals, one that lies below the wholesale monopoly price and one that lies above. This implies that with strictly positive probability retail prices are chosen that are larger than the double marginalization retail price. Moreover, within this range
lower consumer search cost tends to be associated with higher prices (as $\gamma$ decreases). Unlike the DDE in our baseline model, for a given $\lambda > 0$, this type of DDE does not exist when the consumer search cost becomes arbitrarily small. The reason is that as long as manufacturers make positive profits, there will always be a minimum amount of price dispersion in the presence of shoppers (even when $\gamma = 0$) and this is inconsistent with arbitrarily small consumer search cost. However, the lower bound on the consumer search cost for which the equilibrium exists, converges to 0 when lambda goes to zero, showing that our baseline model without shoppers can be considered the limiting model of a model with shoppers. Note also that these equilibrium features are very different from the monopoly manufacturer model with a fraction of shoppers that is studied in Janssen and Shelegia (2015a). In that model, it is the manufacturer that sets a deterministic price while the retailers randomize (as there is no incentive for the manufacturer to undercut herself). Moreover, in that model, an equilibrium where the manufacturer squeezes retailers may exist if the search cost is arbitrarily small.

To illustrate Proposition 6, consider the linear demand case where $D(p) = 1 - p$. The next Figure illustrates the region of consumer search cost for which a reservation price equilibrium with two disjoint intervals as described in Proposition 6 exists. It is clear that the lower bound $s_C(\lambda)$ is very close to 0 and that the upper bound of the search cost region is also increasing in $\lambda$, implying that the DDE features may exist for even higher values of $s_C$ than in our base line model where $\lambda = 0$. This also implies that there may be a region of search cost where the welfare generated in markets with shoppers may actually be lower than the welfare generated in markets without shoppers (as in the latter market the equilibrium would already be characterized by Double Marginalization). Figure 8 describes both the wholesale and retail price distributions for a given fraction of shoppers and three different values of $s_C$. What is interesting
Figure 8: The wholesale (gray-coloured line) and retail (black-coloured line) price distributions for $\lambda = 0.05$ and three different values of $s_C \in \{0.01, 0.03, 0.04\}$. The horizontal axis indicates prices under linear demand $D(p) = 1 - p$.

Figure 9: Both welfare and consumer surplus (weakly) increase with and without shoppers as the consumer search cost $s_C$ increases.

here is that even though the price distributions in both intervals are continuous, the prices in the upper interval, especially at the retail level, are very much concentrated. This effect becomes stronger when $s_C$ increases as in that case $\gamma$ is larger and it is very likely that shoppers will buy from prices in the lower interval, implying a very narrow upper interval.

Finally, it is interesting to consider the welfare effects of introducing shoppers. First, it is clear that the welfare levels, and consumer surplus, are in the same very low range as we have seen before in the DDE. As is to be expected, as shoppers introduce more competition in the market, welfare is generally a little bit larger than in the DDE and DME we have considered in the baseline model. However, what is surprising is that there is an interval of parameter values where the presence of shoppers actually lowers total surplus (see Figure 9). This occurs as the equilibrium with two disjoint intervals of prices exists for a much larger range of consumer search cost parameter values than in the market without shoppers and that when in the DDE without shoppers $\gamma$ is already close to 1, there is still a considerable mass of the probability distribution in the upper interval of the market equilibrium with shoppers.
8 Discussion and Conclusion

This is the first paper dealing with upstream competition in consumer search markets where retailers search for offers from manufacturers. We show that retailer search has important implications for the functioning of markets. From a welfare perspective, our main result is that markets with fairly small consumer search cost will produce outcomes that are not better, and often much worse, than the double marginalization outcome. This is especially true when there are many manufacturers and many retailers. This remains true even if there is a fraction of shoppers in the market that compares all prices.\textsuperscript{32}

The welfare results of our paper are surprising and disturbing. To the best of our knowledge, there is little direct evidence on the magnitude of manufacturers’ margins in vertically related industries.\textsuperscript{33} Further, notice that the evidence suggesting that retail markets are competitive in the sense that retail prices are close to retailers’ marginal cost, whereas manufacturer margins may be substantial. Indeed, retail margins in the Double Diamond Equilibria are lower than those in the Double Marginalization Equilibrium, but the welfare generated by the Double Diamond Equilibria is much lower.

From an empirical point of view, our paper generates a new and interesting way to explain retail price distributions. In a given store, the distribution of prices for a given product tends to be concentrated around two price levels: a regular price and a sales price. Existing models of price dispersion typically fail to generate this bimodal nature of the (retail) price distribution, or when they do (as in dynamic monopoly models) they fail to consider the interaction between wholesale and retail prices. One obvious way to empirically disentangle our explanation of sales and other explanations is to focus on whether or not wholesale and retail prices are positively correlated and whether retail margins are larger in periods of sales. Our model also suggests that it is manufacturers who induce sales by offering discounts to retailers. While there is surprisingly little

\textsuperscript{32}Our model assumes that consumers do not know which retailers are supplied by which manufacturers. If consumers would know, they could direct their search activities to retailers that are supplied by different manufacturers than their current retailer if they have observed a high retail price. This may make the retailers’ position even worse, possibly providing them with an incentive to hide the identity of their supplier.

\textsuperscript{33}An exception is Noton and Elberg (2015), who show that manufacturers actually get a substantial share of the surplus vis-a-vis retailers, even if retailers are large. There are a number of studies that have used indirect identification based on pre-existing models to assess this magnitude. Since they are model-driven, they do not show the size of model-independent empirical margins. See, e.g., Villas-Boas (2009).
empirical research on the interaction between retail and wholesale prices, in the Online Appendix we provide some suggestive evidence that these decisions are very much related. These results draw attention to the importance of simultaneously studying price fluctuations at both levels of the product chain and call for a more ambitious empirical study of pricing along the vertical product chain in markets with search frictions.

Our paper also points in the direction of taking search frictions seriously when doing horizontal merger analysis at both the retail and wholesale level. More concentration at the retail level potentially gives retailers more incentive to continue to search for better price offers from manufacturers. Markets with relatively low consumer search costs are less like to end up in a Double Diamond Equilibrium if the number of retailers is small. Also, the number of manufacturers may affect consumer reservation prices, as with fewer manufacturers it is more likely that alternative retailers will be supplied by the same manufacturer, making it less beneficial for consumers to search and, therefore, not optimal for manufacturers to deviate. It should be noted, though, that our results are derived in a model where it is the manufacturer that has the power to propose the terms of trade with retailers. It would be interesting to reverse this power and to investigate how our conclusions change when retailers have the power to determine wholesale conditions (as is certainly true in some markets).

We have focused attention on a simple vertical industry with linear pricing contracts between independent firms. It is not difficult to see that if the first search is costly, allowing for two-part tariffs will result in a market break down as the manufacturers cannot commit to a contract where retailers make a profit. In a sense, this would strengthen our inefficiency results, but one could also argue that sequential search models are not well-suited to studying non-linear contracts.34 In this sense, future research on simultaneous search by retailers would be of some interest. Allowing for vertical integration provides firms with incentives to charge simple monopoly prices at the retail level. Thus, in markets where retailer search is important, there is a marked difference between a vertical merger (where a retailer no longer needs to search) and a two-part tariff where search is still important. Eliminating retailer search cost is another reason to have a positive attitude towards vertical mergers. The reason not to consider vertical integration in this paper is that there are many markets where retailers carry

34If the first search is costless, two-part tariffs would give rise to double marginalization. However, this would create high risks for the company that is supposed to pay the high fixed fee. These companies may not be willing to bear these risks in a world where there is high uncertainty about demand.
products of many different manufacturers, creating barriers for vertical integration. When retailers carry many different products, multi-product search is important, and it would be interesting to combine the analysis of Rhodes (2015) and Zhou (2014) with ours.

**References**


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A Appendix A: Proofs

Proof of Proposition 1

Consider an arbitrary symmetric (possibly mixed) equilibrium strategy profile of manufacturers, retailers, and consumers. Let \( w \) be the lower bound of the wholesale price distribution. Without loss of generality, we assume that given any symmetric strategies of manufacturers and retailers, there exists a maximum price \( \rho_C \) such that for any retail price that is lower than \( \rho_C \), consumers do not continue to search and buy at that price.

First of all, we show that if retailers observe the minimum wholesale price \( w \leq \rho_C \), it is optimal for retailers to set the optimal retail price \( p_m(w) \) because consumers would not search at that price. This corresponds to Lemma 7 below. Next, given Lemma 7, we show that if \( w < w^m \), manufacturers have an incentive to unilaterally deviate from \( w \) to a slightly higher wholesale price. This corresponds to Lemma 8 below. Thus,
Lemmas 7 and 8 imply that $w \geq w^m$ (resp. $p \geq p^m(w^m)$) holds in any equilibrium strategy profile.

**Lemma 7.** Assume that manufacturers adopt a symmetric strategy whereby they put positive probability on a price $w \in (0, w^m)$. If a retailer visits a manufacturer setting $w$, it is optimal for the retailer to buy the product and set the retail price at $p^m(w)$.

**Proof.** First of all notice that in such a strategy profile, $\rho_C > p(w)$, since consumers should be compensated for their search costs. Therefore, for any price in $(p(w), \rho_C)$ retailers’ profits are just proportional to per-consumer profits. Since per-consumer profits are single-peaked, if $p(w) < p^m(w)$, increasing the price leads to higher profits. On the other hand, any $p(w) > p^m(w)$ is dominated. Thus, $p(w) = p^m(w)$.

**Lemma 8.** In any equilibrium, $w \geq w^m$.

**Proof.** Assume to the contrary that there exists a symmetric equilibrium such that the lower bound of the wholesale prices in the manufacturers’ strategy is $w \in [0, w^m)$. Notice that whenever retailers visit a manufacturer setting the minimum wholesale price $w$, they buy the product from the manufacturer and set the retail price $p^m(w) < \rho_C$ to consumers, and then consumers visiting the retailers buy the product.

Suppose that a manufacturer deviates from $w$ to a slightly higher wholesale price $w$ that is close enough to $w$. Since $w$ is sufficiently close to $w$, $p^m(w) < \rho_C$ holds. Since $w < w < w^m$ and $p^m(w) < p^m(w) < \rho_C$ hold, by single-peakedness of the profit function, it follows that $w_D(p^m(w)) < w(p^m(w))$. Thus, the deviating manufacturer can slightly increase the profit by setting a slightly higher wholesale price $w$.

Since we can repeat the above argument until the minimum wholesale price $w$ reaches the wholesale monopoly price $w^m$, it implies that manufacturers never set a wholesale price $w \in [0, w^m)$.

**Proof of Proposition 2**

By Proposition 1, we do not need to consider a case where a manufacturer sets some wholesale price $w < w^m$ with some probability.

Assume to the contrary that there exists a symmetric equilibrium such that the lower bound of the wholesale prices in the strategy of manufacturers is $w > w^m$. When

\[\text{35The inequality follows because } p^m(w) \text{ is the minimum retail price and consumers have a positive search cost.}\]
a manufacturer sets \( w \), a visiting retailer buys the product at \( p^m(w) \) due to Lemma 7, and then a visiting consumer buys the product at \( p^m(w) \) because of \( p^m(w) < \rho_C \). Given this, each manufacturer has an incentive to deviate to the wholesale monopoly price \( w^m \) for which visiting retailers buy and sell at the retail monopoly price \( p^m(w^m) \), and then visiting consumers buy at \( p^m(w^m) \). This gives the deviating manufacturer the profit given by \( w^m D(p^m(w^m)) \) that is larger than the profit under \( w \) given by \( w D(p^m(w)) \) due to single-peakedness of the profit function. Thus, the deviation is profitable, a contradiction.

Next, we show that manufacturers do not charge \( w' = w^m + \varepsilon \) for a sufficiently small \( \varepsilon > 0 \). From the above, we know that manufacturers set \( w^m \) with some positive probability. Given this, assume to the contrary that there exists a symmetric equilibrium such that manufacturers charge a slightly higher wholesale price \( w' = w^m + \varepsilon \) with a positive probability for a sufficiently small \( \varepsilon > 0 \). Since \( w' \) is sufficiently close to \( w^m \), a visiting retailer buys the product at \( w' \) without continuing to search due to search cost \( s_R > 0 \) and sells to consumers at \( p^m(w') \) that is a little bit above \( p^m(w^m) \) because consumers visiting the retailer buy the product at \( p^m(w') \) without continuing to search due to search cost \( s_C > 0 \). From the above, the manufacturer setting \( w' \) gets the profit given by \( w' D(p^m(w')) \) that is smaller than the profit under \( w^m \) given by \( w^m D(p^m(w^m)) \). Thus, each manufacturer has an incentive to deviate to \( w^m \), a contradiction.

**Proof of Proposition 3**

*Proof.* The main part of the proof is given in the text. To prove that it is not optimal to deviate to wholesale prices larger than \( \rho_C \), first note that since the lower bound of the support of the price distribution in any equilibrium is \( p^m(w^m) \), \( \rho_C \geq p^m(w^m) \). Thus, it suffices to prove that \( p^m(w^m) \geq \arg \max_w w D(w) \). To see this, notice that

\[
D(p^m(w^m)) + p^m(w^m) D'(p^m(w^m)) = D(p^m(w^m)) + \left( w^m - \frac{D(p^m(w^m))}{D'(p^m(w^m))} \right) D'(p^m(w^m))
\]

\[
= w^m D'(p^m(w^m)) < 0,
\]

where the first equality comes from the definition of \( p^m(w^m) \) and the inequality comes from downward-sloping demand. Since \( w D(w) \) is single-peaked, the result follows. \( \Box \)
Proof of Proposition 5

We first restate the Proposition in terms of admissible beliefs. As explained in the text, we shall impose the Minimum Deviation Property. See the Online Appendix for details.

Lemma 9. There exists (at most) one RPE Outcome for any belief system satisfying the Minimal Deviation Property.

Proof. First, in an RPE, there cannot be active search. We show this for the case of consumers, but the same argument works for retailers. For a contradiction, assume that there exists a set of equilibrium retail prices \( P \) such that consumers search at \( P \) and let \( \bar{p} \) be the maximum of those prices. Clearly, by the reservation price property, \( \bar{p} \) is also the maximum equilibrium price. Hence, demand at \( \bar{p} \) is zero. Hence, either the wholesale price is also \( \bar{p} \) or the retailer could profitably deviate to a lower price. But a wholesale price of \( \bar{p} \) cannot be optimal for a manufacturer who could charge the double marginalization price. Thus, there is no active search. We are left with two cases to consider. First, if there is price dispersion in equilibrium, both retailers’ and consumers’ reservation prices are pinned down uniquely in any belief system satisfying the MDP. Since consumers’ demand is \( D(p) \) for any \( p \leq \rho_C \), the retailers’ optimal price is \( \max\{p^m(w), \rho_C\} \) if \( w \leq \rho_R \), \( p = w \) otherwise (provided that \( w \leq \rho_R \) holds and retailers do not continue to search). This implies that retailers’ demand is proportional to \( D(p^m(w)) \) for any \( w < \min\{\rho_R, \rho_C\} \). Clearly, if \( w < \min\{\rho_R, \rho_C\} \), then \( p(w) = p^m(w) \). Otherwise, \( p(w) = \min\{\rho_R, \rho_C\} \). Second, if there is no price dispersion, Proposition 1 implies that the unique equilibrium outcome is the DME outcome. Next, notice that DM is an equilibrium outcome, if and only if, there is no profitable deviation for a manufacturer to charge a higher price than the wholesale monopoly price. Hence, if there exists a pure-strategy equilibrium there does not exist an equilibrium with price dispersion and vice versa. Hence, RPE is unique.

Proof of Proposition 6

Proof. The main part of the proof is in the text. What we will show here is that \( \gamma \) is strictly increasing in \( s_C \) so that the lower and upper bounds of \( s_C \) for which this equilibrium holds coincide with the constraints that \( \gamma \) should be larger than 0 and smaller than 1. For sufficiently small \( \lambda \), let

\[
\begin{align*}
h_1(\gamma) &= \left[ (1 - \lambda) + 2\lambda(1 - \gamma) \right] \frac{\pi_1(w^m)}{2\lambda} = \left( 1 + \frac{\lambda - 2\lambda\gamma}{2\lambda} \right) \frac{\pi_1(w^m)}{\pi_1(w^m)},
\end{align*}
\]

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\[ \mathbb{E}[D_1(w)] = \int_{p_1^*(w)}^{p_2^*(w)} D(p)dp, \quad \mathbb{E}[D_2(w)] = \int_{p_2^*(w)}^{p_C(w)} D(p)dp, \quad \text{and} \]

\[ h_2(\gamma) = \int_{\omega_1}^{\omega_m} (\mathbb{E}[D_1(w)]) \left( \frac{\pi_1'(w)}{(\pi_1(w))^2} \right) dw + \int_{\omega_2}^{p_2^*(w)} (\mathbb{E}[D_2(w)]) \left( \frac{\pi_2'(w)}{(\pi_2(w))^2} \right) dw. \]

where the derivative \( \pi_1'(w) \) for \( w \in [\omega_1, \omega_m] \) and \( \pi_2'(w) \) for \( w \in [\omega_2, \bar{\omega}_2] \) both must be positive under the equilibrium conditions, as long as \( \lambda \) is sufficiently small. We can rewrite the condition (21) by

\[ h_1(\gamma) h_2(\gamma) = s_C. \]  \hspace{1cm} (23)

Denote by \( f(\gamma) \) and \( f'(\gamma) \) the LHS of (23) and its first derivative with respect to \( \gamma \) respectively. We show that \( f'(\gamma) \) is positive for any given \( \gamma \in (0, 1) \).

The lower bound \( \omega_1 \) of the distribution \( G_1 \) provides the condition: \( (1 + \lambda) \pi_2(\omega_1) = (1 + \lambda - 2 \lambda \gamma) \pi_1(\omega_m) \). This leads to

\[ \omega_1'(\gamma) = - \left( \frac{2 \lambda}{1 + \lambda} \right) \left( \frac{\pi_1(\omega_m)}{\pi_1'(\omega_1)} \right). \]

Similarly, the upper bound \( \bar{\omega}_2 = \rho_C \) of the distribution \( G_2 \) satisfies the condition: \( (1 - \lambda) \pi_2(\rho_C) = (1 + \lambda - 2 \lambda \gamma) \pi_1(\omega_m) \) with \( p_2^*(\rho_C) = \rho_C \). It follows that

\[ \rho_C'(\gamma) = - \left( \frac{2 \lambda}{1 - \lambda} \right) \left( \frac{\pi_1(\omega_m)}{\pi_2'(\rho_C)} \right). \]

Since both the lower bound \( \omega_1 \) and the upper bound \( \rho_C \) are functions of \( \gamma \), we denote them by \( \omega_1 = \omega_1(\gamma) \) and \( \rho_C = \rho_C(\gamma) \) respectively if necessary. Similarly, \( p_2^*(w) \) also should be a function of \( \gamma \) because the retail strategy \( p_2^*(w) = w(1 + \ln(\rho_C/w)) \) is a function of \( \rho_C \), and its first derivative with respect to \( \gamma \) is given by

\[ p_2'(\gamma) = (w/\rho_C) p_C'(\gamma). \]

Taking the first derivative of the LHS yields

\[ f'(\gamma) = h_1'(\gamma) h_2(\gamma) + h_1(\gamma) h_2'(\gamma) \]

where

\[ h_1'(\gamma) = - \pi_1(\omega_m) \]
and

\[
    h'_2(\gamma) = -\mathbb{E}[D_1(w_1)] \left( \frac{\pi'_1(w_1)}{(\pi_1(w_1))^2} \right) w'_1(\gamma) + \int_{w_1}^{w_m} D(\rho_C) \rho'_C(\gamma) \left( \frac{\pi'_1(w)}{(\pi_1(w))^2} \right) \, dw \\
    + \mathbb{E}[D_2(\rho_C)] \left( \frac{\pi'_2(\rho_C)}{(\pi_2(\rho_C))^2} \right) \rho'_C(\gamma) + \int_{w_2}^{\rho_C} D(\rho_C) \rho'_C(\gamma) \left( \frac{\pi'_2(w)}{(\pi_2(w))^2} \right) \, dw \\
    - \int_{w_2}^{\rho_C} D(p'_2(w)) \frac{w}{\rho_C} \rho'_C(\gamma) \left( \frac{\pi'_2(w)}{(\pi_2(w))^2} \right) \, dw
\]

where \( \mathbb{E}[D_2(\rho_C)] = 0 \). We can rewrite \( f'(\gamma) \) by

\[
f'(\gamma) = -\left( \frac{2\lambda}{1 + \lambda - 2\lambda\gamma} \right) s_C + h_1(\gamma, \lambda)h'_2(\gamma)
\]

where the condition \( f(\gamma) = s_C \) holds. When taking \( \lambda \) arbitrarily small, the term in the bracket, \( 2\lambda/(1 + \lambda - 2\lambda\gamma) \), is sufficiently small while \( h_1(\gamma, \lambda) \) is sufficiently large.

We rewrite the respective terms in \( h'_2(\gamma) \) as follows.

\[
(i) = \left( \frac{2\lambda}{1 + \lambda} \right) \left( \frac{\pi'_1(w_1)}{(\pi_1(w_1))^2} \right) \mathbb{E}[D_1(w_1)] > 0;
\]

\[
(ii) + (iv) = \left( \frac{2\lambda}{1 - \lambda} \right) \left( \frac{\pi'_1(w_1)}{\pi'_2(\rho_C)} \right) D(\rho_C) \left[ (\pi^{-1}_1(w_1) - \pi^{-1}_1(w_2)) + (\pi^{-1}_2(\rho_C) - \pi^{-1}_2(w_2)) \right] \\
= -\left( \frac{2\lambda}{1 - \lambda} \right) \left( \frac{2\lambda}{1 + \lambda - 2\lambda\gamma} \right) \left( \frac{D(\rho_C)}{\pi'_2(\rho_C)} \right),
\]

where the second equality follows from the three indifference conditions: \( \pi_1(w_1^m) = \pi_2(w_2); \pi_1(w_1^m)/\pi_1(w_1) = (1 + \lambda)/(1 + \lambda - 2\lambda\gamma); \pi_1(w_m)/\pi_2(\rho_C) = (1 - \lambda)/(1 + \lambda - 2\lambda\gamma); \)

\[
(v) = -\left( \frac{\rho'_C(\gamma)}{\rho_C} \right) \int_{w_2}^{\rho_C} (wD(p'_2(w))) \left( \frac{\pi'_2(w)}{(\pi_2(w))^2} \right) \, dw \\
= -\left( \frac{\rho'_C(\gamma)}{\rho_C} \right) \left( \ln \frac{1 + \lambda - 2\lambda\gamma}{1 - \lambda} \right)
\]

where the second equality follows from both \( \pi_2(w) = wD(p'_2(w)) \) and \( \pi_2(\rho_C)/\pi_1(w_1^m) = \)
\[(1 + \lambda - 2\lambda \gamma)/(1 - \lambda).\]

Taken together,

\[
h'_2(\gamma) = (i) + (ii) + (iv) + (v)
\]

\[
= \left( \frac{2\lambda}{1 - \lambda} \right) \left\{ \left( \frac{1 - \lambda}{1 + \lambda} \right) \left( \frac{\pi_1(w^m)}{(\pi_1(w_1))^2} \right) \mathbb{E}[D_1(w_1)] + \rho_C(\gamma) [r(\lambda, \gamma)] \right\}
\]

where

\[
r(\lambda, \gamma) = \frac{1 - \lambda}{1 + \lambda - 2\lambda \gamma} \frac{D(\rho_C)}{\pi_1(w^m)} - \frac{1}{\rho_C} \left( \ln \frac{1 + \lambda - 2\lambda \gamma}{1 - \lambda} \right).
\]

If \(\lambda\) is sufficiently small, it must be that \(\rho'_C(\gamma)\) is sufficiently small, otherwise the equilibrium conditions for both the upper and lower bounds of the distribution \(G_2\) do not hold. This implies that \(h'_2(\gamma)\) is approximately equal to \((i) > 0\) if \(\lambda\) is sufficiently small.

From (24), the derivative \(f'(\gamma)\) equals roughly

\[- \left( \frac{2\lambda}{1 + \lambda - 2\lambda \gamma} \right) s_C + \left( \frac{1 + \lambda - 2\lambda \gamma}{1 + \lambda} \right) \left( \frac{\pi_1(w^m)}{\pi_1(w_1)} \right)^2 \mathbb{E}[D_1(w_1)],\]

which is positive for a sufficiently small \(\lambda\).

Thus, we have shown that the LHS of the condition (21) is strictly increasing in \(\gamma\) if \(\lambda\) is sufficiently small. \(\Box\)