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# Advertising and consumer search in a duopoly model<sup>☆</sup>

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## Abstract

We consider a duopoly in a homogenous goods market where part of the consumers are *ex ante* uninformed about prices. Information can come through two different channels: advertising and sequential consumer search. We arrive at the following results. First, there is no monotone relationship between prices and the degree of advertising. Second, advertising and search are “substitutes” for a large range of parameters. Third, when the cost of either search or advertising vanishes, the competitive outcome arises. Finally, both expected advertised and non-advertised prices are non-monotonic in search cost. One of the implications is that firms actually may benefit from consumers having low (rather than high) search costs. © 2007 Elsevier B.V. All rights reserved.

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## 1. Introduction

Consumers and firms do not find each other costlessly in the marketplace. Consumers spend time, money and effort searching to find firms that offer the price and quality that best suits their tastes. Firms, on the other hand, spend money on trying to attract (new) consumers. There is a considerable amount of economic literature on both advertising (see, *e.g.*, the seminal papers by

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Butters (1977)) and search (see, e.g., seminal papers by Diamond (1971), Burdett and Judd (1983), Stahl (1989)) separately, but very little on the interaction between advertising and search activities.

We study a homogeneous goods duopoly model where firms choose an advertising intensity and a price they charge for their product. We consider a linear advertising technology where the cost of reaching an additional consumer is independent of the fraction of consumers already reached. This allows us to get closed-form solutions. After (not) having observed firms' advertisements, consumers decide whether or not to search for (more) prices in a sequential way, i.e., they first choose whether or not to obtain one price quotation and after having seen the result of this price search they decide whether to continue searching or not. As some consumers want to search despite the fact that they are already informed about one firm's price, we allow consumers to be heterogeneous in their search costs: one group of consumers has a positive search cost, whereas another group has zero search cost either because they enjoy shopping or have a negligibly small opportunity cost of time or because they use search engines.

In the context where one allows for both search and advertising, the interpretation of the search cost parameter deserves some attention. In general, search costs consist of (at least) two components: the *cost of visiting* a shop knowing that the shop carries the product the consumer wants to buy and the *cost of finding* or searching for a shop that carries the product. A firm's advertisement not only informs a consumer of the price the firm charges, it also informs the consumer that the firm carries the product, i.e., it (importantly) eliminates the second component of search costs. As this general formulation is difficult to analyze in detail, we consider the extreme situation, where the costs of visiting a shop are negligibly small compared to the costs of finding a shop that sells the product. The role of advertising is to inform consumers that the shop sells the product and at which price.

We arrive at the following results. First, there is no consistent relationship between advertised and non-advertised prices. In particular, contrary to conventional wisdom advertised prices may be higher than non-advertised prices. The intuition for this result may be understood as follows. In our model, receiving an ad means that a consumer does not have to pay search costs, and so advertised prices can be fairly high. Furthermore, the existence of shoppers is relatively more important for non-advertising firms than for advertising firms as advertising firms reach out for many more consumers, whereas non-advertising firms only receive those consumers that actively search themselves. As shoppers create competitive pressure, this creates a tendency for non-advertised prices to be lower.

Second, in our model advertising and search are 'substitutes' meaning that, roughly speaking, if for a certain parameter constellation the equilibrium has firms advertising a lot, then consumers search relatively little, and *vice versa*. This result is quite intuitive as the main goal of search and of advertising is the same, namely to overcome the problem that without information there cannot be any beneficial form of exchange. If one side of the market is willing to pay for overcoming the information gap, then the other side of the market can free ride.

Third, expected prices are non-monotonic in changes in the search cost parameter, but when search costs approach zero, equilibrium prices converge to the competitive price level. To understand the reason why expected prices are non-monotonic, it is important to understand the reason why there is price dispersion in this model. Price dispersion arises from the fact that in equilibrium there are different types of consumers: those that are informed of all prices (or at least more than one firm's price) and those that have only observed one price (either through advertisement or through search). Over the last group of consumers, firms have monopoly power, but there is competition for the first group of consumers. Price dispersion is the way firms balance these two forces. When search cost declines, it is natural for consumers to search more *ceteris paribus*. This forces firms to lower their

prices. On the other hand, when consumers search more, and with lower prices, firms have an incentive to lower their advertising intensity and thereby to increase prices (as *ceteris paribus* there are fewer consumers who make price comparisons). In some cases, the first tendency is larger than the second; in other cases, the second tendency is larger.

Bakos (1997) argues that firms have an incentive to raise search costs since this will lead to higher prices and profits. The result above shows that this need not be the case if one takes advertising into account. The underlying mechanism of two contrasting forces, less search and more advertising, has some similarity to the literature on switching costs. Initially, one would argue that higher switching costs give firms more market power and so raise prices. Firms, however, also have an incentive to compete for market shares, since consumers who are ‘locked in’ can be profitably exploited. The result of these competing forces can go both ways: switching costs can lead to both higher and lower prices (see e.g. Klemperer (1987) and Dube et al. (2006)).

Robert and Stahl (1993) is the first<sup>1</sup> paper analyzing the interaction between search and advertising incentives. Their model differs in two important ways. First, all consumers are *ex ante* identical even with respect to their search costs. Second, Robert and Stahl interpret the search cost parameter as the cost of visiting a firm. In their model therefore consumers also have to pay search costs when buying from a firm they got an advertisement from, neglecting the fact that receiving an ad reduces the cost of finding a shop that carries the product. In their case, the role of advertising is to inform consumers of the price only.<sup>2</sup> There are also many important qualitative differences in results obtained. For example, in Robert and Stahl’s model search and advertising turn out to be complements to each other in the sense that for those parameter values for which firms do not advertise, consumers do not search either (autarky). Moreover, whenever firms do advertise, consumers also are engaged in search activities. Above, we explained why we think it is more natural to (partly) think of search and advertising as substitutes for each other. Second, lower prices are unambiguously more heavily advertised than higher prices in Robert and Stahl (1993). Finally, our model shows that perfect competition is a limiting case when search cost approaches zero. Robert and Stahl arrive at the surprising conclusion that pricing behavior does not converge to competitive pricing when search cost vanishes.

The remainder of the paper is organized as follows. Section 2 presents the model and Section 3 gives a full characterization of the equilibrium configurations possible. The most important comparative static results are presented in Section 4 and Section 5 concludes. Some proofs are given in the Appendix; the remaining proofs are given in Janssen and Non (2005).

## 2. Model

Our model deals with a homogeneous goods market with two active firms. The production costs of the good are constant and equal across firms. We will normalize the production costs to 0. There are no capacity constraints. Firms have the possibility to advertise. The per consumer advertising costs are  $A$ . Advertising is an ‘all-or-nothing’ decision, that is, a firm either does not advertise or it advertises to the complete market. In an ad the firm informs the consumers that it exists and sells

<sup>1</sup> The only other theoretical paper is Stahl (2000). This paper builds on the model by Robert and Stahl (1993) and analyzes some specific cases. Recently, Cason and Datta (2006) reported on an experimental study of the Robert and Stahl model with a fixed advertising intensity. Overall, the experiment supports the predictions of the (modified) Robert and Stahl model.

<sup>2</sup> Other differences are that Robert and Stahl (1993) consider the general oligopoly case with convex advertising technology, *i.e.*, it requires more and more money to reach one more consumer. This level of generality implies that they cannot get a closed-form solution for the equilibrium strategies.

the product and it mentions the price it charges. It is assumed firms have to stick to the price they announce in their ad, that is, ads never lie, and that they have to set a single price to all consumers.

At the demand side of the market there is a unit mass of consumers. Each consumer demands a single unit of the good and has valuation  $\theta > 0$  for the good. We will concentrate on the case of  $\theta > A$ , otherwise it is clear firms will not advertise. A fraction  $\gamma$ , with  $0 < \gamma < 0.5^3$ , of the consumers is called shoppers. These consumers are assumed to know the prices charged by both firms and they will buy at the firm with the lowest price (provided this price is lower than  $\theta$ ). The other  $1 - \gamma$  consumers *a priori* do not know the prices charged by the two firms. Sometimes they will get an ad from one or both firms, depending on the advertising strategy of the firms. Consumers can also decide to search one or both firms for prices. This search is costly: each search action costs  $c$ , where  $c < \theta$ . Consumers have perfect memory; they know which firms they already searched and also remember which price they found there. Furthermore consumers receive all advertisements that are sent by the firms before they start to search. In our model search is sequential: after each search action the consumer decides whether or not to continue searching.

The timing of the game is as follows. First the firms simultaneously decide on their advertising and pricing strategies. With probability  $\alpha^j$  a firm  $j$  advertises and chooses a price from price distribution  $F_1^j(p)$ , where  $F_1^j(p)$  denotes the probability that a price smaller than or equal to  $p$  is chosen. With probability  $1 - \alpha^j$  a firm does not advertise and chooses a price from price distribution  $F_0^j(p)$ . So a firm  $j$ 's strategy is given by  $\{\alpha^j, F_0^j(p), F_1^j(p)\}$ . After the firms have decided on their strategy, advertisements are realized according to  $\alpha^j$  and prices are drawn from  $F_0^j(p)$  or  $F_1^j(p)$ . We will denote by  $\bar{p}_0^j$  and  $\bar{p}_1^j$  the upper bounds of the supports of price distributions  $F_0^j(p)$  and  $F_1^j(p)$  respectively. In the same way,  $\underline{p}_0^j$  and  $\underline{p}_1^j$  denote the respective lower bounds. The shoppers now buy at the lowest-priced firm (provided the price is lower than  $\theta$ ). The non-shoppers first see the advertisements and then decide on their search strategy. If they decide not to search, they buy at the firm with the lowest advertised price lower than  $\theta$  (or, if there are no ads, they do not buy at all). If they decide to search they pick a non-advertising firm at random and obtain a price quotation from that firm and decide whether to search further or to stop searching. If they decide to stop searching, the product is bought from the firm with the lowest price lower than  $\theta$  in the set of already obtained price quotations.

We analyze the symmetric perfect Bayesian equilibrium of the game described above. In the remainder we will therefore drop the index  $j$ . The profit  $\pi_0(p)$  denotes the profit when a firm does not advertise and charges price  $p$  and  $\pi_1(p)$  denotes the expected profit when advertising and charging price  $p$ . We define  $\pi_0 = \pi_0(p)$  for all  $p$  in the support of  $F_0(p)$  and  $\pi_1 = \pi_1(p)$  for all  $p$  in the support of  $F_1(p)$ . Whenever  $0 < \alpha < 1$   $\pi_0 = \pi_1$ .

The assumption made above that advertising is an ‘all-or-nothing’ decision may seem somewhat unrealistic. Other models incorporating advertising (eg. Butters (1977), Stahl (1994), Robert and Stahl (1993)) assume that firms choose an advertising intensity, indicating the fraction of consumers who are informed by an advertisement. This advertising intensity generally depends on the price chosen, and so a firm’s strategy in this context can be denoted by a price distribution  $F(p)$  indicating the probability that the price chosen by the firm is below  $p$  and an advertising function  $\alpha(p)$ , indicating the advertising intensity conditional on a price  $p$ . In Janssen and Non (2005) we show that the two formulations are equivalent.

<sup>3</sup> The equilibrium analysis is dependent on whether  $\gamma$  is smaller than or larger than 0.5. Denote by  $\mu$  the probability that a consumer searches. One of our equilibria needs  $\mu > 2\gamma$ , which is impossible for  $\gamma > 0.5$ . Therefore, for  $\gamma > 0.5$  this equilibrium disappears. On the other hand, one of our equilibria needs  $\mu < 2\gamma$ . This restriction holds automatically for  $\gamma > 0.5$ . We decided to analyze the case of  $\gamma < 0.5$ , since Moraga-Gonzalez and Wildenbeest (2006) estimate the fraction of shoppers to be clearly below 0.5, but significantly above 0.

### 3. Equilibria

Before focussing on equilibrium behavior, we discuss optimal search behavior and optimal firm behavior. By doing so, we use the fact that firms will never set a price equal to 0 or above  $\theta$  as this yields nonpositive profits, while setting a positive price below  $\theta$  and not advertising always yields strictly positive profits.

#### 3.1. Consumer behavior

As in all sequential search models, the optimal search behavior is characterized by a reservation price  $r_0$ . To define the reservation price assume the lowest price already observed is given by  $\hat{p}$ . The expected gain from searching once more is then given by

$$\int_{\underline{p}_0}^{\hat{p}} (\hat{p}-p)dF_0(p). \quad (1)$$

This expression shows that the expected gain arises when the price found is below  $\hat{p}$ . Note that  $F_0(p)$  is used: only non-advertising firms are searched. The above expression can be integrated in parts and simplifies to

$$\int_{\underline{p}_0}^{\hat{p}} F_0(p)dp.$$

This defines  $r_0$  as  $\int_{\underline{p}_0}^{r_0} F_0(p)dp = c$ . One can easily see it is profitable to search if observed prices are above  $r_0$ .

We implicitly assumed that the consumer already observed a price quotation. If this is not the case, consumers will search for sure if  $r_0 < \theta$ . If  $r_0 = \theta$  they will search with probability  $\mu \leq 1$ , and if  $r_0 > \theta$  they will not search at all. To see this, note that the expected gain from searching when no price has yet been obtained is given by expression (1) where  $\hat{p}$  is replaced by  $\theta$ . This implies that searching is profitable if and only if  $\theta > r_0$ . Hence,  $\mu = 1$  when  $r_0 < \theta$ ,  $0 \leq \mu \leq 1$  when  $r_0 = \theta$  and  $\mu = 0$  when  $r_0 > \theta$ .

#### 3.2. Firm behavior

In this part we will derive some general results on firm behavior that will be helpful in the next part where we derive the equilibria of our model.

**Lemma 3.1.** *In a symmetric equilibrium,  $\alpha < 1$ .*

The main idea behind this lemma can be explained as follows. If both firms advertise for sure, all consumers will be aware of all prices in the market. Price will therefore be driven down to 0 (Bertrand outcome). The firms obtain negative payoffs  $-A$  and so have an incentive not to advertise.

A second observation is that like many search papers, but unlike the paper by Robert and Stahl (1993), the price distributions are atomless. Also, prices that are chosen will never be larger than the reservation price of non-shoppers. A third observation is that for all prices between  $\underline{p}_0$  and  $\bar{p}_0$  it should be that  $\pi_0(p) = \pi_0$ . The same holds for  $\underline{p}_1$ ,  $\bar{p}_1$  and  $\pi_1(p)$ . This implies that whether the price distributions have a gap or not,  $\pi_0(p) = \pi_1(p)$  in the region  $[\max\{\underline{p}_0, \underline{p}_1\}, \min\{\bar{p}_0, \bar{p}_1\}]$ . We will use this in the derivation of our equilibria.

**Lemma 3.2.**  $F_0(p)$  and  $F_1(p)$  are atomless and  $F_0(r_0) = F_1(r_0) = 1$ . Furthermore, for  $\underline{p}_0 \leq p \leq \bar{p}_0$   $\pi_0(p) = \pi_0$  and for  $\underline{p}_1 \leq p \leq \bar{p}_1$   $\pi_1(p) = \pi_1$ .

### 3.3. Characterization of equilibria

The model has four exogenous parameters:  $\theta$ ,  $c$ ,  $A$  and  $\gamma$ . It can be shown that if  $\theta$ ,  $c$  and  $A$  are all multiplied by a scalar  $x \neq 1$  the equilibria do not change except that all prices (including  $r_0$ ) and profits are multiplied by  $x$ . For convenience, we can thus normalize with respect to  $\theta$ , and set  $\theta = 1$ .

We first provide a classification of the different types of equilibria that may arise in our model and then characterize the equilibrium strategies of firms and consumers in each of these cases. The following theorem shows that three types of equilibria may arise in our model.

**Theorem 3.3.** *Each symmetric equilibrium can be classified in one of three different types. Type I has firms not advertising at all ( $\alpha = 0$ ). Type II has firms advertising with strictly positive probability ( $0 < \alpha < 1$ ) and overlapping supports of the price distributions with  $\bar{p}_0 = \bar{p}_1$ . Type III has firms advertising with strictly positive probability ( $0 < \alpha < 1$ ) and price distributions that do not overlap. In particular,  $\bar{p}_0 = \underline{p}_1$  and so advertised prices are higher than non-advertised prices.*

Note that in type III equilibria, advertised prices are always higher than non-advertised prices. The reverse, advertised prices always being below non-advertised prices, cannot arise in equilibrium. In the Appendix we show that if advertised prices are always lower than non-advertised prices, an advertising firm has an incentive to deviate and advertise the highest non-advertised price. This leads to somewhat less sales, but at a much higher price.

Each of the three types of equilibria has a corresponding parameter region where the equilibrium exists. These regions will be expressed in terms of  $A$  and  $c$ , taking  $\gamma$  constant. The derivation of the equilibria and corresponding parameter regions will show that each parameter set  $[A, c, \gamma]$  has a unique corresponding symmetric equilibrium. Each symmetric equilibrium of a certain type can be further classified in one of two cases, dependent on the value of  $\mu$ , the probability that uninformed consumers search. Case *a* has  $0 < \mu < 1$  and case *b* has  $\mu$  equal to 1 (for type I and II) or 0 (for type III); see Janssen and Non (2005) for more details. For each of the three types of equilibrium, we will now derive case *a*. The derivation of case *b* is very similar in nature.

### 3.4. Equilibrium type I: no advertising ( $\alpha = 0$ )

As indicated above when characterizing optimal search behavior,  $0 < \mu < 1$  implies  $r_0 = 1$ . As the no advertising case is identical to the search model of Janssen et al. (2005), we can use the same arguments to show that

$$F_0(p) = 1 - \frac{(r_0 - p)^{\frac{1}{2}} \mu (1 - \gamma)}{\gamma p}.$$

Since  $r_0 = 1$ ,  $\int_{\underline{p}_0}^{r_0} F_0(p) dp = c$  is an expression in  $\gamma$ ,  $c$  and  $\mu$ . This expression provides an implicit definition of  $\mu$ :

$$\mu \ln \frac{\mu(1-\gamma)}{2\gamma + \mu(1-\gamma)} = (c-1) \frac{2\gamma}{1-\gamma}. \tag{2}$$

Note that  $\mu \ln \frac{\mu(1-\gamma)}{2\gamma + \mu(1-\gamma)}$  is strictly decreasing from 0 to  $\ln \frac{1-\gamma}{1+\gamma}$  for  $0 \leq \mu \leq 1$  so that  $\mu$  is uniquely defined.



For  $\alpha=0$  to hold, advertising should not be profitable. The expected profit from advertising a price  $p$  is given by  $p[(1-\gamma) + \gamma(1-F_0(p))]-A = (1-\gamma)(p + \frac{1}{2}\mu r_0 - \frac{1}{2}p\mu) - A$ . This expected profit is maximized for  $p=r_0$  giving profit  $r_0(1-\gamma) - A$ . Advertising is not profitable when  $r_0(1-\gamma) - A < r_0(1-\gamma)\frac{1}{2}\mu$ . Rearranging terms and using  $r_0=1$  gives

$$\mu > 2 \left( 1 - \frac{A}{1-\gamma} \right).$$

The two assumptions  $0 < \mu < 1$  and  $\alpha=0$  together hold when

$$\max \left\{ 0, 2 \left( 1 - \frac{A}{1-\gamma} \right) \right\} < \mu < 1.$$

Using the definition of  $\mu$  given by Eq. (2) and the fact that  $\mu \ln \frac{\mu(1-\gamma)}{2\gamma + \mu(1-\gamma)}$  is strictly decreasing for  $0 < \mu < 1$ , this gives rise to the following parameter restrictions:

$$\beta_1 \equiv 1 + \frac{1-\gamma}{2\gamma} \ln \frac{1-\gamma}{1+\gamma} < c < \min \left\{ 1, 1 + \frac{1-\gamma-A}{\gamma} \ln \frac{1-\gamma-A}{1-A} \right\}. \quad (3)$$

As  $0 < \beta_1 < 1$ , it is clear that this type of equilibrium exists whenever  $\frac{1-\gamma}{2} < A$ .

The above discussion, and the derivation of equilibrium Ib can be summarized as follows.

**Proposition 3.4.** *An equilibrium with  $\alpha=0$  has*

$$F_0(p) = 1 - \frac{(r_0-p)\frac{1}{2}\mu(1-\gamma)}{\gamma p}.$$

If (3) holds,  $r_0=1$  and  $\mu$  is implicitly defined by Eq. (2). If  $c < \min \left\{ \beta_1, \frac{2A\beta_1}{1-\gamma} \right\}$ ,  $\mu=1$  and  $r_0 = \frac{c}{\beta_1}$ .

3.5. *Equilibrium type II: some advertising ( $0 < \alpha < 1$ ) and partially overlapping price distributions ( $\bar{p}_0 = \bar{p}_1$ )*

First note that since  $0 < \mu < 1$  the reservation price for non-shoppers should be equal to the consumers' valuation in this case, i.e.,  $r_0=1$ . Furthermore, the profit equations in the case of non-advertising and advertising are equal to

$$\pi_0(p) = p \left[ \gamma\alpha(1-F_1(p)) + \gamma(1-\alpha)(1-F_0(p)) + (1-\gamma)(1-\alpha)\frac{1}{2}\mu \right],$$

respectively,

$$\pi_1(p) = p[\alpha(1-F_1(p)) + \gamma(1-\alpha)(1-F_0(p)) + (1-\gamma)(1-\alpha)] - A.$$

These equations can be interpreted as follows. A firm only attracts all shoppers if it has the lowest price taking into account that the competitor may charge different prices depending on whether or not he advertises. In case the firm does not advertise, it attracts only half of the non-shoppers that do search themselves and only when the competitor does not advertise. The advertising firm attracts more consumers, namely all non-shoppers, if the competitor does not advertise or if the competitor advertises a higher price, but has to pay the advertising cost  $A$ .

Whenever the upper bounds of the two price distributions are equal we can use standard arguments to show that the upper bound then has to be equal to the reservation price, *i.e.*,  $\bar{p}_0 = \bar{p}_1 = r_0$ , and therefore  $\pi_0(r_0)$  has to be equal to  $\pi_1(r_0)$ , which gives the condition

$$r_0(1-\gamma)(1-\alpha)\left(1-\frac{1}{2}\mu\right) = A. \tag{4}$$

For all prices larger than  $\max\{\underline{p}_0, \underline{p}_1\}$ , we can use  $\pi_0(p) = \pi_1(p)$  (see Lemma 3.2) to derive

$$F_1(p) = 1 - \frac{(r_0-p)(1-\alpha)\left(1-\frac{1}{2}\mu\right)}{\alpha p}. \tag{5}$$

Using  $\pi_0(p) = \pi_0(r_0)$  and the above expression for  $F_1(p)$  we can also derive that

$$F_0(p) = 1 - \frac{(r_0-p)\left(\frac{1}{2}\mu-\gamma\right)}{\gamma p}. \tag{6}$$

We stress that these price distributions only hold for prices larger than or equal to  $\max\{\underline{p}_0, \underline{p}_1\}$ . Thus, we distinguish two cases: (i),  $\underline{p}_0 < \underline{p}_1$  and (ii),  $\underline{p}_0 > \underline{p}_1$ .

In case (i) we can use  $\pi_0(p) = \pi_0(r_0)$  to get the price distribution

$$F_0(p) = 1 - \frac{(r_0-p)(1-\gamma)(1-\alpha)\frac{1}{2}\mu-p\gamma\alpha}{p\gamma(1-\alpha)}, \tag{7}$$

for all  $p < \underline{p}_1$ . Similarly, in case (ii) we get that for all  $p < \underline{p}_0$

$$F_1(p) = 1 - \frac{r_0(1-\gamma)(1-\alpha)-p(1-\alpha)}{\alpha p}. \tag{8}$$

To check under which conditions an equilibrium of type IIa exists, we first note that  $\mu$  should be between 0 and 1. Furthermore,  $\alpha$  as defined by Eq. (4) should also be between 0 and 1. This gives rise to condition  $\mu < 2 - \frac{2A}{1-\gamma}$ . Finally, it should be that  $\mu > 2\gamma$ , since otherwise  $F_0(p)$  is decreasing in  $p$ . Thus, we have that  $\mu$  should satisfy  $2\gamma < \mu < \min\{1, 2 - 2A/(1-\gamma)\}$ . For equilibrium IIai,  $\int_{\underline{p}_0}^{r_0} F_0(p)dp = c$  and  $r_0 = 1$  gives

$$f(\mu, A, \gamma) \equiv 1 + \frac{1-\gamma}{2\gamma} \mu \ln \frac{\frac{1}{2}\mu A}{\frac{1}{2}\mu A + (1-\frac{1}{2}\mu)\gamma} - \left(1 - \frac{1}{2}\mu\right) \ln \frac{A(1-\frac{1}{2}\mu)}{(1-\gamma)(1-\frac{1}{2}\mu)-A\frac{1}{2}\mu} = c, \tag{9}$$

which implicitly defines  $\mu$  as a function of  $A$ ,  $c$  and  $\gamma$ . For equilibrium IIaii,  $\int_{\underline{p}_0}^{r_0} F_0(p)dp = c$  and  $r_0 = 1$  gives

$$g(\mu, \gamma) \equiv 1 - \frac{\gamma - \frac{1}{2}\mu}{\gamma} \ln \frac{\mu - 2\gamma}{\mu} = c, \tag{10}$$

again implicitly defining  $\mu$  as function of  $c$  and  $\gamma$ . The L.H.S. of expressions (9) and (10) are both decreasing in  $\mu$  and so  $2\gamma < \mu < \min\{1, 2 - 2A/(1-\gamma)\}$  can be rewritten as

$$\max\left\{f\left(1, A, \gamma\right), f\left(2 - \frac{2A}{1-\gamma}, A, \gamma\right)\right\} < c < f(2\gamma, A, \gamma) \tag{11}$$



for case (i), and

$$\max \left\{ g(1, \gamma), g \left( 2 - \frac{2A}{1-\gamma}, \gamma \right) \right\} < c < g(2\gamma, \gamma) \quad (12)$$

for case (ii). For equilibrium type IIa to hold either restriction (11) or restriction (12) should hold. However, note that for restriction (11) to be relevant,  $\underline{p}_0$  should be smaller than  $\underline{p}_1$ , while for restriction (12),  $\underline{p}_0$  should be larger than  $\underline{p}_1$ . It can be shown that the resulting conditions can be simplified to the ones mentioned in Proposition 3.5.

**Proposition 3.5.** *An equilibrium with  $0 < \alpha < 1$  and  $\bar{p}_0 = \bar{p}_1$  has  $\alpha = 1 - \frac{A}{r_0(1-\gamma)(1-\frac{1}{2}\mu)}$  and  $F_0(p)$  and  $F_1(p)$  being defined by Eqs. (6) and (5) respectively in the common support  $[\max\{\underline{p}_0, \underline{p}_1\}, r_0]$  and Eqs. (7) and (8) respectively for  $p < \underline{p}_1$  and  $p < \underline{p}_0$ .*

*If either  $A > \frac{1-2\gamma}{2}$  and (11) hold or  $A < \frac{1-2\gamma}{2}$  and  $g(1, \gamma) < c < f(2\gamma, A, \gamma)$  hold, where  $f(\mu, A, \gamma)$  and  $g(\mu, \gamma)$  are as defined in Eqs. (9) and (10), then  $0 < \mu < 1$ ,  $r_0 = 1$  and  $\mu$  is defined by Eq. (9) for the case where  $\underline{p}_0 < \underline{p}_1$  and by Eq. (10) for the case where  $\underline{p}_1 < \underline{p}_0$ .*

*If  $A < \frac{1-2\gamma}{2}$  and  $\frac{2A\beta_1}{1-\gamma} < c < \beta_2$  or  $A > \frac{1-2\gamma}{2}$  and  $\frac{2A\beta_1}{1-\gamma} < c < f(2 - \frac{2A}{1-\gamma}, A, \gamma)$ ,  $\mu = 1$  and  $r_0$  is implicitly defined by  $r_0 f(1, \frac{A}{r_0}, \gamma) = c$  for the case where  $\underline{p}_0 < \underline{p}_1$  and by  $r_0 = \frac{c}{\beta_2}$  for the case where  $\underline{p}_0 > \underline{p}_1$ .*

**3.6. Equilibrium type III: some advertising ( $0 < \alpha < 1$ ) and price distributions that do not overlap ( $\bar{p}_0 = \underline{p}_1$ )**

Finally, we will turn to the last type of equilibrium, namely the one where firms spend some money on advertising and when they do advertise, they set consistently higher prices than when they don't advertise, i.e.,  $\bar{p}_0 = \underline{p}_1$ . Using standard arguments, we first observe that  $\bar{p}_1 = r_0 = 1$ . Profits in case of advertising are now given by the following expression

$$\pi_1(p) = p\alpha(1 - F_1(p)) + p(1-\alpha)(1-\gamma) - A.$$

This expression can be understood as follows. When a firm advertises, it knows it gets all consumers when its competitor also advertises and sets a higher price. When the competitor does not advertise, he always asks a lower price and so shoppers will buy from him. Non-shoppers however do not search after receiving an ad and buy from the advertising firm. In order to derive the equilibrium price distribution in case of advertising, we equate this expression with the profits the firm gets when advertising the reservation price:  $\pi_1(1) = (1-\alpha)(1-\gamma) - A$ . This yields the expression

$$F_1(p) = 1 - \frac{(1-p)(1-\alpha)(1-\gamma)}{p\alpha}. \quad (13)$$

The profits a firm gets from not advertising are given by

$$\pi_0(p) = p\gamma\alpha + p\gamma(1-\alpha)(1 - F_0(p)) + p(1-\gamma)(1-\alpha)\frac{1}{2}\mu.$$

A non-advertising firm in this case gets shoppers if, and only if, the other firm advertises or the other firm does not advertise and sets a higher price. Non-shoppers will come to the shop only when the competitor also does not advertise and in that case, both firms receive half of the non-

shoppers. It is easy to see that setting the highest non-advertised price yields a profit equal to  $\pi_0(\bar{p}_0) = \bar{p}_0\gamma\alpha + \bar{p}_0(1-\gamma)(1-\alpha)\frac{1}{2}\mu$ . Equating  $\pi_0(p)$  and  $\pi_0(\bar{p}_0)$  gives

$$F_0(p) = 1 - \frac{(\bar{p}_0 - p) \left( \gamma\alpha + (1-\gamma)(1-\alpha)\frac{1}{2}\mu \right)}{p\gamma(1-\alpha)}. \tag{14}$$

It is easy to see that advertising a price below  $\underline{p}_1$  is never profitable.<sup>4</sup> It also should not be profitable to refrain from advertising and charge a price above  $\bar{p}_0$ . Note that for  $p \geq \bar{p}_0$ ,  $\pi_0(p) = p(\gamma\alpha(1-F_1(p)) + (1-\alpha)(1-\gamma)\frac{1}{2}\mu)$ . Substituting the expression for  $F_1(p)$  given in Eq. (13) yields an expression that is decreasing in  $p$  whenever  $\mu < 2\gamma$ . Hence, this is a necessary condition for this equilibrium to exist.

For the above equilibrium to hold, there are some other parameter restrictions as well. First,  $0 < \alpha < 1$  requires  $\pi_0 = \pi_1$ , which gives

$$\alpha^2 \frac{1}{2} \mu (1-\gamma) + \alpha \left( (1-\gamma)(1-\mu) + \frac{\gamma}{1-\gamma} A \right) + A - (1-\gamma) \left( 1 - \frac{1}{2} \mu \right) = 0. \tag{15}$$

Furthermore,  $0 < \mu < 1$  gives  $\int_{\underline{p}_0}^1 F_0(p) dp = c$ . Substituting  $F_0(p)$  gives

$$1 + \frac{(1-\alpha)(1-\gamma) - A}{\gamma(1-\alpha)} \ln \frac{\gamma\alpha + (1-\alpha)(1-\gamma)\frac{1}{2}\mu}{\gamma + (1-\alpha)(1-\gamma)\frac{1}{2}\mu} = c. \tag{16}$$

Eqs. (15) and (16) together define  $\alpha$  and  $\mu$  as equations of  $c$ ,  $A$  and  $\gamma$ . For the equilibrium to hold,  $0 < \alpha < 1$  and  $0 < \mu < 2\gamma$ .

Note that Eq. (15) is an increasing function of  $\alpha$ , that reaches  $A - (1-\gamma)(1 - \frac{1}{2}\mu)$  for  $\alpha = 0$  and  $\frac{A}{1-\gamma}$  for  $\alpha = 1$ . The restriction  $0 < \alpha < 1$  therefore reduces to  $A - (1-\gamma)(1 - \frac{1}{2}\mu) < 0$ , which gives  $\mu < 2 - \frac{2A}{1-\gamma}$ .

It can be shown that expression (16) is decreasing in  $\mu$  and so the restrictions  $0 < \mu < \min\{2\gamma, 2 - \frac{2A}{1-\gamma}\}$  can be written as

$$\max \left\{ 1 + (1-\gamma) \ln \frac{(1-\gamma)^2 - A\gamma}{(1-\gamma)^2 + A(1-\gamma)}, 1 + \frac{(1-\gamma) - A}{\gamma} \ln \frac{(1-\gamma) - A}{1 - A} \right\} < c < 1 \\ + ((1-\gamma) - A) \ln \frac{(1-\gamma)^2 - A(1-\gamma)}{(1-\gamma)^2 + A\gamma} = \beta_3. \tag{17}$$

We summarize as follows:

**Proposition 3.6.** *Equilibria with  $0 < \alpha < 1$  and  $\bar{p}_0 = \underline{p}_1$  have  $F_1(p)$  as defined in Eq. (13) and  $F_0(p)$  as defined in Eq. (14) with  $\bar{p}_0 = \frac{(1-\alpha)(1-\gamma)}{\alpha + (1-\alpha)(1-\gamma)}$ . If Eq. (17) holds we have  $0 < \mu < 1$  and  $\alpha$  and  $\mu$  are implicitly defined by Eqs. (15) and (16). If  $c > \beta_3$ , with  $\beta_3$  as defined in Eq. (17),  $\mu = 0$  and  $\alpha$  is given by  $\frac{(1-\gamma)^2 - A(1-\gamma)}{(1-\gamma)^2 + A\gamma}$ .*

<sup>4</sup> To see this, write down  $\pi_1(p)$  for  $p \leq \underline{p}_1$  using the above expression for  $F_0(p)$  and note that  $\pi_1(p)$  is increasing in  $p$ .

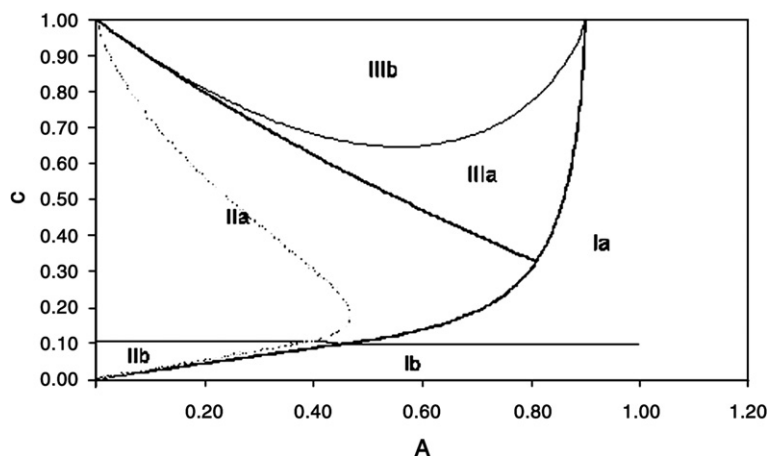


Fig. 1. The parameter regions in terms of  $A$  and  $c$  where the different equilibria exist, assuming  $\gamma=0.1$ . Each region is numbered according to the type of equilibrium that exists in that region.

### 3.7. Discussion

We will now take a closer look at the parameter regions in which the different types of equilibria exist. Fig. 1 depicts for the case where  $\gamma=0.1$  for each equilibrium the region where it exists in terms of  $A$  and  $c$ . We note that the regions do not overlap and that they together fill the complete parameter space. Equilibria IIa and IIb are both divided by a dotted line. Left of this line,  $\underline{p}_1 < \underline{p}_0$ , while right of this line  $\underline{p}_0 < \underline{p}_1$ .

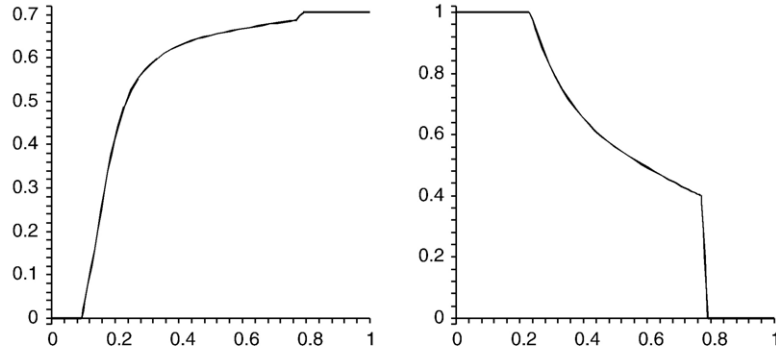
We note that for the equilibria with no advertising (equilibrium type I) to exist, the search costs  $c$  should be low or the advertising costs  $A$  should be high. The intuition is fairly simple: for high  $A$  advertising is too expensive to be profitable. For low search cost, firms cannot ask high prices since otherwise the consumers will search on and so advertising is too expensive relative to the prices that can be asked.

The equilibria with full consumer search (equilibrium types Ib and IIb where  $\mu=1$ ) are in a region with low search costs, whereas their counterparts with partial consumer search ( $0 < \mu < 1$ ) are in a region with higher search cost. The equilibrium with no consumer search (equilibrium type IIIb) only exists for  $c$  relatively high and  $A$  not too low or high. It is clear that higher search cost leads to less search. The intuition for the restriction on  $A$  is as follows. For high  $A$  it is not profitable to advertise and so firms only sell to the shoppers and searching consumers. Note that if consumers do not search at all, prices and profits will be driven to 0, giving consumers an incentive to search. For low  $A$  firms have a large incentive to advertise, which drives prices down. This leads to a higher payoff from search and therefore to some consumer search even if the search costs are large.

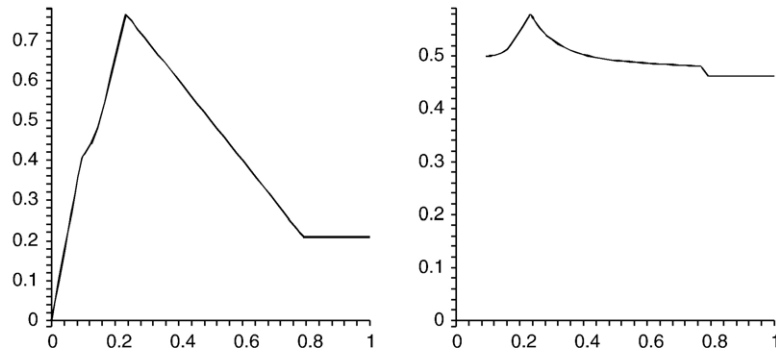
We note that if firms do not advertise ( $\alpha=0$ ), the non-shoppers search with strictly positive probability as long as the search cost  $c$  are below the valuation for the product (see also Janssen et al. (2005)). On the other hand, if the non-shoppers do not search ( $\mu=0$ ), the firms advertise with strictly positive probability as long as the advertising costs are below  $1-\gamma$ . Thus, certainly in these regions advertising and search act as substitutes for each other.

#### 4. Comparative statics

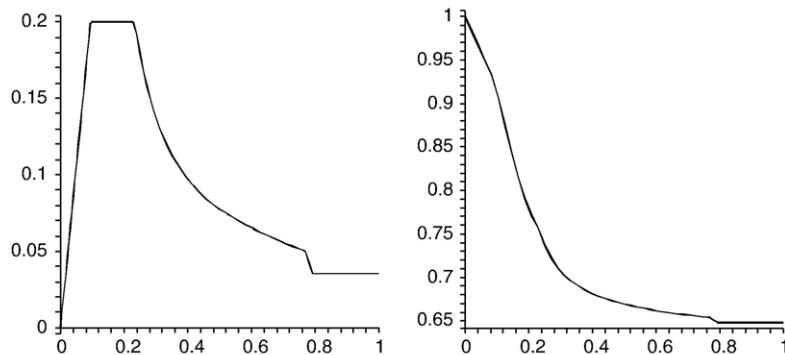
In this section we will give some asymptotic and comparative static results. We are interested in the impact of changes in  $c$  and  $A$  on the variables that are of main interest such as prices, profits and welfare. We start the discussion with some limiting results.



(a) Plots of  $\alpha$  (left) and  $\mu$  (right) as a function of  $c$ .



(b) Plots of  $Ep_0$  (left) and  $Ep_1$  (right) as a function of  $c$ .



(c) Plots of  $\pi$  (left) and welfare (right) as a function of  $c$ .

Fig. 2. Plots of several variables as a function of  $c$ . The figures are drawn for  $\gamma=0.2$  and  $A=0.2$ .

**Theorem 4.1.** *a. When  $c$  becomes arbitrarily small, firms do not advertise and there is full consumer search ( $\mu = 1$ ). The maximum price charged approaches 0.*

*b. When  $A$  becomes arbitrarily small, the advertising intensity  $\alpha$  converges to 1 and the expected advertised price  $Ep_1$  converges to 0.*

These results can be understood as follows. When the search costs approach 0, the Bertrand result arises. This asymptotic result also occurs in the pure search models of Stahl (1989) and Janssen et al. (2005). The intuition is simple: if the search costs are very low, consumers are almost always willing to search and to prevent further search, firms lower their prices, in that way preventing advertising. This Bertrand-like result however does not arise in Robert and Stahl's model. In their model, when search costs approach zero, advertising diminishes, but the minimum price is strictly above zero. Without shoppers, Robert and Stahl obtain a Diamond type of result, namely that when  $c$  vanishes, the equilibrium price distribution converges to a degenerate distribution where all firms charge one particular, strictly positive, price for sure. A fraction of shoppers, however small, causes a breakdown of this result and the traditional Bertrand-type result emerges.

When the advertising costs approach 0 we get the intuitive result that the advertising probability rises to 1 and advertised prices decrease to 0.

We next proceed to some comparative statics results. The analysis provides an assessment of the impact of a change in exogenous parameter values on the economic variables of interest  $\alpha$ ,  $\mu$ , the expected non-advertised price  $Ep_0$ , the expected advertised price  $Ep_1$ , the profit  $\pi$  and welfare conditional on a certain type of equilibrium to be present in the economy. We define expected welfare as the value of all transactions taking place minus the search and advertising costs that have been incurred, *i.e.*, expected welfare equals

$$1 - (1 - \gamma)(1 - \mu)(1 - \alpha)^2 - 2\alpha A - (1 - \gamma)\mu(1 - \alpha)^2 c.$$

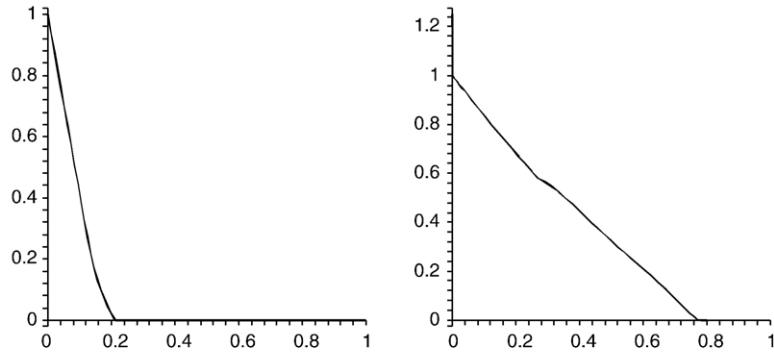
In Figs. 2 and 3 we give a description of how for example, expected price depends on search cost across equilibria for specific values of the other exogenous parameters. Tables 1 and 2 provide an overview of the local comparative statics.

We will first discuss the comparative statics results with respect to changes in search costs. The local comparative statics are given in Table 1.<sup>5</sup>

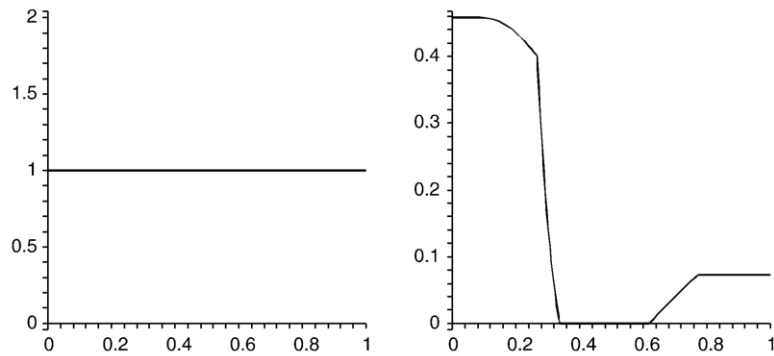
We note that the primary effect of a change in search costs is twofold. In equilibria with full consumer search, the reservation price increases in the search costs, while in equilibria with partial consumer search ( $0 < \mu < 1$ ) the primary effect of a change in search costs is in reducing the amount of search. In the equilibrium with no search there is no effect of a change in search costs. This comes back in Table 1 as the comparative statics results in equilibria Ia, IIa and IIIa are clearly different from the results in equilibria Ib and IIb. In equilibria of type *a*, the decreasing search intensity makes not-advertising less attractive (a non-advertising firm only attracts those non-shoppers that are searching) and so firms advertise more. It is intuitively clear that more advertising leads to lower prices. In equilibria *b*, the reservation price increases in a reaction to increasing search costs. This leads to more advertising. The effect on prices is twofold: a higher

<sup>5</sup> A '+' in these tables denotes a positive impact, a '-' a negative impact, '0' no impact and '+/-' denotes that the impact can be positive as well as negative, depending on specific parameter values. In some cases we have been unable to analytically evaluate the sign. In these cases we resort to a numerical analysis and evaluate the impact by taking  $\gamma$  fixed to 0.01, 0.05, 0.1, 0.15, and so on. These cases are marked by a \*.

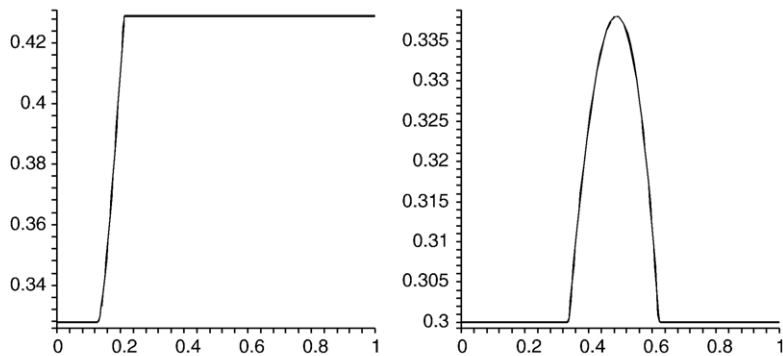
reservation price means higher prices can be asked, but on the other hand there is more competition and so a decreasing effect on prices. These opposite effects are nicely illustrated by the ‘+/-’ for  $E\rho_0$  in equilibrium IIb.



(a) Plots of  $\alpha$  as a function of  $A$ .



(b) Plots of  $\mu$  as a function of  $A$ .



(c) Plots of  $E\rho_0$  as a function of  $A$ .

Fig. 3. Plots of several variables as a function of  $A$ . The left figures are drawn for  $\gamma=0.2$  and  $c=0.1$ , the right figures are drawn for  $\gamma=0.2$  and  $c=0.7$ .

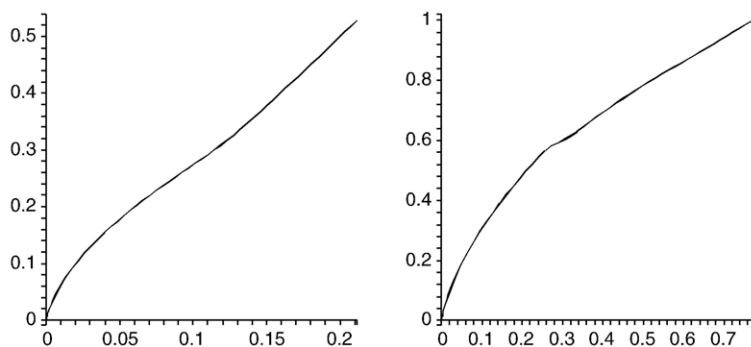
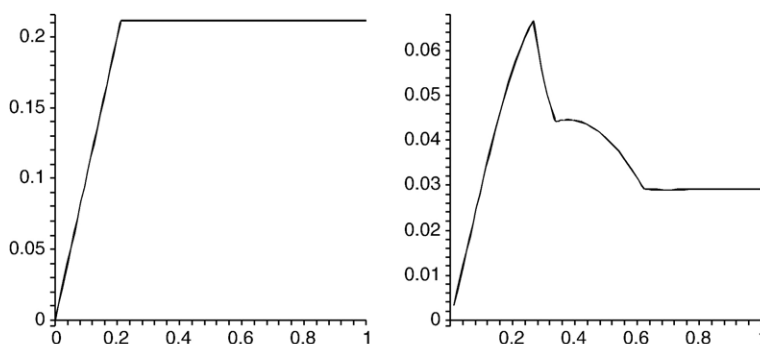
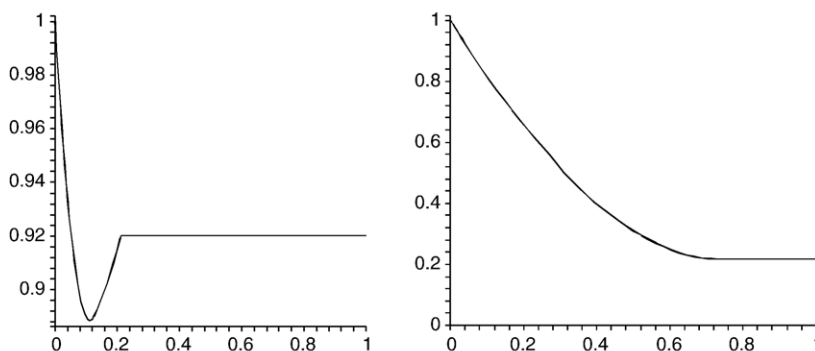
(d) Plots of  $Ep_1$  as a function of  $A$ .(e) Plots of  $\pi$  as a function of  $A$ .(f) Plots of welfare as a function of  $A$ .

Fig. 3 (continued).

An overview of these results is given in Fig. 2, where we depict  $\alpha$ ,  $\mu$ ,  $Ep_0$ ,  $Ep_1$ ,  $\pi$  and welfare as a function of  $c$ , taking  $\gamma$  and  $A$  fixed. The figures are drawn for  $\gamma=0.2$  and  $A=0.2$ . Letting  $c$  increase from 0 to 1 means that equilibria Ib, IIbi, IIbii, IIaii, IIai, IIIa and IIIb hold in this order. The figure clearly shows how the monotonicity of both  $\alpha$  and  $\mu$  with respect to changes in  $c$  may give rise to non-monotonic behavior of both the expected advertised



Table 1  
Local comparative statics results with respect to changes in  $c$

	Ia	Ib	IIa	IIb	IIIa	IIIb
$\alpha$	0	0	+	+	+	0
$\mu$	–	0	–	0	–	0
$Ep_0$	–	+	–	+/-	–	0
$Ep_1$			– <sup>a</sup>	+	–	0
$\pi$	–	+	–	0	–	0
Welfare	–	–	–	–	– <sup>a</sup>	0

<sup>a</sup> Denotes that numerical methods have been used to obtain the result.

and non-advertised price with respect to  $c$ . The figures also show that the profits are non-monotonic with respect to  $c$ , which is quite intuitive given the behavior of prices and advertising.

It is also possible to look at the effects of a change in advertising costs  $A$ . The impact of a change in advertising costs on the economy is through a change in advertising intensity: higher advertising costs lead to less advertising and therefore to higher advertised prices.

An overview of the results is depicted in Fig. 3, where  $\alpha$ ,  $\mu$ ,  $Ep_0$ ,  $Ep_1$ ,  $\pi$  and welfare are represented as a function of  $A$ , taking  $\gamma$  and  $c$  fixed. In the left graphs, we take  $\gamma=0.2$  and  $c=0.1$ , while in the right graphs  $\gamma=0.2$  and  $c=0.7$ . When  $c=0.1$ , letting  $A$  increase from 0 to 1 means that equilibria IIbii, IIbi and Ib hold in this order. For  $c=0.7$  equilibria IIaai, IIai, IIIa, IIIb, IIIa and Ia hold in this order. Note that for  $c=0.1$  we plot  $Ep_1$  only for  $A < 0.21$ . For higher values of  $A$   $Ep_1$  is not defined. The figures show that although  $Ep_0$  is non-monotonic in  $A$ , the expected non-advertised price has only little variation. This is in contrast with the expected advertised price, which monotonically increases from 0 to 0.5–1. Profits and welfare can both increase and decrease in the advertising costs. The effect on profits can be explained by the fact that an increase in advertising costs implies on one hand less competition and therefore higher advertised prices, but on the other hand also leads to more expenditure on advertising if firms decide to advertise.

### 5. Conclusion

In this paper we have analyzed the interaction between two information transferring technologies: advertising and search. In particular we have focused on the interaction between the

Table 2  
Local comparative statics results with respect to changes in  $A$

	Ia	Ib	IIa	IIb	IIIa	IIIb
$\alpha$	0	0	– <sup>a</sup>	–	– <sup>a</sup>	–
$\mu$	0	0	0/–	0	+/-	0
$Ep_0$	0	0	0	+/0	0	+/-
$Ep_1$			+ <sup>a</sup>	+	+ <sup>a</sup>	+
$\pi$	0	0	+/-	+	+/-	+/-
Welfare	0	0	+/-	+/-	+/-	–

<sup>a</sup> Denotes that numerical methods have been used to obtain the result.

incentives for firms to advertise and for consumers to search in relation to the parameters underlying the search and advertising technology.

We reach three important conclusions. First, if the cost of one of these technologies becomes arbitrary small, the model's outcome gets very close to the fully competitive outcome where price equals marginal cost. The fully competitive outcome occurs when consumers are somehow fully informed of all prices. This will necessarily happen if one of the technologies becomes arbitrarily cheap. Second, there are important non-monotonicities in the relations between expected prices on one hand and search and advertising cost on the other hand. An important part of the explanation for this phenomenon to occur is that search and advertising are 'substitutes' over an important domain of the model's parameters in the sense that if consumers (initially) start searching more (because of a decrease in search cost) firms will advertise less. In the aggregate this may mean that consumers actually are less informed about prices when search cost are lower and this gives firms an incentive to raise prices. Third, advertising firms have an advantage since consumers who receive an advertisement do not have to pay search costs. Therefore, advertised prices may actually be higher than non-advertised prices.

## Appendix A

**Proof of Theorem 3.3.** There are three possible types of equilibria not mentioned in the theorem. We will show that each of these possibilities cannot be part of an equilibrium.

(i)  $0 < \alpha < 1$  and  $\bar{p}_0 > \bar{p}_1$ . Using standard techniques, we note that in this case  $F_0(p) = 1 - \frac{(\bar{p}_0 - p)^{\frac{1}{2}\mu(1-\gamma)}}{p^\gamma}$  for  $p \geq \bar{p}_1$ . Whenever  $p \geq \bar{p}_1$ , the profit for an advertising firm equals  $\pi_1(p) = p[\gamma(1-\alpha)(1-F_0(p)) + (1-\gamma)(1-\alpha)] - A$ . Using the expression for  $F_0(p)$  it can be shown that

$$\pi_1(p) = (1-\alpha) \frac{1}{2} \mu (1-\gamma) (\bar{p}_0 - p) + (1-\gamma)(1-\alpha)p - A,$$

which is increasing in  $p$  for  $\bar{p}_1 \leq p \leq \bar{p}_0$ , showing that advertising firms have an incentive to deviate.

(ii)  $0 < \alpha < 1$  and  $\underline{p}_1 < \bar{p}_0 < \bar{p}_1$ . In this case, using  $\pi_0(p) = \pi_1(p)$  in the price region  $[\max\{\underline{p}_0, \underline{p}_1\}, \bar{p}_0]$ , yields the following expression for  $F_1(p)$ :

$$F_1(p) = 1 - \frac{A - (1 - \frac{1}{2}\mu)p(1-\alpha)(1-\gamma)}{\alpha(1-\gamma)p}.$$

Moreover, using this and  $\pi_0(\bar{p}_0) = \pi_0(p)$  we get  $F_0(p) = 1 - \frac{(\bar{p}_0 - p)^{\frac{1}{2}\mu(1-\gamma)}}{p^\gamma}$  on  $[\max\{\underline{p}_0, \underline{p}_1\}, \bar{p}_0]$ . To make sure  $F_0(p)$  is below 1,  $\frac{1}{2}\mu - \gamma$  should be positive. We furthermore can derive an expression for  $F_1(p)$  for  $p \geq \bar{p}_0$  by using  $\pi_1(p) = \pi_1(\bar{p}_0)$ . This gives for  $p \geq \bar{p}_0$ ,

$$1 - F_1(p) = \frac{\alpha \bar{p}_0 [1 - F_1(\bar{p}_0)] - (1-\alpha)(1-\gamma)(p - \bar{p}_0)}{\alpha p}$$

Plugging this expression into  $\pi_0(p)$  for  $p \geq \bar{p}_0$  gives  $\pi_0(p) = \gamma \bar{p}_1 (1-\alpha)(1-\gamma) + p(1-\alpha)(1-\gamma)(\frac{1}{2}\mu - \gamma)$ , which is increasing in  $p$  since  $\frac{1}{2}\mu - \gamma$  is positive. This shows that non-advertising firms have an incentive to deviate.

(iii)  $0 < \alpha < 1$  and  $\bar{p}_0 < \underline{p}_1$ . In this case,  $\pi_0(p) = p(\alpha\gamma + (1-\alpha)(1-\gamma)\frac{1}{2}\mu)$  for  $\bar{p}_0 \leq p \leq \underline{p}_1$ , which is increasing in  $p$  and so non-advertising firms have an incentive to deviate.  $\square$

**Proof of Theorem 4.1.** a. Observe that when  $c \rightarrow 0$ , the equilibrium is in region Ib with  $\alpha = 0$  and  $\mu = 1$ . Furthermore, the maximum price is  $r_0 = \frac{c}{\beta_1} \rightarrow 0$ .  $\square$

b. Observe that when  $A \rightarrow 0$ , either equilibrium IIaii or equilibrium IIbii holds. In equilibrium IIaii,  $\mu$  does not depend on  $A$ , and as  $A \rightarrow 0$ ,  $\alpha = 1 - \frac{A}{(1-\gamma)(1-\frac{1}{2}\mu)} \rightarrow 1$  and  $Ep_1 = A \left( \frac{\frac{1}{2}\mu - \gamma}{\alpha(1-\gamma)(1-\frac{1}{2}\mu)} \ln \frac{\mu - 2\gamma}{\mu} + \frac{1}{\alpha(1-\frac{1}{2}\mu)} \ln \frac{1-\frac{1}{2}\mu}{A} \right) \rightarrow 0$ . In equilibrium IIbii,  $\alpha = 1 - \frac{2A}{(1-\gamma)r_0}$  and  $Ep_1 = A \frac{2}{\alpha} \ln \frac{(1-2\gamma)r_0}{2A} - \frac{A}{(1-\gamma)\alpha} \ln(1-2\gamma)$ , while  $r_0$  does not depend on  $A$ . As  $A \rightarrow 0$ , this yields  $\alpha \rightarrow 1$  and  $Ep_1 \rightarrow 0$ .  $\square$

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