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DO AUCTIONS SELECT EFFICIENT FIRMS?*

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We consider a government auctioning off multiple licences to firms that compete in an aftermarket. Firms have different costs, and cost-efficiency is private information in the auction and in the aftermarket. If only one licence is auctioned, standard results say that the most efficient firm wins the auction as it has the highest valuation for the licence. We analyse conditions under which this result does and does not generalise to the case of auctioning multiple licences and aftermarket competition. Strategic interaction in the aftermarket is responsible for the fact that auctions may select inefficient firms.

In many liberalisation or privatisation processes, governments eventually face the issue how to select firms that will provide the formerly publicly provided service. One of the advantages of using auctions as a selection mechanism, so it is often thought, is that auctions select the most cost-efficient firms. Markets where active firms are more cost-efficient typically yield more efficient market outcomes than when these same markets are served by less cost-efficient firms. Other things being equal, cost-efficiency seems to be good for overall economic welfare.

In a monopoly context, the most cost-efficient firm will win the competition for the market (read: will win the auction). In this article, we refer to this result as the monopoly result. This result has permeated a large literature on procurement issues and, indeed, Laffont and Tirole (2002, pp. 307–8) state that if one ignores the processing, capture and dynamic costs of auctions, it is easy to see that auctions typically select the firm with the lowest cost. They attribute this argument to Demsetz (1968) who argued that competition for the market might be a good substitute for competition in the market.

Recently, many governments have relied on a combination of 'competition for' and 'competition in' the market, indicating that it is not just auction revenue that the government is interested in.[1] Competition in the market guarantees that the inefficiency of the market allocation is as small as possible. This is achieved by auction designs that stipulate that each winning firm can obtain only one licence.[2] Competition for the market guarantees that the government receives a financial return for the licences that is in line with their value. An important case in point is the wave of 3G mobile telephony spectrum auctions that have been organised around the world.[3] In all

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[1] As monopoly profits are higher than total industry profits under oligopoly, governments may consider selling just one licence if auction revenue were the prime objective. When costs are sufficiently low and nearly symmetric across firms, and asymmetries between players do not play a significant role (as they do in Hoppe *et al.*, 2006, for example), the efficiency gains due to scale economies under monopoly are outweighed by the extra consumer surplus under oligopoly, making selling multiple licences the optimal thing to do.

[2] In the upcoming 2.6 GHz auction in the UK and other countries, the government usually imposes a maximum spectrum band one firm can obtain, again guaranteeing that multiple firms win spectrum.

[3] See Klemperer (2002*a,b*), Binmore and Klemperer (2002) and Jehiel and Moldovanu (2003) for overviews of 3G mobile telephony spectrum auctions that have been held around the world.

of the 3G auctions, multiple licences were sold. As typically there were more firms participating in the auction than available licences, firms had to compete to obtain a licence. Many governments formally or informally stated that efficient assignment of frequency spectrum was one of the goals to be achieved. With cost asymmetries between firms, efficient assignment implies that the most cost-efficient firms should win licences, and indeed the Dutch government, among others, mentioned selecting the most efficient firms as one of the reasons for holding an auction.

In this article, we study the conditions under which the monopoly result does and does not carry over to the case of multiple licences where firms compete in an oligopolistic aftermarket. We will show that due to strategic interaction among firms in the aftermarket, it is not always true that auctions select the most cost-efficient firms.

The monopoly result rests on the fact that a more efficient monopolist can make more profit in the aftermarket than any other (less efficient) firm can make. This effect is also present in an oligopoly context if we fix the aftermarket strategies of other winning firms. We call this the direct effect: a more cost-efficient firm will *ceteris paribus* make more profit in the aftermarket and will therefore be willing to bid more in the auction than a less efficient firm.

In addition to the direct effect, a strategic effect arises in oligopoly markets, which works in the opposite direction. The strategic effect originates from two sources. First, a firm's profit in an oligopoly market depends on the cost-efficiency of all other competitors. Efficient firms impose a *negative externality* on other firms as *ceteris paribus* such firms charge lower prices or produce more output than less efficient firms. Second, even if firms' types are *ex ante* statistically independent they are positively correlated *ex post*, conditional on winning a licence, implying that a firm that is more efficient itself expects to compete with other firms that are more efficient. This *ex post* correlation stems from the fact that conditional on winning, a firm knows that it has not been the case that more firms have submitted higher bids than its own bid than the number of licences.

Depending on the market conditions, determining the strength of the aftermarket externality, and on the *ex ante* distribution of firms' efficiency parameters, determining the strength of the *ex post* correlation, the strategic effect can be stronger than the direct effect. We show that this is more likely to happen when the impact of the efficiency parameter on marginal cost is large relative to its impact on average costs. In the simplest case when cost is piecewise linear (as in the Example in Section 2) the impact of the efficiency parameter on marginal cost is constant and, therefore, the strength of the strategic effect is constant, whereas the direct effect becomes weaker when the impact of the efficiency parameter on average cost becomes smaller.

One possibility where marginal cost is more responsive than average cost to the efficiency parameter is when the efficient parameter scales down the cost function in a multiplicative way and the production technology exhibits decreasing returns to scale so that marginal cost is large relative to average cost. Appendix A shows that in markets with congestion effects, such as telecommunications and transportation markets, production cost may be increasingly rising in the number of consumers because of the increasing congestion probability. Alternatively, when a technological improvement leads to a marginal cost reduction and an increase in fixed cost, the efficiency effect on total production cost can be very modest relative to the effect on marginal cost. In such a case, an efficient selection equilibrium may fail to exists independently of the return to scale.

More technically, we consider a standard multi-unit uniform-price auction where firms have private information about their costs and overall economic efficiency requires the most efficient firms to win the auction. A strategy for the firms is a function specifying how a firm's bid depends on its efficiency parameter. The generalisation of the monopoly result to the case where multiple licences are auctioned requires that the more efficient a firm is, the higher it bids in the auction, i.e. a symmetric bidding equilibrium should exist where a firm's strategy is monotonically increasing in the efficiency parameter. We call such an equilibrium an 'efficient selection equilibrium'. Such an equilibrium exists only if firms' expected aftermarket profit conditional on winning the auction increases in their efficiency parameter.

If an efficient selection equilibrium does *not* exist, then it must be the case that

(*i*) asymmetric equilibria exist in which different firms have different bidding functions, and/or
(*ii*) there are equilibria with non-monotone bidding functions, and/or
(*iii*) there are equilibria where firms use random bidding strategies, and/or
(*iv*) a decreasing equilibrium exists.

In all of these cases, there is at least a positive probability that less efficient firms will bid more than more efficient firms and, therefore, obtain the licences. Using standard arguments (Krishna, 2002) we argue that there always are many asymmetric equilibria where the probability of inefficient outcomes is high.

A first, more easily identifiable condition under which an efficient selection equilibrium fails to exist is that firms' efficiency parameters are positively correlated so that learning one's own efficiency parameter provides information about other firms' private information before the auction takes place. In practice, positive correlation of firms' efficiency parameters may arise in sectors where firms use similar production technologies, and prices of inputs fluctuate with (macroeconomic) shocks that are common to all firms. Alternatively, firms may implement cost-saving technologies that arise from an exogenous stochastic process. In both cases, if a firm is more cost-efficient itself, it infers that all other firms are more likely to be cost-efficient as well. Therefore, firms that are more cost-efficient expect to be competing with other more cost-efficient firms and the latter are known to be fierce competitors. We show that for *any* oligopolistic market, no matter how weak the negative externality is, there are distributions of firms' types for which an efficient selection equilibrium does not exist.

A second, more surprising condition under which an efficient selection equilibrium fails to exist is where firms' efficiency parameters are *ex ante* independent. Despite this *ex ante* independence, the types of a winning firm's competitors turn out to be positively correlated with the type of that winning firm. The reason is that in an efficient selection equilibrium, a winning firm rules out the states of the world where there are more firms that are more cost-efficient (than this firm) than the number of available licences as in all these states, the firm would not have won the auction in the first place. As firms compute their bids by considering only states in which they win the licence, they consider this *ex post* correlation when determining their bidding strategy before the auction. In Section 2, we provide an example, which clearly demonstrates the negative externality, the *ex post* correlation and the resulting strategic effect.

Our last result, which we show by means of an example, is that when firms' types are positively correlated *and* the negative externality is sufficiently strong, a unique monotone symmetric bidding equilibrium can be *decreasing* in firms' efficiency parameter. In this case, firms that are most profitable in the aftermarket and that submit the highest bids for licences are the least cost-efficient firms. Thus, despite the auction allocation being efficient, the aftermarket allocation is as inefficient as it can get, implying lower social welfare than from *any other* selection mechanism.

The article is organised as follows. The extensive, relatively recent, related literature is reviewed in Section 1. Section 2 describes a simple example that intends to illustrate the main mechanisms at work. Section 3 discusses the general two-stage model with an auction stage and a market competition stage. Section 4 contains the main results showing the necessary conditions for an efficient selection equilibrium to exist. Section 5 illustrates what these general conditions imply in the case of commonly used aftermarket models when firms' types are independent, whereas Section 6 focuses on the correlated types' case. Section 7 presents an example where the unique monotone symmetric bidding equilibrium is decreasing. Section 8 concludes and discusses some remaining issues. Appendix A shows what cost functions may look like in markets with a (small) probability of congestion. Appendix B contains analytical derivations for Section 2, and Appendix C contains all proofs.

1. Literature Review

There is a relatively large, recent literature on the possibility of inefficient allocation of licences in auctions due to the presence of externalities. First, there is literature where one licence is auctioned and the winner of the auction competes in the aftermarket with non-winners.[4] Moldovanu and Sela (2003) analyse aftermarket Bertrand competition where cost is private information at the auction stage.[5] When in such a situation a patent for a cost-reducing technology is auctioned amongst the competitors, they show that standard auction formats do not exhibit efficient equilibria where bids are increasing in the firms' efficiency parameter. Goeree (2003) and Das Varma (2003) analyse a similar setting but allow for signalling private information through the auction bid. Das Varma (2003) shows that when aftermarket competition is in strategic substitutes an efficient equilibrium does exist as firms have an incentive to overstate their efficiency through higher bids. When aftermarket competition is in strategic complements this is not the case, however, as firms have an incentive to understate their efficiency and will adjust their bid downwards. Therefore, with strategic complements an efficient equilibrium may fail to exist. Goeree (2003) mainly focuses on the comparison of firms' bid with and without signalling under different auction

[4] Jehiel and Moldovanu (2006) provide an overview of existing work in this area and argue that in case firms' aftermarket profits depend on private information in the hands of other winning firms there is an informational externality. See, also, Jehiel *et al.* (1996) and Jehiel and Moldovanu (2000), for related papers where an (informational) externality may lead to inefficiency even in standard single-unit auctions.

[5] To avoid signalling issues, they consider the case where the true production costs of the bidding firms are revealed after the auction. This creates the negative externality that is required for the results.

formats, and on the seller's revenue in case a separating equilibrium exists. Also in his model, there is a possibility for efficient equilibria not to exist, namely if players have an incentive to understate their valuation. Katzman and Rhodes-Kropf (2008) show how different bid-announcement policies affect auction revenue and efficiency.

The above papers show that *auction* inefficiencies may already occur in single-object auctions with interdependent valuations. There are some obvious differences between our article and the papers mentioned above as we consider auctions where multiple licences are offered for sale and as we abstract from signalling issues. The reason why, in Moldovanu and Sela (2003), what we call 'an efficient selection equilibrium' does not exist is that on the margin, due to the strong negative externality, a firm that is more efficient has a lower willingness to pay for a licence than a firm that is less efficient. We use a similar reasoning to show that when types are *ex ante* independently distributed, an efficient selection equilibrium may fail to exist. Also in our context, it is necessary that the strategic effect is sufficiently strong for this result. The results on signalling in Goeree (2003) and Das Varma (2003) suggest that an efficient selection equilibrium may also fail to exist in our framework if the bids are made public after the auction and bidders compete in prices.

Compared to these papers, we offer two other reasons why *market* inefficiencies may arise. First, an efficient selection equilibrium may fail to exist if types are *ex ante* correlated. This failure may even occur when the strategic interaction between firms in the aftermarket is weak. *Ex ante* correlation of types by itself guarantees that the strategic effect is strong. In this case, the *market* inefficiency is due to the inefficiency of the auction. Second, and more importantly, when both *ex ante* correlation of types and the market externality are strong enough, expected profit of the winning firms decreases in their efficiency parameter and the only monotone symmetric equilibrium of the auction is an equilibrium where bids are decreasing in firms' efficiency parameters. This equilibrium guarantees an efficient allocation of licences in the auctions as the firms that value the licences most also win the auction. However, the efficient *auction* allocation leads to an inefficient *market* allocation as the most inefficient firms win the licences. Therefore, our article points to the fact that inefficient market allocations may arise not only due to inefficiencies created in the process of allocating licences.

There is also literature on inefficiencies created by multiple licence auctions if players have interdependent valuations,[6] but these papers usually address issues related to collusion and/or asymmetries between different participants. Hoppe *et al.* (2006), for example, consider an auction with asymmetric players (incumbents and entrants) and focus on the competitiveness induced by the number of licences that is auctioned. Contrary to common wisdom, they show that auctioning more licences does not necessarily result in a more competitive outcome due to the asymmetries between players. Grim *et al.* (2003) study conditions under which firms may collude on a low-price equilibrium. Our article, in contrast, focuses on inefficiencies that may arise when selling multiple licences when players are *ex ante* identical and collusion is not an issue.

[6] Krishna (2002, Example 6.4), shows that it might happen that the bidder with the highest type is the bidder with the lowest valuation and that standard auction formats do not have efficient equilibria.

Finally, there is literature on how allocation of licences affects aftermarket equilibrium prices and welfare; see, for example, Janssen (2006) and Janssen and Karamychev (2007, 2009). The last two papers also rely on a selection aspect of auctions.

2. Aftermarket Externality and *Ex post* Correlation: an Example

The example we present intends to make clear how the different effects we identified in the Introduction work independently and interact with each other to create the overall mechanism our article identifies. To this end, we first consider a situation where a monopoly licence is auctioned. Then, we extend the analysis to a multiple licence case by comparing a random allocation of licences with an allocation by means of an auction.

We always consider an allocation mechanism where three firms compete for one or two licences and an aftermarket where active firms set prices. In all examples, firms differ in their efficiency, indicated by e_i for firm i. Efficiency has an effect on both total cost and marginal cost in accordance with the following cost function:

$$C(q_i) = \begin{cases} (1-e_i)c(q_i-\bar{q}), & \text{if } q_i > \bar{q} \\ 0, & \text{if } q_i \leq \bar{q} \end{cases}$$

where q_i is the production level of firm i, and $c \in (0,1)$ is a parameter. When firms' output is less than a threshold $\bar{q} \in [0, 1/2]$ (which by an appropriate choice of $\bar{q}$ will never happen in equilibrium), firms produce at zero cost. Above this threshold, firms have constant marginal cost of $(1 - e_i)c$. Figure 1 shows the cost function $C(q)$. The larger the value of e_i, the more cost-efficient the firm. Efficiency is private information to firm i at the auction and the aftermarket stage.

Consider first the situation where one licence is auctioned and aftermarket demand is given by $q = 1 - p$. Applying intermediate microeconomic techniques leads to the conclusion that if firm i gets the licence and sets a profit-maximising price, its aftermarket profit equals:

$$\frac{1}{4}[1-(1-e_i)c]^2 + \bar{q}(1-e_i)c \tag{1}$$

for $\bar{q} \leq (1 - c)/2$. (For higher values of $\bar{q}$, the least-efficient firms choose prices such that cost equals 0 and there is no effect of a firm's efficiency level.) Note that profits depend positively on the efficiency parameter, implying that a firm that is more efficient can make more profit in the aftermarket. Consequently, the most efficient firm has the highest valuation for the licence, bids the largest amount and wins any standard auction.

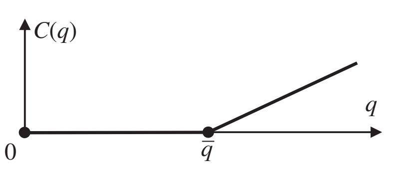

Fig. 1. *Cost Function C(q)*

The direct effect mentioned in the Introduction measures the effect of a firm's efficiency parameter on its aftermarket profit. This effect is the only effect that is relevant in the monopoly case. It is clear from (1) that the threshold level $\bar{q}$ reduces the magnitude of the direct effect.

When more than one licence is allocated, an externality arises. To see this, we augment the example considered above by having two firms compete in prices in an aftermarket with the following demands:

$$q_1(p_1, p_2) = 1 - p_1 + p_2 \text{ and } q_2(p_2, p_1) = 1 - p_2 + p_1.$$

Each firm has a cost function as specified above and knows only the value of its own efficiency parameter but not of its rival.

If the two licences were randomly allocated to two firms, and firms' types are identically and independently distributed, firms 1's expected aftermarket profit equals

$$\left\{1 - \frac{1}{2}[\mathrm{E}(e) - e_1]c\right\}^2 + (1 - e_1)c\bar{q}, \text{ for } \bar{q} < 1 - \frac{c}{2}, \tag{2}$$

where firm 1 computes the expected efficiency level of its aftermarket competitor in assessing the price that firm 2 will charge. As in the monopoly example, the direct effect of a firm itself being more efficient is positive. At the same time, the expected profit of firm 1 decreases in the expected value $\mathrm{E}(e)$, i.e. the externality is negative. When licences are randomly allocated, however, this aftermarket externality plays no role: every firm expects to compete with an average firm and there is no relation between a firm's own efficiency and the efficiency of its competitor.

If, to the contrary, licences are auctioned, the externality does play an important role in determining firms' bidding behaviour. When it has won a licence, firm 1 infers that at least one of the two other firms must necessarily have bid lower than firm 1. If this were not the case, firm 1 would not have won the licence in the first place. In particular, in an efficient selection equilibrium where a firm's bid increases in its own efficiency parameter, firm 1 expects that at least one of the two other firms is less efficient than firm 1 itself.

This implies that, even though firms' types are *ex ante* independent, the types of the winning firms are correlated conditional on winning the auction. We refer to this correlation as *ex post* correlation. As this correlation can already be anticipated at the auction stage, it is taken into account by firms when determining their optimal bid.

Suppose that the efficiency parameters of all firms are *ex ante* independently and uniformly distributed over the unit interval [0,1]. Let e_1 take a value of x. Then, if firm 1 wins a licence, the expected type of its competitor is the largest type amongst types e_2 and e_3, i.e. $\max(e_2, e_3)$, provided at least e_2 or e_3 is smaller than x. In order to see the *ex post* correlation is positive, we compute the expected type of a rival winning firm conditional on the type of firm 1 and that firm 1 has won a licence. This expected value is

$$\mathrm{E}[\max(e_2, e_3) \mid x, \min(e_2, e_3) < x] = \frac{3 - x^2}{3(2 - x)} = \frac{1}{3}\left[x + 2 - \frac{1}{(2 - x)}\right],$$

and it is easy to see that this expression is increasing for all relevant values of x. This implies that, due to the *ex post* correlation, in an efficient selection equilibrium of *any* standard auction, a more efficient firm expects to be competing with a more efficient rival. The positive *ex post* correlation together with the negative aftermarket externality makes up the negative strategic effect: a more efficient firm expects to compete with other efficient firms and this has a reducing effect on profits.

The *ex post* correlation takes on a specific form in the uniform-price auction we consider here. In this auction format, the winners pay an auction fee w that is equal to the highest non-winning bid, and the optimal bid of a firm is determined as if it has to compete for one remaining licence with a firm that is of an identical type. Thus, firm 1 bids its entire expected aftermarket profit conditional on the losing firm having an efficiency level equal e_1 and the other winning firm having an efficiency level that is larger. Thus, the expected efficiency level of firm 1's competitor is $(1 + e_1)/2$ and in the case of a uniform-price auction, (2) turns into the optimal bid for firm 1 being:

$$b(e_1) = \left[1 - \frac{1}{2}\left(\frac{1+e_1}{2} - e_1\right)c\right]^2 + (1 - e_1)c\bar{q}, \text{ for } \bar{q} < 1 - \frac{c}{2}. \tag{3}$$

Equation (3) captures both the direct and strategic effect. The overall effect of a firm's efficiency level on his bid is thus given by the derivative of the bidding function:

$$\frac{\mathrm{d}b(e_1)}{\mathrm{d}e_1} = \left[\frac{1}{2} - \frac{c}{8}(1 - e_1) - \bar{q}\right]c.$$

For an efficient selection equilibrium to exist it must be the case that the derivative is positive for all values of e_1. It is clear that this is the case if $\bar{q} \leq 1/2 - c/8$. If $\bar{q}$ is larger, the derivative is negative for small values of e_1 and an efficient selection equilibrium fails to exist. Hence, when the strategic effect is strong relative to the direct effect, an efficient selection equilibrium does not exist and the aftermarket is inefficient with positive probability.

The example we have presented so far can be extended in different directions. For instance, decreasing return to scale is not required. In fact, the shape of the cost function $C(q)$ for $q \leq \bar{q}$ plays no strategic role in the example. Moreover, if in addition to a reduction in *marginal* cost, the efficiency parameter has a positive effect on firms' *fixed* cost, it is even more unlikely that an efficient selection equilibrium exists as the direct effect becomes weaker due to the increasing fixed cost.

The example also works in the full information scenario where firms' types become public after the auction. In such a case, the optimal bid of firm 1 is given by

$$b(e_1) = 1 - \frac{c}{3}(1 - 3\bar{q})(1 - e_1) + \frac{c^2}{27}(1 - e_1)^2, \text{ for } \bar{q} < 1 - \frac{c}{3},$$

which, like (3), captures both the direct and the strategic effect. If $\bar{q} > 1/3 - 2c/27$ this bidding function is a decreasing function at $e_i = 0$:

$$\frac{\mathrm{d}b(e_1)}{\mathrm{d}e_1} = \left[\frac{1}{3} - \frac{2c}{27}(1 - e_1) - \bar{q}\right]c,$$

and for these values of $\bar{q}$ an efficient selection equilibrium thus fails to exist. An extension to a Cournot competition is more complicated as efficient selection equilibria only fail to exist in the case of non-linear demand and/or cost functions. Section 5 contains an example with linear demand and convex cost functions.

One may wonder what an equilibrium would look like if an efficient selection equilibrium does not exist. Here, we point at the fact that asymmetric equilibria always exist, even in the monopoly case. In this example, one of the asymmetric equilibria looks as follows: firms 1 and 2 bid in accordance with the bidding function $b_1 = b_2 = (1+c/4)^2$ and firm 3 bids $b_3 = 0$. Note that these bids are independent of the efficiency level of the firms. The bids for the first two firms represent the expected profit of a firm with type $e = 1$ competing with a randomly chosen firm. These bidding functions form a Bayes-Nash equilibrium where firms 1 and 2 get licences without paying (as firm 3 bids nothing). No firm wants to deviate. This is obvious for firms 1 and 2. Firm 3, on the other hand, can only win by bidding $(1 + c/4)^2$ or more. In this case, the auction fee is higher than the highest possible expected aftermarket profit it can ever achieve, so that this deviation is not profitable either. The outcome of this equilibrium is inefficient with probability 2/3, which is the *ex ante* probability that firm 1 or firm 2 is the least efficient firm.

One may wonder whether it is possible to characterise the equilibrium set in this simple example fully. It is well known that under interdependent values a second-price auction and its multi-unit generalisation usually have many asymmetric equilibria; see, for example, Krishna (2002, Example 8.4). These equilibria only exist, however, in the presence of *positive* externalities and they do not exist if the externality is negative, as in our case. Therefore, it is not an easy task to provide a full characterisation.

3. The General Model

In the general model, a government allocates $n \geq 2$ licences in a multi-unit auction to the highest bidding firms and we assume that $N \geq (n + 1)$ firms participate in the auction. In the oligopolistic aftermarket, firms compete by simultaneously choosing a value of a strategic variable s. Depending on the market, we interpret s as a price or a quantity, or any other relevant strategic variable. The profit π^i of firm i is determined by the level of s that firm i and the other $(n - 1)$ firms choose and by the firm's efficiency parameter e_i. The function π^i is symmetric in all s_j, $j \neq i$, and we write:

$$\pi^i = \pi(s_i, s_{-i}, e_i),$$

where s_{-i} denotes levels of strategic variables chosen by all other firms j. The function $\pi(s_i,s_{-i},e_i)$ is twice differentiable, and, to shorten notation, we denote the partial derivatives of the function $\pi(s_i,s_{-i},e_i)$ and all other functions by subscripts as follows:

$$\pi_i \equiv \partial\pi/\partial s_i, \pi_j \equiv \partial\pi/\partial s_j \text{ for } j \neq i, \pi_e \equiv \partial\pi/\partial e_i, \pi_{i,j} \equiv \partial^2\pi/\partial s_i \partial s_j, \text{ etc.}$$

The efficiency parameter e_i positively influences the profit of firm i by reducing its total as well as its marginal costs. In order to ensure the existence, uniqueness and stability of the Bayes-Nash equilibrium in the aftermarket, we assume that the function $\pi(s_i, s_{-i}, e_i)$ satisfies a stability requirement.[7]

We analyse the case where the government organises a multi-unit uniform-price auction to allocate n licences, where all the winning firms pay the same licence fee w, which is equal to the highest non-winning bid. This uniform-price auction allows us to simplify the exposition of results while keeping the formulation of the aftermarket competition stage quite general.[8] In the main body of the article we assume that resale of licences is not allowed. In the final Section, we discuss how allowing for different auction formats and resale of licences after the auction will affect our results.

A firm's efficiency parameter e_i is private information. The *prior* distribution of types F is symmetric so that all e_i are identically distributed over the unit interval [0,1]. A firm i submits a bid b_i dependent on e_i. We denote a monotone symmetric equilibrium bidding function by $b(e)$, and firm i bids $b(e_i)$.

Depending on the information revealed immediately after the auction, three different scenarios can be considered: a *private information* scenario (where neither firms' types nor the winning bids become public), an *imperfect information* scenario (where only the bids of the winning firms but not their types become public) and a *full information* scenario (where types of all winning firms become public). In what follows, we focus on the private information scenario. One reason for doing so is to bring out most sharply the result that inefficiencies are due to strategic interaction and not to signalling. Signalling plays an important role if the imperfect information scenario is analysed and the auction stage then is an N-player signalling game with each firm being a sender and a potential receiver of signals.[9] The full information scenario, on the other hand, does not seem to be realistic. Moreover, this scenario can be analysed in a way similar to the private information scenario we analyse here.

4. When an Increasing Bidding Equilibrium Exists

We are now ready to discuss the oligopoly case and analyse necessary and sufficient conditions for an efficient selection equilibrium to exist. We first derive these conditions for the general model. Then we analyse two sets of circumstances (firms' types being independently distributed, and the case of correlated types) where the necessary conditions cannot be satisfied.

Let $b(e)$ be a strictly increasing symmetric equilibrium bidding function. We denote by x the type of firm i and by z the type of firm that submits the nth highest bid among $(N-1)$ firms other than firm i so that the auction fee w is given by $w = b(z)$. We use subscripts k and l to refer to winning and losing firms, respectively. For any realisations of x and z, the aftermarket stage has a unique symmetric Bayes-Nash equilibrium in

[7] In particular, we assume in the case of strategic complements ($\pi_{i,j} > 0$) that $(\pi_{i,i} + \sum_{k \neq i} \pi_{i,k}) < 0$ and in the case of strategic substitutes ($\pi_{i,j} < 0$) that $(\pi_{i,i} - \pi_{i,j}) < 0$; cf. Bulow *et al.* (1985).

[8] Other auctions' formats and other informational scenarios (see further) in our model can only be effectively analysed when the efficiency parameter has a vanishingly small variation.

[9] The results of Das Varma (2003) suggest that the signalling effect helps the efficient equilibrium to survive in the case of strategic substitutes but destroys its existence in the case of strategic complements.

which a winning firm i follows a strategy $s_i^* = s(x, z)$. Using this notation, we write the expected profit of type x conditional on winning a licence as

$$v(x, z) \equiv \mathrm{E}[\pi(s_i^*, s_{-i}^*, x) \mid x, e_l \leq z < e_k]. \tag{4}$$

The function $v(x, z)$ is a firm's valuation function, which is used in the auction stage to determine the optimal bidding strategy. It is important to note that the function $v(x,z)$ is implicitly based on the hypothesis that $b(e)$ is an increasing function so that an efficient selection equilibrium exists but is not based on the specific form of $b(e)$.[10] The function $v(x, z)$ is the maximum willingness to pay of a firm with type x when type z determines the auction fee w, all winning firms have an efficiency level higher than z, while all losing firms have an efficiency level lower than z.

Following the methodology of the standard auction model with interdependent valuations and correlated signals (Milgrom and Weber, 1982), the existence and uniqueness of an efficient selection equilibrium is shown in Proposition 1.

Proposition 1. *If $v_x(x, z) > 0$ and $v_x(x, x) + v_z(x, x) > 0$ for all $x,z \in [0, 1]$, then an efficient selection equilibrium exists and is unique and given by $b(x) = v(x, x)$. If such an equilibrium exists, then it must be that $v_x(x, x) \geq 0$ and $v_x(x, x) + v_z(x, x) \geq 0$.*

The shape of the bidding function follows from the fact that each bidder bids as if it is competing for one remaining licence with a firm who has the same efficiency level as it has itself. The necessary and sufficient conditions follow from the fact that the bidding function so defined should be an increasing function. It is also worth noting that in a uniform-price auction, the distribution of types of winning firms *conditional* on z has the same correlation structure as the *ex ante* distribution but it is truncated at z. In the auction stage, however, when the value of z is not yet realised, the distribution of types of winning firms *conditional* on the type of a given firm exhibits a higher degree of correlation than the *ex ante* correlation.

If an efficient selection equilibrium does not exist, then other types of equilibria exist. It is standard (Krishna, 2002, Example 8.4) that symmetric auction formats have asymmetric equilibria and our setting does not differ in this respect. There are always asymmetric equilibria where n firms bid very high and the remaining firms bid 0 (see also the example provided in Section 2). As any given asymmetric equilibrium basically picks n out of the N firms, the chance that the efficient selection is made is very small. Other equilibria (in mixed strategies and in non-monotone strategies) may exist but are very difficult to characterise as the valuation function as defined above depends on the hypothesis that the bidding function is monotone.

5. Statistically Independent Types

Generally, the value of $v(x, x)$ and its partial derivatives that are used in the necessary and sufficient conditions of Proposition 1 depend on the distribution of types over the interval $[x, 1]$ and on firms' aftermarket equilibrium strategies for any realisation of types from that interval, so that a general further analysis of the necessary and sufficient

[10] The fact that $b(e)$ is an increasing function is used to condition the expected profit on the efficiency types of all losing firms to be smaller than z and all winning firms to be more efficient than z.

conditions is intractable.[11] However, analytical results can be obtained for the special case with $x = 1$, in which all winning firms have the same type $e_j = 1$, have equal bids, and choose the same value of their aftermarket strategy.

In Proposition 2, we analyse the necessary condition mentioned in Proposition 1 for the case of statistically independent types evaluated in this special case where $x = 1$, and derive a sufficient condition under which this necessary condition fails to hold. In this case, an efficient selection equilibrium does not exist.

PROPOSITION 2. *If the condition*

$$\pi_e - \frac{1}{2}\frac{(n-1)\pi_j\pi_{i,e}}{[\pi_{i,i} + (n-1)\pi_{i,j}]} < 0 \tag{5}$$

holds at all $e_i = 1$, then an efficient selection equilibrium does not exist.

Condition (5) can be interpreted as follows.[12] In the absence of the coefficient $\frac{1}{2}$, the LHS of (5) represents the full derivative of the equilibrium profit function $\pi(s_i^*, s_{-i}^*, x)$ at $x = 1$ when firms' types are perfectly correlated, i.e., $e_i = x$ for all i. In particular, if condition (5) holds, then an increase in all firms' efficiency levels leads to a decrease in their profits. The coefficient $\frac{1}{2}$ arises from the *ex post* correlation of types. For a general *ex ante* distribution of types, the *ex post* correlation can be approximated in the special case we consider here by a uniform distribution for which the *ex post* correlation equals $\frac{1}{2}$ (see also Section 2). To understand better when (5) is satisfied, let the efficiency parameter only affect firms' costs but not their revenues, so that

$$\pi(s_i, s_{-i}, e_i) = R(s_i, s_{-i}) - C(e_i, q_i),$$

where $R(s_i,s_{-i})$ is the firm i's revenue and $C_{e,q} < 0$. In this case, the condition of Proposition 2 can be interpreted as a restriction on the cost function:

$$\frac{AC_e}{MC_e} \equiv \frac{C_e/q}{C_{e,q}} < \frac{1}{2}\frac{(n-1)\pi_j}{[\pi_{i,i} + (n-1)\pi_{i,j}]q}\frac{\partial q}{\partial s_i}. \tag{6}$$

In other words, the larger the impact of the efficiency parameter on marginal cost relative to its impact on average cost, the less likely it is that an efficient selection equilibrium exists. In particular, when cost is proportional to (a function of) the efficiency parameter, the ratio AC_e/MC_e equals AC/MC and is fully determined by the convexity of the cost function: $AC/MC > 1$ for concave cost functions whereas $AC/MC < 1$ for convex cost functions. In the example of Section 2, it is easily verified that the LHS of (6) is simply equal to $(1 - \bar{q}/q_i)$, whereas the RHS is not dependent on $\bar{q}$. Thus, if an efficient equilibrium does not exist in that example for some value of $\bar{q}$, then it certainly does not exist for all larger values of $\bar{q}$.

[11] The specification of cost and demand functions in the example provided in Section 2 is the only case where we have been able to get closed-form solutions for the bidding function.

[12] Condition (5) can be relaxed for specific forms of the *ex ante* distribution, especially when the distribution density function equals to zero at the upper bound $e_i = 1$.

In Section 2, we have illustrated that an efficient selection equilibrium may fail to exist in settings with price competition. Here, we illustrate Proposition 2 by considering an example with quantity competition. Let firms' cost function be $C(e,q) = q^{\alpha}(1 - e)$, with $\alpha \geq 1$ so that a firm's efficiency parameter is a multiplicative scaling factor.[13] For this cost function, the ratio AC_e/MC_e mentioned above is given by $1/\alpha$ so that the condition of Proposition 2 is easier to satisfy when the cost function is more convex, i.e., for larger values of α. In Appendix A, we argue that convex cost functions may be relevant to the mobile telephony sector due to congestion effects. Even though the probability of congestion in a network is very small, the fact that it is exponentially increasing in the number of calls, creates a very convex cost function. Let firms' efficiency parameters be uniformly and independently distributed over the [0, 1] interval and let market demand be linear and given by $Q = 1 - p$. Under Cournot competition, firms' aftermarket profit is then given by

$$\pi(q_i, q_{-i}, e_i) = q_i\Big(1 - \sum_j q_j\Big) - q_i^{\alpha}(1 - e_i)$$

and their best response function $q(q_{-i}, e_i)$ is implicitly defined by

$$1 - 2q_i(q_{-i}, e_i) - \sum\nolimits_{j \neq i} q_j - \alpha[q_i(q_{-i}, e_i)]^{\alpha-1}(1 - e_i) = 0.$$

It is then easy to see that equilibrium output levels at $e_i = e_j = 1$ are given by

$$q_i = q_j = 1/(n+1).$$

An inspection of the partial derivatives of the profit function at this point yields:

$$\pi_{i,i} = -2, \quad \pi_{i,j} = -1, \quad \pi_j = -1/(n+1), \quad \pi_e = 1/(n+1)^{\alpha}, \text{ and } \pi_{i,e} = \alpha/(n+1)^{\alpha-1}.$$

Substituting these values into the condition of Proposition 2 yields that if

$$\alpha > 2\frac{n+1}{n-1},$$

an efficient selection equilibrium does not exist. It is easy to see that when $\alpha > 6$ so that the cost function is sufficiently convex, this condition is satisfied for all values of n. With smaller values of α,n has to be sufficiently large. It is also worth noting that in the case of linear demand and constant marginal cost ($\alpha = 1$), an efficient selection equilibrium always exists.

In both Bertrand and Cournot examples presented so far, market demand functions have been assumed linear. The examples indicate that in this case, cost needs to be convex (decreasing returns to scale). Similar (although slightly more extensive) derivations show that for convex demand functions the corresponding sufficient conditions can be relaxed so that they also cover increasing returns to scale technologies. The examples also indicate that an efficient selection equilibrium is more likely to fail, the larger the number of licences to be allocated.

[13] A similar cost function can be used in the case of Bertrand competition. However, closed-form solutions of the bidding function can only be obtained in the case of a piecewise linear cost function.

6. Correlated Types

When firms' types are statistically independent, the negative externality must be sufficiently strong in order for an efficient selection equilibrium to fail to exist. The reason is that the *ex post* correlation of firms' types can never be very large when firms' types are *ex ante* independently distributed. If, on the other hand, firms' types are *ex ante* correlated, an increasing equilibrium may fail to exist even if the externality is weak. We illustrate this case by means of the following example.

We assume conditions that commonly hold in oligopoly markets, namely that firms prefer being more cost-efficient themselves (a positive direct effect) and that firms prefer to compete with less cost-efficient competitors (a negative externality). Further, we assume that the economy can be in either of the following two states with equal probability: a high-variation state σ^H in which firms' efficiency parameters are independently and uniformly distributed over the [0, 2] interval with mean 1 and a low-variation state σ^L in which they are independently and uniformly distributed over an interval $[\delta - \varepsilon, \delta + \varepsilon]$ with mean $\delta < 1$. For simplicity, we consider the case of two licences, and assume that δ is small.

If the realised type of firm i is $e_i^1 = \delta$, the probabilities that the economy is in either of the states conditional on $e_i = e_i^1$ are as follows

$$\Pr(\sigma^H \mid e_i = e_i^1) = \frac{\varepsilon}{\varepsilon + 1}, \quad \Pr(\sigma^L \mid e_i = e_i^1) = \frac{1}{\varepsilon + 1}.$$

The expected type of firm i's competitor conditional on $e_i^1 = \delta$ is, therefore,

$$E(e_j \mid e_i = e_i^1) = 1 \cdot \Pr(\sigma^H \mid e_i = e_i^1) + \delta \Pr(\sigma^L \mid e_i = e_i^1) = \frac{\varepsilon + \delta}{\varepsilon + 1} = \delta + \frac{\varepsilon(1 - \delta)}{\varepsilon + 1}.$$

If, to the contrary, firm i is of type $e_i^2 = \delta + 2\varepsilon > e_i^1$, the economy is in the high-variation state with probability one, and the expected type of its competitor is

$$E(e_j \mid e_i = e_i^2) = 1 = \delta + \frac{\varepsilon(1 - \delta)}{\varepsilon + 1} + \frac{(1 - \delta)}{\varepsilon + 1} > E(e_j \mid e_i = e_i^1) = \delta + \frac{\varepsilon(1 - \delta)}{\varepsilon + 1}.$$

In other words, the more efficient type e_i^2 faces (in expected terms) more intense competition but is also more efficient itself than the low-efficient type e_i^1.[14]

It is easy to see that when the variation parameter ε becomes very small so that e_i^2 approaches e_i^1 from above, the direct effect for both types becomes equally strong as both types become equally efficient. The strength of the strategic effect, however, remains very different. Indeed, the conditional probability, $\Pr(\sigma^L \mid e_i = e_i^1)$ becomes very close to 1 so that both types expect to compete with rivals of different types: type e_i^1 expects to compete with a type $e_j \approx \delta$ whereas type e_i^2 expects to compete with type $e_j = 1$. Due to the negative externality, no matter how small, type e_i^1 receives in the limit a strictly higher expected profit (for sufficiently small δ) than type e_i^2, even though $e_i^1 < e_i^2$. This implies that the valuation and, hence, the bid of type e_i^2 is lower than of type e_i^1, and the bidding function cannot be monotonically increasing.

[14] Here, the distribution of types is such that the expected efficiency level of a rival firm conditional on one's own efficiency level is very sensitive to the own efficiency level. Any distribution with a conditional distribution function $F(e_j \mid e_i)$ that is 'almost' discontinuous in e_i will deliver this result.

7. On Decreasing Bidding Equilibria

The analysis in the previous Section leads us to a natural question, namely whether there exist market structures and distributions of types for which not only an increasing equilibrium fails to exist but instead a symmetric decreasing equilibrium does exist. In such an equilibrium, we arrive at a perverse situation where the least efficient firms submit the highest bids and obtain the available licences. In this Section, we first argue that both a negative externality and *ex ante* correlation of firms' types are necessary for a decreasing equilibrium to exist. Next, we provide an example of specific market conditions where a decreasing equilibrium exists indeed.

In what follows, we assume that $b(e)$ is a strictly decreasing symmetric bidding function, and the corresponding valuation function is given by

$$v(x,z) \equiv \mathrm{E}[\pi(s_i^*, s_{-i}^*, x) \mid x, e_l \geq z > e_k].$$

Similar to an increasing equilibrium analysed in Proposition 1, the unique symmetric monotonically decreasing equilibrium is given by $b(x) = v(x, x)$ and it exists if $v_x(x, z) < 0$ and $v_x(x, x) + v_z(x, x) < 0$. A necessary condition for the equilibrium to exist is $v_x(x, x) \leq 0$ and $v_x(x, x)+v_z(x, x) \leq 0$ for all $x \in [0, 1]$.

Consider first the condition $v_x(x, x) \leq 0$. When firms' types are statistically independent, then $v_x(x, x) = \mathrm{E}(\pi_e \mid x, e_l \geq x > e_k) > 0$, because $\pi_e > 0$ and the distribution of competitors' types does not depend on x. Hence, a decreasing equilibrium never exists if types are statistically independent. If, on the other hand, there is no strategic effect so that $\pi_j = 0$ and firms have local monopolies, we have $v_x = \pi_e > 0$. Hence, also in this case a decreasing equilibrium does not exist.

In the remainder of this Section, we provide an example of market conditions where a unique symmetric bidding equilibrium exists and it is decreasing. To this end, we take the following joint distribution F^* of firms' efficiency parameters. Let a macroeconomic fundamental (e.g. interest rate, oil price, the growth rate or a state of the economy etc.) β be distributed over the interval [0, 1] in accordance with an arbitrary twice differentiable distribution function $F_\beta(t) \equiv \Pr(\beta < t)$. Then, for any given β, let e_i be independently and uniformly distributed over the interval $[\underline{x}, \bar{x}]$, where $\underline{x} = \beta - \varepsilon\beta(1 - \beta), \bar{x} = \beta + \varepsilon\beta(1 - \beta)$, and $\varepsilon \in (0,1)$ is a parameter. Figure 2

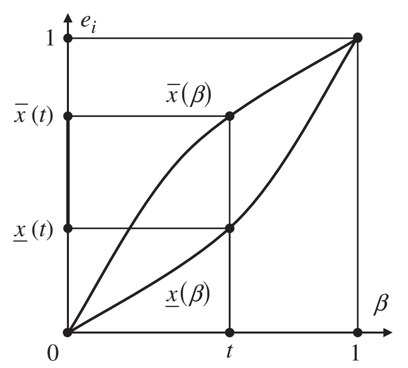

Fig. 2. *Support of the Conditional on $\beta = t$ Distribution* $F_e(x \mid t) = \Pr(e_j < x \mid \beta = t)$

shows the support $[\underline{x}, \bar{x}]$ of the conditional distribution of a rival's efficiency parameter $F_e(x \mid t) \equiv \Pr(e_j < x \mid \beta = t)$.

This distribution has the following property: for small values of ε, if a firm i has a type x , the distribution of other firms' types conditional on x is concentrated on a small neighbourhood of x. Thus, all firms competing in the aftermarket have approximately the same type, the aftermarket Bayes-Nash equilibrium outcome is almost symmetric, and a decreasing equilibrium bidding function can be analytically calculated in the limit when ε converges to zero. Using this technique, we show the existence of a decreasing equilibrium in Proposition 3 below.

PROPOSITION 3. *Let $N = n + 1$ firms with constant marginal costs $c - e_i > 0$ compete in an auction for $n \geq 2$ licences. Let the winning firms compete in quantities in a market with constantly elastic demand $Q = p^{-r}$, and let firms' efficiency parameters e_i be distributed in accordance with the distribution F^*. If the price elasticity r satisfies*

$$\frac{1}{n} < r < \bar{r}(n) \equiv \frac{1}{3}\left(1 + \frac{8}{3n^2 + 3n - 2}\right),$$

then there exists an $\tilde{\varepsilon}(r, n) > 0$ such that for all $\varepsilon \in [0, \tilde{\varepsilon})$ the auction stage has a unique symmetric bidding equilibrium that is decreasing.

In the example considered here, the demand elasticity r must not be too small $(r > 1/n)$ in order to ensure that the aftermarket Bayes-Nash equilibrium exists and is stable. On the other hand, r must be small enough $(r < \bar{r}(n))$ so that the strategic effect is sufficiently strong. The minimum number of licences for this decreasing equilibrium to exists is $n = 3$. As the strategic effect gets stronger, the larger the number of firms competing in the aftermarket, a minimum number of licences is required. The condition of Proposition 3 can be relaxed even further by assuming convex production costs.

Proposition 3 shows that even when the auction is efficient in that the most profitable firms win the licences, the overall market allocation might be inefficient. The reason is that if firms' types are *ex ante* affiliated, the strategic effect can be such that auction efficiency requires the least cost-efficient firms to win a licence.

8. Discussion and Conclusion

We have analysed the conditions under which auctions select efficient firms. When multiple licences to operate in an aftermarket are allocated, there is a direct and a strategic effect related to a firm's own efficiency level. The direct effect basically says that a firm's aftermarket profit is increasing in its own efficiency level. The strategic effect depends on two forces. First, a firm typically prefers to compete with inefficient (instead of efficient) rivals. Second, a firm that is more efficient itself expects to compete with other firms that are also more efficient. We have identified conditions under which efficient firms downsize their bid so much more than less efficient firms that an efficient selection equilibrium does not exist.

The model developed in this article does not fit into the now standard assumptions of the affiliated valuation model (Milgrom and Weber, 1982). In the affiliated valuation model, a player's valuation is an increasing function of his own signal *as well as* of the private signals received by all other players. In our case, where firms receive a signal of their cost efficiency level, a firm's valuation is an increasing function of its own signal but a decreasing function of the signals of other firms. Moreover, but less important, firms only care about the signals received by other *winning* firms.

In this article, we focus our attention on a multi-unit uniform-price auction. It can be shown, however, that the same effects are also present in other simultaneous-bid multi-unit auction formats, such as a pay-your-own-bid auction. The analysis is, however, much more complicated in the case of sequential auctions, where licences are sold one-by-one. It is easy to see that the last licence ends up in the hands of the most efficient remaining firm. Nevertheless, the strategic effect might create inefficient market allocations in selling preceding licences.

We have not allowed for resale in this article. Resale opens up the possibility that in the case of an inefficient allocation of licences, an efficient firm buys a licence from a less efficient firm. Such a single transaction would be mutually beneficial because, for given efficiency levels of the competitors, a more efficient firm makes more profit than a less efficient firm does. However, it is much less clear whether such a transaction is feasible in case other firms are also allowed to transact so that a sequential resale market would emerge. The result of Gomes and Jehiel (2005) suggests that allowing resale can make the outcome even worse. Apart from the fact that reselling is sometimes not allowed or prohibitively costly, there is another good reason not to consider the possibility of reselling. If, together with reselling, firms could make side payments to other firms for not selling their licences, licence holders (together) may be able to 'outbid' an offer of a more efficient firm as the profits they would lose when this new firm competes in the market are larger than the profit the newcomer could make.

We have not considered the question whether a mechanism exists that always selects efficient firms. This is an interesting, non-trivial question for further research.[15]

Appendix A: Cost under a Congestion Externality

In (6) in the main body of the article, we have stated a condition under which an efficient selection equilibrium does not exist. In this Appendix, we show why decreasing returns may be relevant to the mobile telephony sector. Let a mobile telephony operator have its own network with a capacity for K channels, i.e. the network allows for K independent and concurrent connections. The firm serves q customers and, at a certain moment, each customer makes a call (connection) with probability p. The probability of making a call is statistically independent across customers and S denotes a random variable, which is the number of customers trying to connect at a given time.

When $S > K$ customers try to communicate, the firm has to buy extra capacity to be able to handle these calls and, for simplicity, let this extra capacity also come in bundles of K independent communication channels and costs c per bundle. As S follows a binomial distribution

$$b(x, q, p) \equiv \Pr(S = x) = C_n^x p^x (1-p)^{q-x} \text{ and } B(x, q, p) \equiv \Pr(S \le x) = \sum_{t=0}^{x} b(t, q, p),$$

a firm's cost function $C(q)$ can be written as:

[15] It is clear that the results of Ausubel (2004), and Perry and Reny (2002) do not apply, as the present model does not satisfy the monotonicity assumptions of the affiliated valuation model (as indicated above).

$$C(q) = 0\Pr(S \le K) + c\Pr(K < S \le 2K) + 2c\Pr(2K < S \le 3K) + \ldots$$
$$= c\sum_{t=1}^{\infty} t\Pr[tK < S \le (t+1)K] = c[\Pr(K < S) + \Pr(2K < S) + \ldots]$$
$$= c\sum_{t=1}^{\infty} \Pr(tK < S) = c\sum_{t=1}^{\infty}[1 - \Pr(S \le tK)] = c\sum_{t=1}^{\infty}[1 - B(tK, q, p)]$$

The summation in the above expression, although infinite, terminates at t when $tK \ge q$ so that $B(tK, q, p) = 1$. Figure 3 shows the cost function $C(q)$ for $c = 1$, $p = 0.01$, $K = 10$ and $q = 1,\ldots,1000$.

The Figure shows that for $q \approx 100$, the cost is extremely small, but the ratio of marginal cost to average cost is around 10; for $q \approx 700$, the cost is larger and the ratio of marginal cost to average cost is around 5. The cost function we use in the Cournot example is a power function $C(e, q) = q^{\alpha}(1 - e)$. For this cost function, α measures the ratio of marginal cost to average cost. The Figure shows that the convexity (the level of α) can be large indeed.[16]

Appendix B: Derivations for Section 2

In this Appendix we provide more detail to the equations given in Section 2. For the one licence case, we have that maximising monopoly profit

$$\pi(p_i) = p_i q_i(p_i) - C[q_i(p_i)] = (1 - p_i)p_i - (1 - e_i)c(1 - p_i - \bar{q})$$

with respect to p_i yields the optimal price $p_i^* = [1 + (1 - e_i)c]/2$. The (reduced-form) profit function is thus given by $\bar{\pi}(e_i) \equiv \pi(p_i^*) = [1 - (1 - e_i)c]^2/4 + (1 - e_i)c\bar{q}$. The direct effect is

$$\frac{\mathrm{d}\bar{\pi}(e_i)}{\mathrm{d}e_i} = \left\{\frac{1}{2}[1 - (1 - e_i)c] - \bar{q}\right\}c > 0, \text{ for } \bar{q} \le (1 - c)/2.$$

When $\bar{q}$ is larger, the monopolist with type $e_i \in [0, 1 - (1 - 2\bar{q})/c]$ produces $q_i = \min(1/2, \bar{q})$ at zero cost and sells at price $p_i = 1 - \bar{q}_i$, independently of its type and there is no relation between profits and a firm's efficiency so that the direct effect is absent. For more efficient types $e_i \in (1 - (1 - 2\bar{q})/c, 1]$, the direct effect is positive.

With two licences and a random allocation of licences, maximising the expected profit of firm i

$$\pi(p_i) = \mathrm{E}[p_i q_i(p_i, p_j) - C(q_i)] = \mathrm{E}[p_i(1 - p_i + p_j) - (1 - e_i)c(1 - p_i + p_j - \bar{q})]$$

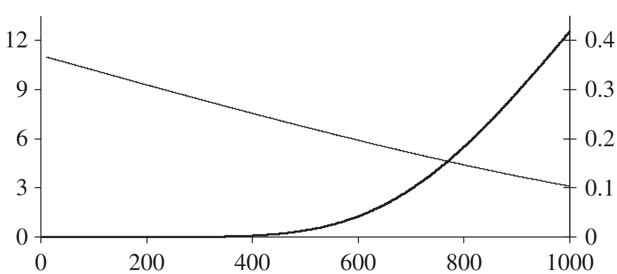

Fig. 3. *The Cost Function C(q) (bold line) Measured on the Right Axis and the Ratio of Marginal to Average Cost Measured on the Left Axis*

[16] A similar example applies to road and railway transportation, where K reflects traffic capacity of different routes and c reflects the lower use value of a second route when the first route is congested.

with respect to p_i yields the first-order condition $p_i^* = [1 + (1 - e_i)c + \mathrm{E}(p_j^*)]/2$. Let firms' Bayes-Nash equilibrium strategy be given by $p_i^* = p(e_i)$. Then

$$p(e_i) = \frac{1}{2}\{1 + (1 - e_i)c + \mathrm{E}_i[p(e_j)]\}, \tag{7}$$

where E_i denotes the expectations taken by firm i. Then, for firm j:

$$\begin{aligned} p(e_j) &= \frac{1}{2}\{1 + (1 - e_j)c + \mathrm{E}_j[p(e_i)]\} = \frac{1}{2}\left[1 + (1 - e_j)c + \mathrm{E}_j\left(\frac{1}{2}\{1 + (1 - e_i)c + \mathrm{E}_i[p(e_j)]\}\right)\right] \\ &= \frac{1}{4}(3 + [3 - \mathrm{E}_j(e_i) - 2e_j]c + \mathrm{E}_j\{\mathrm{E}_i[p(e_j)]\}). \end{aligned}$$

Taking expectations E_i on both sides of the equation yields:

$$\mathrm{E}_i[p(e_j)] = \frac{1}{4}[3 + [3 - \mathrm{E}_j(e_i) - 2E_i(e_j)]c + E_i(\mathrm{E}_j\{\mathrm{E}_i[p(e_j)]\})]$$

Now, as $\mathrm{E}_i[p(e_j)]$ does not depend on information of firm j, the law of iterative expectations says that $\mathrm{E}_i(\mathrm{E}_j\{\mathrm{E}_i[p(e_j)]\}) = \mathrm{E}_i[p(e_j)]$ so that

$$\mathrm{E}_i[p(e_j)] = \frac{1}{4}\{3 + [3 - \mathrm{E}_j(e_i) - 2\mathrm{E}_i(e_j)]c + \mathrm{E}_i[p(e_j)]\}$$

and

$$\mathrm{E}_i[p(e_j)] = 1 + \frac{1}{3}[3 - \mathrm{E}_j(e_i) - 2\mathrm{E}_i(e_j)]c.$$

Substituting this into (7) yields the following equilibrium pricing strategy:

$$p(e_i) = 1 + \frac{1}{6}[6 - 3e_i - \mathrm{E}_j(e_i) - 2\mathrm{E}_i(e_j)]c.$$

Taking into account that $\mathrm{E}_j(e_i) = \mathrm{E}_i(e_j) = \mathrm{E}(e)$, yields the equilibrium pricing strategy:

$$p(e_i) = 1 + \frac{1}{2}[2 - e_i - \mathrm{E}(e)]c,$$

so that a firms' profit function is given by

$$\bar{\pi}(e_i) \equiv \pi[p(e_i)] = \left\{1 - \frac{1}{2}[\mathrm{E}(e) - e_i]c\right\}^2 + (1 - e_i)c\bar{q} \text{ for } \bar{q} < 1 - \frac{c}{2}.$$

The direct effect is

$$\frac{\mathrm{d}\bar{\pi}(e_i)}{\mathrm{d}e_i} = \left\{1 - \frac{1}{2}[\mathrm{E}(e) - e_i]c - \bar{q}\right\}c > 0.$$

As in the monopoly case, higher values of $\bar{q}$ result in the disappearance of the direct effect for the least-efficient types.

On the other hand, if two licences are allocated in a uniform-price auction and an efficient selection equilibrium exists, each winning firm in the aftermarket stage knows that its competitor's type is above the type of the firm that has not won the licence. Conditional on z, the type of the losing firm, the winners' types are i.i.d. variables and, therefore, firms' Bayes-Nash equilibrium strategies and profit functions are given by the same expressions as under random allocation case with the difference that the expectation $\mathrm{E}(e_j)$ is taken conditional on $e_j > z$ (by neglecting ties):

$$p(e_i) = 1 + \frac{1}{2}[2 - e_i - \mathrm{E}(e_j \mid e_j > z)]c \text{ and}$$

$$\bar{\pi}(e_i, z) = \left\{1 - \frac{1}{2}[\mathrm{E}(e_j \mid e_j > z) - e_i]c\right\}^2 + (1 - e_i)c\bar{q}, \text{ for } \bar{q} < 1 - \frac{c}{2}.$$

In Proposition 1 we show that firms bid amounts $\bar{\pi}(e_i, e_i)$ in equilibrium, so that

$$b(e_i) = \bar{\pi}(e_i, e_i) = \left\{1 - \frac{1}{2}[\mathrm{E}(e_j \mid e_j > e_i) - e_i]c\right\}^2 + (1 - e_i)c\bar{q},$$

where $\mathrm{E}(e_j \mid e_j > e_i) = (1 + e_1)/2$ for the uniform distribution.

If the same situation is analysed under perfect information firm i's aftermarket profit is given by

$$\tilde{\pi}(e_i, e_j) = \left[1 - \frac{1}{3}(e_j - e_i)c\right]^2 + (1 - e_i)c\bar{q}, \text{ for } \bar{q} < 1 - \frac{c}{3}.$$

Taking the expectation conditional on $e_j > z$ yields (for the uniform distribution):

$$\begin{aligned}
\bar{\pi}(e_i, z) &= \mathrm{E}[\tilde{\pi}(e_i, e_j) \mid e_j > z] = \mathrm{E}\left\{\left[\left(1 + \frac{c}{3}e_i\right) - \frac{c}{3}e_j\right]^2 \mid e_j > z\right\} + (1 - e_i)c\bar{q} \\
&= \left(1 + \frac{c}{3}e_i\right)^2 - 2\left(1 + \frac{c}{3}e_i\right)\frac{c}{3}\mathrm{E}(e_j \mid e_j > z) + \frac{c^2}{9}\mathrm{E}(e_j^2 \mid e_j > z) + (1 - e_i)c\bar{q} \\
&= \left(1 + \frac{c}{3}e_i\right)^2 - 2\left(1 + \frac{c}{3}e_i\right)\frac{c}{3}\frac{1+z}{2} + \frac{c^2}{9}\frac{1 + z + z^2}{3} + (1 - e_i)c\bar{q} \\
&= \left[1 - \frac{c}{3}\left(\frac{1+z}{2} - e_i\right)\right]^2 + \frac{c^2}{108}(1 - z)^2 + (1 - e_i)c\bar{q}.
\end{aligned}$$

Hence, if an efficient selection equilibrium exists, firm i must bid

$$\begin{aligned}
b(e_i) = \bar{\pi}(e_i, e_i) &= 1 - \frac{c}{3}(1 - e_i) + \frac{c^2}{36}(1 - e_i)^2 + \frac{c^2}{108}(1 - e_i)^2 + (1 - e_i)c\bar{q} \\
&= 1 - \frac{c}{3}(1 - 3\bar{q})(1 - e_i) + \frac{c^2}{27}(1 - e_i)^2.
\end{aligned}$$

Appendix C: Proofs

Proof of Proposition 1

Given the definition of z, let Z be the nth highest order statistics among e_j, $j \neq i$. We denote the distribution of Z conditional on $e_i = x$ by $G(z \mid x) \equiv \Pr(Z < z \mid e_i = x)$, and the corresponding density function by $g(z \mid x)$.

Suppose that all firms other than i follow the bidding function $b(x)$ and Z takes a value z. We consider a firm i, which has a cost parameter $e_i = x$ and which bids $b(y)$. If $y < z$, then $b(y) < b(z)$ and firm i loses the auction and receives no profit. If, on the other hand, $y > z$, then $b(y) > b(z)$ and firm i gets a licence at the auction fee $w = b(z)$, which yields the firm the conditional (on z) expected profit $v(x, z)$. The unconditional expected profit $V(x,y)$ of a firm which has cost parameter x and a bids $b(y)$ is

$$V(x, y) \equiv \int_{z<y} [v(x, z) - b(z)]g(z \mid x)\mathrm{d}z.$$

In equilibrium, the maximum of $V(x,y)$ w.r.t. y must be attained at $y = x$ (incentive compatibility or truth-telling condition). Maximising $V(x, y)$ yields the first-order condition $\partial V(x,x)/\partial y = 0$ which simplifies to $b(x) = v(x, x)$.

This bidding function $b(x)$ is an increasing function only if $v_x(x, x) + v_z(x, x) \geq 0$. The second necessary condition is the second-order condition $\partial^2 V(x, x)/\partial y^2 \leq 0$, which simplifies to $v_x(x, x) \geq 0$.

Suppose now that $v_x(x, z) > 0$ and $v_x(x, x) + v_z(x, x) > 0$. In order to check that these conditions are sufficient for $b(x) = v(x, x)$ to be an equilibrium bidding function, we first note that $b(x)$ is indeed increasing: $b'(x) = v_x(x, x) + v_z(x, x) > 0$. Then, we evaluate $V(x, x) - V(x, y)$ for any $y \neq x$:

$$\begin{aligned} V(x,x) - V(x,y) &= \int_{z<x} [v(x,z) - b(z)]g(z\,|\,x)\mathrm{d}z - \int_{z<y} [v(x,z) - b(z)]g(z\,|\,x)\mathrm{d}z \\ &= \int_{y<z<x} [v(x,z) - v(z,z)]g(z\,|\,x)\mathrm{d}z = \int_y^x g(z\,|\,x)\mathrm{d}z \int_z^x v_x(t,z)\mathrm{d}t \geq 0. \end{aligned}$$

This shows that firm i does not have a profitable deviation from $b(x) = v(x, x)$. □

Proof of Proposition 2

This proof consists of two parts. Part 1 derives firms' aftermarket equilibrium strategy $s(x, z)$ and its derivatives at $x = z = 1$. Part 2 derives firms expected aftermarket profit function $v(x, z)$, its partials at $x = z = 1$ and the sufficient condition for $v_x(1, 1) + v_z(1, 1) < 0$.

Part 1. In the limit when $(x, z) \to (1,1)$, the firms' Bayes-Nash equilibrium strategy $s(x, z)$ can be written as $s(x, z) = s_0 - (1 - x)s_x - (1 - z)s_z$, where

$$s_0 \equiv s(1,1), \quad s_x \equiv \frac{\partial s}{\partial x}(1,1) \text{ and } s_z \equiv \frac{\partial s}{\partial z}(1,1).$$

Dropping arguments in all functions evaluated at $(x, z) = (1, 1)$, the first-order condition $0 = \mathrm{E}[\pi_i(s_i^*, s_{-i}^*, e_i)\,|\,e_i = x, e_l \leq z < e_k]$ for independent types in the first-order approximation becomes

$$\begin{aligned} 0 &= \mathrm{E}\{\pi_i[s_0 - (1-x)s_x - (1-z)s_z, s_0 - (1-e_k)s_x - (1-z)s_z, x]\,|\,z < e_k\} \\ &= \mathrm{E}\left\{\pi_i - [(1-x)s_x + (1-z)s^{(2)}]\pi_{i,i} - \pi_{i,j}\sum_k [(1-e_k)s_x + (1-z)s_z] - (1-x)\pi_{i,e}\,|\,z < e_k\right\} \\ &= \pi_i - (\pi_{i,i}s_x + \pi_{i,e})(1-x) - [\pi_{i,i}s_z + (n-1)\pi_{i,j}(\tilde{E}s_x + s_z)](1-z). \end{aligned}$$

where

$$\tilde{E} \equiv \lim_{z\to 1}\frac{\mathrm{E}(1-e_k\,|\,z<e_k)}{\bar{e}-z} = \lim_{z\to 1}\frac{\int_z^1 (1-x)f(x)\mathrm{d}x}{(1-z)[1-F(z)]} = \lim_{z\to 1}\frac{(1-z)f(z)}{[1-F(z)] + (1-z)f(z)} = \frac{1}{2},$$

if $f(1) > 0$, and

$$\tilde{E} = \lim_{z\to 1}\frac{\mathrm{E}(1-e_k\,|\,z<e_k)}{\bar{e}-z} = \frac{1}{2} - \frac{1}{2}\lim_{z\to 1}\frac{(1-z)f'(z)}{2f(z) - (1-z)f'(z)} \geq \frac{1}{2},$$

if $f(1) = 0$. The first-order condition implies the following two equations, which determine $s^{(1)}$ and $s^{(2)}$:

$$\begin{cases} 0 = \pi_{i,i}s_x + \pi_{i,e} \\ 0 = \pi_{i,i}s_z + (n-1)\pi_{i,j}(\tilde{E}s_x + s_z). \end{cases}$$

Solving them yields

$$s_x = -\frac{\pi_{i,e}}{\pi_{i,i}} \text{ and } s_z = \frac{(n-1)\pi_{i,j}\pi_{i,e}}{\pi_{i,i}[\pi_{i,i} + (n-1)\pi_{i,j}]}\tilde{E}.$$

Hence, firms' aftermarket equilibrium strategy can be written in the limit when $(x, z) \to (1, 1)$ as follows

$$s(x,z) = s_0 + \frac{\pi_{i,e}}{\pi_{i,i}}\left\{(1-x) - \frac{(n-1)\pi_{i,j}\tilde{E}(1-z)}{[\pi_{i,i} + (n-1)\pi_{i,j}]}\right\}.$$

Part 2. Firms' valuation function $v(x, z)$ in the limit $(x, z) \to (1, 1)$ can be written as

$$\begin{aligned} v(x,z) &= \mathrm{E}\{\pi - (n-1)[(1-e_k)s_x + (1-z)s_z]\pi_j - (1-x)\pi_e \mid z < e_k\} \\ &= \pi - (n-1)(\tilde{E}s_x + s_z)\pi_j(1-z) - (1-x)\pi_e \\ &= \pi + \frac{(n-1)\pi_j\pi_{i,e}\tilde{E}}{[\pi_{i,i} + (n-1)\pi_{i,j}]}(1-z) - (1-x)\pi_e. \end{aligned}$$

Finally, the necessary condition $v_x(1,1) + v_z(1,1) \geq 0$ for an efficient selection equilibrium to exist is

$$\pi_e - \frac{(n-1)\pi_j\pi_{i,e}\tilde{E}}{[\pi_{i,i} + (n-1)\pi_{i,j}]} \geq 0.$$

Hence, if inequality (5) holds, then the above inequality holds for any $\tilde{E} \in [1/2, 1]$ (and, therefore, for any distribution with a differentiable over [0,1] density function) and an efficient selection equilibrium does not exist. □

Proof of Proposition 3

The necessary and sufficient conditions for a decreasing equilibrium can be derived in a way similar to the proof of Proposition 2. Here we first derive firms' aftermarket Bayes-Nash equilibrium strategy, which we denote by $s(x, z; \varepsilon)$ to emphasise its dependence on ε, under the assumption that in the auction stage all they follow an increasing bidding function $b(e)$. Under the assumption that the profit function π is twice continuously differentiable, we represent $s(x, z; \varepsilon)$ as a first-order Tailor expansion. We show that this bidding function does not satisfy the second order condition, hence, an increasing symmetric bidding equilibrium does not exist. Second, we repeat the previous exercise for a decreasing bidding function $b(e)$ and analyse conditions under which it is indeed an equilibrium bidding function.

From now on we denote by x a type of a firm i. Types of all other (winning and losing) firms are e_j (e_k and $e_l = z$ respectively) and we define

$$\lambda_j \equiv (e_j - x)/\varepsilon, \quad \underline{t}(x) \equiv \left[(1+\varepsilon) - \sqrt{(1+\varepsilon)^2 - 4\varepsilon x}\right]/(2\varepsilon),$$

and

$$\bar{t}(x) \equiv \left[\sqrt{(1-\varepsilon)^2 + 4\varepsilon x} - (1-\varepsilon)\right]/(2\varepsilon),$$

so that $\underline{x}[\bar{t}(x)] = \bar{x}[\underline{t}(x)] = x$ for any $x \in [0,1]$.

For given x, conditional distribution $F_\beta(t \mid x) \equiv \Pr(\beta < t \mid e_i = x)$ has the support$[\underline{t}, \bar{t}]$, and conditional distribution $F_e(\lambda \mid x) \equiv \Pr(\lambda_j < \lambda \mid e_i = x)$ has the support$[\underline{\lambda}, \bar{\lambda}]$, where both $\underline{\lambda}(x)$ and $\bar{\lambda}(x)$ are bounded:[17]

$$\underline{\lambda}(x) = (\underline{t} - x)/\varepsilon - \underline{t}(1 - \underline{t}) \geq -0.5 \text{ and } \bar{\lambda}(x) = (\bar{t} - x)/\varepsilon + \bar{t}(1 - \bar{t}) \leq 0.5.$$

Denoting the value of $s(x, z; \varepsilon)$ and its partial derivatives evaluated at $(x, x; 0)$ by

$$s_0(x) \equiv s(x, x; 0), \quad s_x(x) \equiv \frac{\partial s}{\partial x}(x, x; 0), \quad s_z(x) \equiv \frac{\partial s}{\partial z}(x, x; 0), \text{ and } s_\varepsilon(x) \equiv \frac{\partial s}{\partial \varepsilon}(x, x; 0),$$

allows us to write $s(x, z; \varepsilon)$ and $s(e_j, z; \varepsilon)$ in the first-order approximation as

$$s(x, z; \varepsilon) = s_0 + (s_z\lambda_z + s_\varepsilon)\varepsilon \text{ and } s(e_j, z; \varepsilon) = s_0 + (\lambda_j s_x + \lambda_z s_z + s_\varepsilon).$$

Dropping arguments lists in all functions evaluated at $z = x$, we write the first-order condition

$$0 = \mathrm{E}\{\pi_i[s(x, z; \varepsilon), s(e_k, z; \varepsilon), e_i] \mid e_i = x, e_l = z < e_k\}$$

as follows:

$$\begin{aligned}
0 &= \mathrm{E}\{\pi_i[s_0 + (s_z\lambda_z + s_\varepsilon)\varepsilon, s_0 + (\lambda_k s_x + \lambda_z s_z + s_\varepsilon)\varepsilon, x] \mid e_i = x, e_l = z < e_k\} \\
&= \mathrm{E}\left(\pi_i + \left\{\pi_{i,i}(s_z\lambda_z + s_\varepsilon) + \pi_{i,j}\left[s_x \sum\nolimits_k \lambda_k + (n-1)(s_z\lambda_z + s_\varepsilon)\right]\right\}\varepsilon \mid e_i = x, e_l = z < e_k\right) \\
&= \pi_i + \{[\pi_{i,i} + (n-1)\pi_{i,j}](s_z\lambda_z + s_\varepsilon) + (n-1)\pi_{i,j}s_x \lim_{\varepsilon\to 0} \mathrm{E}(\lambda_k \mid e_i = x, e_l = z < e_k)\}\varepsilon.
\end{aligned}$$

If $\varepsilon = 0$, the first-order condition implies $0 = \pi_i$, which for a given profit function

$$\pi(s_i, s_{-i}, e_i) = s_i\left[\left(\sum\nolimits_j s_j\right)^{-1/r} - c + e_i\right]$$

yields the symmetric aftermarket Nash equilibrium strategy:

$$s(x, x; 0) = s_0 = \frac{1}{n}\left[\frac{(nr-1)}{nr(c-x)}\right]^r.$$

In this case

$$\pi = \frac{(c-x)s_0}{(nr-1)}, \quad \pi_j = -\frac{(c-x)}{(nr-1)}, \quad \pi_{i,i} = -\frac{[2nr - (1+r)](c-x)}{(nr-1)nrs_0},$$
$$\pi_{i,j} = -\frac{[nr - (1+r)](c-x)}{(nr-1)nrs_0} \text{ and } s_x + s_z = \frac{rs_0}{(c-x)}.$$

Using $0 = \pi_i$, we rewrite the first-order condition for $\varepsilon \neq 0$ and $\lambda_z = 0$ as follows:

$$0 = [\pi_{i,i} + (n-1)\pi_{i,j}]s_\varepsilon + (n-1)\pi_{i,j}s_x \lim_{\varepsilon\to 0} \frac{\mathrm{E}(e_k \mid e_i = x, e_l = z < e_k) - x}{\varepsilon},$$

so that

$$s_\varepsilon = -\frac{(n-1)\pi_{i,j}}{[\pi_{i,i} + (n-1)\pi_{i,j}]} s_x \lim_{\varepsilon\to 0} \frac{H(x,x) - x}{\varepsilon},$$

where $H(x,z) \equiv \mathrm{E}(e_k \mid e_i = x, e_l = z, z < e_k)$.

[17] These inequalities can be obtained by minimising $\underline{\lambda}(x)$ and maximising $\bar{\lambda}(x)$ with respect to x.

In Lemma 1 of Janssen and Karamychev (2009) it is proved that $H(x,z)$ can be written for small values of ε as

$$H(x,z)=\frac{nx+(n+2)z}{2(n+1)}+\frac{nx(1-x)}{(n+1)}\varepsilon+o(\varepsilon).$$

Using this fact, we rewrite s_ε as follows:

$$s_\varepsilon=-\frac{n(n-1)\pi_{i,j}x(1-x)}{(n+1)[\pi_{i,i}+(n-1)\pi_{i,j}]}s_x.$$

Substituting this expression into the first-order condition yields:

$$\begin{aligned}s_z&=-\frac{(n-1)\pi_{i,j}}{(n+1)[\pi_{i,i}+(n-1)\pi_{i,j}]}s_x\lim_{\varepsilon\to 0}\frac{(n+1)[\mathrm{E}(e_k\,|\,e_i=x,e_l=z<e_k)-x]-nx(1-x)\varepsilon}{(z-x)}\\&=-\frac{(n+2)(n-1)\pi_{i,j}}{2(n+1)[\pi_{i,i}+(n-1)\pi_{i,j}]}s_x.\end{aligned}$$

Solving $s_x+s_z=rs_0/(c-x)$ together with the above expressions for s_z and s_ε finally yields:

$$s_x=\frac{2r(n+1)[\pi_{i,i}+(n-1)\pi_{i,j}]}{(c-x)[2(n+1)\pi_{i,i}+n(n-1)\pi_{i,j}]}s_0,$$

$$s_z=-\frac{r(n+2)(n-1)\pi_{i,j}}{(c-x)[2(n+1)\pi_{i,i}+n(n-1)\pi_{i,j}]}s_0$$

and

$$s_\varepsilon=-\frac{2rn(n-1)\pi_{i,j}x(1-x)}{(c-x)[2(n+1)\pi_{i,i}+n(n-1)\pi_{i,j}]}s_0.$$

Hence, the aftermarket Nash equilibrium strategy $s(x,\ z;\ \varepsilon)$ for small ε and $(z-x)$ is

$$s(x,z;\varepsilon)=\left\{1-\frac{r(n-1)\pi_{i,j}[(n+2)(z-x)+2nx(1-x)\varepsilon]}{(c-x)[2(n+1)\pi_{i,i}+n(n-1)\pi_{i,j}]}\right\}s_0.$$

This ends the analysis of the aftermarket stage of the game.

In order to show that an increasing symmetric bidding equilibrium does not exist for small ε, we compute firms' valuation function $v(x,\ z;\ \varepsilon)$ and verify that for given parameters' restrictions, $v_x(x,\ x;\ 0)<0$. Using the first-order approximation for $s(x,\ z;\ \varepsilon)$, firms' valuation function $v(x,\ z;\ \varepsilon)$ in the first-order approximation can be written as follows:

$$\begin{aligned}v(x,z;\varepsilon)&=\mathrm{E}\{\pi[s(x,z;\varepsilon),s(e_k,z;\varepsilon),z]\,|\,e_i=x,e_l\le z<e_k\}\\&=\mathrm{E}\{\pi[s_0+(s_z\lambda_z+s_\varepsilon)\varepsilon,s_0+(\lambda_k s_x+\lambda_z s_z+s_\varepsilon)\varepsilon,x]\,|\,e_i=x,e_l=z<e_k\}\\&=\mathrm{E}[\pi+\pi_j\textstyle\sum_k(\lambda_k s_x+\lambda_z s_z+s_\varepsilon)\varepsilon\,|\,e_i=x,e_l=z<e_k]\\&=\pi+(n-1)\pi_j(\varepsilon\{\lim_{\varepsilon\to 0}[H(x,z)-x]s_x/\varepsilon+s^{(3)}\}+s_z(z-x))\\&=\pi+\frac{r(n-1)\pi_j\pi_{i,i}[(n+2)(z-x)+2nx(1-x)\varepsilon]}{(c-x)[2(n+1)\pi_{i,i}+n(n-1)\pi_{i,j}]}s_0.\end{aligned}$$

Substituting expressions for π, π_j, $\pi_{i,i}$, $\pi_{i,j}$, and s_0 finally yields the following valuation function $v(x,\ z;\ \varepsilon)$:

$$v=\frac{1}{n^2r}\left[\frac{nr(c-x)}{(nr-1)}\right]^{1-r}\left(1-\frac{r(n-1)[2nr-(1+r)][(n+2)(z-x)+2nx(1-x)\varepsilon]}{(c-x)\{2(n+1)[2nr-(1+r)]+n(n-1)[nr-(1+r)]\}}\right).$$

But then

$$v_x(x,x;0) = -\left[\frac{nr(c-x)}{(nr-1)}\right]^{1-r} \frac{(nr-1)[(n^2+n+2)-(3n^2+3n-2)r]}{n^2(c-x)\{(nr-1)[r(n^2+2n+3)+1]+(1-r)(1+2r)\}} < 0,$$

provided $r < \bar{r}(n)$. This implies that $v_x(x, x; \varepsilon) < 0$ for sufficiently small (but strictly positive) ε, so that an increasing symmetric bidding equilibrium does not exist.

Suppose now that there exists a symmetric decreasing equilibrium bidding function $b(e)$. In a similar way as above, firms' valuation function $v(x, z; \varepsilon)$ can be written as follows:

$$v = \frac{1}{n^2 r}\left[\frac{nr(c-x)}{(nr-1)}\right]^{1-r}\left(1 - \frac{r(n-1)[2nr-(1+r)][(n+2)(z-x)-2nx(1-x)\varepsilon]}{(c-x)\{2(n+1)[2nr-(1+r)]+n(n-1)[nr-(1+r)]\}}\right).$$

Then,

$$v_x(x,x;0) = -\left[\frac{nr(c-x)}{(nr-1)}\right]^{1-r} \frac{(nr-1)[(n^2+n+2)-(3n^2+3n-2)r]}{n^2(c-x)\{(nr-1)[r(n^2+2n+3)+1]+(1-r)(1+2r)\}} < 0,$$

for $r < \bar{r}(n)$ and

$$v_z(x,x;0) = -\left[\frac{nr(c-x)}{(nr-1)}\right]^{1-r} \frac{(n+2)(n-1)[2nr-(1+r)]}{n^2(c-x)\{2(n+1)[2nr-(1+r)]+n(n-1)[nr-(1+r)]\}} < 0.$$

By continuity argument, there exists an $\tilde{\varepsilon} > 0$ so that $v_x(x, z; \varepsilon) < 0$ and $v_z(x, z; \varepsilon) < 0$ for all $\varepsilon \in (0, \tilde{\varepsilon})$ and feasible z. For $\varepsilon = 0$ the game is of complete information, and firms bid their entire aftermarket profits $v(x, x; 0)$. Therefore, the proposed function $b(x) = v(x,x)$ is indeed decreasing for all $\varepsilon \in [0, \tilde{\varepsilon})$ and is a unique symmetric equilibrium bidding function. □

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