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European Economic Review 50 (2006) 517–532

EUROPEAN ECONOMIC REVIEW

www.elsevier.com/locate/econbase

Auctions as coordination devices☆

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Received 16 October 2003; accepted 12 January 2005
Available online 19 February 2005

Abstract

This paper develops an economic argument relating auctions to high market prices. At the core of the argument is the claim that market competition and bidding in an auction should be analyzed as part of one game, where the pricing strategies in the market subgame depend on the bidding strategies during the auction. I show that when there are two licenses for sale the only equilibrium in the overall game that is consistent with the logic of forward induction is the one where firms bid an amount (almost) equal to the profits of the cooperative market outcome and follow a cooperative pricing strategy in the market game resulting in high prices. With three or more licenses the auction format co-determines whether or not the forward induction argument works.

JEL classification: L50

Keywords: Auctions; Market prices; Coordination

☆ An earlier version of this paper was entitled *Auctions as Collusion Devices*. I thank Esther Gal-Or and two referees, Geir Asheim, Ken Binmore, Martin Dufwenberg, Paul Klemperer, Benny Moldovanu, Giancarlo Spagnolo, Elmar Wolfstetter and participants of presentations at ESEM 2003, Erasmus University and the AMD 2003 conference (Milan) for helpful comments.
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0014-2921/$ - see front matter
doi:10.1016/j.euroecorev.2005.01.003

1. Introduction

One of the most debated questions concerning the recent wave of spectrum auctions held around the world is whether auctions give rise to higher prices in the market after the auction. Firms tend to stress that they have to recover the money they spend on obtaining a license and therefore tend to set higher prices when auction revenues are high. Economists tend to the view that payments during an auction should be considered as a sunk cost at the moment firms compete in the market place. According to the economic point of view, there is, or should not be, any relation between auction revenues and market prices (see, e.g., Binmore and Klemperer, 2002). Recent experiments show, however, that auctioning rights to compete in the market does lead to higher market prices (see, Offerman and Potters, 2000).

Formally, the sunk cost argument is based on the notion of subgame perfection. This notion basically says that at the beginning of each subgame (read: After some auction outcome is observed and market competition starts) players should look at the future and choose strategies that form an equilibrium in the subgame. The behaviour that has lead to a particular subgame is not relevant for the strategic analysis of that subgame. With multiple equilibria in a subgame, subgame perfection does not give clear guidance to players what to choose. In this case, the notion of *forward induction* (see, e.g., Kohlberg and Mertens, 1986; Van Damme, 1989; Osborne, 1990; Ben-Porath and Dekel, 1992) may complement an analysis in terms of subgame perfection. Under forward induction, past behaviour may signal which future play is intended. Using this idea, this paper argues that market competition and bidding in an auction should be analyzed as part of one game, where bidding behaviour in the auction may signal intended pricing behaviour in the market game.

The paper analyzes a situation with N bidders. The prize that the winners of the auction get is the right to play the market game. Players can obtain one license at most. In its most simple form, the market competition game is analyzed as a coordination game with Pareto-ranked equilibria: Firms can either follow a competitive strategy resulting in relatively low profits if both decides to do so or a more cooperative strategy with relatively high profits if they coordinate. What is important for the argument to hold is that the market game has multiple equilibria and that the strategy space is finite.[1]

The formal results are as follows. First, as a preliminary result I show that the game has very many symmetric equilibria. The only restriction on behaviour implied by the equilibrium notion is that a very high bid at the auction stage followed by a competitive market strategy cannot be part of an equilibrium as players would make losses and it is better not to win the auction at all. More interestingly, the notion of subgame perfection does not impose any further restrictions and any behaviour that

[1] In theoretical IO models, price, quantity and other decision variables are typically modelled as continuous variables. However, considering discrete variables is more realistic in most cases: Prices have to be set in cents and firms in the real world tend to consider only certain ‘‘psychological’’ prices (ending on 5 or 9). Therefore, restricting the analysis to finite strategies is a reasonable restriction to impose.

can be rationalized in terms of Nash equilibrium can also be rationalized in terms of subgame perfect equilibria. I then proceed to the analysis of the game in terms of forward induction. With *two licenses* for sale the claim is that the only equilibrium in the overall game that is consistent with the logic of *forward induction* is the one where firms bid an amount (almost) equal to the profits of the cooperative market outcome and follow the cooperative strategy in the market game. In other words, the auction solves the coordination game at the market level in favour of the high profit equilibrium at the market stage.

With *three or more licenses* for sale, the validity of the forward induction argument depends on the type of auction that is chosen. For open auctions such as the simultaneous ascending auctions,[2] where bidders can react on each others' bids and players have to pay their own bid, the above argument can be adapted and the result carries over. When the licenses are allocated using some type of sealed-bid auction such as a discriminatory auction,[3] the forward induction logic does not apply, however, and I obtain the more standard argument that bidding behaviour in the auction and strategic behaviour in the market are not connected. The differences in results indicate, once more, that details of auction design may have important implications for the final outcome.[4]

Intuitively, what the forward induction argument establishes is that by integrating the auction game and the market game into one larger game, auction expenditures are no longer sunk in the larger game. By looking at the market game separately, auction expenditures are indeed sunk, but at the auction stage they are not! Therefore auction expenditures may signal the intention to play a high profit equilibrium in the market game.

Forward induction has first been used in the ''burning money'' argument in game theory (see, e.g., Van Damme, 1989; Osborne, 1990; Ben-Porath and Dekel, 1992; Rubinstein, 1991). The basic idea in this literature is that coordination problems like the ones in Battle-of-the-Sexes games can be resolved if one of two players before playing the game has the option to burn some money. The *forward induction* argument that is used in this literature is quite similar to the one applied in this paper although there are some major differences. First, in an auction all players, and not just one, have the possibility of ''burning money''. Second, the competitive pressure present in auctions makes that in the resulting equilibrium, players *do* burn money, whereas the equilibrium that is selected in the burning money/Battle-of-the-Sexes

[2] In a simultaneous ascending auction (Milgrom, 2004, pp. 5–6), multiple heterogeneous goods are auctioned in multiple rounds and in each round bidders make sealed bids for as many units they want. A unit is allocated to the player with the highest bid on that unit only after there has been a round in which no bid has been made on any unit. Players (again) have to pay their own bid(s) for the units they obtain.

[3] In a discriminatory auction (Krishna, 2002, pp. 166–169), multiple homogeneous goods are auctioned, bidders express their bids in terms of a bid vector telling the auctioneer how much they are willing to pay for the first and each subsequent unit, units are allocated to the players with the highest bids and players have to pay their own bid(s) for the units they obtain.

[4] An interesting side issue that comes out of the proof is that during the bidding stage in the game, firms may find it optimal to introduce a ''jump bid'' (see also Avery, 1998). The jump is necessary to convince others that they will follow the cooperative strategy in the aftermarket.

example, players do *not* burn money. The fact that nothing is burnt, leads to a fundamental issue regarding game theoretic modelling (see Osborne and Rubinstein, 1994, p. 113): One may always argue that players have the possibility of burning money, but as *nothing is burnt* it is difficult to perceive this as a signal as it is not clear whether the other player considered the possibility of burning money in the first place.[5] My argument is not prone to this objection as the equilibrium that is chosen has players ‘‘burning money’’.

The forward induction argument has been used in a loose way in two closely related papers discussing auctioning the rights to play a coordination game. van Huyck et al. (1993) discusses an experiment where the right to play a series of coordination games between nine players is auctioned off between 18 players. In the auction, an auctioneer keeps on raising the stakes until nine bidders remain. They find strong evidence that auctioning the rights to play the coordination game makes players coordinate on the Pareto-superior equilibrium. Crawford and Broseta (1998) provides a theoretical model explaining the experimental evidence. The model is based on a history-dependent learning dynamics. Even though both papers mention the similarity of their work to the intuition of the forward induction logic, they explicitly reject the formal forward induction argument as the explanation for the experimental evidence.[6] The main difference, therefore, between the present paper and these two papers is that I show that the formal forward induction argument can be used. In addition, details of the auction design do matter (see above).[7]

The paper is also related to recent literature on the interaction between auctions and aftermarkets (Klemperer, 2002a, b). Jehiel and Moldovanu (1996a, b, 2001) study the way externalities in the aftermarket have an impact on bidding behaviour in the auction. They show that, depending on the specific context, standard properties of auctions do not hold when bidding firms also interact after the auction outcome has been established (see also, e.g., Das Varma, 2002). Signalling does not play a role in these papers. Signalling does play a role in Goeree (2003). In that paper players have private information that effects aftermarket competition. He shows that players may have an incentive to overstate their private information in an attempt to influence the behaviour of competitors in the aftermarket. This paper, therefore, falls in the tradition of signalling models where actions (in this case, firms bidding behaviour) may reveal a player's type. In contrast, I look at a situation where private

[5]Note that this critique does not apply to the analysis of Asheim and Dufwenberg (2003a, b) of the same game.

[6]In the abstract to their paper, for example, Crawford and Broseta (1998, p. 198) argue that ‘‘the efficiency enhancing effect of auctions is reminiscent of forward induction, but it is not explained by equilibrium refinements’’.

[7]It is true that forward induction does not have the necessary bite in the formal game analyzed by Crawford and Broseta (1998), see p. 205 of their paper for more details. Another difference relates to the interpretation of the results. Crawford and Broseta (1998) use the term ‘‘efficiency-enhancing’’ as the Pareto-superior equilibrium is a high effort equilibrium in a game where players choose effort in a production process, which presumably is good for both everyone in the model economy. In our setting, the Pareto-superior equilibrium is a high profit equilibrium, which generally leads to low levels of consumer surplus and to overall inefficiencies.

information does not play a role and past actions signal future actions, instead of a player's private information.

The rest of the paper is organized as follows. Section 2 specifies the model in case two licenses are auctioned. Section 3 contains the main proposition and its proof for this case of two licenses. Section 4 discusses details of the $k \geqslant 3$ license case and Section 5 concludes with a discussion. Technical proofs are contained in the Appendix.

2. The model and solution concept

There are $N > 2$ firms. The game the firms play is a two-stage game. In the first stage the firms bid in an auction. The two firms with the highest bid continue to the second stage where they play a market competition game. If there is a tie for the first and/or second highest bid, a lottery will determine the ranking of the bids. The bids of the two players that continue to play the market competition game are denoted by x_1 and x_2, respectively, where $x_1 \geqslant x_2$. Players have to pay their own bid in case they continue to the second stage of the game. Each firm can win at most one license. For simplicity, but without loss of generality, the next section analyzes a sealed-bid discriminatory auction where each firm submits only one (sealed) bid. For the formal part of the argument it is convenient to have a discrete strategy space and therefore, I assume firms can bid any amount $x = \varepsilon, 2\varepsilon, 3\varepsilon, \ldots$. The grid ε measures the bidding increment and I assume that ε is small.[8] In the second stage, the two winning firms play a market competition game. Firms can choose to play competitively (aggressively), denoted by A, or cooperatively, denoted by C. The 2×2 game is described in the matrix below:

$$\begin{array}{cc} & \begin{array}{cc} C & A \end{array} \\ \begin{array}{c} C \\ A \end{array} & \begin{pmatrix} a, a & c, d \\ d, c & b, b \end{pmatrix}, \end{array} \quad \text{where } a > d \geqslant b > c > 0.$$

So, the value v of winning the auction is uncertain, and can be equal to a, b, c or d. For simplicity, the pay-offs of the market game are supposed to be multiples of ε so that there exists a natural number n such that for example a can be written as $n\varepsilon$. Note that the restrictions on the pay-off parameters imply that there are two symmetric pure-strategy equilibria: (C, C) and (A, A), where the first equilibrium Pareto-dominates the second.[9] There is also a symmetric mixed strategy equilibrium where players play C with probability $\theta = (b - c)/((a - d) + (b - c))$. The profit, this mixed strategy equilibrium generates is equal to $(ba - dc)/((a - d) + (b - c))$ and it is easy to see that this expression is larger than or equal to b and smaller than or equal to d. The market game is not fully specified in order to allow for many different

[8]Note that in most auction designs a bid increment of some kind is implemented.

[9]For the argument that follows it is not necessary that $d \geqslant b$. The assumption is made to avoid writing $\max(b, d)$ each time. Moreover, in line with the collusion and trigger strategy interpretation, the assumption $d \geqslant b$ is more natural than the reverse.

interpretations. According to most interpretations, high profits and high market prices go together. One interpretation is that the market game is a static game with multiple Pareto-ranked equilibria as the search model described in Janssen and Moraga-Gonzalez (2004). Another interpretation of the market game is as a simplified version of repeated interaction in the market, where cooperative play can be supported as an equilibrium outcome.[10] In line with this second interpretation, I denote the action (price) of player i in the market game by p_i. The overall strategy of player i is then denoted by $(x_i, p_i(x_1, x_2))$.

I will first show that any type of market behaviour and many types of bidding behaviour can be part of a subgame perfect equilibrium. The next proposition shows that there are very many equilibria and that the notion of subgame perfection does not impose any further restrictions.

Proposition 1. *The following strategies are part of a symmetric equilibrium*:

(a) *For any* $\alpha \in [b-\varepsilon, a)$, $x_i = \alpha$ *and* $p_i(\alpha,\alpha) = C$ *and* $p_i(x_1, x_2) = A$ *if, and only if,* x_1 *or* x_2 *is not equal to* α.

(b) *For any* $\alpha \in [b-\varepsilon, (ba-dc)/((a-d)+(b-c)))$, $x_i = \alpha$ *and* $p_i(\alpha,\alpha) = C$ *with probability* θ *and* $p_i(\alpha,\alpha) = A$ *with probability* $1-\theta$ *and* $p_i(x_1, x_2) = A$ *if, and only if,* x_1 *or* x_2 *is not equal to* α.

(c) *For* $\alpha \in [b-\varepsilon, b)$, $x_i = \alpha$ *and* $p_i(x_1, x_2) = A$ *always.*

All these equilibria are also subgame perfect. The lowest bid that can be observed as part of a symmetric subgame perfect equilibrium equals

$$\max\left\{\frac{N(b-\varepsilon)-2a}{N-2}, 0\right\}.$$

Proof. All strategies are constructed in such a way that following the equilibrium strategies yields a positive pay-off. Unilaterally reducing the bid implies that the player will not win a license and get a pay-off of 0. Increasing the bid always implies bidding more than b and this also brings about lower overall pay-offs as the strategies are constructed in such a way that the off-the-equilibrium-path pay-off equals b. Therefore, deviating in the first stage of the game is not optimal. It is easy to see that the strategies used in the market stage are equilibrium strategies of the stage game. Deviating at this stage is thus also not optimal. Hence, the strategies are part of a subgame perfect equilibrium.

Let us denote by $\underline{x}$ the lowest bid in any symmetric subgame perfect equilibrium. The lowest bid is obtained if the equilibrium path prescribes playing cooperatively at the market stage and a deviation is followed by playing aggressively. The equilibrium pay-off then equals $2/N(a-\underline{x})$ and the best possible deviation pay-off equals $b-\underline{x}-\varepsilon$. The proposed equilibrium pay-off is larger than or equal to the best possible deviation pay-off if $\underline{x} \geqslant (N(b-\varepsilon)-2a)/(N-2)$. This gives the lower bound on the subgame perfect equilibrium bids. □

[10] When we take the fact that licenses are auctioned for a fixed period of time literally, cooperation can still be an equilibrium outcome for some periods of time if we allow uncertainty à la Kreps et al. (1982).

The equilibria described in the Proposition above are not all the subgame perfect equilibria of the game. The proof shows how to construct subgame perfect equilibria where players bid less than $b - \varepsilon$ and they play cooperatively if, and only if, the others stick to the equilibrium bid. For certain low values of N, it is also possible to construct subgame perfect equilibria where players always play aggressively in the market game, but nevertheless bid somewhat less than $b - \varepsilon$. I have not described these equilibria in detail as they rather specific conditions to hold. The main purpose of the Proposition is to show that very many subgame perfect equilibria exist.

Underlying the notion of subgame perfection is the view that deviations from a proposed equilibrium strategy are considered mistakes which are not informative about future behaviour (cf., Selten, 1975). The only requirement the notion imposes is that the strategies in the market game form an equilibrium in the market subgame and in our case there are two pure-strategy equilibria and one mixed-strategy equilibrium. The claim made in Section 1 that in the present context the notion of backward induction formalizes the view of "auction revenues are sunk cost" can now be investigated more deeply by considering the question why an equilibrium of type b or c mentioned in the Proposition above is subgame perfect? If a player deviates by bidding a relatively high amount, for example an amount higher than the (equilibrium) pay-off in the market stage, other players do not consider this as a signal that the deviating player wants to play C in the market stage, but instead analyze the market stage as an independent game where playing A is an equilibrium strategy. The fact that deviating by bidding high and subsequently playing aggressively yields negative pay-offs, and is a strictly dominated strategy in the overall game, is irrelevant as far as the notion of subgame perfection is concerned.

The argument I made in Section 1, namely that a deviation of a proposed equilibrium bid in the auction game should *not* be interpreted as a random mistake, but rather as a signal of future actions in the market game is what motivates the notion of forward induction (see, e.g., Kohlberg and Mertens, 1986; Van Damme, 1989, and others). Kohlberg and Mertens (1986, pp. 1013) argue that "a subgame should not be treated as a separate game, because it was preceded by a very specific form of preplay communication-the play leading to the subgame". A general definition of the forward induction argument cannot be found in the literature, however, and it is not the purpose of this paper to develop one.[11] Underlying the notion of forward induction is the idea that deviations from a proposed equilibrium should be interpreted as signals of future actions, if possible. The solution concept that is typically used in the context of forward induction arguments is iterative

[11]For a class of *two-player* games, Fudenberg and Tirole (1991, pp. 464) provide the following definition of the notion of forward induction. They say that an equilibrium is consistent with forward induction if it is not the case that some player, by deviating from the equilibrium path, can ensure that a proper subgame is reached where all solutions but one give the player strictly less than the equilibrium pay-off, and where exactly one solution gives the player strictly more. One may easily construct an example showing that an application of this definition to games with more than two players gives unreasonable results.

elimination of weakly dominated strategies (IEDS). It is this concept that I will use in the proofs.[12]

3. Analysis for two licenses

In this section, I prove one of the main results of the paper. The result says that provided the number of competing firms in the auction is large enough, the forward induction argument selects only one of the many subgame perfect equilibria. The equilibrium that is selected has firms coordinating on the high profit (price) equilibrium in the market game. Moreover, during the auction phase firms "burn" all their future profits, i.e., their bids are close to the profits obtained in the market game. There are three important steps in the proof, namely Steps (i)–(iii), and I will describe them informally here. First, the strategy "bid an amount in the auction game that is larger than the profits one can maximally achieve by choosing a competitive market strategy *and* choose a competitive market strategy" is dominated as it always leads to a negative profit. This in turn implies that if one of the firms that wins the auction has made a relatively high bid, the other firm can safely assume that this firm will choose to play cooperatively in the market game. Thus, the second step of the argument argues that strategies of the form "bid an amount x in the auction game and choose a competitive market strategy whenever the other winning firm has bid an amount that is larger than the profits one can maximally achieve by choosing a competitive market strategy" is dominated by a similar strategy where cooperative play in the market game is recommended. These two steps together assure that *if* someone bids relatively high in the auction, firms play cooperatively in the market game. The last step in the argument shows that given this anticipation, competition between a large enough number of contestants in the auction assures that it is indeed optimal to bid higher than the profits one can maximally achieve by choosing a competitive market strategy.

Theorem 1. *For N large enough, in particular when*

$$N \geqslant \max\left\{\frac{2a}{a-d}, 4\right\},$$

the unique equilibrium that is consistent with forward induction has $x_i = a - \varepsilon$ *and* $p_i(x_1, x_2) = C$, $i = 1, \ldots, N$.

The proof, especially Step (iii), highlights the use of a "jump bid".[13] Given the two earlier steps of the proof, informally described above, a bidder can only *guarantee* himself the highest possible continuation pay-off in the aftermarket, if he chooses a

[12] Recent papers by Asheim and Dufwenberger (2003a, b) show that the forward induction argument can also be based on the concept of fully permissible sets. Their arguments can also be applied in the present context.

[13] Of course, taking literally jump bids cannot take place in a sealed-bid auction. However, a similar analysis applies to a multi-unit ascending auction.

bid that is *larger than or equal to* the *maximal* pay-off of d one could get by following market strategy A. Up to that moment in the auction (proof) no bid is eliminated. The size of the jump bid is thus at least equal to d as the player has to make sure that there is only one way his bid can be interpreted.

One issue that remains to be discussed is why the argument only works when the number of firms is larger than a specific lower bound on the number of firms participating in the auction. The reason is the following. There is a possibility that bidding stops at the moment all participants to the auction do not bid at all. The two firms that are randomly selected face a coordination problem in the market game: Both playing cooperatively and both playing aggressively are both Nash equilibria. Even though there is no specific reason to do so, it may thus happen that both firms coordinate on playing cooperatively. The total pay-off for the two firms of following this strategy is then smaller than or equal to a. The chance of being selected is $2/N$. Note that the expected pay-off decreases in N as the chance of being selected in the lottery decreases. Each firm then faces the following decision problem: Being satisfied with this chance of getting a relatively large pay-off or "jump bidding" to a bid larger than or equal to d, which guarantees a pay-off of a in the market game. For "jump bidding" to be profitable, N has to be relatively large. The reason that N has to be larger than equal to 4 stems from the fact that if everyone bids $a-2\varepsilon$, the expected pay-off to each player is $4\varepsilon/N$. Unilaterally deviating to the next highest bid yields a pay-off of ε. The latter is higher than the former when $N \geqslant 4$.

4. Auctioning $k \geqslant 3$ licenses

In this section, we analyze to what extent the result of the previous section can be generalized to the case where $k \geqslant 3$ licenses are auctioned off. In order to discuss the implications of this generalization, we first need to generalize the market stage pay-offs to the case where k firms compete. The pay-offs when everyone behaves cooperatively or aggressively do not need to be modified. When $n<k$ players play cooperatively and the remaining $k-n$ players play aggressively, one may denote the pay-off to the aggressors and cooperators by d_n and c_n, respectively. It is natural to assume that $d_n<d_{n+1}$ and $c_n<c_{n+1}$, i.e., the more cooperators, the higher the pay-offs to both cooperators and aggressors. Moreover, I assume that the structure of the coordination game is unaffected, i.e., for every n the following holds: $a>d_n \geqslant b>c_n>0$. The bids of the k players that continue to play the market competition game are denoted by $x_1, \ldots, x_k$, respectively, where $x_1 \geqslant x_2 \geqslant \cdots \geqslant x_k$. As indicated in Section 1, the results for the case of $k \geqslant 3$ licenses crucially depend on the auction format. To see this, I will consider a sealed-bid discriminatory auction and an open format such as the simultaneous ascending auction in turn.[14]

In a discriminatory sealed-bid auction, the forward induction argument does not work. The overall strategy of player i in such an auction can be denoted by

[14]Some of the arguments presented below are related to the ideas expressed in Ben-Porath and Dekel (1992, pp. 44) who argue that the timing of the signaling is crucial in n-person games of common interest.

$(x_i, p_i(x_1, x_2, \ldots, x_k))$. To see why the forward induction arguments fails, consider the equilibrium in which every player chooses a bid just below b and play aggressively in the market game. The reason that no firm wants to deviate and "signal" the intention to play C in the market game (bidding high), is the following. Suppose one firm did deviate and bid higher. The other $k-1$ firms then face the following coordination "game" at the market stage. If $h<k-2$ out of the other $k-2$ winners cooperate, the pay-off in the market game to a player who did not bid above d_{k-1} is equal to c_{h+2} if he himself cooperates and equal to $d_{h+1}>c_{h+2}$ if he himself plays aggressively. On the other hand, if all the other $k-2$ winners cooperate, the pay-off in the market game to a player who did not bid above d_{k-1} is equal to a if he himself cooperates and equal to $d_{k-1}<a$ if he himself plays aggressively. Thus, for these players it is optimal to choose C in the market game if, and only if, all others play C. If only one player bid high in the auction, it is not clear what other players who have not bid high will do in the market game. As one aggressive player in the market game is sufficient for everyone to be better off with A, some players who have not bid high may well play A. So, if one player deviates in a sealed-bid auction and bid high, it may well be that the coordination problem at the market stage cannot be resolved. This in turn implies that bidding high (i.e., signalling the intention to play C in the market game), may not be followed by everyone playing C in the market game. As in a simultaneous auction bidders cannot react on each other, it may well be that only one player bids high and that then the above argument holds that the coordination problem at the market stage cannot be resolved. The result is that nobody may signal the intention to play C in the market game as players fear that others will not coordinate on the high pay-off equilibrium. Hence, the equilibrium in which every player chooses a bid just below b and play aggressively in the market game is consistent with forward induction when a simultaneous auction is held.

In an open format such as the simultaneous ascending auction players can react on each other and this makes the situation different. To analyze this type of auction design, I use the following notation: $x_i(x_i^0, \underline{x}^k)$ denotes the bid of player i at a certain moment during the auction when her highest bid so far is x_i^0 and the k-highest bid so far is $\underline{x}^k$; moreover, the notation $p_i(\overline{x}_1, \overline{x}_2, \ldots, \overline{x}_k))$, indicates that market behaviour is conditional on the final k-highest bids, where $\overline{x}_i$ denotes player i's final bid. Player i's strategy is then denoted by $(x_i(x_i^0, \underline{x}^k), p_i(\overline{x}_1, \overline{x}_2, \ldots, \overline{x}_k))$. In this case of a simultaneous ascending auction, the following result extends the analysis of the previous section to the case where there are three or more licenses for sale.

Theorem 2. *In a simultaneous ascending auction of $k \geqslant 3$ licenses the following holds. For any $N \geqslant \max\{ka/(a-d_{k-1}), 2k\}$ the unique equilibrium that is consistent with forward induction has $\widetilde{x}_i = a - \varepsilon$ and $p_i(\overline{x}_1, \overline{x}_2, \ldots, \overline{x}_k) = C$, $i = 1, \ldots, N$.*

The difference between the sealed-bid result and the result of an open format can be seen as follows. A crucial argument in the proof of Theorem 2 is the following induction argument. With k licenses, if $k-1$ players have bid an amount larger than or equal to d_{k-1}, $k-1$ players have signalled their intention to play C in the market game and therefore the cooperative equilibrium is selected in the market game. Now suppose that so far in the simultaneous ascending auction $k-2$ players have bid an amount larger than or equal to d_{k-1}. All remaining players can now reason that if they also will place a high bid, there will be $k-1$ high bids and then the above argument comes into effect. Therefore, these players will also bid higher than d_{k-1}. The iterative argument can then be extended to the case where $k-3$ players have bid an amount larger than or equal to d_{k-1}, and so on. For this iterative argument to work, players have to observe how many bids larger than d_{k-1} have been placed so far in the auction. It is clear that this information can be available in open auctions, but not in sealed-bid auctions.

The fact that the condition on N becomes tighter, when comparing Theorems 1 and 2, is understood if one realizes that with more licenses being auctioned, the chance of getting one of them increases if players bid relatively low amounts. The "jump" that has to be made in order to signal future cooperative behaviour remains the same, however, and therefore, the cost of making such a "jump bid". The main argument remains the same: If someone bids an amount during the auction that he cannot hope to receive in the market game by playing aggressively, i.e., if he bids more than d_{n-1}, then he signals future cooperative behaviour. Also, during the auction stage firms compete away their future profits, like in the Theorem stated in the previous section.

5. Discussion and conclusion

In this paper, I have shown in the context of a simple model how auctions may lead to high prices in the after-auction market. The main idea is that by bidding more than the profits a firm could possibly make by playing a competitive strategy in the market game, a firm signals that he will act cooperatively in the market game. Other firms pick up this signal and play cooperatively as well if they take part in the market game. As during the auction, firms compete to get a license to operate in the after-auction market, firms outbid each other during the auction game. Thus, all firms bid more than they possibly could make by competing in the after-auction market. When two licenses will be sold, this argument holds true for both sealed-bid auctions and simultaneous ascending auctions; when three or more licenses are sold, the argument fails to hold for sealed-bid auctions, but continuous to hold for simultaneous ascending auctions.

It is important to note that some conditions are necessary to make the argument work: (i) there should not be too much uncertainty about future market pay-offs, (ii) equilibrium behaviour in the market should be indetermined in the sense that multiple equilibria in the market game exists, (iii) the winning bids should be publicly observable, and (iv) the number of contestants in the auction should be relatively large. I will briefly comment on the first three points below; the fourth issue has already been discussed in Sections 3 and 4.

Concerning uncertainty, the argument made allows for some form of pay-off uncertainty as long as the maximum pay-off from competing aggressively in the market stage is smaller than the minimum possible pay-off of all firms cooperating in the market place. In case there is too much uncertainty about future pay-offs, auction fees cannot be interpreted as a signal of future market stage behaviour. In the case of the European UMTS-auctions, one may argue that they were held at such an early point in time that it was highly uncertain how much profits were to be gained. If this is so, the above argument does not apply.

When there is a unique equilibrium in the market stage, signalling future behaviour does not make much sense as future behaviour is fully determined by the market constellation itself. Hence, a necessary condition for our argument to work is the existence of multiple market equilibria. Finally, when the winning bids are *not* made public, firms cannot condition their market behaviour on these winning bids. Accordingly, the firms cannot use their bids to signal future intentions in this case.

It is not crucial to the argument, however, that firms are identical. For example, one could introduce a private value component in the following way: For any $v = a, b, c$ or d, one may write $v_i = v + \varepsilon_i^v$, where ε_i^v is private knowledge and drawn from some distribution F with support $[\underline{\varepsilon}^v, \overline{\varepsilon}^v]$. This makes clear that the pay-offs of cooperating or competing in the market place may depend on the firms' identity. It is relatively easy to see that if the private value component is not too large, more precisely if $a + \underline{\varepsilon}^a > d + \overline{\varepsilon}^d$, the core results hold true, i.e., firms will bid an amount close to the highest equilibrium pay-off and cooperate in the market stage. In this case, the winning firms are likely to make some profits, however, as we cannot eliminate bids higher than $a + \underline{\varepsilon}^a - \varepsilon$ and the firms with the highest valuations do not have to bid $\varepsilon + \underline{\varepsilon}^a$ close to their cooperation pay-off.

There are several interesting policy issues concerning auction design that come out of this paper. One issue that arises is that by announcing the winning bids, the government may facilitate coordinating on the high profit (price) equilibrium in the sense discussed in this paper. If only the identity of the winning bidders is revealed, but not their bids, firms cannot directly infer what the other firm has paid and therefore they cannot condition market behaviour on the bids. In this case, the argument developed in this paper breaks down.

A second issue is that coordinating on the high profit (price) equilibrium may be difficult to detect by competition authorities as no explicit communication is needed. Moreover, the firms may argue that the auction has forced them to pay so much that if they do not coordinate on the high profit equilibrium, they will go bankrupt. If bankruptcy of crucial firms in an economy is a serious concern for competition authorities, there is not much the authorities can do *after* the auction has taken place. Of course, the authorities may threaten ex ante that they will introduce severe punishments, but one may wonder whether this is a credible threat given the observation that ex post the authorities may not find it optimal to punish.

Appendix: Proofs

Proof of Theorem 1. The proof eliminates sets of strategies in four consecutive stages.

Step (i): Any strategy $(x_i, p_i(x_1, x_2))$ with $x_i \geqslant d$ and $p_i(x_1, x_2) = A$ whenever $x_i \geqslant x_2$ is weakly dominated by $(\widetilde{x}_i, \widetilde{p}_i(x_1, x_2))$ with $\widetilde{x}_i < b$ and $\widetilde{p}_i(x_1, x_2) = p_i(x_1, x_2)$.

To prove this claim, I will denote the first strategy by s_1 and the second one by s_2. When player i sets strategy s_1, her pay-off is either 0 or negative. I will show that by choosing strategy s_2, she can never do worse and sometimes better. There are three possibilities: $x_2 > x_i$, $\widetilde{x}_i \leqslant x_2 \leqslant x_i$, $\widetilde{x}_i > x_2$. In the first case, both s_1 and s_2 yield a pay-off of 0. In the second case, $\pi_i(s_1, s_{-i}) \leqslant 0 \leqslant \pi_i(s_2, s_{-i})$. In the third case, $\pi_i(s_1, s_{-i}) < 0 < \pi_i(s_2, s_{-i})$. Thus, s_2 weakly dominates s_1.

Step (ii): Any strategy $(x_i, p_i(x_1, x_2))$ that assigns $p_i(x_1, x_2) = A$ for some value of $x_1 \geqslant d$ is weakly dominated by $(\widetilde{x}_i, \widetilde{p}_i(x_1, x_2))$ with $\widetilde{x}_i = x_i$ and $\widetilde{p}_i(x_1, x_2) = C$ whenever $x_1 \geqslant d$ and otherwise $\widetilde{p}_i(x_1, x_2) = p_i(x_1, x_2)$.

To prove this claim, note that all strategies $(x_i, p_i(x_1, x_2))$ with $x_i \geqslant d$ that survived IEDS up to this stage have $p_i(x_1, x_2) = C$ because of Step (i). There are two cases then to consider: $x_i < x_2$ and $x_i = x_2$.[15] In the first case, both strategies yield a pay-off of 0. In the second case, let us denote by m the number of players with a bid equal to x_i. There are two subcases: x_1 equals a value larger than or equal to d to which the first strategy assigns $p_i(x_1, x_2) = A$ and all other values of x_1 including $x_1 < d$. In the first subcase, the overall pay-off of the first strategy is $(d - x_i)/m$, whereas the pay-off of the second strategy is $(a - x_i)/m$. In the second subcase, the actions prescribed by both strategies are identical and, therefore, the pay-offs are equal.

Steps (i) and (ii) together assure that if one player bids an amount larger than or equal to d in the auction, both players proceeding to the second stage of the game will choose to play cooperatively. The next step argues that all strategies that prescribe players to bid less than d in the auction are iteratively dominated. To this end let us denote by $\widehat{S}^C(0)$ the set of strategies $\{(x_i, p_i(x_1, x_2)) | x_i \geqslant 0$ and $p_i(x_1, x_2) = C$ if $x_1 \geqslant d\}$. Note that this class leaves the second stage action unspecified whenever $x_1 < d$. Let us also define $\widetilde{S}^C(0)$ as the subset of $\widehat{S}^C(0)$ with the lowest bid x_i, i.e., $\widetilde{S}^C(0) \equiv \{(x_i, p_i(x_1, x_2)) | x_i = 0$ and $p_i(x_1, x_2) = C$ if $x_1 \geqslant d\}$. Using these two notions, we can define $\widehat{S}^C(1) \equiv \widehat{S}^C(0) \backslash \widetilde{S}^C(0)$ and similarly to defining $\widetilde{S}^C(0)$, one can define $\widetilde{S}^C(1)$ as the subset of $\widehat{S}^C(1)$ with the lowest bid x_i, i.e., $\widetilde{S}^C(1) \equiv \{(x_i, p_i(x_1, x_2)) | x_i = \varepsilon$ and $p_i(x_1, x_2) = C$ if $x_1 \geqslant d\}$. Proceeding iteratively, I define for all $k > 1$, $\widehat{S}^C(k) \equiv \widehat{S}^C(k-1) \backslash \widetilde{S}^C(k-1)$ and $\widetilde{S}^C(k)$ as the subset of $\widehat{S}^C(k)$ with the lowest bid x_i. In each round the lowest bid itself in $\widetilde{S}^C(k)$ is $k\varepsilon$. Finally, I define K by $(K+1)\varepsilon = d$.

Step (iii): Fix a $0 \leqslant k \leqslant K$ and $\widehat{S}^C(k)$. Given that players' strategies are restricted to $\widehat{S}^C(k)$, all strategies in $\widetilde{S}^C(k)$ are weakly dominated by the strategy $(x_i, p_i(x_1, x_2))$ with $x_i = d$ and $p_i(x_1, x_2) = C$ for all pairs (x_1, x_2).

To prove this step, let us call the dominating strategy s_3. For each $k \geqslant 0$ there are three situations to consider. Either $x_2 \geqslant d + \varepsilon$, or $k\varepsilon < x_2 \leqslant d$, or $x_2 = k\varepsilon$. In the first case, all strategies in $\widetilde{S}^C(k)$ as well as strategy s_3 itself yield a pay-off of 0. In the second case, all strategies in $\widetilde{S}^C(k)$ yield a pay-off of 0, whereas strategy s_3 yields a positive pay-off (of either $a - d$ or the chance to get this pay-off) due to the fact that

[15]Note that the case $x_i > x_2$ is covered by (i) above as it implies that $x_i = x_1 \geqslant d$.

Step (ii) implies that if someone bids an amount higher than d, players play C. In the third case, the pay-off of strategy s_3 is equal to $a-d$, whereas the pay-off of choosing a strategy in $\widetilde{S}^C(k)$ cannot be larger than $2a/N$. When $N \geqslant 2a/(a-d)$, the first pay-off is not smaller than the second.

Steps (i)–(iii) imply that all strategies with bids $x_i < d$ are iteratively eliminated. The last step of the argument then is a conventional auction type of argument. To this end, define K^A as the integer such that $(K^A+1)\varepsilon = a$.

Step (iv): Fix an integer k with $K+1 \leqslant k \leqslant K^A - 1$. Given that players bidding strategies are restricted to $x_i \geqslant k\varepsilon$, any strategy $(x_i, p_i(x_1, x_2))$ with $x_i = k\varepsilon$ and $p_i(x_1, x_2) = C$ for all pairs (x_1, x_2) is iteratively dominated by $(\widetilde{x}_i, \widetilde{p}_i(x_1, x_2))$ with $x_i = (k+1)\varepsilon$ and $\widetilde{p}_i(x_1, x_2) = p_i(x_1, x_2)$.

Given Steps (i)–(iii) firms always bid $x_i \geqslant d$ and play cooperatively in the market game, which guarantees a pay-off of a of winning the auction. Firms would like therefore, to outbid each other, which drives the bids in the auction up. The last step of this stage of the elimination procedure is the most stringent and gives a good idea about the previous steps. So, let us briefly consider the argument for this last step. In this case I have to argue that any strategy $(x_i, p_i(x_1, x_2))$ with $x_i = (K^A - 1)\varepsilon$ is iteratively dominated by $(\widetilde{x}_i, \widetilde{p}_i(x_1, x_2))$ with $\widetilde{x}_i = K^A\varepsilon$ and $\widetilde{p}_i(x_1, x_2) = p_i(x_1, x_2) = C$. There are two possible situations to consider: Either $x_2 = K^A\varepsilon$, or $x_2 = (K^A - 1)\varepsilon$. In the first case, the strategy with $x_i = (K^A - 1)\varepsilon$ yields a pay-off of 0, whereas the strategy with $x_i = K^A\varepsilon$ yields a positive expected pay-off. In the third case, the pay-off of the strategy with $x_i = (K^A - 1)\varepsilon$ yields a pay-off of at most $2(a - (K^A - 1)\varepsilon)/N$. The pay-off of the strategy with $x_i = K\varepsilon$ yields a pay-off of $a - K^A\varepsilon$. As $(K^A - 1)\varepsilon = a - 2\varepsilon$, this latter expression is not smaller than the first expression if $\varepsilon \geqslant 4\varepsilon/N$, or $N \geqslant 4$.

Finally, as it is clear that bidding a is weakly dominated by bidding $a - \varepsilon$, the result follows. □

Proof of Theorem 2 (*Sketch*).[16] The proof eliminates sets of strategies in several consecutive stages. The first and the last step are similar to Steps (i) and (iv) of the proof of Theorem 1 and are, therefore, not formally given here. The first step results in that bidding more than one ever could get by playing aggressively in the market game, i.e., bidding more than d_{k-1}, and playing aggressively in the market game is dominated. Steps (ii) and (iii) require some modifications and basically have to be replaced by an iterative procedure.

To this end, suppose we have executed the first step of the elimination procedure and that the auction has proceeded so far that x_i^0 and $\underline{x}^k$ are well-defined.[17] I then

[16]When I mention pay-offs in this proof, I implicitly assume that the pay-offs to a player under consideration are not affected by future bidding in the auction by himself or any other player. Future bidding will never make the lower bid better than the higher bid and as I claim that higher bids dominate lower bids, the argument will never be reversed when future bids would be taken into account.

[17]In case bidder i has not bid yet or if less than k different players have bid, one can set $x_i^0 = 0$ and/or $\underline{x}^k = 0$.

claim that any strategy $(x_i(x_i^0, \underline{x}^k), p_i(\overline{x}_1, \overline{x}_2, \ldots, \overline{x}_k))$ that assigns $p_i(\overline{x}_1, \overline{x}_2, \ldots, \overline{x}_k) = A$ for some value of $\underline{x}^{k-1} \geqslant d_{k-1}$ is weakly dominated by $(\widetilde{x}_i(x_i^0, \underline{x}^k), \widetilde{p}_i(\overline{x}_1, \overline{x}_2, \ldots, \overline{x}_k))$ with $\overline{x}_i = x_i$ and $\widetilde{p}_i(\overline{x}_1, \overline{x}_2, \ldots, \overline{x}_k) = C$ whenever $\underline{x}^{k-1} \geqslant d_{k-1}$ and $\widetilde{p}_i(\overline{x}_1, \overline{x}_2, \ldots, \overline{x}_k) = p_i(\overline{x}_1, \overline{x}_2, \ldots, \overline{x}_k)$ otherwise.

To prove this claim, note that all strategies $(x_i, p_i(\overline{x}_1, \overline{x}_2, \ldots, \overline{x}_k))$ with $x_i \geqslant d_{k-1}$ that survived IEDS up to this stage have $p_i(\overline{x}_1, \overline{x}_2, \ldots, \overline{x}_k) = C$ because of Step (i). There are two cases then to consider: $x_i < \underline{x}^k$ and $x_i = \underline{x}^k$.[18] In the first case, both strategies yield a pay-off of 0. In the second case, let us denote by m the number of players with a bid equal to x_i. There are two subcases to be considered: $\underline{x}^{k-1} \geqslant d_{k-1}$ which according to the first strategy is followed by $p_i(\overline{x}_1, \overline{x}_2, \ldots, \overline{x}_k) = A$ and all other values of $\underline{x}^{k-1}$. In the first subcase, the overall pay-off of the first strategy is $(d_{k-1} - x_i)/m$, whereas the pay-off of the second strategy is strictly larger, namely $(a - x_i)/m$. In the second subcase, the actions prescribed by both strategies are identical and, therefore, the pay-offs are equal.

The above argument assures that if $k-1$ players bid an amount larger than or equal to d_{k-1} in the auction, all k players proceeding to the second stage of the game will choose to play cooperatively. The next step, similar to Step (iii) of the proof of Theorem 1, argues that if there are already $k-2$ bids above d_{k-1} all strategies that prescribe players to bid less than d_{k-1} in the auction are iteratively dominated. To make the claim more precise, I use notation similar to that in the proof of Step (iii) of Theorem 1 with, e.g., $\widetilde{S}^C(0) = \{(x_i(x_i^0, \underline{x}^k), p_i(\overline{x}_1, \overline{x}_2, \ldots, \overline{x}_k)) | x_i(x_i^0, \underline{x}^k) = 0$ and $p_i(\overline{x}_1, \overline{x}_2, \ldots, \overline{x}_k) = C$ if $\overline{x}_{k-1} \geqslant d_{k-1}\}$. So, the next claim, similar to Step (iv) of the proof of Theorem 1, is the following: Fix a $0 \leqslant l \leqslant K^d$, where K^d is such that $(K^d + 1)\varepsilon = d_{k-1}$, and $\widehat{S}^C(l)$. Given that players' strategies are restricted to $\widehat{S}^C(l)$, all strategies in $\widetilde{S}^C(l)$ are weakly dominated by the strategy $(x_i(x_i^0, \underline{x}^k), p_i(\overline{x}_1, \overline{x}_2, \ldots, \overline{x}_k))$ with $x_i(x_i^0, \underline{x}^k) = d_{k-1}$ and $p_i(\overline{x}_1, \overline{x}_2, \ldots, \overline{x}_k) = C$ for all $(\overline{x}_1, \overline{x}_2, \ldots, \overline{x}_k)$.

To prove this claim we should consider for each $l \geqslant 0$ three situations. Either $\underline{x}^k \geqslant d_{k-1} + \varepsilon$, or $l\varepsilon < \underline{x}^k \leqslant d_{k-1}$, or $\underline{x}^k = l\varepsilon$. In the first case, all strategies in $\widetilde{S}^C(l)$ as well as the dominating strategy itself yield a pay-off of 0. In the second case, all strategies in $\widetilde{S}^C(l)$ yield a pay-off of 0, whereas the dominating strategy yields a positive pay-off due to the fact that Step (ii) implies that if $k-1$ bidders bid an amount larger than or equal to d_{k-1}, players play C. In the third case, the pay-off of the dominating strategy is $a - d_{k-1}$, whereas the pay-off of choosing a strategy in $\widetilde{S}^C(l)$ cannot be larger than ka/N. When N satisfies the condition mentioned in the Theorem, the first pay-off is not smaller than the second.

The rest of the proof of Step (iii) proceeds by induction on j: If there are already $k-j$ bids above d_{k-1} all strategies that prescribe players to bid less than d_{k-1} in the auction are iteratively dominated. The above two claims together argue that this induction claim is true for $j = 2$. Arguments similar to these two claims, and using the fact that players know that if they signal others will follow, can be used to argue that it also holds for $j = 3, \ldots, k$. □

[18]Note that the case $x_i > \underline{x}^k$ is covered by (the not explicitly treated) Step (i) above as it implies that $x_i \geqslant \underline{x}^{k-1} \geqslant d_{k-1}$.

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