Bargaining in Spectrum Auctions: A Review of the German Auction in 2015

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Abstract

Germany was the first country in Europe that auctioned off spectrum in the valuable 700 MHz band for mobile telecommunication usage. The German regulator decided to sell this spectrum together with spectrum in the 900 MHz and 1800 MHz legacy bands. With only the three incumbent operators bidding in the auction and a relatively transparent auction design, it was possible to get a very clear impression of the actual bidding behaviour. We show that in the beginning of the auction, bidders were actively searching for a way to allocate the available spectrum that all bidders could agree to at low prices. Bidders were teaching each other what they should bid and were providing carrots and sticks. When the excess demand was concentrated in one band only, bidders started to compete head on leading to a war of attrition. This competition only stopped when bidders started to raise prices in bands that were already cleared. We interpret this bidding behavior in terms of bidders expressing allocative externalities and conclude that the ability to do so may be regarded as a positive aspect of the transparent design.

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1 Introduction

Economists have long argued that auctions will promote efficiency when governments award spectrum licenses to private companies (Coase, 1959).[1] Theoretical research on the properties of auctions played a key role and provided central arguments for the use of auctions for spectrum sales.[2] To understand whether or not auctions in general, and specific auction designs in particular, have lived up to this expectation of promoting efficient allocations, we should not only understand the theoretical properties of these auction designs, but especially see how bidders behave in particular auctions in the field. If bidders actually behave quite differently from what the theoretical model assumes, the value of theoretical predictions concerning auction properties is limited.

In this paper, we analyze bidder behaviour in the recent 2015 German auction in detail. One advantage of studying bidding behaviour in the German auction is that the regulator made the highest standing bids after each round publicly available. Bidders even learned about all bids submitted after each round. This allows us to use publicly available data to analyze actual bidding behavior in detail. We show that simple straightforward bidding does not explain bidder behaviour. Our analysis identifies different kinds of deviations from straightforward bidding and we rationalize these deviations with economic arguments and a simple model.

The German auction used the SMRA format.[3] If the blocks are substitutes for each other and bidders bid straightforwardly in each round, then the SMRA terminates in a Walrasian equilibrium (Milgrom, 2000). Gul and Stacchetti (1999) showed that even if the substitutes condition is satisfied, then ascending and linear-price auctions such as the SMRA cannot implement the VCG outcome. In contrast, demand reduction has widely been observed in the field and analyzed game-theoretically (Ausubel et al., 2014). The SMRA is a family of auction rules, where the common feature is a multi-object version of the English auction: bidders express bids in increasing fashion, the auctions stops when no bidder expresses new bids and winning bidders pay their last own bid(s). SMRAs differ along many dimensions, but an important differentiation is the level of transparency after each auction round. In many countries, bidders in an SMRA only learn the minimum bid price for a license in a new round, but they do not learn what other bidders are winning or even what they have bid on in the last rounds. The SMRA implementation used in Germany is different in this respect. Bidders learn the entire bid history of other bidders from the start.

Our description of bidding behaviour in the 2015 auction and the simple game theoretic explanations

[1]The first implementation had to wait until 1994 when the US Federal Communications Commission (FCC) used the simultaneous multi-round auction (SMRA) for the sale of radio spectrum for *personal communication services* (PCS). See Porter and Smith (2006) for a historical overview that finally led to this auction.

[2]For example, Milgrom (2000) writes that "the auction design made detailed use of the ideas of economic theory and the recommendations of economic theorists."

[3]Theoretically, equilibrium bidding strategies are not easy to derive for an SMRA. A full Bayesian analysis for general valuations is quite challenging (Goeree and Lien, 2014) and theoretical models need to make restrictive assumptions, which do not always carry over to the field.

of different behaviours suggest that *bargaining about the full allocation* serves as an adequate description of bidding behaviour. In an initial *cooperation phase*, bidders try to implicitly coordinate on an allocation at low cost. If bidders cannot agree on a focal point they bargain about different allocations with costly signals in a *competition phase*. The bidding behaviour indicates that bidders care about the full allocation and not only about the share of the spectrum supply they themselves acquire. They do so by indicating to others ("teaching others") that they should reduce their spectrum demand. Whether the spectrum is equally distributed among the operators in a market or whether there is a dominant player who owns a large share has a substantial impact on the competition in the downstream market. So, it is not surprising that the relative size of a bidder's package compared to that of the other bidders matters.

Our paper not only provides new insights into how bidders bid in real-world auctions. It also adds a new perspective on questions regarding auction design. As we have indicated above, and we will show in more detail in the main body of the paper, the transparent auction design of the German auction(s) allows bidders to express their preferences over the full allocation. Some authors have argued that there is too much transparency in these auctions, as it enables bidders to implicitly coordinate their behaviour on a low-revenue outcome (cf., Grimm et al. (2003) and Cramton and Ockenfels (2016)). Although, we agree with this argument (and we show there are clearly some elements of this present in the 2015 German auction), we also think that a transparent design has certain advantages in the presence of significant allocative externalities. If bidders do not learn during the auction how the spectrum is divided among all players, they face a substantial strategic problem. In an intransparent auction a bidder might expect the final allocation to be fairly balanced. If, however, one strong player emerges instead, who wins most of the complementary licenses, then the net present value for his package would be much lower, because end consumers will often decide to switch to the operator with the best network, i.e., the one with most spectrum. Transparency may help bidders to express these allocative externalities as they can veto undesirable allocations with their bids. One insight we provide is that there is a trade-off when deciding on the level of transparency in an auction. On one hand, allowing for some transparency enables bidders to express their allocative externalities in a simple way. On the other hand, allowing for too much transparency may facilitate implicit coordination among bidders. However, such implicit coordination would hardly lead to an asymmetric allocation of spectrum with a dominant player. Since regulators aim for a sustainable competition in the end consumer market, transparency might actually be positive in markets where allocative externalities matter.

Allocative externalities are particularly important in national markets with only a few competitors (which is what we have in most countries worldwide), and considering these externalities sheds a different light on the question how efficient auction markets should be designed. Thus, this paper suggests taking allocative externalities more serious in the spectrum auction design literature.[4]

[4] There is some literature on the fact that bidders' bidding behaviour is affected if allocative externalities play a role (see e.g., Jehiel and Moldovanu (2000); Goeree (2003); Jehiel et al. (1996); Janssen and Karamychev (2009, 2010); Klemperer

At this point it is important to understand the primary design goals of a regulator. European Directives are clear that regulators should mainly aim at creating a competitive and innovative market after the auction. For example, Article 8 of Directive 2002/21/EC states in paragraph (2): "The national regulatory authorities shall promote competition [...] by [...] (a) ensuring that users, including disabled users, derive maximum benefit in terms of choice, price, and quality; (b) ensuring that there is no distortion or restriction of competition in the electronic communications sector; (c) encouraging efficient investment in infrastructure, and promoting innovation; (d) encouraging efficient use and ensuring the effective management of radio frequencies and numbering resources." [5] These ideas are also reflected in and enforced by the German telecommunication law where it is stated in Section 2, paragraph 2: "The aims of regulation shall be: 1. to safeguard user, most notably consumer [...]; 2. to secure fair competition and to promote telecommunications markets with sustainable competition [...]; 3. to encourage efficient investment in infrastructure and to promote innovation;" Thus, revenue cannot be the main objective of an auction and implicit coordination on a low price outcome therefore does not necessarily point at a major failure of the auction.[6] In a transparent auction design, bidders can submit bids to avoid a dominant position of competitors. Such bids reflect the strategic value for a bidder of a more symmetric spectrum allocation, and here bidders' incentives to prevent others to become dominant, are aligned with the regulatory objectives as outlined in the European Directives.

Due to its high level of transparency, the German auction design is specific and one may wonder what one can learn from studying this auction for other markets. We argue that the German auction is not only interesting in its own right, but that it is also interesting for other markets for several reasons. First, the fact that bidders have preferences over the full auction allocation is independent of the auction format. The findings from the German auction are probably relevant for many other spectrum auction markets, as allocative externalities and the relative spectrum allocation of bidders almost always play a role. Second, regulators and mobile companies often treat relative auction prices across different spectrum bands in important auctions as reflecting the relative value of spectrum in the different bands. The 2015 German auction is no exception. For example, in a recent decision (September 2015) on so-called annual license fees, the British regulator Ofcom used relative auction prices in various auctions, including the 2015 German auction as indicators for the market value of spectrum licenses in the UK. We argue that prices in different bands in the 2015 German auction can largely be explained by specific bidding strategies and they reveal little about the underlying valuations that operators may have for these spectrum licenses. Due to the high degree of transparancy in the bidding process, this paper is able to establish this for the German auction, but a similar conclusion is likely to hold for other auctions as well.

(2002). However, when it comes to auction design, the traditional assumption that bidders only value their own share and not the full allocation is predominant.

[5] See also Article 13 of Directive 2002/20/EC (the Authorisation Directive or Auth D).

[6] Note that the goals described in the telecommunication law are different from allocative efficiency of an auction, which might well be a monopoly or dominant position of one of the players (Bichler et al., 2014)

There are a number of papers with descriptions of European spectrum auctions around the year 2000 (Illing, 2005; Klemperer, 2002). Two papers have a similar flavour to ours, also analyzing individual bidder behavior in detail. Börgers and Dustmann (2005) analyze bidding behaviour in the 2000 UK auction and show that the bidding behaviour of several companies significantly deviated from straightforward bidding. They mention several possible reasons for these deviations. An important difference between their paper and ours is that the auction designs have significantly progressed since 2000. In the 2000 UK auction and in many 3G auctions that were held around that time in Europe (except Austria and Germany), the combinations of licenses were determined ex ante by the regulator and each mobile operator could win at most one such combination. Since then it has become more common to design the allocation process such that multiple units are for sale and mobile operators can combine their own packages. This allows companies to develop more sophisticated bargaining strategies where bidders can signal how they think the available supply should be allocated.

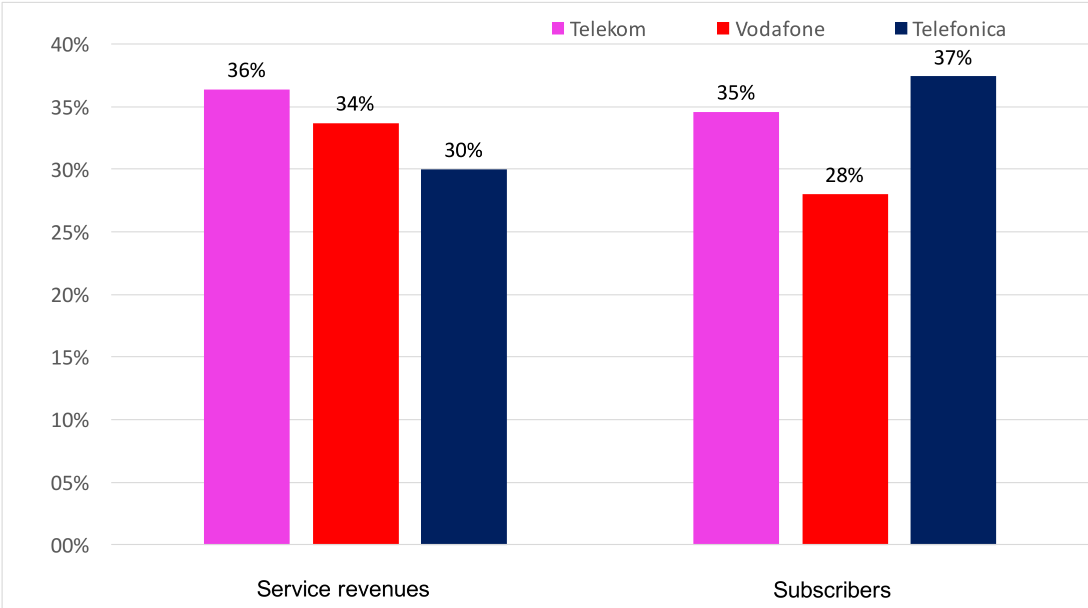
Cramton and Ockenfels (2016) perform a similar analysis to ours for the 2010 German auction. The strategic situation in the 2015 auction differed, however, substantially from the 2010 auction. In the 2010 auction, there was one bidder (EPlus) that could drive up the prices of competitors at little cost for himself, because it was possible for EPlus to drop out of the most valuable 800 MHz band and become a low-cost operator with a lower quality network. The possibility for EPlus to act as if it was a strong bidder bidding for 800 MHz band blocks at relatively high prices (similar to a pooling strategy in a signaling game), was an important consideration in the 2010 auction. In this 2010 auction, even a weak EPlus could pretend to be a strong bidder to raise rivals' cost. Raising rivals' costs is a wide-spread strategy in auctions where bidders meet again in the downstream market, as it weakens competitors and binds their budgets in the downstream market. In contrast, in the 2015 auction all bidders needed spectrum in all bands and price driving strategies always implied a cost increase for the bidder himself.[7] As we will argue, the resulting strategic situation in the 2015 auction resembles that of a war-of-attrition game. Another important difference between the two auctions is that with three bidders, one can deduce much better the actual bidding behaviour from the publicly available bid data. This allows us to go into more detail concerning actual bid strategies that bidders followed than Cramton and Ockenfels (2016).

The rest of the paper is organized as follows. Section 2 describes the market environment of the German mobile telephony sector at the time of the 2015 auction and the auction rules in detail. Section 3 provides an in-depth overview of the bidding behaviour in the auction. Section 4 provides an extensive interpretation of that behaviour in terms of game theoretic bidding strategies. To answer the question what is specific about the 2015 German auction, Section 5 compares this auction to other SMRAs. Section 6 concludes with a discussion on auction design issues.

[7] With the exception that Telefonica (TEF) did not want to acquire licenses in the ancillary downlink only 1500 MHz band.

Figure 1: Market shares of the involved bidders in terms of service revenues and number of subscribers at the end of 2014.

Source: Financial reports of the respective companies



2 Auction Rules and Market Environment

To understand the strategic possibilities of the bidders in this auction it is important to first discuss the relevant competitors, the market environment and the spectrum that was for sale.

Shortly before the auction the third largest Mobile Network Operator (MNO) in the market, Telefonica, merged with the former fourth largest MNO, EPlus. As a consequence, there were three active MNOs in the German market before the auction: Telekom Deutschland (DT), Vodafone (VOD), and Telefonica (TEF). These were also the only three bidders in the auction: no new competitors were admitted to the auction. Before the start of the auction DT was the market leader in terms of service revenues, followed by VOD and TEF. Moreover, according to Connect, a magazine testing the mobile network quality in Germany every year, DT had also the highest quality network.[8] In terms of number of customers, TEF was the market leader followed by DT and VOD (after the merger between TEF and EPlus). The differences between the three players were not very large, however. The overview of the market is depicted in Figure 1.[9]

The spectrum holdings of each operator prior to the auction are depicted in Figure 2. Due to the merger, TEF had the largest spectrum holdings. It is important to note, however, that as part of the merger remedies, the German regulator BNetzA reserved the right to reallocate some of the TEF owned spectrum, especially in the 2.1 GHz band, after observing the results of the auction. Thus, the auction was conducted under a high degree of uncertainty for all bidders whether, and if so how, the allocation

[8]See http://www.connect.de/vergleich/netztest-2014-bestes-handynetz-connect-DT-o2-vodafone-eplus-2744431.html.
[9]The numbers are taken from the publicly available yearly financial reports of the respective companies.

of spectrum in the Germany would be reorganized after the auction (and maybe as a consequence of the outcome of the auction).

BNetzA offered spectrum for mobile use in four different bands to be auctioned in the 2015 auction: 700, 900, 1800, and 1500 MHz.[10] The spectrum in the 700, 900, and 1800 MHz band was offered in 2x5 MHz paired blocks (FDD spectrum), whereas the spectrum in the 1500 MHz band was offered in 1x5 MHz downlink only blocks (DL spectrum). Some of the blocks were specific. That is, the winner of a specific block wins that specific frequency block. The rest of the blocks was generic. In what follows we give a brief description of important characteristics of each band.

- *700 MHz:* The 6 blocks were previously used for television broadcasting and were divided into one specific block and 5 generic blocks. The specific block was less valuable, i.e., debased, because it came with special obligations. The block could only be used if additional investments are made by the winner of this block. Spectrum in the 700 MHz band is most likely to be used for the new LTE (Long Term Evolution, 4G) technology.

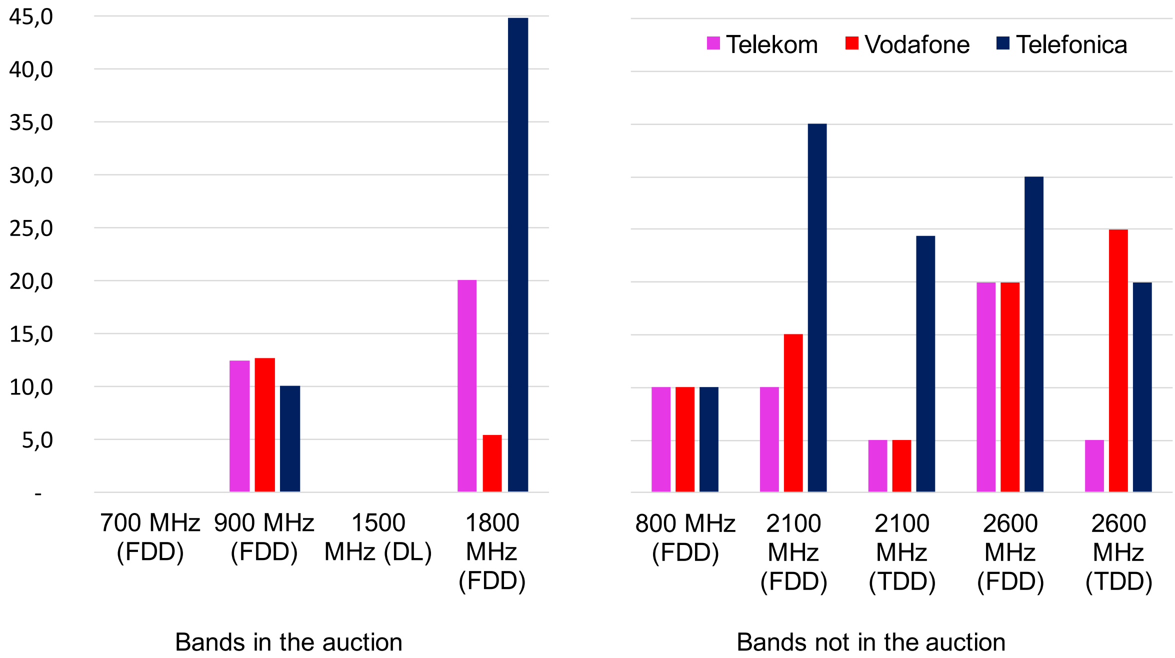
- *900 MHz:* The 7 blocks were already in use by the bidders for GSM (2G) and were divided into one specific block and 6 generic blocks. The specific block is debased in that it faces interference with the GSM network of the German railroad (GSM-R). Using this block for LTE would require additional investments. Each bidder was only allowed to purchase up to 3 blocks of 900 MHz spectrum (spectrum cap) to ensure that each operator at least receives one block of spectrum to seamlessly continue their operation. If a bidder would acquire 3 blocks of 900 MHz spectrum, he would have the possibility to use two blocks for LTE, otherwise he would likely continue to use all of his 900 MHz spectrum for GSM. Thus, for any of the bidders acquiring three blocks of 900 MHz is most likely to have a large incremental value compared to acquiring two blocks.

- *1800 MHz:* Of the 10 blocks for sale, 9 were already in use by the bidders for mobile services, while one block was the guard band to secure not to have interferences between the mobile services and the cordless DECT telephones. This block was specific, while the rest of the blocks were generic. The specific block is debased in that it may face interference with cordless phones, leading again to additional investments being needed from the winner of this block to use this block effectively. Importantly, in addition to the spectrum for sale at auction, DT was holding 3 blocks and TEF 2 blocks in the 1800 MHz band. These MNOs acquired this spectrum in the 2010 spectrum auction. If the spectrum in the 1800 MHz band is used for LTE technology, then it is most valuable if after the auction the bidder can either assemble 4 or 6 adjacent blocks. In particular, acquiring a 5th block of 1800 MHz spectrum has a relatively small intrinsic incremental value (apart from

[10] All the details of the auction regulative framework including the rules of the auction can be found in the official documents provided by the Bundesnetzagentur available at `https://www.bundesnetzagentur.de/SharedDocs/Downloads/DE/Sachgebiete/Telekommunikation/Unternehmen_Institutionen/Frequenzen/OffentlicheNetze/Mobilfunk/DrahtloserNetzzugang/Projekt2016/EntscheidungProjekt2016_pdf.pdf`

Figure 2: Allocation of spectrum before the auction.

Source: Bundesnetzagentur



preventing another MNO to acquire a 4 or 6 adjacent blocks). This spectrum could also be used, however, for GSM technologies and some of the operators probably intended to use some of their spectrum in this band for GSM. For GSM usage, any block of 2x5 MHz is valuable.[11]

- *1500 MHz:* The 8 blocks that were for sale in this band were previously used for radio broadcasting. All blocks were auctioned as generic blocks. This band was considered to be a non-mainstream band, inferior to existing bands. Moreover, at the time of the auction there were no mobile devices available that would make use of the 1500 MHz band.

The regulator implemented a simultaneous multiround auction (SMRA) using the same rules that were used in the 2010 German spectrum auction. Most of the rules are standard[12] and include an activity rule, click-box bidding, waivers, minimum increments etc.[13]. The staged activity rule requires bidders to bid actively from the start. Bidders had activity points, and if they did not bid on a certain percentage of these points in each round or win a certain number of licenses, they lost activity points. In principle,

[11] At the time of the auction DT was the only operator that was not running any other services besides LTE on the 1800 MHz band. Thus, for VOD and TEF who were using 1800 MHz for GSM it is of some value to hold more than 4 blocks of 1800 MHz spectrum after the auction to have some maneuvering room when shifting customers from 1800 MHz GSM to other bands. In particular, due to the E-Plus merger TEF was using 2x25 MHz in the 1800 MHz band for GSM services. Thus, if TEF would acquire merely 2 blocks of 1800 MHz spectrum it would not be able to use their current holdings of 2 blocks in the 1800 MHz band to assemble 4 adjacent blocks of 1800 MHz spectrum for LTE in the first couple of years following the auction.

[12] For a general description of SMRAs see Cramton (2006).

[13] The rules of the auction can be found in the official documents of the Bundesnetzagentur available at `https://www.bundesnetzagentur.de/SharedDocs/Downloads/DE/Sachgebiete/Telekommunikation/Unternehmen_Institutionen/Frequenzen/OffentlicheNetze/Mobilfunk/DrahtloserNetzzugang/Projekt2016/EntscheidungProjekt2016_pdf.pdf`

BNetzA could increase the activity requirement in three stages, where in the last stage, any reduction of demand would imply a loss in activity points. In earlier stages with lower activity requirements, bidders can still increase demand to some extent, in spite of a demand reduction in an earlier round. Throughout most of the auction the activity rule was set at 65%. A bidder keeps his bid rights for the subsequent auction round, if he satisfies this minimum activity level.

To actually loose activity points bidders had to express demand that is below 65% of their current level of activity points.

An important specific feature of the German auction format is that in each band the blocks were named alphabetically[14] and even if the blocks were generic, bidders had to bid on blocks with specific letters rather than just specify the quantity demanded of the generic blocks: if a bidder wanted to acquire more spectrum than they are holding at the end of a certain auction round, they had to overbid other bidders on specific spectrum blocks. For example, a bidder demanding 2 generic blocks in the 700 MHz band had to place specific bids on specific blocks (for example, the B and D block). Moreover, in every round and for each block the identity of the highest standing bidder and his bid was made public.[15] As compared to the publicly available information the bidders received additional information. They were able to see not only the highest bid on a particular block but also all unsuccessful bids for this block and potential bid withdraws. That is, bidders received more information than the public and the information available to the bidders during the auction was never made public. Thus, the following analysis is solely based on the publically available data. However, with only three bidders in the auction, the publically available data is sufficient to extrapolate all of the relevant bids. An example for the publicly available data can be found in Figure 3.[16] This kind of transparency is what distinguishes the German auction from other SMRAs.[17] Transparency allows the bidders to use bids to communicate with their competitors and such communication is facilitated by the relaxed activity rule of 65%.

3 A Description of the Bidding Behaviour

In this section we will describe the main developments in the auction in terms of three phases: an initial *cooperation phase*, followed by a lengthy *competition phase* and a short *end phase*. In the cooperation phase, the demands of the bidders reflect a search for a mutually compatible set of bids that are considered appropriate for each of the three bidders individually. At the end of the cooperation phase it became clear that there is no allocation that is acceptable for all bidders without a fierce price war. Thus, the cooperation phase is followed by a competition phase where bidders fight for one or more blocks to achieve

[14]For example, in the 900 MHz band the blocks were named from A to G.

[15]The publicly available data can be found at `https://www.bundesnetzagentur.de/cln_1411/DE/Sachgebiete/Telekommunikation/Unternehmen_Institutionen/Frequenzen/OeffentlicheNetze/Mobilfunknetze/Z_Auktion2016.html?nn=268128`

[16]Even though we will argue below that transparency can be beneficial for efficiency, there seems to be no reason at all to provide bidders and the public with different amounts of information.

[17]See also Cramton and Ockenfels (2016) in their analysis of the 2010 German spectrum auction.

Figure 3: Example for publicly available bid data from round 11 showing for each block the current highest standing bidder and the current price for the bands in the auction. The last three rows show (from top to bottom) the sum of all valid bids, the sum of all currently withdrawn bids and the overall sum.

Source: Bundesnetzagentur

Rundenergebnis der Runde 11

Frequenzbereich	Block	Ausstattung	Höchstbieter	Höchstgebot (€ in Tsd.)
700 MHz (gepaart)	700 A	2x5 MHz konkret	TEF DE	75.050
	700 B	2x5 MHz abstrakt	TEF DE	75.050
	700 C	2x5 MHz abstrakt	Vodafone	75.020
	700 D	2x5 MHz abstrakt	Vodafone	75.020
	700 E	2x5 MHz abstrakt	Telekom	75.000
	700 F	2x5 MHz abstrakt	Telekom	75.000
900 MHz (gepaart)	900 A	2x5 MHz konkret	TEF DE	75.050
	900 B	2x5 MHz abstrakt	TEF DE	78.812
	900 C	2x5 MHz abstrakt	TEF DE	82.687
	900 D	2x5 MHz abstrakt	Vodafone	82.709
	900 E	2x5 MHz abstrakt	Vodafone	82.709
	900 F	2x5 MHz abstrakt	Telekom	78.771
	900 G	2x5 MHz abstrakt	Telekom	82.687
1,8 GHz (gepaart)	1800 A	2x5 MHz abstrakt	TEF DE	45.603
	1800 B	2x5 MHz abstrakt	Vodafone	45.640
	1800 C	2x5 MHz abstrakt	TEF DE	43.467
	1800 D	2x5 MHz abstrakt	Vodafone	43.467
	1800 E	2x5 MHz abstrakt	Vodafone	45.640
	1800 F	2x5 MHz abstrakt	Telekom	45.640
	1800 G	2x5 MHz abstrakt	Telekom	45.604
	1800 H	2x5 MHz abstrakt	Telekom	45.604
	1800 I	2x5 MHz abstrakt	TEF DE	45.604
	1800 J	2x5 MHz konkret		
1,5 GHz (ungepaart)	1500 A	1x5 MHz abstrakt	Telekom	18.750
	1500 B	1x5 MHz abstrakt	Telekom	18.750
	1500 C	1x5 MHz abstrakt	Telekom	18.750
	1500 D	1x5 MHz abstrakt	Telekom	18.750
	1500 E	1x5 MHz abstrakt	Telekom	19.708
	1500 F	1x5 MHz abstrakt	Telekom	19.708
	1500 G	1x5 MHz abstrakt	Vodafone	18.770
	1500 H	1x5 MHz abstrakt	Vodafone	18.770
Summe aller gehaltenen Höchstgebote (€ in Tsd.)				1.571.790
Zahlungsverpflichtung aufgrund zurückgenommener Höchstgebote (€ in Tsd.)				0
Summe (€ in Tsd.)				1.571.790

a certain allocation. The competition phase ends when prices rise to a sufficiently high level that one or more bidders reduce their demand. The 2015 auction ran over the course of 4 weeks or 181 rounds. Auction revenues totaled over EUR 5 billion.[18]

In what follows we will use a 3-tuple as a short hand describing how many licenses a bidder demanded in his bid in the 700 MHz, 900 MHz, and 1800 MHz band. For most of the time we will thereby ignore the 1500 MHz band which was less valuable and not decisive for the strategic situation. Whenever we explicitly consider demand in the 1500 MHz band we will denote demand as a 4-tuple. For example, if DT was bidding on 2 licenses in the 700 MHz band, 3 licenses in 900 MHz band, and 3 licenses in the 1800 MHz band, we will denote this by 2-3-3, if we want to point out that DT was also bidding on 4 licenses in the 1500 MHz band, we will denote this by 2-3-3-4.

3.1 The Cooperation Phase (Rounds 1-30)

(i) *Opening bids(rounds 1 to 6).* The first bid (together with the bids in the first couple of rounds) revealed that TEF bids for 2-2-5 including the specific blocks in the 700 MHz and the 900 MHz band. Given it could keep two blocks in the 1800 MHz band, this could be considered a relatively aggressive bid in the 1800 MHz band, but TEF compensated this by immediately bidding on the debased blocks in both the 700 and 900 MHz bands, even though the starting prices probably did not fully reflect the value of the restriction.[19] VOD and DT demanded 2-3-4. The overall excess demand amounted to 0-1-3. Remarkable was the position of VOD's bids in the 900 MHz band and TEF's bids in the 1800 MHz band in the first round.[20] In the 900 MHz band VOD bid on blocks D to F leaving open blocks A to C and block G. In the 1800 MHz band TEF bid on block B to F leaving open block A and blocks F to J. One way to interpret such bids is that VOD was offering to divide the 900 MHz band in a way such that one bidder receives 1 block of 900 MHz spectrum and two bidders receive 3 blocks each. In the 1800 MHz band the bids of TEF can be interpreted as an offer to divide the 1800 MHz band in a way that one of the bidders receives 1 block, one of the bidders receives 4 blocks and TEF receives 5 blocks.[21] After the opening bid, the bidders continued in the first 6 rounds by bidding on the cheapest undebased blocks without changing the overall demand.

(ii) *Establishing the claims (rounds 7 to 18).* In the rounds following the opening bids, the bidders did not change their demands. However, in contrast to the opening rounds, bidders sometimes did not bid on the cheapest undebased blocks after they were outbid on one of their blocks. Bidders were

[18] The following analysis is based on the publicly available data as described in Section 2

[19] Moreover, TEF was not bidding in the 1500 MHz band.

[20] The publicly available bid data only allows to deduce the demands of VOD and TEF in those bands with certainty as bids on some of the blocks are not shown if another bidder bid earlier or higher on one of those blocks.

[21] Given the spectrum holdings in the 1800 MHz band that DT and TEF hold outside the auction, this would imply that both DT and VOD were offered a total of 4 blocks each.

making sure that their competitors understand their demand and the potential overall allocations. For example, at the end of round 6, VOD was the highest standing bidder on only 2 blocks in the 900 MHz and needed to bid on a third block. However, in round 7, TEF raised the bid on one of the blocks in the 900 MHz band for which they were already the highest standing bidder after round 6. This prevented VOD from outbidding TEF on this particular block in round 7.[22] In round 13, TEF outbid VOD instead of DT who was holding the cheapest block in the 900 MHz band after round 13, probably intending to indicate that TEF thought that VOD had to reduce demand in the 900 MHz band to avoid price increases.

(iii) *Demand reductions (round 19 to 30).* In round 19, it appears that DT reduced their initial demand of 2-3-4 to 2-3-3. DT placed jump bids on the 3 blocks in the 900 MHz band and on the 3 blocks in the 1800 MHz band for which they were the highest standing bidder after round 18. Thus, DT increased prices on blocks it already "owned".

TEF followed in round 21 by reducing its initial demand of 2-2-5 to 2-2-4. However, in the same round, instead of bidding on the cheapest undebased blocks it outbid DT on one of its blocks, which due to the jump bid from round 19, was one of the significantly more expensive blocks. At this point the overall excess demand was reduced to 0-1-1.

In rounds 22 to 29 the overall demand of the bidders did not change. TEF and VOD were consistently overbidding each other in the 900 MHz band.[23]

In round 30, VOD changed its demand from 2-3-4 to 2-2-5 by not bidding in the 900 MHz band and bidding for an additional license in the the 1800 MHz band.[24] After round 30, the overall excess demand was 0-0-2, and the auction would have ended if two bidders would have reduced their demand by one more block in the 1800 MHz band. The total revenue at this point was less than EUR 2 billion.

3.2 The Competition Phase (Rounds 31 to 154)

As none of the bidders wanted to give up an additional license in the 1800 MHz band, it became clear that the competitors could only resolve the conflict by outbidding their opponents. The phase of the auction where bidders searched for mutually compatible demands was over and a competition phase started.

(i) *Price war in the 1800 MHz band (rounds 31 to 73).* All three bidders employed tactics in order to

[22] Even though VOD's bid is not visible in the publicly available data it is highly likely that it was bidding on this particular block in round 7.

[23] DT's jump bid from round 19 still ensures that DT holds the most expensive blocks in this band.

[24] This can be interpreted as a demand reduction strategy as by reducing from three to two blocks in the 900 MHz band VOD foregoes the option to develop an LTE network in the 900 MHz band (see Section 2 for details). As explained in the previous section, the marginal value of the 5th block in the 1800 MHz band in itself is likely to be rather low as only multiples of 10 MHz ensure major speed and capacity improvements of the network. At this point in the auction, the relative prices of blocks in the 1800 MHz and 900 MHz band are not such that it is likely that VOD's marginal value for 2-3-4 versus 2-2-5 was reached at this point.

signal to their competitors that they have deep pockets and can outbid them in a bidding war trying to convince their opponents that they will be better off by giving in immediately. Bidders, however, apparently had different views who had to give in: with three bidders and an excess demand of two blocks only, there was no obvious solution. Two kinds of signals were used. First, bidders used jump bids on the desired bundle in the 1800 MHz band including their "own" blocks for which they were the highest standing bidder. Second, bidders outbid a specific competitor on one of their blocks even though this block was more expensive than a block held by the other competitor.

Examples. In round 33, TEF temporarily increased demand in the 1800 MHz band from 4 to 5 blocks and overbid DT on the most expensive block. In round 34, DT responded by jump bidding on three blocks in the 1800 MHz band including one of its "own" blocks and two blocks of TEF, even though cheaper blocks were held by VOD.

In round 41, TEF overbid DT and VOD with jump bids on one block of each in the 1800 MHz band, even though those blocks were not the cheapest available blocks. DT responded in round 42 by placing 10 Million EUR jump bids on three (including two "own") blocks in the 1800 MHz band. At this point the strategic bidding in the 1800 MHz band lead to price difference of around 23 million EUR between identical blocks of 1800 MHz spectrum.

(ii) *Price war in the 900 MHz band to solve conflict in 1800 MHz band (rounds 74 to 84).* In round 74, VOD changed its demand from 2-2-5 back to 2-3-4. DT responded in round 76 by increasing demand in the 1800 MHz band from 3 blocks to 4 blocks. TEF responded by increasing demand in the 900 MHz band from 2 blocks to 3 blocks. These moves by all three bidders could be interpreted as signals towards the competitors that if a price war cannot be avoided anyway, the bidders might as well go for their most desired packages.

(iii) *Demand reduction (rounds 84 and 85).* In round 84, TEF reduced its demand to 2-2-3. In the same round, DT reduced demand from 2-3-4 back to 2-3-3. VOD responded in round 85 by switching its demand back from 2-3-4 to 2-2-5 such that at this point the overall excess demand was 0-0-1. The total revenue at this point was approximately 2.8 billion EUR.

(iv) *Steps (i) and (ii) repeat themselves (rounds 86 to 154).* Even though the overdemand dropped to one block in the 1800 MHz band (and sometimes to one block in the 900 MHz band), none of the bidders was willing to reduce the demand further and the auction did not end. A price war in the 1800 MHz band started again with several rounds of demand expansion and demand reduction, which led to strong price increases in the 1800 MHz band relative to the other bands. All of the bidders tried to convince their competitors to reduce an additional block.[25] As a solution in the

[25] The bids of DT seemed to be geared towards convincing TEF to reduce in the 1800 MHz band. The bids of VOD and TEF seemed to be geared towards convincing DT to reduce the last block in the 1800 MHz band. In the 900 MHz band the bids of VOD seemed to be geared towards convincing TEF to reduce demand from 2 blocks to 1 block.

1800 MHz band was not reached the bidders started to drive up prices in the 900 MHz band again. Bidding in this phase is accompanied by aggressive jump bids by all of the competitors.

Examples. In round 123, VOD switched demand from 2-2-5 back to 2-3-4, overbid TEF on a relatively expensive block in the 900 MHz band and placed jump bids on all blocks in the 900 and 1800 MHz bands.

In round 134, DT bid on 8 blocks in the 1800 MHz band by placing asymmetric jump bids of up to 20 million EUR.

In round 138, TEF increased demand to 6 blocks in the 1800 MHz band by overbidding DT on all 1800 blocks for which DT was the highest standing bidder.[26] At this point the prices of generic blocks in the 1800 MHz band exceeded the prices in the 900 MHz band by up to 90 millon EUR and exceeded the prices in the 700 MHz band by 150 million EUR even though generally lower frequency bands are considered to be more valuable.

In round 154 the total revenue was at approximately 4.2 billion EUR.

3.3 The End Phase (Rounds 155 to 181)

So far, prices in the 700 MHz had not increased from the start of the auction when the bidders had immediately found an allocation of each acquiring 2x10 MHz (including a specific block for TEF) that was likely to be acceptable to all three bidders (as no one was bidding anymore). When the severe price competition in the 900 and 1800 MHz bands did not reach a solution, however, this started to change.

(i) *Spillover to the 700 MHz band (rounds 155 to 168).* In round 155, VOD placed bids on 4 blocks in the 700 MHz band (including the debased block and overbidding TEF) thereby also raising the prices of blocks for which it was already the highest standing bidder. Thus, VOD shifted its demand from 2-3-4 to 4-2-4. There are two ways to interpret this bid. Either VOD offered a new way to resolve the excess demand in the 900 and 1800 MHz bands or by jeopardizing the low price outcome in the 700 MHz band VOD sends another signal that it really has deep pockets.[27] In the subsequent rounds, TEF and DT reacted by bidding in the 700 MHz band and establishing that they are both not willing to reduce their demand in this band.[28]

(ii) *End of the auction (rounds 169 to 181).* In round 169, VOD placed 20 million EUR jump bids on all 6 blocks in the 700 MHz band and simultaneously reduced their demand in the 900 MHz band to 1 block in the following round. By bidding on only one block in the 900 MHz band it stopped the price increases in the 900 MHz band and allowed the relative prices between the 700 MHz and 900

[26] Due to the jump bids in round 134 those were by far the most expensive blocks in the 1800 MHz band.

[27] The second interpretation seems more likely as it seems unlikely that TEF would have walked away from the auction without acquiring any of the particularly potent spectrum in the 700 MHz band.

[28] The signals of the bidders became hard to read at this point. For example, bidder sometimes bid on 3 blocks in 700 MHz band or DT and TEF increased demand in the 900 MHz and 1800 MHz band without a conceivable pattern.

MHz band to adjust. In the subsequent rounds, TEF continued bidding on 2-3-3 and DT reacted by increasing their demand to 5 blocks in the 1800 MHz band.[29]

In round 172, VOD repeated their bid from round 169 and placed 20 million EUR jump bids on 6 blocks in the 700 MHz band. TEF reacted in round 173 by withdrawing its bid on one block in the 1800 MHz band, withdrawing its bid on two blocks in the 900 MHz band and simultaneously bidding on the debased block in the 900 MHz band. Thus, it dropped its demand to 2-2-2 and it became clear that there is no excess demand in the 1800 MHz band anymore. If VOD and DT would return to their "initial" claims of 2-2-5 and 2-3-3, an equilibrium could be found in the major bands and the auction could end if VOD and DT also would find a solution to divide the 1500 MHz band (as TEF had never expressed a positive demand in this band).

This is exactly what happened. After TEF had reduced its demand to two blocks in the 1800 MHz band, VOD and DT immediately returned to their demands of 2-2-5 and 2-3-3. After a few more rounds, they also found a resolution in the 1500 MHz band and the auction ended.

In the end, DT acquired a package of 2-3-3-4, VOD a package of 2-2-5-4 and TEF a package of 2-2-2-0. The prices in the 700 MHz band ranged from 163 to 171 million EUR, the prices in the 900 MHz band ranged from 180 to 211 million EUR, the prices in the 1800 MHz band ranged from 180 to 255 million EUR, and the prices in the 1500 MHz band ranged from 39 to 43 million EUR. Especially in the 900 and 1800 MHz bands, bidders paid considerably different prices for the same generic lots, with price differences up to 35 percent and more.[30] The total revenue after round 181 amounted to approx 5.1 billion EUR. Interestingly, comparing the final revenue with the revenue in round 84, where it became clear that the overall excess demand is only one block in the 1800 MHz band, reveals that 2.3 billion EUR were spent by the bidders just to determine who will receive this one block. In all likelihood, this may well exceed the marginal value of this block. We view this, apart from the bidding behavior, as strong evidence that bidders actually did care about the full allocation and not merely for which spectrum they are going to win themselves.[31] Even though it is impossible to asses whether the final allocation was efficient or the revenue sufficient, it is apparent that all of the bidders won a significant share of the spectrum and none of the bidders was marginalized. The bidders competed fiercely in all of the bands and none of the blocks was sold close to the reservation price. A summary of the spectrum holdings after the auction can be found in Figure 4.

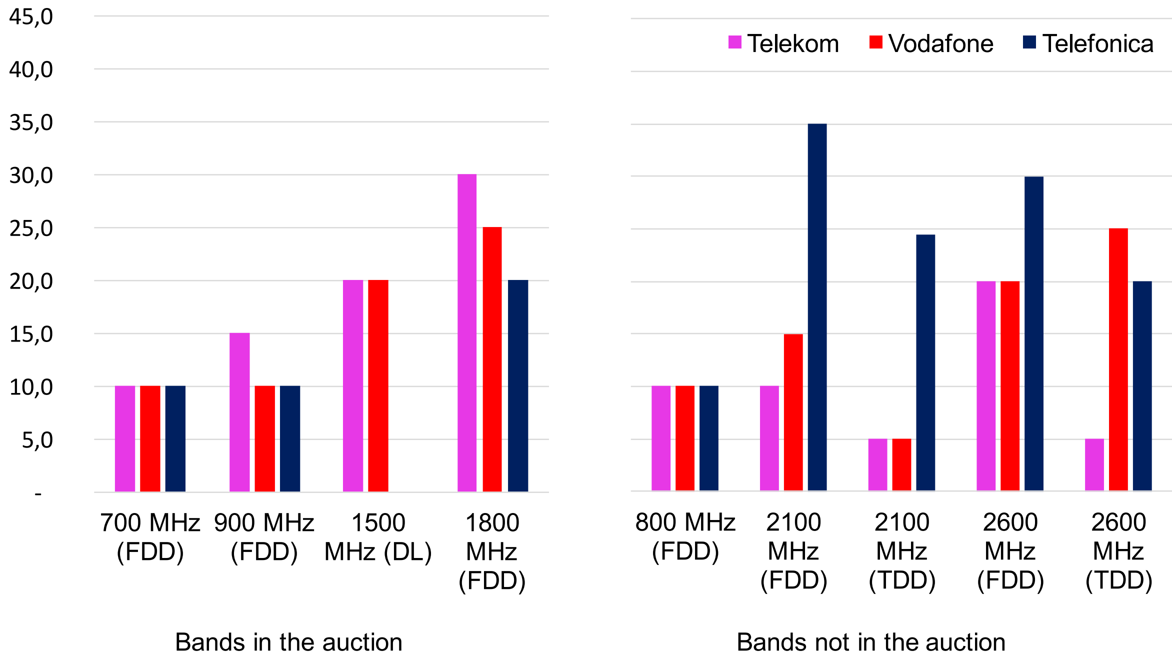
[29] DT's demand and thereby price increase in the 1800 MHz band can be interpreted as a punishment of VOD as VOD demanded the largest number of blocks in the 1800 MHz band and thus is most hurt by price increases in that band.

[30] A discussion of this can be found in Section 4

[31] In particular, if comparing the allocation of round 32, in which the auction could have reasonably ended if VOD reduced to 2-2-4, with round 181, VOD spent approximately 1.4 billion EUR just to acquire a 5th block of 1800 MHz spectrum. Even if taking into account that VOD wanted to have some maneuvering room when switching from GSM to LTE in the 1800 MHz band, this amount seems to exceed the marginal value of a 5th block of 1800 MHz spectrum. However, bidding for this block prevented TEF from acquiring a third block of 1800 MHz spectrum. As mentioned in footnote 11, TEF needed, due to the much larger 1800 MHz GSM legacy network, to acquire strictly more than 2 blocks of 1800 MHz spectrum to be able to deploy full speed LTE in the 1800 MHz band. In combination with their large 2100 MHz holding, this could have given TEF a large advantage in the market.

Figure 4: Allocation of spectrum after the auction

Source: Bundesnetzagentur



4 An Interpretation of the Bidding Behaviour

It is clear that many bids in the 2015 German auction are not in line with straightforward bidding: bidders were increasing demand at different moments in different bands when prices increased, they were overbidding the most expensive blocks even when the blocks are generic and they could have bid on another block at much lower prices and they were engaged in jump bidding even if it was highly likely they could have been the highest standing bidder for that block also with a (much) lower bid. The high transparency of the auction allowed for a fairly explicit communication among the competitors. In this section, we interpret the main developments in the 2015 German auction in terms of simple game theoretic ideas. To describe the competition phase, we develop a simple game theoretic model and provide an equilibrium analysis. By doing so, we want to emphasize that the strategic choices made can well be described by known game theoretic arguments. However, these arguments are quite different from the standard bidding behaviours that are typically assumed in the auction literature on the SMRA.

4.1 Cooperation Strategies

In this subsection we will focus on cooperative strategies that were used by the bidders during the auction. The goal of these strategies was to find an acceptable overall allocation early in the auction. Most of the strategies described in this subsection were used during the opening rounds of the auction which we described as the cooperation phase. However, some of the strategies were also employed at later stages of the auction.

Searching for a Focal Point

Most of the behaviour in the first cooperation phase of the auction serves the purpose of finding a focal point of the game (see also the discussion in Cramton and Ockenfels (2016)), but with the important caveat that in the first few rounds all bidders are claiming a little more than what they would eventually be willing to settle down on.

Naturally, bidders would always want to win a package that is as large as possible at the lowest possible price. They realize, of course, that given the same wish of the others and the limited supply, they have to consider packages that they reasonably may acquire if they want to avoid a price war. Without the possibility to communicate before and during the auction, bidders will naturally try to signal their intentions through their bids.

From this perspective, the initial rounds of the auction are very important. Bidders want to appear to have reasonable demands (in the light of the overall quantity of spectrum for sale). Thus, despite the fact that at the opening prices, bidders probably would have preferred larger packages, they did not bid on such larger packages. Instead, they bid on packages that could possibly form a feasible allocation under certain assumptions about the demands of competitors. This can also be considered a form of strategic demand reduction, a phenomenon that has received much attention in the literature on multi-unit auctions (Ausubel et al., 2014; Weber, 1997).

On the other hand, bidders do not want to express a demand that, given the demands of the others, would appear to be "too small", in the sense that at the end of the auction they could have obtained a larger package if they would have started off by demanding that larger package. The opening bid is regarded by competitors as a claim to a share of the total supply that a bidder would feel comfortable with. It is very difficult to credibly claim a larger share of the supply at a later auction round.[32] At least, a demand increase at a later point in time will be perceived as an aggressive signal by the competitors.

In the 2015 multi-band auction, finding a focal point was more difficult than in some of the previous auctions in Germany.[33] First, there were several bands on sale with different propagation properties. Second, the competitors already owned spectrum and as their current spectrum holdings were asymmetric, other bidders' preferences were hard to predict. Third, especially the 7 blocks in the 900 MHz band, and to a lesser extent the 10 blocks in the 1800 MHz band, were difficult to "divide" among three bidders. Fourth, some of the spectrum blocks in the 700 and 900 MHz band were debased (as explained in Section 2). Finally, the merger between TEF and EPlus before the auction led to a new player in the market whose strength was uncertain. This was amplified by the regulatory uncertainty created by the German

[32] This is very similar to a bargaining situation where if you have expressed that you are willing to buy an item at x, it is difficult later to credibly say you are willing to pay at most 0.9 x.

[33] The German spectrum auction in 1999 is a well-known example where bidders quickly found a focal point (see Grimm et al. (2003) for a game theoretic treatment).

regulator BNetzA as it allowed itself the liberty to re-allocate spectrum in the 2.1 GHz band and thus added to the uncertainty concerning the demand of TEF. In what follows we briefly describe in more detail the issues in each of the main bands that complicated the search for a focal point.

- *700 MHz:* It may seem that for the most valuable spectrum band, the 700 MHz band, it was fairly natural to split the six blocks into three packages of two blocks each. However, one dedicated block in this band (the A block) was debased. TEF accepted this downside, however, by bidding on the A license from the start. This behaviour, together with the bid on the debased 900 MHz block should be considered as TEF's part of trying to reach an early solution and it was combined with a "more than fair share demand" in the 1800 MHz band. It is as if TEF was telling the other bidders "I am ready to go for debased blocks, and forego getting the 3rd block in 900 saving all of us lots of money, if I get rewarded in 1800 MHz band". Indeed, the bidders were able to immediately find the focal point allocation in this 700 MHz band and there was no bidding in this band until late in the auction when the bidding was probably more inspired by a threat than by a reasonable wish to acquire more spectrum in this band.

- *900 MHz:* It was much more difficult for the bidders to coordinate in the 900 MHz band. As in the 700 MHz band, the A block was of lower value, but, as in the 700 MHz band, TEF already bid on this block in the first round indicating that it would be willing to accept this disadvantage. However, a focal point in the 900 MHz band was not easily found as in the initial rounds. Both DT and VOD claimed three blocks, implying an excess demand of one block in this band. It is highly likely that the third block did have a high strategic value, because the winner of the third block would be able to reach a better LTE performance in a quality band.[34]

- *1800 MHz:* In the 1800 MHz band there were 10 licenses available. Given the spectrum holding of DT (3 blocks of 2x5 MHz) and TEF (2 blocks of 2x5 MHz) that were not in auction, it seems that a focal point could have been easily found in this band. An equal division of the spectrum would have led to 5 blocks for each bidder (including the ones owned by DT and TEF that was not for sale). However, the 5th block was of little additional intrinsic value, while a 6th block in this band had technical advantages. The LTE network of the bidder that could string together six blocks of 2x5 MHz would have a significant speed advantage in LTE over their competitors. The fifth block has a value in carrying some GSM traffic, but arguably this value is much lower and mostly only relevant in the years immediately following the auction. Moreover, TEF owned significantly more spectrum in the 2.1 GHz band compared to its competitors. The 2.1 GHz band was technically comparable with the caveat that it could not be fully used for LTE due to legacy 3G. Moreover, as mentioned above, the 2.1 GHz licenses would end in 2020 and were possibly subject to a redistribution by

[34]The fact that VOD reduced from 3 blocks of 900 MHz spectrum to 2 blocks in round 30 already can be viewed as an example of an initial demand expression that is designed to strengthen the bargaining position in later stages of the auction.

the BNetzA. Thus, there was no easy way to divide the 1800 MHz spectrum, and indeed DT and TEF indicated a demand beyond 5 blocks in total. After the first few rounds, there was an excess demand of three licenses in the 1800 MHz band.

Overall, the opening bids are important throughout the entire auction. Increases in demand beyond the opening bid were likely not perceived as real demands, but instead as threats attempting to convince other bidders to reduce their demand. Opening bids thus serve as a benchmark against which to interpret further demands. This makes the choice of an opening bid difficult. On one hand, bidders want to claim what they believe was a fair share of the overall supply. On the other hand, a too high demand may easily lead to a long and costly price war.

Teaching

During the search for a focal point bidders not only seemed to have "voluntarily" reduced their demand but in other occasions also tried to tell ("teach") competitors that (and what) they had to reduce. In what follows we describe two episodes in which bidders tried to teach their competitors who has to reduce demand. The first episode lead successfully to a demand reduction, whereas the second episode did not. We claim that the main difference between those episodes was that the teaching in the first episode was done by a fairly impartial observer, whereas in the second episode one of the interested parties tried to teach a competitor.

Successful teaching. In round 19 and 21 DT and TEF both reduced their demand by one block in the 1800 MHz band each such that the excess demand at this point amounted merely to one block in both the 900 MHz and the 1800 MHz band. One could have expected that VOD would also reduce its demand. This did not happen immediately, however. Instead, TEF increased demand to three blocks in the 900 MHz band and overbid VOD even though VOD were not holding the cheapest block. This can be seen as telling VOD to reduce demand to two licenses in the 900 MHz band. This was successful in round 30, when VOD did reduce demand in the 900 MHz to two licenses. At the same time VOD increased demand in the 1800 MHz band to five licenses. Overall, this move by VOD should probably be interpreted as a form of demand reduction as the marginal value of a third block in the 900 MHz band was probably much larger than the round 30 prices and the marginal value of a fifth block in the 1800 MHz was likely not to be in the same order of magnitude.[35] In the 900 MHz band, given that TEF had already shown that it could live with two units in the 900 MHz band, it was an impartial observer to the fight between VOD and DT about who would get the third block. Impartial in the sense that they probably did not care too much about who gets the third block. However, TEF was not a disinterested party, as it was clearly interested that the bidding competition would quickly stop: if the bidding would continue this would also

[35] This is due to the fact that a third block of 900 MHz spectrum allows a bidder to implement LTE in the 900 MHz band and keep 5 MHz for GSM traffic. A fifth block in the 1800 MHz band is most likely of low marginal value as LTE speed increases discontinuously with at least 10 MHz of additional adjacent spectrum blocks.

lead to higher prices for TEF. At the same time VOD and DT may have been unsure who of them would be the strongest and they both may have had an incentive to continue the fight for some time. On the other hand, they themselves would also not be interested in driving up prices and would prefer to reduce their demand to two blocks if they would have a clear belief that they would be the party that in the end would loose the bidding war. If it was considered that TEF may have had some relevant impartial information concerning the likelihood of one of them winning, then they were certainly in the position as impartial observer to reveal their information in a credible way. The temporary increase in demand to three blocks can be interpreted as TEF teaching VOD their information. Given TEF's behaviour, VOD and DT could update their beliefs about the marginal values of the opponent. VOD had a reason to follow up on this advice as, from the moment it becomes clear to both, it is likely that DT would at the end of a price war acquire the third license anyway. Thus, the incentives of TEF and VOD are aligned and both do not want VOD to continue bidding.

Unsuccessful teaching. While TEF teaching VOD to reduce demand in the 900 MHz band seems to have some effect, a similar episode in the 1800 MHz band was not effective. After DT reduced its demand to the package 2-3-3 with jump bids on three blocks in both the 900 and the 1800 MHz band in round 19, TEF overbid DT on the most expensive block in the 1800 MHz band in round 21 indicating that DT should reduce demand by more than one license. However, DT did not follow. There were a number of similar moments where others tried to convince DT to reduce its demand in the 1800 MHz band, but instead DT kept its demand of three licenses in the 1800 MHz band until the end of the auction. There was also a moment, namely in round 134, where DT tried to tell TEF to accept two blocks in the 1800 MHz band at lower prices than the other blocks, when making uneven jump bids on 8 blocks in the 1800 MHz band, while leaving two blocks for TEF with low prices. Five blocks on which it had made jumps were lower priced so that VOD could acquire these and pay similar prices to DT. If it had accepted, TEF could have claimed it acquired two blocks 1800 MHz spectrum at much lower prices than the others.

A possible reason for the fact that these teachings were unsuccessful is that the situation in the 1800 MHz band was different as compared to the situation in the 900 MHz band. There was no impartial observer as any of the three bidders could have reduced demand to stop the fight. Given the demands around round 84 of DT (3), TEF (3) and VOD (5) each of them would have had in total at least 4 blocks (considered to be the minimum necessary amount) even if they reduced demand by one block, and DT could have even reduced two blocks and still would have had four blocks in total. As each of them wanted the others to reduce, the "teaching others what to do" was not only a pure transmission of information, but was mixed with self-interest. This made the teaching not effective and others did not follow suit.

Providing Carrots

Instead of just trying to provide information through "teaching" bids, bidders were sometimes also engaged in providing (small) carrots to try to seduce competitors to give in on other counts. For example, we understand the following bids in that way.

- TEF opened with bidding on package 2-2-5 coupling an aggressive bid in the 1800 MHz band with immediately taking debased blocks in the 700 MHz and 900 MHz bands.
- VOD provided another example, where after an early fight in the 900 MHz band, it proposed to take the debased blocks in the 900 MHz and 1800 MHz band, while keeping its demand at three blocks in the 900 MHz band.
- In the competition phase, VOD withdrew its bid on two blocks in the 1500 MHz band and overbid DT in the 1800 MHz band indicating it can live with VOD getting not four, but two blocks in the 1500 MHz band (leaving 6 blocks to DT), if DT is willing to reduce one more block in the 1800 MHz band so that the latter would acquire 2-3-2-6.

The moves providing (small) carrots were, however, never successful in this auction, i.e., they were not considered acceptable by the competitors.

4.2 Competition Strategies

After round 30, it was more or less common knowledge that the auction would end with an outcome where VOD would obtain a package 2-2-x, DT would obtain a package 2-3-y, and TEF would acquire 2-2-z, with $x > max(y, z)$. All bidders might still have hoped to quickly resolve the remaining excess demand of two licenses in the 1800 MHz band. All bidders faced the choice between continuing their demand for the same number of units, or to reduce demand. Given that all bidders have strong beliefs that the end allocation is such that all would buy spectrum in all bands, each of them would have an incentive to strategically reduce demand early if they have strong beliefs that in the end, they would loose a bidding war. However, given the uncertainty concerning valuations, it is not clear to them who should give in, and more importantly, if they can pretend to be strong, then it may be that others would like to give in, even if under full information this would not be the efficient allocation.[36] In what follows, in order to understand the basic strategic situation in the competition phase, we introduce a simple model.

[36] Jump bids in the auction could be interpreted as a signal of (pretended) strength with the purpose of convincing opponents to reduce demand by another license. With jump bids, a bidder could show he is willing to accept a significant difference in the prices for the licenses (in case the auction ended) and indicates that he is willing to raise prices even higher. Jump bids were not cost-free in this auction, where every bidder wanted spectrum in every band. Some rounds with jump bids even raised the cost to a bidder by more than EUR 200 million.

A War of Attrition

To illustrate the main issue, we consider a simpler strategic situation with two bidders competing for three homogeneous licenses where both bidders know they acquire at least one license.[37] Bidders have a strictly higher value for two licenses than for one. We index the bidders by $i = 1, 2$. For simplicity, the value of m licenses is $m \cdot v_i$ for bidder i, i.e., there are no complementarities between the blocks. Also for simplicity, we focus on the valuations of one bidder being uncertain in order to highlight what uncertainty concerning bidders' valuations implies for the possibility of a bidder's bid strategy. The bidders have common beliefs about the ranking of the valuations, but they do not know the exact valuation. Bidder 1 plays a pivotal role and has two possible types, v_{1s} (strong) and v_{1w} (weak) with $v_{1s} > v_{1w}$, and an ex ante probability $0 < \theta = Pr\{V_1 = v_{1s}\} < 1$ to be of the strong type. To have an interesting strategic situation, the values of the different types are ordered as follows: $v_{1s} > v_2 > v_{1w}$. Bidders have the choice between either F (fight for two licenses) or A (to accommodate and reduce to one license only).

One way to interpret this toy model is with TEF in the role of the bidder whose valuations are unknown. Given the merger just before the auction and the uncertainty concerning whether TEF would be able to keep its large holdings in the 2.1 GHz band created quite some uncertainty concerning how much spectrum TEF really wanted to acquire or how deep its pockets were. We focus on the equilibrium incentives TEF has to fight or to accommodate and at how other bidders are responding (and interpreting the signals).[38] If both bidders fight, then they believe that they will bid up to their valuations and the strongest bidder wins two licenses and the other wins either one license or drops out of the auction altogether (which gives an equivalent pay-off of 0). If both accommodate, then we assume that the one that gives in first gets one unit and the other gets two at the current price p. If one accommodates and the other fights, then it is obviously the case that the fighting bidder wins two licenses at the current price p. In the real auction, bidders will play this game again and again until one bidder accommodates (and in the 2015 German auction after round 30 until two bidders have reduced one unit; whereas after round 85, only one bidder had still to reduce). The dynamic Bayesian game is illustrated in Figure 5 with N being nature, deciding whether bidder 1 is of a strong or a weak type and bidder 2 observing bidder 1 before making his own choice. In this sense, this is a standard signaling game.

We first show that there does not exist a pure strategy *separating equilibrium* in this model. If Bidder 1 plays a *separating strategy* with F_1 if, and only if, he is of a strong type, then the auction price would not rise in case bidder 1 is of a weak type leaving two licenses to bidder 2. Given this strategy, bidder 2 prefers to accommodate and bid for one license only in case he observes bidder 1 fighting as he interprets the fighting of bidder 1 as a signal of the latter being strong. But if bidder 2 accommodates even at low

[37] Adding a third player does not really add to our understanding of the main strategic issue, while adding a little to its complexity.

[38] Different bidders could potentially take different roles in this toy model during the course of the auction. However, it seems reasonable that due to the merger and the uncertainty about the 2.1 GHz spectrum TEF is the most natural fit for the bidder with the unknown valuations for the most time during the auction.

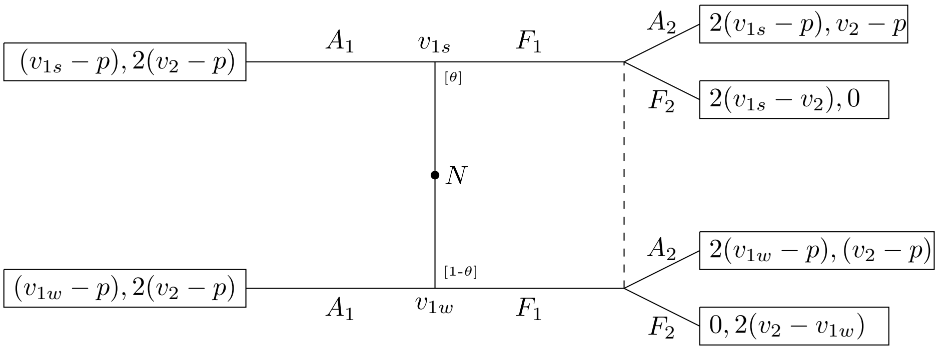


Figure 5: The signaling game illustrating the essence of the competition strategies

prices in case he observes bidder 1 choosing to Fight, then both types of bidder 1 prefer to Fight. Thus, there does not exist a *separating equilibrium*, because if only the strong bidder shows strength, bidder 2 will want to accommodate and bidder 1 does not want to choose the separating strategy.

Next, we consider the conditions under which a pooling equilibrium exists. In a pooling equilibrium bidder 1 chooses to always Fight no matter whether he is of a strong or a weak type. If the ex ante probability θ of bidder 1 being of the strong type (v_{1s}) is large enough, then bidder 2 would find it optimal to accommodate as the chances he will win a bidding war are considered too low and the observed behaviour of bidder 1 does not help him to discriminate between a truly strong bidder and a bluffing bidder. The formal conditions are as follows. If both types of bidder 1 choose to Fight, then bidder 2 will accommodate if, and only if,

$$\theta(v_2 - p) + (1 - \theta)(v_2 - 2v_2 + 2v_{1w} - p) \geq 0,$$

which reduces to

$$(1 - 2\theta)v_2 \leq 2(1 - \theta)v_{1w} - p.$$

This condition is satisfied if p is small and θ is large enough (a value larger than a half will certainly be sufficient, but even smaller values may do, depending on the valuations). If bidder 2 accommodates, then both types of bidders 1 will want to Fight. Thus, a pooling equilibrium exists if θ is relatively large or if the value of the weak bidder 1 is close to the value of bidder 2. Moreover, if p increases, this pooling equilibrium fails to exist.

Overall, the pooling equilibrium is not the best fit to explain the observed behaviour in the 2015 German auction. The pooling equilibrium would predict that a bidder like TEF (who is here taken as the role model of bidder one) pretends to be strong (it continues bidding) and the other bidders react by reducing demand. We have seen, however, that ultimately it was TEF that reduced demand. To explain this behaviour we will resort to a second type of equilibrium. In this equilibrium, bidder 2 randomizes with probability $q_2 = 0.5$ over Fight and Accommodate. This probability $q_2 = 0.5$ is chosen such that the weak bidder 1 is indifferent between Fight and Accommodate, while the strong bidder 1 wants to choose Fight. Bidder 2 is willing to randomize if he is indifferent between Fight and Accommodate given his belief that he will face the weak type of bidder 1. Suppose the weak bidder 1 chooses to Fight with probability q_1. After observing Fight, bidder 2 updates his belief about whether bidder 1 is strong or weak as follows:

$$\Pr(V_1 = v_{1w}|\text{Fight}) = \frac{(1-\theta)q_1}{(1-\theta)q_1+\theta}.$$

Given this updated belief, bidder 2 is willing to randomize if

$$\frac{2(1-\theta)q_1}{(1-\theta)q_1+\theta}(v_2 - v_{1w}) = v_2 - p.$$

In the region where a pooling equilibrium does not exist, this equilibrium does exist, with q_1 adjusting to make this indifference condition being satisfied. Moreover, if p increases, the probability that the weak type of bidder 1 Accommodates and the auctions ends increases and goes to 1.

This equilibrium nicely captures important aspects of the competition phase of the 2015 auction: TEF pretended to be the strong type for quite some time (in the static model: with some probability), and bidder 2 (DT and/or VOD) were unsure about whether TEF (bidder 1) was strong or weak. They reacted by choosing to Fight for some time as well. In the end, after a sufficient price increase, TEF gave in and turned out to be the weaker bidder, and the auction ended immediately.

Controlling the Relative Prices

One particular feature of a SMRA is that even when holding the overall spending constant, the prices for a particular package can vary substantially with the relative prices in the bands. For example, if the prices in the 1800 MHz band are relatively larger than in the 700 MHz band, packages with more 1800 MHz spectrum are relatively more expensive as compared to the situation where prices in the 700 MHz are relatively larger than in the 1800 MHz band. This has two important implications. First, as soon as it becomes clear what packages are realistic final outcomes for a particular bidder and how many blocks of each band those packages will contain, this bidder can be punished by concentrating price increases in the bands, in which he wants to buy relatively more spectrum than his competitors. Second, if a particular bidder has a fixed spending limit, then the same strategy by his competitors will lead to a demand reduction of this bidder at a lower overall price level. Thus, the observation that the demand in the 1800 MHz band was only further reduced after bidders started increasing their demand in one of the other bands to drive up the prices there, can be interpreted as a punishment strategy of the bidders. In particular after round 21 it was clear that, in relative terms, VOD will be most hurt by price increases in the 1800 MHz band, DT by price increases in the 900 MHz band and TEF by price increases in the

700 MHz band. For example, we interpret the following bids in this way.

- Prior to rounds 84 and 85 VOD and TEF increased their demand in the 900 MHz band to increase the prices in that band even though prices had been stabilized already for quite some periods. By overbidding DT with jump bids, certainly TEF was likely to indicate to DT that by getting three blocks in the 900 MHz band (and thereby DT being relatively more hurt by price increases in this band) it could not expect to also get three blocks in the 1800 MHz band. When the punishment did not work, TEF probably wanted to set a good example by reducing demand in the 1800 MHz itself by one block.
- In round 134 DT increased its demand in the 1800 MHz band from 3 blocks to 8 blocks. This could be interpreted as a punishment of VOD for not reducing its demand in the 1800 MHz band from 5 blocks to 4 blocks, even though, absent of externalities, the fifth block of 1800 MHz spectrum has only a small marginal value. Price increases in the 1800 MHz band hurt VOD more than DT as VOD is trying to acquire 5 blocks whereas DT is merely trying to acquire 3 blocks.
- When it became apparent that DT was not about to reduce demand in the 1800 MHz band and prices were unexpectedly much higher than the prices in other bands, VOD felt it should hurt TEF in the band that in relative terms was the most crucial one for TEF, the 700 MHz band. Ultimately, this punishment was effective and the eventual price driving in the 700 MHz band lead TEF to reduce one more time in the 1800 MHz band.

4.3 Final Prices

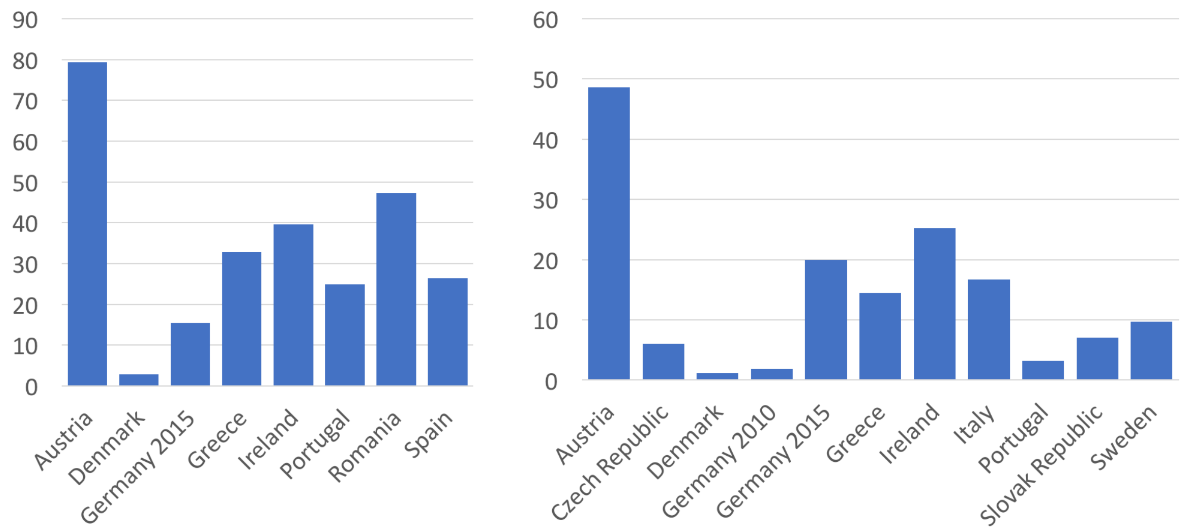
The combination of cooperation and competition led to the surprising result that at the end of the auction the prices in the lower frequency bands were lower than the prices in the higher frequency bands. Due to the better propagation properties, generally, lower frequencies are considered to be more valuable and in most auctions, lower frequency licenses also sell at higher prices than higher frequency licenses.

It should be clear from the above that we do not think that the relative prices of the different spectrum bands in the 2015 German auction reflect relative marginal values. The dynamics in the auction was such that an early solution to the allocation problem was found for both the 700 MHz and the 900 MHz frequency bands. In both bands, one of the bidders made a sacrifice in exchange for a larger demand in the 1800 MHz band: TEF offered from the opening round onward to accept the debased blocks in the 700 MHz band and 900 MHz band, but demanded initially five blocks in the 1800 MHz; VOD offered in round 30 to accept two blocks in the 900 MHz band if it could get five blocks in the 1800 MHz band. Both of these offers were part of an attempt to find an early solution.

When the allocation problem was reduced to the 1800 MHz band, there was no way for the bidders to compromise their 1800 MHz demand without starting price wars again in the other bands. In the 1800

Figure 6: Prices in auctions since 2010, normalized with respect to duration, delayed availability, PPP, CPI, and population to UK-equivalent prices in million £ per MHz. 900 MHz on the left, 1800 MHz on the right.

Source: Ofcom, "Annual licence fees for 900 MHz and 1800 MHz spectrum Provisional decision and further consultation", February 19th 2015



MHz band, it was simply a matter of outcompeting the others. This had nothing to do with the relative value of the different bands, but more about how offers were made to find solutions to the allocation problem. The subsequent price wars in the 700 and 900 MHz bands were punishments to stimulate other bidders to reduce their demands in the 1800 MHz band. As soon as the punishment was successful, the bidder went back to the package it claimed before starting the punishment. It is difficult to interpret this as truthful bidding. Accordingly, relative prices in the 2015 German auction are likely not reflecting relative valuations of the 900 and 1800 MHz bands. Ofcom, the British regulator, recognizes this by stating during a consultation on license fees for 900 and 1800 MHz spectrum: "We note that 900 MHz sold at a significantly lower price than 1800 MHz in the German 2015 auction and we do not observe this outcome in any other auction in our dataset."[39] The comparison of prices per MHz in different auctions since 2010, normalized with respect to duration, delayed availability, PPP, CPI, and population to UK-equivalent prices can be found in Figure 6.[40] The description of the bidding tactics, in particular the strategic jump bids and the agreement not to compete in 700 MHz, should make it obvious that the final prices of the auction are a poor signal of the underlying net present values of the spectrum.

The strategic use of bids as signals also led to different prices of generic blocks within a band. In particular, the use of significant jump bids to signal own demand, like DT's bids in the 1800 and 900

[39] See the "Update on Annual licence fees for 900 MHz and 1800 MHz spectrum: German 2015 auction" by Ofcom (July 9th 2015). Available at `https://www.ofcom.org.uk/__data/assets/pdf_file/0020/81317/2015-07_alf_update_germany.pdf`

[40] The data is taken from "Annual licence fees for 900 MHz and 1800 MHz spectrum Provisional decision and further consultation" by Ofcom (February 19th 2015). Available at `https://www.ofcom.org.uk/__data/assets/pdf_file/0022/83146/annual-licence-fees-900MHz-1800-further-consultation.pdf`. Data on 700 MHz and 1500 MHz auctions is not available as Germany was the first European country to auction those bands.

MHz band, leads to significant price differences within a band. At the end of the auction, the bidders agreed not only on the allocation of spectrum, but also on the different prices that bidders had to pay and the bidders that paid higher prices in the end apparently agreed that this was the price they had to pay for getting an agreement on the allocation.

5 Discussion and Conclusion

In this paper we have described and analyzed bidding behaviour in the 2015 German auction. The transparent auction format, combined with the fact that there were only three bidders, allows us to get a good understanding of the actual bidding behaviour. We have shown that bidder behaviour significantly departs from the straightforward bidding that is typically assumed in the auction literature and that bidders tried to teach each other what they thought a fair overall allocation of spectrum would be. As the evaluation of different auction designs should depend on how bidders actually bid in an auction, we will conclude with a few observations on auction design and different auction mechanisms.

It is important to understand that in many instances where multiple licenses are sold to bidders who compete in a market after the auction, the overall allocation and their relative position is what matters for bidders. In telecommunication markets the overall allocation of spectrum is decisive for the prices mobile operators can charge in the downstream (consumer retail) market. Telecoms with more spectrum relative to their competitors can provide more bandwidth and a better service for end customers. In such markets, the value of a package in the auction is relative to what the other bidders get and does not only depend on a bidder's own allocation. Allocative externalities can explain many instances of the bidding behaviour we have identified. In choosing an auction design a regulator should ideally take allocative externalities into account and allow bidders to express a bid for a package dependent on the full allocation. This is not easy, as we discuss below in more detail. A regulator can also provide bidders with information about the overall allocations in a round such that they can veto an unwanted allocation with their bids. The German auction design provides such information.

Without information on the full allocation and without the possibility to express bids dependent on the full allocation, bidders face a significant exposure risk of ending up with an allocation that is worth much less than what they expected. Note that this exposure risk is different from the classical exposure risk that is known in the auction literature. The classical exposure risk is that a bidder may get only a part of the licenses it is bidding on. In case of complementarities, it may well be that the value of a small number of licenses is lower than what the bidder has to pay. The exposure problem that we discuss here is that even though a bidder may know it will win a certain package, the value of that package heavily depends on how the remaining spectrum is allocated. If the bidder does not know this allocation, he runs the risk of paying more than what the package is worth to him.

Cramton and Ockenfels (2016) argue that the transparency in the German auction design could have led to implicit coordination and low-revenue equilibria in this auction, although "coordination was practically complicated by the fact that there was a multiplicity of focal points to implicitly coordinate on." We agree that implicit coordination is easier in the transparent German auction design. Note that even though there is evidence for implicit coordination in the early rounds of the 2015 auction, the bidders were not able to find a focal point of the auction.

To assess the German design for markets with significant allocative externalities, let us compare it to two wide-spread alternative auction designs for spectrum sales: (i) an SMRA with less transparency than the German auction, and (ii) a two-stage Combinatorial Clock Auction (CCA) that has been popular in recent years. Most SMRA implementations in other countries are less transparent than the German design. Typically, bidders learn the new ask prices in each round and on which licenses they are the highest standing bidder. In some markets with only a few bidders it might still be possible to infer relevant bids from competitors from the new ask prices.[41] Clearly, with less transparency, it is much more difficult to send clear signals of the form discussed in Section 4. Jump bids can be derived from the new ask prices, but they cannot always be attributed to a single bidder unless the specific position of a license in the band layout allows to infer this information. In a pure independent private values model without allocative externalities, such signals would be of little value in finding an efficient allocation. If there are allocative externalities, however, these signals may be important to arrive at an efficient allocation.

In a CCA, the allocation is determined in a final sealed-bid supplementary round (very much like a VCG mechanism), and bidders are uncertain what the final allocation will be when submitting the set of their final bids. Theoretically, one could allow bidders in a combinatorial auction to reveal their valuations for all possible allocations. Note, however, that the number of ways to partition a set of n objects into k subsets (packages for the k bidders) is already the Stirling partition number. The number of allocations needs to consider also the identity of bidders and it grows quickly in the number of licenses and bidders. In practical terms, an auctioneer cannot hope to fully elicit all this information in a single sealed-bid auction. Thus, it is difficult to adjust a CCA in a direction that helps avoiding inefficiencies due to the existence of strong allocative externalities in the bidders' valuation functions.[42] Note that the CCA is only one specific combinatorial auction format. Similar to the German SMRA design, a regulator could also use a transparent and ascending combinatorial auction to address the exposure problem arising from the lack of package bids, and the exposure problem arising from allocative externalities. Interestingly, early combinatorial auction designs put much emphasis on the coordination of bidders to a mutually desirable allocation out of the many possible allocations (Banks et al., 1989; Kelly and Steinberg, 2000).

[41] For example, in the 2013 Czech auction, bidders could infer information about the competitors' bids from the time stamps of the bids.

[42] There is some research on bid languages and alternative combinatorial auction formats considering allocative externalities, which might lead to alternative spectrum auction designs in the future; see, e.g., Krysta et al. (2010).

In summary, from the perspective of the European directive, transparent, ascending auctions such as the German SMRA design have advantages over other formats in that over the course of the auction, bidders learn what the likely allocation is going to be and (in the case of strong allocative externalities) are able to veto unwanted allocations with high bids. The transparency leads to a final assignment of spectrum user rights which is accepted by all auction participants given the available spectrum, the auction prices, the initial spectrum allocations and the expected competition in the aftermarket. In case a participant stops bidding, he accepts the result of the former auction round as a potential end result. If the auction ends, all bidders accept the result in this sense. In a fully transparent SMRA auction when making these decision all participants are aware of the prices they and the others have to pay and the resulting spectrum user rights of each participant. This is not the case in other auction formats like the Combinatorial Clock Auction. This implies that it is unlikely that all bidders would agree to allow a dominant position for one of the bidders or to agree on a very asymmetric distribution of the available spectrum. Thus, the transparent design implicitly ensures a healthy competition in the downstream market, which is what regulators should aim for. Of course, transparency always bears the risk of tacit collusion. However, tacit collusion would only be possible if all competitors implicitly agree on an allocation they all find "acceptable." If there is no agreeable (feasible) solution in the cooperation phase, bidders will start competing, driving up prices. A very asymmetric distribution of spectrum with a dominant player would only be possible, if this bidder actually has a much higher valuation for the spectrum compared to his competitors. Ultimately, whether transparency is a vice or a virtue depends on the importance of allocative externalities.

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