Price Matching Guarantees and Consumer Search*

Maarten Janssen[†] Alexei Parakhonyak[‡]

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Abstract

This paper examines the effect of price matching guarantees (PMGs) on market outcomes in a sequential search model. PMGs are simultaneously chosen with prices and some consumers (shoppers) know the firms' decisions before buying, while others (non-shoppers) enter a shop before observing the price and whether or not the firm has a PMG. In such an environment, PMGs increase the value of buying the good and therefore increase consumers' reservation prices. This increase is so large that even after accounting for the possible execution of PMGs, firms' profits are larger in an equilibrium where PMGs are offered than in an equilibrium without PMGs. We also consider the incentives of firms to choose PMGs and show that an equilibrium where all firms offer PMGs does not exist because of a free-riding problem.

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†Department of Economics, University of Vienna and State University Higher School of Economics (Moscow). Email: maarten.janssen@univie.ac.at

‡Center for Advanced Studies, State University Higher School of Economics. Email: parakhonyak@yandex.ru

1 Introduction

It is well known that price matching guarantees (PMGs) of one sort or the other are found in many sectors and industries. In retail markets, these guarantees often take the form that sellers offer consumers who buy their products to match any other price a competitor charges for identical products, provided that they have a proof that an identical product is sold by a competitor at a nearby shop within a well-defined time period. Most firms do not commit to change list prices and only give the price discount to the customer that executes the PMG.[1]

The effect of PMGs on the (pricing) behavior of competitors has been discussed in economic as well as in business and law literatures. The main conclusion from these literatures is that despite the appearance of creating additional competitive pressure on the pricing behaviour of firms, PMGs are in fact highly anticompetitive by greatly reducing the incentives of rival firms to undercut. In two empirical papers, Arbatskaya et al. (2004, 2006) observe which firm has the lowest price in any pair of firms in their dataset where one firm has and the other does not have a PMG. If the discouraging price undercutting theory is correct, then in a pairwise comparison test, the firm with a PMG should have been among the firms with the lowest prices (and in equilibrium all firms should have the same price). Moreover, if the discouraging price undercutting theory is correct all firms should offer PMGs. Arbatskaya et al. (2006) find empirically, however, that in the same market, firms offering PMGs tend to be among the firms with higher advertised prices. Moreover, in the same market some firms do and others do not offer PMGs.

This paper aims at contributing to a better understanding of the fact that firms offering PMGs tend to set higher prices by casting this marketing instrument in a consumer search perspective. We argue that PMGs have an important effect on the search behaviour of consumers. This effect has largely been neglected in previous literatures. From a consumer search perspective, the main decision consumers have to make concerns their stopping rule: at which price do they continue to search and when do they stop searching and buy. This decision is usually characterized by a reservation price, that is by a maximal price at which consumers will buy instead of continuing to search for lower prices. A PMG increases this reservation price as consumers not only buy the commodity under consideration, but in addition also buy *an option* that if they are later informed of lower prices, they get the price difference back. Consumers value this option and this increases their reservation prices. Higher reservation prices, in turn, give a firm that offers a PMG the opportunity to raise their list prices, thereby increasing their profits. There is, however, also an important indirect strategic effect on the prices charged by others firms in that they react to the higher prices of firms

[1] There are, however, some firms that commit to lowering list prices if competitors offer lower prices (see, e.g., Comet Services at comet.co.uk).

with PMGs by raising their own prices.

To study this search perspective on PMGs, we use a conventional sequential search setup based on Stahl (1989). As PMGs stipulate that goods have to be identical for the guarantee to become effective, the model of Stahl (1989) is the most appropriate search model to use as it is by now the standard search model for markets with homogeneous goods. We add to the Stahl model that after the purchase and with a certain probability a consumer will be informed about another price quotation. This probability represents the level of information communication among consumers (see Galeotti (2010)).

On the different strategic effects of PMGs we have the following results. First, the support of the equilibrium price distribution of a firm that provides a PMG is always above the support of the distribution of a firm without PMG. This explains the finding of Arbatskaya et al. (2006) that firms setting PMGs have prices that are not lower than the prices of rival firms without a PMG. To understand this result note that if firms with PMGs sometimes set the same price as a firm without PMG, dropping the PMG would not affect the firm's demand, but the firm would have a higher expected *effective* price as it never has to provide a discount. Thus, deviating by not offering PMGs would be profitable.

Second, when we consider the interaction between firms in a market where firms can either choose to offer PMGs, or not to offer them, two types of symmetric equilibria exist: one where firms do not set PMGs at all, and one where firms set PMGs with a certain positive probability, which is strictly smaller than one. This explains an implicit finding of Arbatskaya et al. (2006), namely that in markets where firms do offer PMGs, there are likely to be other firms that do not. An equilibrium where all firms offer PMGs does not exist.

Third, to understand the proper effect of PMGs empirically, our paper suggests that one should not just compare prices in stores with and without PMGs, but one also should inquire whether the prices in stores without PMGs are shifted upwards. We show that in the equilibrium where PMGs are offered, even the firms that do not offer them charge higher expected prices than in the equilibrium where no PMGs are offered. Thus, comparing prices across firms with and without PMGs underestimates the true effect of PMGs. Comparing the average equilibrium prices in markets where PMGs are allowed to the average equilibrium prices in markets where PMGs are forbidden (which basically is the Stahl (1989) benchmark) shows that a firm without PMGs in the former equilibrium on average may offer more than 300% higher prices than a firm operating in the latter equilibrium (see Table 1 in the text), whereas a comparison between the prices of a firm offering PMGs to those of a firm not offering PMGs may show that the first prices are only 30% higher. Thus, the underestimation effect may be very significant. Moreover, the more consumers communicate with each other (that is, the more dense their network is), the higher the equilibrium prices in the equilibrium where PMGs are offered and the more the comparison of prices at firms offering PMGs to those that do not, underestimates the true effect of PMGs.

Finally, we also have some results on asymmetric equilibria. Asymmetric equilibria are inherently difficult to characterize as consumers' reservation prices are nonstationary. We show that in the case of a duopoly asymmetric equilibria do not exist, while with three firms they do exist. In the asymmetric equilibrium we characterize, one firm does not have PMGs, one firm offers PMGs for sure and one firm randomizes between offering and not offering PMGs. The three qualitative symmetric equilibrium properties mentioned above, continue to hold in this asymmetric equilibrium. Interestingly, our numerical analysis shows that for given exogenous parameters, multiple asymmetric equilibria exist and that profits can be as much as ten times higher in an equilibrium where some firms offer PMGs compared to the equilibrium without PMGs.

There is now a reasonably large literature on the effect of low price guarantees on the (pricing) behavior of competitors. As said before, the main conclusion that arises from this literature is that PMGs are anticompetitive. One argument that has been made (cf., Salop (1986)) is that PMGs facilitate collusion as they remove the incentives to undercut and act as a trigger strategy. Although some PMGs take the form that firms ex ante commit to change their list prices if they are informed that a competitor has a lower price (see above), most PMGs restrict the PMG to the client that executed the PMG, i.e., list prices are unaffected. This means that most PMGs actually do not act as trigger strategies.

Png and Hirschleifer (1987) argue that PMGs are an effective way to price discriminate between shoppers and non-shoppers. In the absence of PMGs, the activity of shoppers forces firms to reduce prices market-wide. Shoppers provide a positive externality to non-shoppers and force firms to set more competitive prices. With PMGs, however, the effect of the disciplining power of shoppers is limited to these shoppers themselves and PMGs act as a price discrimination mechanism (see, also, Edlin (1997)). Our paper does share some features with the existing literature that views PMGs as a strategy firms can adopt to price discriminate between consumers. As Corts (1997) has argued a PMG may act like a coupon that is exercised by some consumers (who have small hassle cost), and not by others. In our model, consumers that are informed through friends are likely to pay a different price than those that do not. Under price discrimination it may happen that some consumers benefit from PMGs, but in our model consumers are always worse off. The main difference between our paper and the previous literature is the effect of PMGs on consumers' reservation prices, which together with the fact that prices are strategic complements, raises all prices. This effect does not arise in Png and Hirschleifer (1987) or in Corts (1997) as they do not model the search behavior of non-shoppers explicitly and they do not consider the possibility that consumers are informed about prices via friends.[2]

[2]Chen et al. (2001) also show that price matching policies may have pro-competitive effects in case they are pre-announced and there are consumers who prefer to shop at a particular store but are mindful of saving opportunities. In Chen et al. (2001)

Moorthy and Winter (2006) have argued that PMGs may actually have a pro-competitive effect in the case when products are horizontally differentiated and firms have different production costs. In such a context PMGs may signal to consumers that the firm under consideration really has a lower price. Moorthy and Winter's model nicely illustrates how PMGs may work in markets with product heterogeneity. Most PMGs clauses, however, stipulate that the guarantee only comes into effect if prices of *identical* products at nearby shops are compared.

We study an environment where prices and PMGs are chosen simultaneously. This is also the set-up of other theoretical contributions studying the effects of low price guarantees such as Corts (1995, 1997) and Kaplan (2000). In this set-up, the shoppers are fully informed about prices and whether or not firms offer low price guarantees, while the nons-hoppers are uninformed about all aspects of the firms' strategies until they arrive at the shop. This set-up is relevant in some, but not all markets where PMGs are offered. The set-up does not capture markets where firms have built up a reputation for being a PMG shop (like Walmart), but captures markets where firms announce PMGs over a changing part of their product assortment (like many electronics shops, or some supermarkets). Our set-up can be interpreted in two different ways. First, one can think of firms as being able to advertise prices and/or PMGs in a local newspaper. The local newspaper is read by a certain fraction of consumers, while others are uninformed about the ads. Varian (1980) interprets the shoppers in his model as those consumers who receive all ads. This setting where information about PMGs is advertised simultaneously with prices fits major consumer markets.[3] A second interpretation of our set-up that is close to the Stahl (1989) model is where PMGs and prices are not announced and shoppers always buy at the lowest price after having visited all the stores, while non-shoppers visit a store and then discover whether the firm has a PMG. This interpretation best fits markets where firms (e.g., supermarkets) often put a label "low price guarantee" on some of their price labels, but not on all their products. Moreover, at different points in time these stores attach a PMG to different products.

There is a recent paper by Yankelevich (2010) that also studies the search theoretic implications of PMGs. Yankelevich studies a different environment where firms first advertise PMGs before they set prices. This implies that the rival firm and all consumers are aware of which firm has and which firm does not have a PMG and consumers may therefore direct their search activity to firms that do or do not offer a PMG. The main effect of PMGs in his model comes from the fact that shoppers are also assumed to search sequentially

the "search" behaviour of consumers is also exogenously given as in Png and Hirschleifer (1987) and Varian (1980).

[3]For example, Dixons is an electronic store in the Netherlands, which provides PMGs on a part of its products. It advertises PMGs together with the prices using their own leaflet. Many people, however, opt to not receive ads, or they simply do not read them. These people will realize that Dixons offers PMGs on some products only when they enter the shop. Other examples include White Fence offering home services (such as internet, TV and telephone) and utilities (see, http://www2.whitefence.com/popfaq/bpg_popup.htm) and some Russian internet shops selling consumer electronics (http://www.tehnosila.ru/services/best_price/) and medical equipment (http://www.nv-lab.ru/garantee.php).

and that some of them prefer to go back to a previously visited firm if it has a lower price, while others prefer to activate their PMG.

Finally, the spirit of the paper is also related to the analysis of other consumer policies, such as Most Favoured Costumer (MFC) clauses. An MFC clause informs consumers that if after buying, the firm from which they bought the product lowers the price, they are entitled to the lower price as well. Cooper (1986) shows that MFC clauses, like PMGs, tend to raise market prices, even of those firms that do not offer their customers such a clause. Moreover, there are equilibria in which some firms do, while others do not offer such clauses. The main difference with our analysis is that a PMG relates a firm's price to the competitor prices and not to its own future price. Moreover, our model studies the impact of these consumer policies on consumer search behaviour.

The structure of the paper is as follows. In the next section we present the setup of the model. Section 3 contains the analysis of symmetric equilibria, while Section 4 contains the welfare analysis of these equilibria. Section 5 provides some results on asymmetric equilibria. Section 6 concludes with a discussion that includes the possible roles of price beating guarantees in this framework. Formal proofs can be found in the appendix.

2 The Model

Consider a market where N firms produce a homogeneous good and have identical production costs, which we normalize to zero. Firms set prices and decide whether or not to provide price matching guarantees (PMGs). By providing a PMG, a firm commits to compensate the difference between its price and the price of a competitor, if the consumer who has bought the product from the firm provides evidence that a lower price exists.

We essentially adopt the model by Stahl (1989) and add the possibility for firms to offer PMGs and also add the fact that consumers may get information through friends. In the Stahl model there is a fraction $\lambda \in (0,1)$ of "shoppers" that have zero search costs and a fraction $1-\lambda$ of non-shoppers is uninformed. Non-shoppers get their first price quotation for free and pay a search cost c for each subsequent price quotation.[4] We also make the standard assumption that consumers have perfect recall.[5] Unlike Stahl (1989) we restrict attention to consumers having unit (inelastic) demand, and assume they have identical valuation v for the good. We assume v to be sufficiently large not to influence the consumers' decisions. Whether a firm provides PMGs or not is revealed simultaneously with observing the price quotation of that firm. After the consumer

[4] We follow here the standard assumption in the literature on consumer search. An alternative specification where consumers also have to pay for obtaining a first price quote has been analyzed by Janssen et al. (2005).

[5] Janssen and Parakhonyak (2011) have analyzed search markets where consumers have to pay a cost to re-activate a price offer they have obtained in one of their past searches. They show that the Stahl model is robust to revisit costs.

has bought the good there is an exogenous probability $\mu \in (0,1)$ that she observes (costlessly) the price of one randomly chosen other firm that is different from the currently visited firm, but that may have been observed already at a previously visited firm.[6] This information can come either from friends (as in Galeotti (2010)) or just accidentally because she noticed the price in another store.

The timing in the model is as follows. First, firms simultaneously decide on their prices and whether to provide PMGs. Firm i decides to set PMGs with probability α^i, and then sets prices with a probability distribution $F_0^i(p)$ if it chooses not to offer a PMG, and with $F_1^i(p)$ if it provides a PMG. Thus, the strategy of firm i is a tuple $\{\alpha^i, F_0^i(p), F_1^i(p)\}$. We denote by $\underline{p}_j^i$ and $\overline{p}_j^i$ the lower and upper bounds of $F_j^i(p)$, $j = 0,1$ and $\underline{p}^i = \min\{\underline{p}_0^i, \underline{p}_1^i\}$. Second, consumers decide after the outcome of the possible mixed strategies has been realized. Shoppers buy at the lowest price in the market,[7] while non-shoppers first randomly go to one of the shops and then use an optimal sequential search strategy.[8] After all purchasing decisions have been made, a consumer obtains a price discount if she bought from a firm providing a PMG and is informed about a lower price elsewhere.

It is well known that the optimal sequential search strategy is characterized by a reservation price property: at a particular search round, a consumer buys if the best current price offer that she has encountered so far is smaller than or equal to a certain cut-off value. At prices above this cut-off value, non-shoppers continue to search. In the context of this paper, this cut-off value (or reservation price) may depend on the stage t of the search process and on whether or not the firm offers a PMG. Normally, the reservation price applies to the price offer at a particular store that is visited. With PMGs, however, non-shoppers still buy if the best price offer is below the reservation price, but they do not necessarily buy at the store that makes this offer. Indeed, if the best offer comes from a no-PMG store, and there is at least one store left to search, and there is also a store in the sample searched so far that offers PMGs, then the non-shopper is better off buying at the PMG store, knowing that she can always claim the lowest price sampled so far and hoping to get an even lower price observation from a friend. Thus, at any given search round t the optimal search strategy is characterized by two reservation prices: one when the non-shopper has at least one store in her

[6] Obviously in the absence of recall costs, consumers do not want to engage in costly search after the purchase (and before hearing from a friend). This action is weakly dominated by continue to search before purchasing. Formally, if a consumer would have the option to continue searching after she knows that she will not hear from a friend anymore, then she may want to do so if the price quote is relatively high and she would have this option. We assume, however, that such a moment never arises. This is a realistic approximation of reality where one never knows exactly when information arrives and whether one's own search activities result in a new price quote.

[7] In principle, if one of the firms charges a price below its competitor, while the competitor offers a PMG, shoppers are indifferent between buying from any of the two firms. We take as a tie-breaking rule that shoppers buy at the firm with the lowest price. One can rationalize this assumption by considering there is an infinitely small cost of claiming PMGs.

[8] In some markets, there may be a third category of consumers, namely those that do know whether a firm has a PMG or not, but not their price. These consumers can direct their first search to a firm that has or does not have a PMG. If this third category of consumers would be present in our economy, and their fraction would be small, our results will not be very different from what they are now.

sample offering PMGs (and then they may not buy from the lowest priced store if this is a store not offering PMGs), denoted by $r_1(t)$, and the other when none of the stores in the sample so far offered PMGs, denoted by $r_0(t)$. For easy reference, we denote the reservation prices at the first search round simply by r_1 and r_0, i.e., $r_k = r_k(1), k = 0, 1$.

An equilibrium in this game is a set of firms' strategies $\{\alpha^i, F_0^i(p), F_1^i(p)\}, i = 1, .., N$ and a set of search strategies $\{r_0(t), r_1(t)\}, t = 1, .., N-1$ for non-shoppers, such that the search strategy is sequentially rational given the strategies of the firms and the strategies of the firms are optimal given the strategies of the other firms and of consumers. Due to the presence of shoppers, any equilibrium will have firms choose a mixed strategy price distribution, independent of whether they offer a PMG or not. The mixed price strategy $F_1^i(p)$ chosen by firm i offering PMGs has a different support than the price distribution $F_0^j(p)$ of a firm j not offering PMGs. In most of the paper we characterize symmetric equilibria and drop the superscript i whenever this does not create confusion.

3 Analysis of Symmetric Equilibria

We start our analysis by demonstrating the pivotal role of the reservation price r_0 on equilibrium prices. The first Proposition shows that prices below r_0 will be set only by firms not offering a PMG, while prices above r_0, if at all, will only be set by firms offering PMGs. This result provides a first explanation for the empirical evidence provided by Arbatskaya et al. (2006) showing that firms offering PMGs tend to set higher prices than those that do not. To prove this Proposition we first need, however, to prove a few auxiliary results.

First, the reservation prices $r_0(t)$ and $r_1(t)$ are monotonic in the sense that if non-shoppers are indifferent to buy at a certain price in search round t, then they weakly prefer to buy at the same price in the next search round $t + 1$.

Lemma 1. *For both types of reservation prices $r_k(t), k = 0, 1$, in a symmetric equilibrium $r_k(t-1) \leq r_k(t)$, for all $t \leq N - 1$.*

The proof of the lemma mainly exploits the fact that the benefits of a search should be larger (at least not lower) the more search alternatives one still has to explore (as one can always forget about the current price offer and imitate the search behaviour on the next search round). Note that, unlike Stahl (1989) where the reservation price is stationary, reservation prices are not stationary in our model as the fewer new search alternatives left, the lower is the chance one learns of a new price through friends.

The next result says that in any symmetric equilibrium at any search round t, the reservation price $r_1(t)$ in the case when a consumer has sampled at least one firm offering PMGs is larger than the reservation price

$r_0(t)$ in the case when the consumer has not sampled a PMG firm. Thus, if a consumer happens to visit a PMG firm it is willing to buy at higher prices than when it visits a firm not offering PMGs. This is because if a firm has offered a PMG, it not only offers the product for a certain price, but it also offers the option of getting a price discount later. In fact, in a symmetric equilibrium this is the only relevant difference between the two types of firms. To see the latter, note that in a symmetric equilibrium all firms follow identical strategies and from observing whether or not a particular firm offers a PMG, consumers cannot infer any *new* information about the likelihood of other firms offering PMGs along their future search path: if the equilibrium has firms never choosing PMGs or choosing PMGs with probability one, then consumers already expect this and no learning takes place, while if firms symmetrically randomize their PMG decision, then again there is no learning. Thus, the only relevant difference between a firm offering PMGs and a firm that does not, is that the former offers an additional option value. The lemma is therefore stated without formal proof.

Lemma 2. *In any symmetric equilibrium the reservation prices satisfy* $r_1(t) > r_0(t)$.

Finally, firms will never charge prices above the reservation prices in the first search round. The argument underlying this lemma is similar in nature to the argument in Stahl (1989) that no one searches beyond the first firm, but in our context the argument has to be executed with some care as the reservation prices are non-stationary and an induction argument on the number of search rounds has to be made.

Lemma 3. *Whether or not a firm offers a PMG, it will always choose a price that is immediately accepted by uninformed consumers, i.e.,* $\overline{p}_k \leq r_k = r_k(1),\ k = 0, 1$.

We are now then ready to state and prove, the first important result of this paper.

Proposition 1. *In any symmetric equilibrium, a firm offering PMGs will choose prices* $p \in [\underline{p}_1, \overline{p}_1]$*, where* $\underline{p}_1 > r_0$.

The Proposition exploits the following basic facts. First, shoppers always buy from the firm with the lowest price in the market irrespective of whether they offer a PMG. Hence, firms with and without PMGs have the same probability of attracting the shoppers if they set the same price. Second, given that uninformed consumers always buy at prices below r_0, a firm setting a price $p \leq r_0$ and offering a PMG can strictly increase its profit by not offering a PMG as at these prices offering a PMG does not affect demand, but lowers the expected effective price that non-shoppers pay. Thus, firms will never choose to combine low prices with offering PMGs. As the formal statement replicates this more informal argument, the proof is omitted.

It follows from Proposition 1 and Lemma 3 that in any symmetric equilibrium any price $r_1 \geq p > r_0$ will only be offered by a firm offering PMGs. If uninformed consumers observe these prices at a firm that does not offer PMGs, they will continue to search, while when the firm offers a PMG they buy in the hope of being informed of a lower price later. Thus, given a price above r_0 a PMG will lead to higher demand, although possibly yielding a lower revenue per unit on these additional sales as the consumers may execute their PMG. As only the uninformed consumers may pay at lower prices later (as the informed consumers anyway only buy if the firm has the lowest price in the market), PMGs yield higher profits at prices larger than r_0.

To characterize the possible equilibria further we first consider for general N whether there exist equilibria where all firms either offer PMGs or where they do not. The next Proposition shows there is only one such an equilibrium, namely one where firms do not offer PMGs, i.e., $\alpha^i = 0$ for all firms i. This is the equilibrium that corresponds to the equilibrium analyzed in Stahl (1989). Thus, the Stahl equilibrium is robust to firms having the option of offering PMGs.

Proposition 2. *An equilibrium where all firms i choose $\alpha^i = 0$ always exists. The equilibrium price distribution in this case is*

$$F_0(p) = 1 - \left(\frac{1-\lambda}{\lambda N}\frac{r_0 - p}{p}\right)^{\frac{1}{N-1}}, \quad p \in \left[\frac{1-\lambda}{1+\lambda(N-1)}r_0, r_0\right] \tag{1}$$

where r_0 is defined as $\int_{\underline{p}}^{r_0} F_0(p)dp = c$. An equilibrium where all firms choose PMGs does not exist.

The equilibrium price distribution when no firm offers a PMG balances the fact that at higher prices, a firm has higher margins, but lower demand. The proof (in the Appendix) is, however, somewhat more involved as it still has to be checked that a firm does not find it optimal to deviate and offer a PMG, while charging higher prices.

The second part of Proposition 2 follows from Proposition 1 and the main idea is as follows. If all firms offer PMGs, the minimum price non-shoppers expect to find in the market is given by $\underline{p}_1$. If they then happen to encounter a (deviating) firm not offering PMGs they are willing to buy at this firm up to a reservation price r_0 that is at least equal to $\underline{p}_1 + c$. Thus, if all firms offered PMGs some of the prices charged with positive probability by firms with a PMG are strictly smaller than r_0. Proposition 2 then says that a firm can profitably deviate by setting a price equal to r_0, and remove the PMG. Thus, any individual firm can free ride on the PMGs offered by the other firms.

To further explain the empirical evidence offered by Arbatskaya et al. (2006) we need to show that there exist equilibria where some firms may offer PMGs, while others do not. Two types of these equilibria come to mind. There may be asymmetric equilibria according to which some firms offer PMGs with probability one, while others do not. In these equilibria, firms may still randomize their pricing decisions as is common in Stahl type search models. On the other hand, there may be symmetric equilibria where firms choose to offer PMGs with some positive probability strictly smaller than one. In the latter case, we also observe with strictly positive probability markets where both behaviours co-exist. In this section we will focus on the second type of equilibrium, while Section 5 discusses asymmetric equilibria.

Consider the profit function $\pi_1(p)$ of a firm offering PMGs and setting price $p \leq r_1$:

$$\pi_1(p) = \lambda(1 - F(p))^{N-1}p + \frac{1-\lambda}{N}\left[(1-\mu)p + \mu(1-F(p))p + \mu\int_{\underline{p}}^{p} qf(q)dq\right],$$

where $F(p) = (1-\alpha)F_0(p) + \alpha F_1(p)$ is the weighted average of the two equilibrium price distributions. That is, if a firm offers a PMG and sets a price p its profit consists of two parts. The first part represents the expected profit it makes from the shoppers: shoppers only buy if this firm has the lowest price (which happens with probability $(1-F(p))^{N-1}$). This is the regular expression in Stahl type models. The second part represents the fact that a fraction $1/N$ of the non-shoppers visits this firm and buys immediately as (by Lemma 3) the price is smaller than the reservation price (and no other non-shopper will visit this firm as they will all buy immediately at the firm they visit first. The effective price the non-shoppers pay is determined as follows. There is a probability $1-\mu$ that a consumer is not informed about another price and therefore effectively pays p. With the remaining probability μ the consumer is informed of another price and with probability $1-F(p)$ this second price will be higher than p, while with the remaining probability $F(p)$ the second price will be lower and then she will effectively pay the expected price, conditional on this price being smaller than p.

Integrating by parts,[9] this profit function can be written as

$$\pi_1(p) = \lambda(1-F(p))^{N-1}p + \frac{1-\lambda}{N}\left[p - \mu\int_{\underline{p}}^{p} F(q)dq\right].$$

To characterize the mixed pricing strategy $F(p)$ for the firms, firms have to be indifferent over an interval of prices. Unfortunately, symmetric mixed strategy equilibria are difficult to characterize for general N. For $N = 2$ solving the firms' indifference equation is already nontrivial (see the proof of Proposition 3), but for general N it becomes very complicated.[10] Therefore we only characterize the equilibrium for the case where $N = 2$. To do that, we start by investigating the optimal search behaviour of uninformed consumers.

Lemma 4. *Consider the case where $N = 2$. Uninformed consumers accept all prices smaller than or equal to r_0 at a firm that does not provide a PMG, and continue to search otherwise; they accept all prices smaller*

[9]Due to the fact that $F(\underline{p}) = 0$,if we integrate $\int_{\underline{p}}^{p} qf(q)dq$ by parts we get $pF(p) - \int_{\underline{p}}^{p} F(q)dq$.

[10]See our web appendix on this.

than or equal to r_1 at a firm with a PMG, and continue to search otherwise, where r_0 and r_1 are defined by

$$\begin{aligned}\int_{\underline{p}}^{r_0} F(p)dp = c \\ \int_{\underline{p}}^{r_1} F(p)dp = \frac{c}{1-\mu}\end{aligned}, \tag{2}$$

The reservation price r_0 is defined as the price at which a non-shopper is indifferent between buying now and continuing to search, where in this context the consumer who continues to search does not know whether the next firm offers a PMG or not. Using similar techniques to Stahl (1989) the reservation price turns out to be identical to Stahl (1989). Concerning the reservation price r_1 we already know that $r_1 > r_0$, i.e., a consumer is willing to buy at a higher price if the firm happens to provide a PMG. When μ is close to one, consumers visiting a firm with a PMG prefer to stop searching even if its price is very high price since they will almost surely also observe the price of the other firm for free. Hence, with $N = 2$ a costly search after visiting a PMG store is not attractive.

Note that the characterization of both reservation prices r_0 and r_1 in Lemma 4 is only valid for $N = 2$. At the reservation prices r_0 and r_1, the non-shopper is, by definition, indifferent between buying now and continuing to search. To determine the reservation price, one therefore has to evaluate the consumer's continuation pay-off if she would continue to search. With $N = 2$ this is easy as the consumer is informed of all the prices if she continues to search. For $N > 2$ one has to take into account that a non-shopper may buy at a PMG firm even if it does not have the lowest price observed so far.

Given the characterization of the reservation prices when $N = 2$, Proposition 3 shows that indeed, for certain parameter values, a symmetric equilibrium exists where under a duopoly some firms may offer PMGs, while others do not.

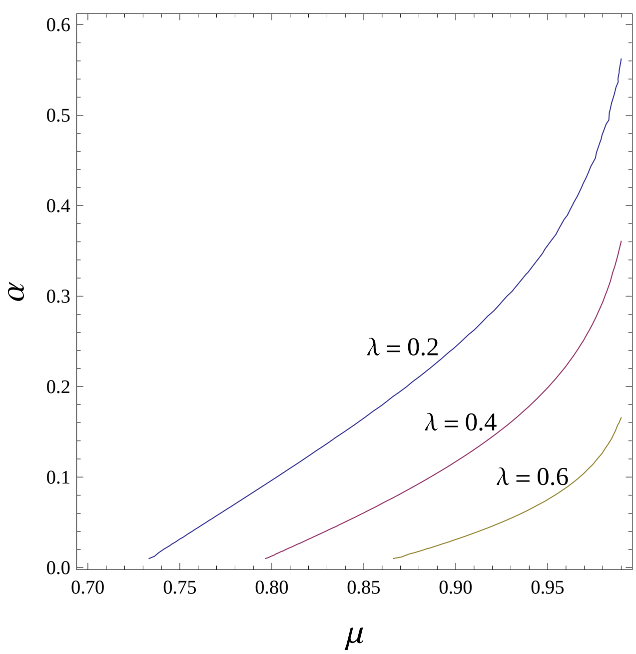
Proposition 3. *Consider $N = 2$. An equilibrium where firms offer PMGs with a strictly positive probability that is smaller than one, that is where $\alpha \in (0, 1)$, exists if and only if*

$$1 > \mu > \frac{4\lambda^2}{(1-\lambda)^2 \ln \frac{1-\lambda}{1+\lambda} + 2\lambda(1+\lambda)} > \frac{2}{3} \tag{3}$$

This result may seem to be somewhat counterintuitive: firms offer (with some probability) PMGs only if there is a sufficiently large probability that consumers would exercise them. The explanation, of course, is that if μ is sufficiently large, consumers would accept much higher prices at the store offering PMGs. The effect of an increase in μ on the reservation price r_1 is hyperbolic (as equation (2) shows), whereas the probability that consumers are informed of lower prices is only lineraly affacted by an increase in μ. This increase in prices set by firms offering PMGs, more than offsets the adverse effect of consumers exercising PMGs on firms' profits.

Equation (3) shows that the probability μ of being informed about the price set by the other firm should be relatively large. Figure 1 depicts the relation between the equilibrium probability of firms offering PMGs and the probability with which consumers observe another price quotation. The figure shows that this relationship is positive: α is increasing with μ. Although high values of μ imply that *ex post* most of the consumers are informed, uninformed consumers are willing to buy at higher prices for higher μ. If μ is close to one, consumers are willing to accept virtually any price. Therefore firms are more likely to set PMGs when μ is large. Not surprisingly, the larger the fraction λ of shoppers, the lower the probability with which firms offer PMGs. The precise numbers of the parameters for which the equilibrium does exist should not be taken too literally as (i) these parameter values are only valid for the case of duopoly and (ii) the model assumes that there are no hassle costs of exercising PMGs. There is nothing in our equilibrium analysis for the case of $N = 2$ that makes that this type of symmetric equilibrium does not exist when $N > 2$. In Section 5 we show that for $N = 3$ asymmetric equilibria exist for much lower values of μ.

Figure 1: How equilibrium probability of PMG depends on μ



4 Welfare Analysis of Symmetric Equilibria

In the previous Section we shoed that there is a region of the parameter space where multiple symmetric equilibria exist: the Stahl equilibrium where firms do not offer PMGs and an equilibrium where some firms offer PMGs with a strictly positive probability and charge higher prices when they do offer PMGs. For this region where multiple equilibria exist, Proposition 4 compares expected prices paid by consumers and expected profits by firms across the two equilibria.

Proposition 4. *Expected profits for firms in the equilibrium where PMGs are offered with positive probability are higher than the expected profits in the equilibrium without PMGs. As a consequence, in the equilibrium where PMGs are offered with positive probability consumers pay higher expected prices (after a possible execution of their PMG) than in the equilibrium without PMGs.*

Proposition 4 shows the “anticompetitive” effect of PMGs in a search environment. In the equilibrium with PMGs the expected price is higher than in the equilibrium where PMGs are not offered. The source of the anticompetitive effect is, however, different from that so far studied in the literature. It is not the case here that there is some type of collusive behaviour between the firms where PMGs play the role of a monitoring device and list prices will be automatically adjusted downwards if firms undercut a rival with a PMG. In our case the result is fully driven by consumers’ search behaviour, namely by the willingness of consumers to accept higher prices when firms do offer PMGs. Another interesting observation is that the higher expected price paid in the equilibrium with PMGs comes from two sources. The *direct* effect is that a firm offering PMGs can set higher prices on average because of the higher reservation price at firms offering a PMG. The *indirect* effect shows that firms without a PMG react to these *possibly* higher prices by setting higher prices themselves. In other words, in the equilibrium where PMGs are chosen with positive probability, but where the realization is such that none of the firms actually do offer them, the expected prices are still higher than in the equilibrium where PMGs are not offered at all (described in Proposition 2). Table 1 shows how prices depend on μ in the equilibrium where PMGs are offered in comparison with the equilibrium prices in markets where PMGs are never offered (marked as “Stahl”). In the latter case, expected prices are, of course, a constant, whereas they are exponentially increasing in μ whenever the PMG equilibrium exists. In this sense, consumers are “punished” for being better informed. The table shows that an empirical investigation estimating the effect of PMGs on market prices by comparing in the same market the average price of firms offering PMGs with the average price of firms not offering PMGs significantly underestimates the true effect of PMGs. Depending on the parameter values, and mainly on μ, a direct comparison within the same market may show that PMG firms offer prices that are 30% higher (41.1 versus

31.9 for $\mu = 0.95$, $\lambda = 0.1$), whereas with the full effect prices have increased by 340% (41.1 versus 9.3).

Table 1: Expected and effective (including exercise option) prices depending on λ and μ ($c = 1$)

	$\mathbb{E}p$ without PMG			$\mathbb{E}p$ with PMG			Effective Price PMG			Stahl
μ	0.75	0.85	0.95	0.75	0.85	0.95	0.75	0.85	0.95	
$\lambda = 0.05$	24.6	36.6	74.2	28.4	42.0	84.1	25.8	38.5	77.9	19.3
$\lambda = 0.10$	11.2	16.4	31.9	15.0	21.6	41.1	12.4	18.1	35.0	9.3
$\lambda = 0.20$	4.6	6.6	11.8	8.5	11.6	20.1	5.7	8.0	14.2	4.3

5 Asymmetric Equilibria

We now briefly consider the possible existence of asymmetric equilibria where some firms always offer PMGs and other firms do not offer them at all. By doing so, we achieve two new types of results. First, we obtain that PMGs can be offered with probability one by a fraction of firms so that it is not required that in an equilibrium where some firms offer PMGs, it has to be the case that all firms randomize their PMG decision. Second, we show that unlike the case analyzed in Section 3, equilibria with PMGs may occur when the probability μ of being informed about other prices via friends is small.

Asymmetric equilibria are difficult to analyze in full as the search behaviour of consumers is non-stationary and reservation prices can be non-monotone. To see that consider a potential asymmetric equilibrium where k out of N firms do not offer PMGs. After observing the price at a firm offering PMGs, the consumer knows (given the equilibrium strategies of the firms) that out of the remaining $N-1$ firms k do not offer PMGs and have lower prices, so that the probability of observing a lower price is larger than before visiting the first firm. Thus, if the consumer was indifferent between buying and continuing to search in the first search round after observing r_1, she would definitely prefer to continue searching if she would observe again r_1 at a firm offering PMGs at the second search round. Thus, the lemmas characterizing the reservation prices at the beginning of Section 3 fail to hold in case of asymmetric equilibria.

Despite these complications, one can show that asymmetric equilibria do not exist for the case where $N = 2$, while we can compute an asymmetric equilibrium for $N = 3$. Let us first consider the case where $N = 2$. If an asymmetric equilibrium existed in this case, then it would be true that one firm offers a PMG and the other does not. Taken together, the following observations demonstrate that an asymmetric equilibrium in case $N = 2$ does not exist. First, it should be the case that after observing the reservation price r_0 the consumer is indifferent between buying and continuing to search the PMG firm, implying that the firm offering a PMG should set prices strictly below r_0 with some strictly positive probability. But like in the case of symmetric equilibria, the firm offering PMGs would be better off dropping the PMG clause and setting the same price. If the firm offering a PMG did not set prices below r_0, there could not be such a reservation price.

We next discuss the numerical results for asymmetric equilibria with three firms, where firm 1 does not offer PMG, firm 2 offers PMG with probability α and firm 3 always offers PMG.[11] The Tables below represent the main findings. Table 2 presents for some values of the exogenous parameter values λ and μ the probability α with which firm 2 chooses to offer PMGs. From the table it follows that there may co-exist two equilibria for the same exogenous parameter values: when two values are printed below one another in a cell of the table, then these two values represent the two different equilibrium values of α. Note that for a given value of λ, the equilibria exist for relatively low (but, not too low) values of μ. Moreover, the lower the value of λ, the larger the range of values of μ for which at least one such an asymmetric equilibrium exists.

Table 2: How α depends on λ and μ

μ	0.1	0.2	0.3	0.4	0.5	0.6
$\lambda = 0.01$	0.042	0.016	0.009	0.005	0.003	0.002
	0.450	0.588	–	–	–	–
$\lambda = 0.05$	–	–	0.061	0.032	0.017	0.007
	–	–	0.329	–	–	–
$\lambda = 0.10$	–	–	–	0.108	0.041	0.016
	–	–	–	0.185	–	–
$\lambda = 0.20$	–	–	–	–	–	0.048
	–	–	–	–	–	–

Tables 3 and 4 represent market outcomes. First, we present the reservation prices. Then, we compute industry profit, which is effectively the weighted price paid by all the consumers. The last column in each table provides the corresponding outcomes of the equilibrium without PMGs being offered (which essentially is the Stahl (1989) model).

We can derive the following interesting observations from the tables. First, reservation prices for firms with PMGs are higher than for those without. This finding is in line with our findings for the symmetric equilibrium. Second, reservation prices and industry profits are higher than in (the Stahl) equilibrium without PMGs. This is also in line with our results for the symmetric case. Thirdly, for some parameter values there are two equilibria and those with higher values of α are characterized by (much) higher prices. Profits and reservation prices are approximately twice as high in the asymmetric equilibrium with low values of α compared to the equilibrium without PMGs, whereas this ratio can go up to 10 and more for the

[11] The details concerning the derivation of this equilibrium are available upon request.

Table 3: Reservation prices $(r_0; r_1)$ depending on λ and μ $(c = 1)$

μ	0.1	0.2	0.3	0.4	0.5	0.6	Stahl
$\lambda = 0.01$	145; 146	128; 129	120; 121	114; 115	108; 109	104; 105	67.3
	776; 787	1760; 1795	–	–	–	–	
$\lambda = 0.05$	–	–	29.4; 30.1	25.7; 26.3	23.6; 24.2	22.0; 22.7	14.0
	–	–	82.0 ; 87.2	–	–	–	
$\lambda = 0.10$	–	–	–	17.3; 18.5	13.3; 14.1	11.8; 12.6	7.30
	–	–	–	22.9; 25.1	–	–	
$\lambda = 0.20$	–	–	–	–	–	7.10; 8.13	3.94
	–	–	–	–	–	–	

Table 4: Total industry profit depending on λ and μ $(c = 1)$

μ	0.1	0.2	0.3	0.4	0.5	0.6	Stahl
$\lambda = 0.01$	144	127	119	113	108	103	66.7
	779	1773	–	–	–	–	
$\lambda = 0.05$	–	–	28.2	24.5	22.5	20.9	13.3
	–	–	81.9	–	–	–	
$\lambda = 0.10$	–	–	–	16.2	12.1	10.7	6.57
	–	–	–	21.9	–	–	
$\lambda = 0.20$	–	–	–	–	–	5.89	3.15
	–	–	–	–	–	–	

equilibrium with a high value of α. In the high α equilibrium, it is more likely that firms offer PMGs and as in the analysis of symmetric equilibria this has a direct effect (PMG firms offer higher prices) and an indirect effect (the firm not offering PMG reacts by increasing prices as the chance it has to compete for the shoppers decreases in α). Unlike the analysis regarding symmetric equilibria, prices can be either increasing or decreasing in μ.

6 Discussion and Conclusion

This paper has analyzed the effect of firms offering PMGs in a consumer search model where reservation prices are endogenously determined. The main points of the paper are as follows. First, firms offering PMGs offer higher prices than firms not offering PMGs as consumers are willing to buy at higher prices from firms offering PMG as there is a possibility that they get a price discount later. Second, firms that do not offer PMGs free ride on other firms offering PMGs and also set higher prices.

This paper has not considered the strategic effects of price beating strategies (PBGs). A price beating strategy promises to give the consumer a strictly lower price than any competitor, if the consumer provides proof that a competitor indeed has a lower price. In principle, PBGs could be incorporated into our framework, and if we did so, their strategic effect will be very different from the effect of PMGs. Allowing for PBGs will yield that all equilibria have firms choosing PBGs with strictly positive probability and all firms choose price equal to the monopoly price. This is basically because with arbitrarily small hassle costs all shoppers will buy at a price beating firm (even if it has a higher price) and then execute the PBG as this gives them an effective purchase price lower than any of the list prices. As a consequence, PBGs neutralize the incentive of rival firms to undercut and sell to the shoppers. There is a continuum of these equilibria with monopoly pricing and they only differ in the fraction of firms setting PBGs, PMGs and no low price guarantees at all. This could partially explain why Arbatskaya et al. (2006) found that the strategic effects of PBGs are quite different from those of PMGs and that empirically firms offering PBGs tend not to have higher prices than other firms in the market.

These results for PBGs would critically depend, however, on the assumption that there are no hassle costs. If hassle costs are non-negligible, shoppers may continue to buy at the lowest price firm as we assumed in this paper. Our results for PMGs do not critically depend on the assumption of no hassle costs, as the shoppers in our model will continue to buy from the lowest price firm in the market if we allowed for hassle costs. An interesting topic for future research is to simultaneously allow for PBGs and PMGs in a search framework based on Hviid and Shaffer (1999) where hassle costs are explicitly modelled.

Appendix: Proofs

Lemma 1. *For both types of reservation prices* $r_k(t), k = 0, 1, r_k(t-1) \leq r_k(t)$, *for all* $t \leq N-1$.

Proof. In a symmetric equilibrium, consumers cannot infer any new information (that they do not know already from the equilibrium strategies) about the likelihood that other firms offer or do not offer PMGs from the fact that they happen to encounter a particular firm with or without PMGs now. That is, there is no (Bayesian) updating along the equilibrium path. Moreover, in search round t consumers can always imitate the optimal continuation search behaviour in search round $t+1$ and therefore, the continuation cost of searching in round t should not be larger than the continuation cost of searching in round $t+1$. As the benefits of buying at a certain price are independent of the search round, it should be the case that reservation prices cannot be strictly lower in later search rounds. $\square$

Lemma 3. *Whether or not a firm offers a PMG, it will always choose a price that is immediately accepted by uninformed consumers, i.e.,* $\overline{p}_k \leq r_k, k = 0, 1$.

Proof. First of all, note that in a symmetric equilibrium both $F_0(p)$ and $F_1(p)$ are atomless. If some price would be charged with strictly positive probability, then a standard argument can be used to demonstrate that, because of the presence of shoppers, each firm would have an incentive to deviate and charge a slightly lower price instead.[12]

The rest of the proof is by induction on the number of search rounds t, starting at the last search round. It cannot be the case that $\overline{p}_0 > r_0(N-1)$. To see this consider a non-shopper coming to a firm that charges $\overline{p}_0$ and does not offer a PMG. If a non-shopper sampled at least one firm not offering PMGs before, she will not buy from this firm for sure (as the price distribution does not have a mass point). If she sampled at least one PMG firm and searched the firm charging $\overline{p}_0$ before the last round she prefers to buy from the PMG firm and claim the PMG (as she has the additional option of being informed of an even lower price). Thus, the only case a firm charging $\overline{p}_0$ and not offering a PMG can make a profit is when all other firms offer PMGs and choose higher prices: in this case the firm sells to all the shoppers. However, the firm can be better off by offering PMGs in that case: as it has the lowest price the effective price to shoppers is the same, but there is an additional chance that some non-shoppers will buy.

It cannot be the case that $\overline{p}_1 > r_1(N-1)$ either. Indeed, by charging such a price a PMG firm cannot make a sale if the non-shopper had observed a PMG firm in a previous search round as a PMG firm has a lower price for sure (as there are no mass points). It could make a sale to non-shoppers, who previously encountered only firms not offering PMGs (and due to the previous paragraph and Lemma 2) have prices smaller than $r_1(N-1)$. These non-shoppers will buy from the PMG firm and then execute their PMG and effectively buy at a price not larger than $r_1(N-1)$ (either the best price they have sampled, or the one which comes with probability μ afterwards). However, by deviating and charging $r_1(N-1)$ the firm will sell to these consumers at the same *effective* price, and in addition has a chance to sell to consumers, who sampled only PMG firms, and search the current firm on round $N-1$.

Thus, we proved the base of induction and proceed with the induction step. Suppose that $\overline{p}_1 \leq r_1(t+1)$ and $\overline{p}_0 \leq r_0(t+1)$. We want to show that then $\overline{p}_1 \leq r_1(t)$, $\overline{p}_0 \leq r_0(t)$. We know from Lemma 1 that $r_1(t) < r_1(t+1)$ and $r_0(t) < r_0(t+1)$. Consider a firm without PMG, offering $r_0(t) < \overline{p}_0 \leq r_0(t+1)$. By the same argument as before this firm cannot sell to non-shoppers and thus is better off by offering a PMG at $\overline{p}_0$. Therefore, $\overline{p}_0 \leq r_0(t)$. Now consider a firm offering PMG and $r_1(t) < \overline{p}_1 \leq r_1(t+1)$. This firm never sells to shoppers. It cannot sell to non-shoppers, who already have a PMG offer in their sample, either.

[12] See, for example, Stahl (1989) for a detailed discussion.

Thus, sales only come from non-shoppers who only sampled firms that do not offer a PMG. Repeating the argument of the previous paragraph shows that it is then better to charge a price equal to $r_1(t)$. Thus, a profitable deviation exists and $\overline{p}_1 \leq r_1(t)$.

By iteratively applying the induction step, we get that $\overline{p}_i \leq r_i(1)$.

□

Proposition 2. *An equilibrium where all firms i choose $\alpha^i = 0$ always exists. The equilibrium price distribution in this case is*

$$F_0(p) = 1 - \left(\frac{1-\lambda}{\lambda N}\frac{r_0 - p}{p}\right)^{\frac{1}{N-1}}, \quad p \in \left[\frac{1-\lambda}{1+\lambda(N-1)}r_0, r_0\right] \tag{4}$$

where r_0 is defined as $\int_{\underline{p}}^{r_0} F_0(p)dp = c$. Moreover, an equilibrium where all firms choose PMGs does not exist.

Proof. As consumers expect all firms not to offer MPGs, we have that $F(p) = F_0(p)$. As no firm offers MPGs in equilibrium, we can define r_1 implicitly by

$$\begin{aligned}(1-\mu)r_1 \quad &+ \quad \mu\left(F_0(r_1)\int_{\underline{p}_0}^{r_1} q\frac{f_0(q)}{F_0(r_1)}dq + (1-F_0(r_1))r_1\right) = \\ &= \quad c + \left(F_0(r_1)\int_{\underline{p}_0}^{r_1} q\frac{f_0(q)}{F_0(r_1)}dq + (1-F_0(r_1))r_1\right)\end{aligned}.$$

Integrating by parts, it follows that the two reservation prices are given by

$$\int_{\underline{p}}^{r_0} F_0(p)dp = c$$
$$\int_{\underline{p}}^{r_1} F_0(p)dp = \frac{c}{1-\mu}.$$

In the equilibrium where no firm offers a PMG, the firms' price distribution has to be such that they are indifferent between setting any price in the support of the mixed strategy distribution, including r_0. If they set r_0, they will only sell to a fraction $1/N$ of the non-shoppers. In equilibrium each firm thus makes a profit of $\pi_0 = \frac{1-\lambda}{N}r_0$. Assume, one firm deviates and offers an MPG. Then the highest possible profit that can be obtained is by charging $p = r_1$. Indeed, it is clear that a firm only benefits from the deviation if $p > r_0$, but in that case the shoppers would not buy from this firm anyway, so the firm has to extract maximum profits from the uninformed consumers, which is attained by charging $p = r_1$. Then

$$\pi_1 = \frac{1-\lambda}{N}\left((1-\mu)r_1 + \mu\mathbb{E}(p|p<r_1)\right) = \frac{1-\lambda}{N}\left((1-\mu)r_1 + \mu(r_0 - c)\right)$$

so that

$$\pi_1 > \pi_0 \Leftrightarrow r_1 - r_0 > \frac{\mu c}{1-\mu}.$$

But we have

$$r_1 - r_0 = \int_{r_0}^{r_1} 1dp = \int_{r_0}^{r_1} F_0(p)dp = \int_{\underline{p}}^{r_1} F_0(p)dp - \int_{\underline{p}}^{r_0} F_0(p)dp =$$
$$\frac{c}{1-\mu} - c = \frac{\mu c}{1-\mu}.$$

Thus, the best possible deviation gives the same payoff and a firm cannot strictly benefit from deviating.

We then show that an equilibrium where all firms offer a PMG does not exist.,Suppose to the contrary that all firms i choose $\alpha_i = 1$. We will argue that each firm individually can profitably deviate and choose $\alpha = 0$ and price at r_0. Note that given the equilibrium strategies of other firms, a nons-hoppers encountering a (deviating) firm not offering a PMG, has reservation price r_0 defined by $\int_{\underline{p}_1}^{r_0} F_1(p)dp = c$. Therefore, it has to be the case that $\underline{p}_1 < r_0 < r_1$ and as r_0 lies in the support of $F_1(p)$ we get

$$\pi(r_0) = \lambda(1 - F_1(r_0))r_0 + \frac{1-\lambda}{N} r_0 >$$
$$> \lambda(1 - F_1(r_0))r_0 + \frac{1-\lambda}{N}\left((1-\mu)r_0 + \mu\left(\int_{\underline{p}_1}^{r_0} q f_1(q)dq + (1 - F_1(r_0))r_0\right)\right) = \pi_1.$$

Therefore, there is a profitable deviation and an equilibrium with all firms choosing PMGs does not exist. □

Lemma 4. *Consider $N = 2$. Uninformed consumers accept all prices at or below r_0 at a firm that does not provide a PMG, and continue to search otherwise; they accept all the prices at or below r_1 at a firm with a PMG, and continue to search otherwise, where $\{r_0, r_1\}$ are defined by*

$$\begin{aligned} \int_{\underline{p}}^{r_0} F(p)dp &= c \\ \int_{\underline{p}}^{r_1} F(p)dp &= \frac{c}{1-\mu} \end{aligned} \tag{5}$$

Proof. After observing price r_0 at a firm without PMGs, a consumer has to be indifferent between buying now and continuing to search. If the consumer continues to search, she proceeds to the next firm. The next firm does not have an PMG with probability $1 - \alpha$, and in this case the consumer can choose the smallest price of r_0 and a random price p that is distributed according to F_0. A similar expression holds in case she continues to search and happens to visit a store with an PMG, which occurs with probability α. Therefore, the reservation price should satisfy the following equation:

$$r_0 = c + (1-\alpha)\left(\int_{\underline{p}_0}^{r_0} q f_0(q)dq + (1 - F_0(r_0))r_0\right) + \alpha r_0$$

since $\underline{p}_1 > r_0$ conditional expectation in the case the other firm offers PMG is just zero.

Using integration by parts, as in Stahl (1989), and realizing that for prices $p < r_0, F(p) = (1-alpha)F_0(p)$ this expression can be simplified to the usual rule determining reservation prices,

$$\int_{\underline{p}}^{r_0} F(p)dp = c.$$

Now consider the case where the consumer finds herself at a shop that provides a PMG. In this case if she accepts the price there is a probability μ that later she observes another price. This new price is either set by a no-PMG firm (with probability $1 - \alpha$) or from a firm offering PMG (with probability α). If she decides to continue searching, the situation is similar to the case described above. Therefore, the reservation price is defined by

$$\begin{aligned} (1-\mu)r_1 &+ \mu\Big((1-\alpha)\left(\textstyle\int_{\underline{p}_0}^{r_1} q f_0(q)dq + (1 - F_0(r_1))r_1\right) \\ &+ \alpha\left(\textstyle\int_{\underline{p}_1}^{r_1} q f_1(q)dq + (1 - F_1(r_1))r_1\right)\Big) = \\ &= c + (1-\alpha)\left(\textstyle\int_{\underline{p}_0}^{r_1} q f_0(q)dq + (1 - F_0(r_1))r_1\right) \\ &+ \alpha\left(\textstyle\int_{\underline{p}_1}^{r_1} q f_1(q)dq + (1 - F_1(r_1))r_1\right) \end{aligned},$$

which, after integrating by parts, simplifies to

$$\int_{\underline{p}}^{r_1} F(p)dp = \frac{c}{1-\mu}.$$

□

Proposition 3. *Consider $N = 2$. An equilibrium where firms offer PMGs with a strictly positive probability that is smaller than one, i.e. where $\alpha \in (0, 1)$, exists if and only if*

$$1 > \mu > \frac{4\lambda^2}{(1-\lambda)^2 \ln\frac{1-\lambda}{1+\lambda} + 2\lambda(1+\lambda)} > \frac{2}{3} \tag{6}$$

Proof. To prove the proposition we explicitly construct an equilibrium and then show that there is such a value of α that all the equilibrium conditions are satisfied.

Equilibrium price distribution support contains two parts: $[\underline{p}, r_0] \cup [\underline{p}_1, r_1]$.

The lower part of the support. The lower part of the support is defined by three equations:

$$\pi(p) = \lambda(1 - F(p))p + \frac{1-\lambda}{2}p$$

$$F(\underline{p}) = 0$$

$$F(r_0) = 1 - \alpha.$$

These three equations allow us to write the relevant endogenous parameters as a function of (r_0, α) as

$$\pi(r_0, \alpha) = \frac{2\alpha\lambda + 1 - \lambda}{2} r_0 \tag{7}$$

$$\underline{p}(r_0, \alpha) = \frac{2\alpha\lambda + 1 - \lambda}{1 + \lambda} r_0$$

$$F(p; r_0, \alpha) = \frac{(1+\lambda)p - (1 - \lambda + 2\alpha\lambda)r_0}{2\lambda p}.$$

Combining this with the optimal search rule

$$\int_{\underline{p}}^{r_0} F(p)dp = c$$

gives an expression for r_0:

$$r_0 = \frac{2\lambda c}{2(1-\alpha)\lambda + (1 - \lambda + 2\alpha\lambda)\ln\left(\frac{1-\lambda+2\alpha\lambda}{1+\lambda}\right)}. \tag{8}$$

Thus, if the value of α is known the probability distribution on the lower part of the support is fully described.

The upper part of the support. Let us then consider the upper part. The profit function is here defined by:

$$\pi(p) = \lambda(1-F(p))p + \frac{1-\lambda}{2}\left(p - \mu\int_{\underline{p}}^{p} F(q)dq\right). \tag{9}$$

Since $\int_{\underline{p}}^{r_1} F(p)dp = \frac{c}{1-\mu}$ we get

$$\pi = \frac{1-\lambda}{2}\left(r_1 - \frac{\mu c}{1-\mu}\right).$$

Similarly, since $\int_{\underline{p}}^{p_1} F(p)dp = c + (1-\alpha)(\underline{p}_1 - r_0)$ we get

$$\pi = \alpha\lambda\underline{p}_1 + \frac{1-\lambda}{2}\left(\underline{p}_1 - \mu(c + (1-\alpha)(\underline{p}_1 - r_0))\right).$$

Now, using equations (7) and (8) we can get expressions for r_1 and $\underline{p}_1$ as functions of α.

$$r_1 = \frac{2\lambda(1-\lambda(1-(2-\mu)\alpha)+\alpha\mu) + (1-\lambda)(1-\lambda+2\alpha\lambda)\mu\ln\frac{1-\lambda+2\alpha\lambda}{1+\lambda}}{(1-\lambda)(1-\mu)\left(2(1-\alpha)\lambda + (1-\lambda+2\alpha\lambda)\ln\frac{1-\lambda+2\alpha\lambda}{1+\lambda}\right)}c$$

and

$$\underline{p}_1 = \frac{(1-\lambda+2\alpha\lambda)\left(2\lambda + (1-\lambda)\mu\ln\frac{1-\lambda+2\alpha\lambda}{1+\lambda}\right)}{(1-\lambda(1-(2-\mu)\alpha-\mu)-\mu(1-\alpha))\left(2(1-\alpha)\lambda + (1-\lambda+2\alpha\lambda)\ln\frac{1-\lambda+2\alpha\lambda}{1+\lambda}\right)}c.$$

Note, that $\underline{p}_1 \geq r_0$ since $\mu\int_{\underline{p}}^{p_1} F(p)dp \geq 0$.

Determination of α. To determine the value of α we use the following approach. We solve for the probability distribution on the upper part of the support using differential equation (9). The solution requires the determination of a constant, say Q, using a boundary condition. as we have two boundary conditions, namely $F(\underline{p}_1) = 1-\alpha$ and $F(r_1) = 1$, this gives us two values of the constant Q_1 and Q_2. Since the solution must satisfy both boundary conditions, it should be that $Q_1 = Q_2$ which gives us the equation determining α. Note, that we do not calculate the optimal search integral here, since it is already incorporated in the determination of r_1[13].

We start with the following differential equation:

$$Ay(x) + Bxy'(x) + Cx + D = 0. \tag{10}$$

[13]Another, may be more natural approach, is to use just one boundary condition and then explicitly calculate the search integral to get the equation for r_0 and α, as we did for the lower part of the support. However this approach results in more analytical complications, so we use the one presented in the text.

The solution of this equation is

$$y(x) = Qx^{A/B} - \frac{Cx}{A+B} - \frac{D}{A}.$$

Now, if we compare (10) with (9) we observe that it is the same equation with $x = p$, $y(x) = \int_{\underline{p}}^{p} F(p)dp$, $y'(x) = F(p)$, $A = -\frac{1-\lambda}{2}\mu$, $B = -\lambda$, $C = \frac{1+\lambda}{2}$, $D = \pi$.

Thus, the equilibrium price distribution is defined by

$$F(p) = \frac{1+\lambda}{2\lambda + (1-\lambda)\mu} - Q\frac{(1-\lambda)\mu}{2\lambda} p^{-\frac{2\lambda+(1-\lambda)\mu}{2\lambda}}$$

where Q is determined by the initial conditions $F(r_1) = 1$ and $F(\underline{p}_1) = 1 - \alpha$. These two values Q_1 and Q_2 have to be equal.

$$\begin{aligned} Q_1 &= -2\lambda(1-\lambda)\mu\left(1 - \frac{1+\lambda}{2\lambda+(1-\lambda)\mu}\right) \\ &\quad \left(\frac{c\left(2\lambda(1-\lambda(1-\alpha(2-\mu)-\mu)-\mu(1-\alpha))-(1-\lambda)(1-(1-2\alpha)\lambda)\mu\ln\frac{1-\lambda+2\alpha\lambda}{1+\lambda}\right)}{(1-\lambda)(1-\mu)\left(2(1-\alpha)\lambda+(1-(1-2\alpha)\lambda)\ln\frac{1-\lambda+2\alpha\lambda}{1+\lambda}\right)}\right)^{1+\frac{(1-\lambda)\mu}{2\lambda}} \\ Q_2 &= -2\lambda(1-\lambda)\mu\left(1 - \alpha - \frac{1+\lambda}{2\lambda+(1-\lambda)\mu}\right) \\ &\quad \left(\frac{c(1-(1-2\alpha)\lambda)\left(2\lambda+(1-\lambda)\mu\ln\frac{1-\lambda+2\alpha\lambda}{1+\lambda}\right)}{(1-\lambda(1-\alpha(2-\mu)-\mu)-\mu(1-\alpha))\left(2(1-\alpha)\lambda+(1-(1-2\alpha)\lambda)\ln\frac{1-\lambda+2\alpha\lambda}{1+\lambda}\right)}\right)^{1+\frac{(1-\lambda)\mu}{2\lambda}}. \end{aligned}$$

Equation $Q_1 = Q_2$[14] can be reduced to:

$$\begin{aligned} \left(\frac{(1-\lambda)(1-\mu)}{1-\lambda(1-\alpha(2-\mu)-\mu)-(1-\alpha)\mu}\right)^{\frac{(1-\lambda)\mu}{2\lambda}} &= \\ \left(\frac{2\lambda(1-\lambda+2\alpha\lambda-\alpha(1+\lambda)\mu)+(1-\lambda)(1-\lambda+2\alpha\lambda)\mu\ln\frac{1-\lambda+2\alpha\lambda}{1+\lambda}}{(1-\lambda+2\alpha\lambda)\left(2\lambda+(1-\lambda)\mu\ln\frac{1-\lambda+2\alpha\lambda}{1+\lambda}\right)}\right)^{\frac{2\lambda+(1-\lambda)\mu}{2\lambda}} & \end{aligned} \tag{11}$$

First, we evaluate (11) at $\alpha = 0$. It is easy to see that both LHS and RHS take the values of 1 for all (λ, μ). Second, we claim that the LHS of equation (11) evaluated at $\alpha = 1$ is larger than the RHS. Indeed, after canceling some terms the equation can be rewritten as $\left(\frac{1-\lambda}{1+\lambda}\right)^{\frac{(1-\lambda)\mu}{2\lambda}} = (1-\mu)$. Thus, the LHS is increasing in λ and as $\lambda \to 0$ it goes to e^{μ} which is larger than $(1-\mu)$. Finally, we examine the behaviour both of LHS and RHS around $\alpha = 0$. Obviously, if LHS decreases faster than the RHS, there must be an intersection point at $\alpha \in (0, 1)$. The derivative of the LHS with respect to α evaluated at $\alpha = 0$ equals to $-\frac{\mu(\lambda(2-\mu)+\mu)}{2\lambda(1-\mu)}$. The derivative of RHS evaluated at $\alpha = 0$ equals to $-\frac{(1+\lambda)(\lambda(2-\mu)+\mu)\mu}{(1-\lambda)\left(2\lambda+(1-\lambda)\mu\ln\frac{1-\lambda}{1+\lambda}\right)}$. Solving

[14] Note, that Q_1 is always greater than 0, so the distribution function is increasing

$$-\frac{\mu(\lambda(2-\mu)+\mu)}{2\lambda(1-\mu)} < -\frac{(1+\lambda)(\lambda(2-\mu)+\mu)\mu}{(1-\lambda)\left(2\lambda+(1-\lambda)\mu\ln\frac{1-\lambda}{1+\lambda}\right)}$$

gives $\mu \in (-\frac{2\lambda}{1-\lambda}, 0) \cup (\frac{4\lambda^2}{(1-\lambda)^2\ln\frac{1-\lambda}{1+\lambda}+2\lambda(1+\lambda)}, \infty)$. Given that μ is between 0 and 1 we get (3).

Now we show that $f(\lambda) \equiv \frac{4\lambda^2}{(1-\lambda)^2\ln\frac{1-\lambda}{1+\lambda}+2\lambda(1+\lambda)} > 2/3$. First, this expression is increasing in λ with $f(1) = 1$. Second, we take a limit $\lim_{\lambda\to 0} f(\lambda)$. By applying l'Hopital's rule twice we get:

$$\lim_{\lambda\to 0} f(\lambda) = \lim_{\lambda\to 0}\frac{8}{4\frac{3+\lambda(3+\lambda)}{(1+\lambda)^2}+2\ln\frac{1-\lambda}{1+\lambda}} = \frac{2}{3}.$$

To prove that (3) is also a necessary condition we show that if the derivative of LHS of (11) is larger than the derivative of the RHS at $\alpha = 0$ than the LHS is larger than the RHS for any other α. That implies that there is no such a value of α which can equate both sides of the equation. Note, that both LHS and RHS of (11) are smooth functions in α, λ, μ. Therefore, we can directly verify the result on a grid for $(\alpha, \lambda, \mu) \in (0, 1)^3$. Numerical verification shows that (3) is indeed not only sufficient, but a necessary condition as well. The structure of the solution is presented in Figure 4. The intersection of the solution surface and surface $\alpha = 0$ is exactly the equality $\mu = \frac{4\lambda^2}{(1-\lambda)^2\ln\frac{1-\lambda}{1+\lambda}+2\lambda(1+\lambda)}$.

□

Proposition 4. *Expected profits for firms in the equilibrium where PMGs are offered with positive probability are higher than the expected profits in the equilibrium without PMGs. As a consequence, in the equilibrium where PMGs are offered consumers pay higher expected prices (after a possible execution of their PMGs) than in the equilibrium without PMGs.*

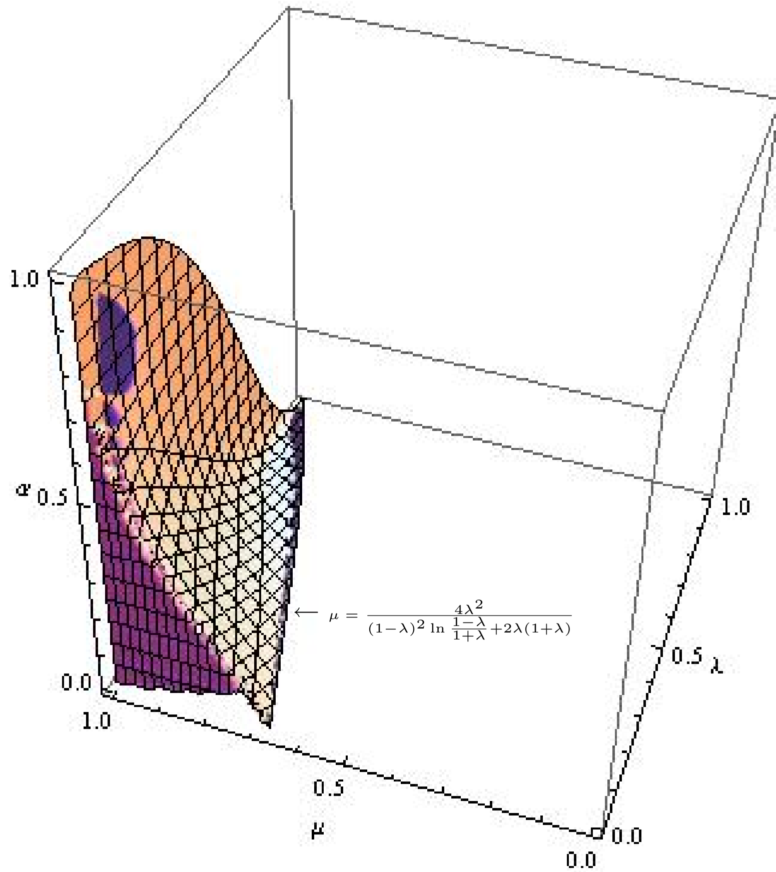
Proof. In fact, the equilibrium without PMG described by the same formulas as the equilibrium with PMG when α is set to be zero. The level of equilibrium profits for the equilibrium with PMG is

$$\pi(\alpha) = \frac{\lambda(1-\lambda+2\alpha\lambda)}{2(1-\alpha)\lambda+(1-\lambda+2\alpha\lambda)\ln\left(\frac{1-\lambda+2\alpha\lambda}{1+\lambda}\right)}c$$

Then

$$\frac{\partial\pi}{\partial\alpha} = \frac{4(1-\alpha)\lambda^3}{\left(2(1-\alpha)\lambda+(1-\lambda+2\alpha\lambda)\ln\left(\frac{1-\lambda+2\alpha\lambda}{1+\lambda}\right)\right)^2}c > 0$$

Thus, profits are strictly increasing in α, so the lowest level of profits is attained when there are no PMGs ($\alpha = 0$). Since we have unit demand and full participation of consumers in the market, the same result holds n of (λ, μ)



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for prices. □

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