

**DYNAMIC TRADING IN A DURABLE GOOD MARKET
WITH ASYMMETRIC INFORMATION***

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We analyze a dynamic version of the Akerlof–Wilson “lemons” market in a competitive durable good setting. There is a fixed set of sellers with private information about the quality of their wares. The price mechanism sorts sellers of different qualities into different time periods—prices and average quality of goods traded increase over time. Goods of *all* qualities are traded in finite time. Market failure arises because of the waiting involved—particularly for sellers of better quality. The equilibrium path may exhibit intermediate breaks in trading.

1. INTRODUCTION

The difficulties associated with trading under asymmetric information due to *adverse selection* were first pointed out by Akerlof (1970), who analyzed trading possibilities in a static Walrasian market where each seller is privately informed about the quality of his endowment and the valuations of both buyers and sellers depend on quality. In such a market only low-quality goods are traded, if at all, even if the buyers are willing to pay more than the reservation price of sellers for each individual quality (see also Wilson, 1979). This “lemons problem,” as it has come to be known, afflicts not only competitive markets but a wide class of trading arrangements (including “nonmarket” mechanisms). The primary contribution of the Akerlof model, however, is that it provides an information-based theory of inefficiency in *competitive* markets.

When the commodity being traded is durable and we allow for trading over time, goods not traded in any period can be offered for sale in the future. Sellers with goods of higher quality are more willing to wait and trade at higher prices in later periods (relative to those with lower-quality goods). It follows that the kind of

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prediction obtained in the static model and, in particular, the insight gained about the nature of market failure may be significantly altered in dynamic settings. This article aims to re-examine the nature of the so-called “lemons problem” when durable goods are traded in a competitive market and repeated opportunities for trading occur over time. The existing literature has focused on analysis of dynamic trading in comparable settings when trading occurs under “nonmarket mechanisms” such as bargaining and auctions with strategic price setting. In contrast, we focus on the way in which the classical price mechanism functions.

Our specific model is as follows: We consider a Walrasian market for a durable good with a continuum of traders having perfect foresight. There is a fixed set of potential sellers, each endowed with a unit of the good. The quality of the good varies across sellers and is known only to the seller who owns it. There are a large number of identical buyers with unit demand. The valuations of both buyers and sellers depend on quality and for each specific quality, a buyer’s valuation exceeds that of a seller, so that there is always a positive gain from trading (under full information). A seller chooses to wait until the period in which he maximizes his discounted net surplus from selling. As in the literature on bargaining and auctions, our enquiry confines attention to the possibility of trading a *given* set of goods with a fixed set of sellers; that is, no new seller enters the market after the initial period. Finally, we assume that buyers leave the market after trading; that is, there is no scope for reselling the good.²

A buyer’s willingness to pay depends on her expectation about quality. In equilibrium, buyers’ expectations must be matched by the average quality of goods traded. The latter is not well defined in periods where no trade occurs. If we do not impose any restriction on buyers’ beliefs about quality in such periods, then there is a large set of possible outcomes, including one in which trade never occurs no matter how favorable the distribution of quality. We impose a mild consistency requirement, viz, that buyers do not expect the quality in any period to be lower than the lowest unsold quality at the beginning of that period (on the equilibrium path).

We show that a dynamic equilibrium exists and, more importantly, *in every equilibrium, all goods, including those of the highest possible quality, are traded in finite time.* The support of the distribution of quality can be partitioned into consecutive intervals, successive higher intervals being traded in later periods. Prices increase over time—reflecting increases in the average quality of goods traded. The main implication of this result is that the prevailing perception about the “lemons problem” as being a problem of *trading* higher-quality goods does not readily extend to dynamic environments. Instead, the problem manifests itself in the fact that sellers

² Note that even if cohorts of new sellers or new goods emerge over time, as long as the goods (or their sellers) can be distinguished by their year of entry into the market, goods entering the market in different periods would trade in separate markets (e.g., markets where goods of a particular vintage are traded). Also, if the number of times a good is traded is observable, whenever a good is retraded, such trading takes place in a separate market. These features are present in some used car markets. For an analysis of dynamic equilibria when cohorts of new sellers enter the market over time and the goods cannot be distinguished by period of entry, see Janssen and Roy (1999) and Janssen and Karamychev (2000).

wait in order to trade and sellers with goods of higher quality wait more than those with lower quality. Even though all goods are traded, market failure arises as future gains from trade are discounted.

The extent of inefficiency is related to the length of waiting involved and the rate of impatience plays an important role here. At lower rates of impatience, traders have a higher incentive to wait. Therefore, in order to reduce the incentive to wait for low-quality sellers, price differences across periods need to be smaller and, as prices reflect average quality traded, the size of the intervals of quality traded is also smaller; that is, it takes longer to realize the gains from trading higher-quality goods. Sometimes, even this is not enough to ensure the right incentives for sorting, in which case the equilibrium path may involve no trading for some intermediate periods (which also reduces the incentive to wait for lower-quality sellers). We develop an example of a market where all dynamic equilibria are characterized by intermediate breaks in trading.

There are several important strands in the existing literature that relate to our article. First, there is a large volume of literature on functioning of markets where the price mechanism is augmented by other nonmarket institutions or technologies that enable signaling or screening of information by allowing agents to choose actions which change the information structure endogenously. In particular, there is a growing literature on how institutional innovations (such as costly inspection of quality, certification intermediaries, and leasing rather than selling) and intermarket interactions between primary and secondary markets reduce the severity of the “lemons problem” in used goods markets.³ It is important to emphasize that, in contrast to this literature, our analysis does *not* focus on the actual working of specific markets or the institutions observed in these markets. Rather, we wish to understand the problems of resource allocation when we rely exclusively on the price mechanism within a single market and there are no other institutions, technologies, or processes that can modify the information structure.

The second strand of literature, which is actually closer to our analysis, is that of strategic dynamic trading under asymmetric information when prices are explicitly set by traders. Uninformed traders setting prices may use the price sequence as a way of screening the private information of informed traders. On the other hand, prices as well as the decision to wait may be used by informed traders to strategically manipulate the beliefs of uninformed traders.

In models of durable goods monopoly (where prices are set by a seller who is uninformed about the valuation of the buyer and the valuations of the seller and the buyers are uncorrelated), it is well known that the equilibrium path is characterized by the “skimming property” (higher valuation buyers buy earlier) and intertemporal price discrimination with prices *decreasing* over time.⁴ While these features are somewhat similar to the properties of the equilibria in our model, one important result of these models is that as the real time difference between offers goes to zero

³ See, among others, Guha and Waldman (1997), Hendel and Lizzeri (1999a,b), Lizzeri (1999), Taylor (1999), and Waldman (1999).

⁴ See, for example, Fudenberg et al. (1985) and Gul et al. (1986).

(effectively, the rate of impatience goes to zero), in the limit all trade takes place in the initial instant. In our model, as the rate of impatience goes to zero, the time required for all goods to be traded tends to infinity.

In a more general setting, the literature on sequential bargaining with one-sided incomplete information analyzes situations where a buyer and a seller bargain in order to arrive at an agreement over trading a unit of an indivisible good and the valuation of either the buyer or the seller is private information. The broad result obtained in this class of models is that the possibility of trading over time and waiting to trade may lead to intertemporal price discrimination and signaling or sorting of private information.⁵ Vincent (1989) analyzes sequential bargaining when (as in Akerlof's model) the valuations of the buyer and the seller are correlated and the seller's valuation is private information (see also, Evans, 1989). As in our model, higher-quality sellers trade later and the trading process ends in finite time (when valuations are bounded). Further, there is delay to agreement as sellers use waiting over real time to signal their information.⁶

Closer to our competitive market mechanism are models of auctions. Directly comparable to our model is the dynamic auction game analyzed by Vincent (1990) where two uninformed buyers engage in Bertrand-like competition in order to purchase a perfectly durable object of uncertain quality from an informed seller. There is a close correspondence between the equilibrium outcomes of this game and the dynamic equilibria of our model—though the actual solution concept used there is, in effect, much stronger than ours.

In order to clarify the concept of equilibrium and the nature of refinement implicit in the restriction on beliefs adopted in our article, we consider a strategic signaling version of our model where sellers set prices every period, following which buyers decide whether or not to buy. The dynamic equilibrium outcomes in our Walrasian model are closely related to outcomes in perfect Bayesian equilibria (of this dynamic game) that meet the refinement induced by the "intuitive criterion."

Finally, it should be pointed out that at a fundamental level, our analysis is closely related to Wilson (1980), where it was pointed out that the clue to sorting of types in an anonymous market suffering from adverse selection is through price dispersion. In our model, time offers a natural way to segment the market and create price variations.⁷

The plan of the article is as follows: Section 2 sets out the model and the equilibrium concept. In Section 3, we state our main results about existence and char-

⁵ See, among others, Fudenberg and Tirole (1983), Sobel and Takahashi (1983), Cramton (1984), Gul and Sonnenschein (1988), and Ausubel and Deneckere (1989).

⁶ When the buyer offers prices, she wants to keep prices paid to sellers of different qualities far enough apart so as to reduce the possibility of buying a low-quality good at a high price. As a result, even if the time difference between price offers goes to zero, delay can persist; this is a property that we also find in our model.

⁷ In Wilson's article, in a version of the model where sellers set prices, the sorting device used is the probability of trade; sellers with higher quality quote a higher price and sell with a lower probability because they are more willing to be stuck with the good. In our model, the sorting device is waiting; higher-quality sellers sell at higher prices but wait longer.

acterization. Section 4 outlines a sufficient condition under which there is “no break in trading.” Section 5 outlines an example where every equilibrium path is characterized by breaks in trading. Section 6 relates our model to comparable strategic models of signaling and dynamic auction. Section 7 concludes. Proofs of the main results are contained in the Appendix.

2. THE MODEL

Consider a Walrasian market for a perfectly durable good whose quality, denoted by θ , varies between $\underline{\theta}$ and $\bar{\theta}$. Time is discrete and is indexed by $t = 1, 2, \dots, \infty$. All agents discount their future return from trading using a common discount factor δ , $0 \leq \delta < 1$.⁸ There is a continuum of (potential) sellers; the set of all potential sellers is the unit interval, denoted by I . Each seller i is endowed with one unit of the durable good in the initial time period; seller i knows the quality $\theta(i)$ of the good he is endowed with. Seller i 's valuation (reservation price) of the good is his infinite horizon discounted sum of gross surplus derived from ownership of the good, and we assume that it is exactly equal to $\theta(i)$. Thus, the per-period gross surplus derived by seller i from owning the good is $(1 - \delta)\theta(i)$.⁹ Ex ante, sellers are distributed over quality according to a probability measure μ and an associated distribution function F . No seller enters the market after the initial time period; that is, the total stock of goods to be traded is fixed. We assume that the distribution of quality is continuous with no mass point. In particular,

A1. The support of μ is an interval $[\underline{\theta}, \bar{\theta}]$, $0 < \underline{\theta} < \bar{\theta} < +\infty$. The distribution function F is continuous and strictly increasing on this support.

There is a continuum of (potential) buyers of measure greater than 1. All buyers are identical and have unit demand. A buyer's valuation of a unit of the good with quality θ is equal to $v\theta$ where $v > 1$. Thus, for any specific quality, a buyer's valuation exceeds the seller's. Buyers know the *ex ante* distribution of quality, but do not know the quality of the good offered by any particular seller.¹⁰ Once trade occurs, the buyer leaves the market with the good she has bought; that is, there is no scope for reselling.¹¹

In the static version of our model, goods of all qualities are traded in the market if $vE(\theta) \geq \bar{\theta}$; that is, there is no “lemons problem.” In this article, we confine attention

⁸ In our model, all buyers earn zero surplus in equilibrium and therefore our results remain unaffected even if buyers use a discount factor different from sellers; it is, however, important that all sellers use the same discount factor.

⁹ An alternative interpretation of the model (suggested by G. Mailath) is that each seller can produce a unit of the good in any period he wishes; the quality of the good produced by seller i as well as its cost of production is $\theta(i)$.

¹⁰ The assumption that $v > 1$ and $\underline{\theta} > 0$ implies that in the one-period version of our model, some low-quality goods are always traded (as in Wilson, 1979). So the static version of our model differs from the specific case contained in Akerlof (1970) where no trade occurs.

¹¹ As noted earlier, if the number of transactions a particular commodity has undergone is publicly observable, then the market where a buyer resells the good is separate from the one where she initially buys.

to situations where there is a problem of trading high-quality goods in the one-period version of the model and assume that

A2. $vE(\theta) < \bar{\theta}$

Observe that A1 implies that any nondegenerate subinterval of $[\underline{\theta}, \bar{\theta}]$ has strictly positive measure. For x, y such that $\underline{\theta} \leq x < y \leq \bar{\theta}$, let $\eta(x, y)$ denote the *conditional expectation of quality* θ given that $\theta \in [x, y]$. It is easy to show that $\eta(x, y)$ is continuous on $\{(x, y): \underline{\theta} \leq x < y \leq \bar{\theta}\}$. For any $y \in (\underline{\theta}, \bar{\theta}]$, $\eta(x, y)$ is strictly increasing in x on $[\underline{\theta}, y)$ and for any $x \in [\underline{\theta}, \bar{\theta})$, $\eta(x, y)$ is strictly increasing in y on $(x, \bar{\theta}]$.

Given a sequence of anticipated prices $\mathbf{p} = \{p_t\}_{t=1,2,\dots,\infty}$, each seller chooses whether or not to sell and if he chooses to sell, the time period in which to sell. A seller with quality θ can always earn gross surplus θ by not selling. If he sells in period t , his *net* surplus is

$$\sum_{i=0,\dots,t-1} [(1 - \delta)\theta]^i + \delta^t p_t - \theta = \delta^t(p_t - \theta)$$

The set of time periods in which a potential seller with quality θ (facing prices \mathbf{p}) finds it optimal to sell is denoted by $T(\theta, \mathbf{p})$. Formally,

$$T(\theta, \mathbf{p}) = \{t \geq 1: \theta \leq p_t \text{ and } \delta^{t-1}(\theta - p_t) \geq \delta^{t'-1}(\theta - p_{t'}) \text{ for all } t' \geq 1\}$$

We set $T(\theta, \mathbf{p}) = \{\infty\}$ if the above set is empty; that is, a potential seller with quality θ (facing prices \mathbf{p}) finds it optimal not to sell in any period. Each potential seller $i \in I$ chooses a particular time period (including “infinity”) $\tau(i, \mathbf{p}) \in T(\theta(i), \mathbf{p})$ in which to sell. This, in turn, generates a certain distribution of quality among goods offered for sale in each time period t . Given \mathbf{p} , the average quality of goods offered for sale in period t is $E\{\theta(i) \mid i \in I, \tau(i, \mathbf{p}) = t\}$.

Market equilibrium requires that in periods in which trade takes place, buyers’ expected quality should equal the average quality sold. As all potential buyers are identical, we will assume that their belief about quality is symmetric. Further, as we have assumed that there are always more buyers than sellers, in any period in which trade occurs, buyers must earn zero expected net surplus; that is, the price must equal the buyers’ valuation of the average quality traded in that period. This implies that even though buyers can choose to trade in any period, in equilibrium they are indifferent between trading in any of the periods in which positive trading takes place and not trading at all. A dynamic equilibrium is defined as a situation where all agents maximize their objectives, expectations are fulfilled, and markets clear every period.

DEFINITION. A dynamic equilibrium is given by a price sequence $\mathbf{p} = \{p_t\}_{t=1,2,\dots,\infty}$, a set of selling decisions $\tau(i, \mathbf{p}), i \in I$, and a sequence $\{E_t(\mathbf{p})\}_{t=1,2,\dots,\infty}$, where $E_t(\mathbf{p})$ is the (symmetric) expectation of quality in period t held in common by all buyers, such that

- (i) *Sellers maximize:* $\tau(i, \mathbf{p}) \in T(\theta(i), \mathbf{p})$; that is, it is optimal for seller i with quality $\theta(i)$ to sell in period $t(i, \mathbf{p})$.
- (ii) *Buyers maximize and markets clear:* If $\mu\{\theta(i) \mid i \in I, \tau(i, \mathbf{p}) = t\} > 0$, then $p_t = vE_t(\mathbf{p})$; that is, if strictly positive measure of trade occurs, then buyers

must be indifferent between buying and not buying so that the market clears. If $\mu\{\theta(i) \mid i \in I, \tau(i, \mathbf{p}) = t\} = 0$, then $p_t \geq vE_t(\mathbf{p})$; that is, if no trade occurs, then it must be optimal for buyers to not buy in that period.

- (iii) *Expectations are fulfilled*: The expectations of quality in periods of positive trading must exactly equal the average quality of goods sold in that period; that is,

$$E_t(\mathbf{p}) = E\{\theta(i) \mid i \in I, \tau(i, \mathbf{p}) = t\}, \quad \text{if } \mu\{\theta(i) \mid i \in I, \tau(i, \mathbf{p}) = t\} > 0$$

- (iv) *Minimal consistency of beliefs*: Even if no trading occurs in a period, as long as there is a positive measure of unsold goods in the market, buyers never expect quality to fall below the *lowest unsold quality*; that is, if $\mu\{\theta(i) \mid i \in I, \tau(i, \mathbf{p}) = t\} = 0$ and $\mu\{\theta(i) \mid i \in I, \tau(i, \mathbf{p}) > t - 1\} > 0$, then $E_t(\mathbf{p}) \geq \inf\{\theta(i) : i \in I, \tau(i, \mathbf{p}) > t - 1\}$.

While conditions (i)–(iii) are standard, condition (iv) requires further explanation as it is a constraint on the belief about quality held by buyers (i.e., their willingness to pay) in periods in which no goods are traded. To see the reasoning behind condition (iv), suppose for a moment that equilibrium is defined only by conditions (i)–(iii). First, without condition (iv), the autarkic outcome where no trade occurs in any period is always sustainable as an equilibrium outcome (no matter how large v is or how good the distribution of θ is). To construct such an equilibrium, set prices equal to zero in every period. No seller would wish to trade in any period. As there is no restriction on expectation about quality in periods of no trading, buyers can expect quality to be zero in every period and so they are indifferent between buying and not buying at zero price. A moment’s introspection will reveal that there is something odd about the equilibrium specified above. If it is common knowledge that the distribution of quality is $[\underline{\theta}, \bar{\theta}]$, so that all tradeable goods have quality at least as large as $\underline{\theta} > 0$, a buyer’s expectation about quality should not be below $\underline{\theta}$; that is, her willingness to pay should not be below $v\underline{\theta}$ in any period.¹² Therefore, it appears reasonable that we should impose a restriction such as

$$(2.1) \quad E_t(\mathbf{p}) \geq \underline{\theta} \quad \text{for all } t$$

so that equilibrium prices are bounded below by $v\underline{\theta}$. However, that does not quite get rid of the basic problem. Consider a quality level θ^s defined by

$$(2.2) \quad \theta^s = \sup\{\theta \in [\underline{\theta}, \bar{\theta}] : v\eta(\underline{\theta}, \theta^s) = \theta^s\}$$

θ^s is the highest quality that would be sold in the one-period version of this model. As $vE(\theta) < \bar{\theta}$, we have $\theta^s < \bar{\theta}$. We shall refer to θ^s as the *static quality*. It is easy to check that under a restriction like (2.1), an outcome where sellers with quality lying in $[\underline{\theta}, \theta^s]$ trade at price θ^s in period 1, and trade never occurs after that, is sustainable as an equilibrium. The reason is simple. Once goods of quality in the range $[\underline{\theta}, \theta^s]$ are

¹² In fact, if we allow for this kind of equilibrium, then no trade would always be a market outcome even in a static model and, in a limiting case, even if the distribution of quality degenerates to “no uncertainty about quality” (no information problem).

traded in the market in period 1, we can set prices equal to $v\underline{\theta}$ from period 2 onwards and, by definition, $v\underline{\theta} < \theta^s$ so that no seller with an untraded good (whose quality must be greater than θ^s) will be willing to sell. We can set $E_t(\mathbf{p}) = \underline{\theta}$ for $t \geq 2$ so that restriction (2.1) is satisfied and all buyers are indifferent. However, the same reasoning that makes us doubt the autarkic outcome and impose a restriction like (2.1) also suggests that there is something unreasonable about the equilibrium outlined. For if buyers anticipate the equilibrium price sequence, they can easily see that all sellers with quality below θ^s will have sold their goods in period 1 so that in periods $t > 1$, while there is a positive measure of unsold and potentially tradeable goods in the market, none of them are of quality below θ^s . Hence, in periods $t > 1$, even if no trader actually offers to sell, it is not reasonable for buyers to expect quality of the good to be below θ^s . As price-taking buyers, their expected net surplus from buying at a price like $v\underline{\theta}$ (which is less than θ^s) should be strictly positive. This is what motivates condition (iv) of the definition of equilibrium. It requires that buyers' expectations of quality in any period should not lie below the minimum unsold quality as long as there is a positive measure of goods unsold.

Note that similar "refinements" have been used in other dynamic trading models with price-taking agents in order to rule out trivial equilibria and to incorporate rational conjectures by agents in periods of no trading that are not inconsistent with the incentives that agents have in equilibrium (see, e.g., Dubey et al., 1988).

3. EXISTENCE AND CHARACTERIZATION: ON THE POSSIBILITIES OF DYNAMIC TRADING

In this section, we present the main results of this article relating to existence of dynamic equilibrium and the nature of dynamic equilibria.

It can be shown that in any equilibrium, prices and quality traded increase over time. This is easy to see once we realize that the incentive of a seller with quality θ to wait for a future price is strictly increasing in θ .¹³ This implies that in any period, the set of quality traded until that period is an interval $[\underline{\theta}, x]$ where the seller of quality x is indifferent between selling and not selling in period t , while every seller with quality less than x (for $x > \underline{\theta}$) strictly prefers to sell before period t . The support of quality $[\underline{\theta}, \bar{\theta}]$ can be partitioned into nondegenerate intervals such that sellers with quality lying in the first interval sell in period 1 and successively higher intervals are traded in later periods. Increasing prices simply reflect the increase in average quality of goods traded.

More interestingly, it can be shown that *in any dynamic equilibrium all goods (no matter how high the quality of such goods) must be traded in finite time*. The reasoning behind why the entire range of quality must be traded in finite time is in two steps.

¹³ In the literature on bargaining under incomplete information, this is just the standard *skimming property* that, among other things, forms part of the Coasian dynamics characterizing perfect Bayesian equilibria (see, e.g., Fudenberg et al., 1985).

First, if trading were to occur for infinite periods, then given our earlier discussion, along the sub-sequence of time periods in which positive trading occurs, the highest and lowest qualities traded must be increasing and convergent (as they are bounded above)—converging to some quality level θ^* (say), while the prices in those periods must converge to $v\theta^*$. As $v > 1$, this would mean that the surplus earned by sellers is bounded away from zero. However, in each period of trading, the (marginal) seller selling the highest quality traded in that period must be indifferent between selling in the current and in the next period of trading. Therefore, the ratio of surplus earned by such a seller from selling in the current period to his surplus from selling in the next period of trading cannot exceed $\delta < 1$; this is contradicted by the fact that both these surpluses converge to the same positive number. Therefore, trade can occur only for a finite number of time periods.

Next, observe that there cannot be any subset of $[\underline{\theta}, \bar{\theta}]$ that has positive measure and that is never traded for in that case; under condition (iv) of the definition of equilibrium, buyers' valuations, and hence prices, must eventually exceed v times the highest quality that is traded (identical to the lowest unsold quality), which would create incentives for further trading.

However, trade need not occur in all periods. The seller with the marginal quality traded in a particular period is indifferent between selling in that period and the next period in which trade occurs. If there are intermediate periods where no trade occurs, then the prices must be such that this seller prefers not to sell in these periods. Of course, condition (iv) of the definition of equilibrium ensures that the price in any such intermediate period must be at least as large as v times the reservation price of this seller.

We summarize our results in the following proposition:

PROPOSITION 3.1. *Every dynamic equilibrium path is characterized by a partition of the range of possible quality into intervals such that goods of quality lying in higher intervals are traded in later periods and at higher prices. All goods are traded in finite time. More formally, for any dynamic equilibrium $[\mathbf{p} = \{p_t\}, \tau(i, \mathbf{p}), i \in I, \{E_t(\mathbf{p})\}]$, there exist finite integers T, N where $1 \leq N \leq T$, a set of increasing constants $\gamma_0, \gamma_1, \dots, \gamma_N$ where $\gamma_k < \gamma_{k+1}$, $k = 0, 1, \dots, N - 1, \gamma_0 = \underline{\theta} < \gamma_1, \gamma_N = \bar{\theta} > \gamma_{N-1}$, and a set of time periods $\{t_1, \dots, t_N\}$, $1 = t_1 < t_2 < t_3 \dots < t_N = T$, such that the following hold:*

- (i) *All potential sellers trade their goods by the end of period T .*
- (ii) *There are N periods $\{t_1, \dots, t_N\}$ in which strictly positive measure of trade occurs.*
- (iii) *For $n = 1, 2, \dots, N$, $\tau(i, \mathbf{p}) = t_n$ if $\theta(i) \in (\gamma_n, \gamma_{n+1})$; that is, all sellers whose endowed quality lies between γ_n and γ_{n+1} sell in period t_n . The price in period t_n is equal to $v\eta(\gamma_n, \gamma_{n+1})$; further, if $1 < n < N$, any seller with quality γ_n is indifferent between selling in period t_n and period t_{n-1} :*

$$(3.1) \quad \mathbf{p}_{t_{n-1}} - \gamma_n = \delta^{t_n - t_{n-1}} [\mathbf{p}_{t_n} - \gamma_n]$$

that is,

$$(3.2) \quad v\eta(\gamma_{n-1}, \gamma_n) - \gamma_n = \delta^{t_n - t_{n-1}} [v\eta(\gamma_n, \gamma_{n+1}) - \gamma_n]$$

(iv) Consider a period t , where $1 \leq t \leq T$ such that $t \neq t_n$ for any $n = 1, 2, \dots, N$, that is, at most zero measure of trade occurs in period t . If $t_{n-1} < t < t_n$, then

$$(3.3) \quad v\gamma_n \leq p_t, \quad (p_t - \gamma_n) \leq \delta^{n-t} [v\eta(\gamma_n, \gamma_{n+1}) - \gamma_n]$$

A formal proof of this proposition is contained in the Appendix.

Proposition 3.1 provides a strong characterization of the nature of dynamic equilibria and the possibilities of trading in competitive markets. In contrast to the static models where only low-quality goods are traded, the possibility of waiting and trading later allows the market prices to give incentives to sellers of higher-quality goods to sell in later periods, so that goods of all quality—no matter how high—are eventually traded. Further, all trade takes place in a finite number of periods. The result is particularly strong as it holds for all *dynamic equilibria*. The implication is that in markets where sellers can wait to sell, the “lemons problem” caused by adverse selection due to asymmetric information among traders is not really one of being unable to trade, but rather the fact that higher-quality sellers need to wait and, in fact, wait more than lower-quality sellers in order to realize the gains from trade. The welfare loss from such waiting is the main index of market failure caused by asymmetric information.

Given a certain distribution of quality, the length of the waiting time before “high”-quality sellers sell in the market depends on the rate at which equilibrium prices increase over time. The latter, in turn, is constrained by (3.2); that is, the fact that between any two successive periods in which trade takes place, prices must be such that the seller trading the marginal quality must be indifferent between selling in either of the two periods. The higher the value of δ (lower the impatience), the smaller the rate at which prices can increase between any two given time periods. Loosely speaking, we would expect that the total length of time before goods of all quality can be traded is likely to increase with δ and, in fact, becomes infinitely large as $\delta \uparrow 1$. We actually show that a stronger result holds. For any given $\delta \in [0, 1)$ and $\theta \in [\underline{\theta}, \bar{\theta}]$, let $\tau(\delta, \theta)$ denote the minimum number of time periods it takes to trade quality θ (over all dynamic equilibria).

PROPOSITION 3.2. *For any $\theta^* \in (\theta^*, \bar{\theta}]$, $\tau(\delta, \theta^*) \rightarrow \infty$ as $\delta \uparrow 1$; that is, for any quality higher than the static outcome θ^* , the length of time before a unit of such quality is actually traded becomes infinitely large as the rate of impatience goes to zero.*

The proof of this proposition is contained in the Appendix.

These characterization results are vacuous unless we show that a dynamic equilibrium, as we have defined it, actually exists. The next proposition states the existence result.

PROPOSITION 3.3. *A dynamic equilibrium exists.*

The proof of this existence result (contained in the Appendix) is constructive. We define a level of quality $\beta < \bar{\theta}$ with the intention of constructing an equilibrium where the set of quality traded in the last period of trading would be an interval $[y, \bar{\theta}]$ for some y lying between β and $\bar{\theta}$. For each $y \in [\beta, \bar{\theta}]$, we set $y_0(y) = \bar{\theta}$, $y_1(y) = y$

and then define a decreasing set of points $y_t(y)$ such that if the intervals $[y_t(y), y_{t-1}(y)]$ are traded t periods before the last, then it is incentive compatible for sellers with quality in the interval $[y_{t+1}(y), y_t(y)]$ to trade $(t + 1)$ periods before the last period. Sometimes, we run into the following problem: having defined $y_0(y), \dots, y_t(y)$, we may find that for any choice of $x < y_t(y)$, if the interval $[x, y_t(y)]$ is traded $(t + 1)$ periods before the last period, then the price in that period is such that the seller with quality y_t is not indifferent but rather strictly prefers to sell in the next period. This happens whenever

$$(3.4) \quad (v - 1)y_t(y) > \delta[v\eta(y_t(y), y_{t-1}(y)) - y_t(y)]$$

In that case, we cannot have positive trading in the period that is $(t + 1)$ periods before the last period of trading. However, because of discounting, there exists some $\tau > 1$ such that the seller with quality $y_t(y)$ can be made indifferent between selling in time periods that lie t and $(t + \tau)$ periods before the last period of trading; the equilibrium path is one where no trade occurs in the intermediate periods. Under condition (iv) of the definition of equilibrium, prices in the intermediate periods of no trading must not be less than $vy_t(y)$. It can be shown that no seller would want to sell in these intermediate periods even if the price equals $vy_t(y)$. Finally, we use continuity of the functions $y_t(y)$ to show that there exists some y^* and T such that $y_T(y^*) = \underline{q}$; that is, the entire support of quality is traded in T periods.

Before concluding the section, it is worth noting that our results can be extended to the case where the good is less than perfectly durable. To see this, suppose that for both buyers and sellers the good depreciates at a constant multiplicative rate d , $0 < d < 1$, per period. In Proposition 3.1 we argued that the equilibrium (when there is no depreciation) is characterized by constants $\gamma_i, i, i = 0, \dots, N$ such that

$$v\eta(\gamma_{n-1}, \gamma_n) - \gamma_n = \delta^k[v\eta(\gamma_n, \gamma_{n+1}) - \gamma_n]$$

where $k = t_n - t_{n-1}$. When we allow for such depreciation and the good is not traded for k periods, both buyers' as well as the sellers' valuations are reduced to $(1 - d)^k$ times their valuation k periods ago. Hence any equilibrium of the model without depreciation where the discount factor is $[(1 - d)\delta]$ is also an equilibrium of the model with depreciation rate d and discount factor δ ; the converse is also true. Allowing for depreciation is equivalent to an increase in impatience.

4. EQUILIBRIUM WITH NO "BREAK" IN TRADING

In the previous section, we have shown that a dynamic equilibrium always exists though the equilibrium path may be such that there is no trading for some period(s) before trade resumes again. Are there conditions under which we can ensure the existence of a dynamic equilibrium where trade occurs in successive periods with no breaks until all goods are sold? In this section, we attempt to answer this question.

For $z \in (\underline{q}, \bar{\theta}]$, let $\alpha(z) < z$ and $\beta(z) < z$ be defined by

$$\begin{aligned} \alpha(z) &= \sup\{y: \underline{\theta} \leq y \leq z, [(v-1)y] < \delta[v\eta(y, z) - y]\}, & \text{if } [(v-1)\underline{\theta}] < \delta[v\eta(\underline{\theta}, y) - \underline{\theta}] \\ &= \underline{\theta}, & \text{otherwise} \\ \beta(z) &= \inf\{y: \underline{\theta} \leq y \leq z, [v\eta(y, z) - z] \geq \delta(v-1)z\}, & \text{if } [v\eta(\underline{\theta}, z) - z] < \delta(v-1)z \\ &= \underline{\theta}, & \text{otherwise} \end{aligned}$$

To understand what $\alpha(z)$ represents, suppose for a moment that the range of quality traded next period is $[y, z]$ for some $y \leq z$. To be part of an equilibrium path with no break in trading, we would need that there exists $x \leq y$ such that when the range $[x, y]$ of quality is traded in the current period, the seller of quality y is indifferent between selling in the current period and selling in the next period. Does such an x exist? The answer is in the affirmative, if y lies in $[\alpha(z), z]$. More particularly, let $\delta_0 > 0$ be defined by

$$[(v-1)\underline{\theta}] = \delta_0[v\eta(\underline{\theta}, \bar{\theta}) - \underline{\theta}]$$

It is easy to check that for $\delta \leq \delta_0$, $\alpha(z) = \underline{\theta}$ for all $z \in [\underline{\theta}, \bar{\theta}]$. If $\delta > \delta_0$, $\alpha(\bar{\theta}) > \underline{\theta}$ and there exists some $z^0 \in (\underline{\theta}, \bar{\theta})$ such that $\alpha(z) = \underline{\theta}$ for $z \in [\underline{\theta}, z^0]$, while for $z \in (z^0, \bar{\theta}]$, $\alpha(z) > \underline{\theta}$ and

$$(4.1) \quad [(v-1)\alpha(z)] = \delta[v\eta(\alpha(z), z) - \alpha(z)]$$

Further, it can be shown that $\alpha(z)$ is strictly increasing in z on $[z^0, \bar{\theta}]$.¹⁴

The interpretation of $\beta(z)$ is similar to $\alpha(z)$. For any quality level $y \in [\beta(z), z]$ if the range of quality traded in the current period is $[y, z]$, then there exists $x \geq z$ such that if the range of quality traded next period is $[z, x]$, the seller with quality z is indifferent between selling in the current and the next period. It is easy to check that as $vE(\theta) < \bar{\theta}$, as $\beta(\bar{\theta}) > \underline{\theta}$. Further, there exists $z^1 \in (\underline{\theta}, \bar{\theta})$ such that $\beta(z) = \underline{\theta}$ for $z \in [\underline{\theta}, z^1]$, while for $z \in (z^1, \bar{\theta}]$, $\beta(z) > \underline{\theta}$ and

$$(4.2) \quad [v\eta(\beta(z), z) - z] = \delta(v-1)z$$

We now state the main condition under which there is an equilibrium with no break in trading:

CONDITION C. If $\alpha(\theta) > \underline{\theta}$, then $\beta(\theta) > \alpha(\theta)$.

Note that Condition C imposes no restriction if $\delta \leq \delta_0$; on the other hand, if $\delta > \delta_0$, then it requires

$$(4.3) \quad \beta(\theta) > \alpha(\theta) \quad \text{for } \theta \in (z^0, \bar{\theta}]$$

PROPOSITION 4.1. *Suppose Condition C holds. Then there exists a dynamic equilibrium where strictly positive measure of trade occurs in every period until all goods are sold.*

¹⁴A proof of this is contained in a working-paper version of this article; see Janssen and Roy (1998).

Note that whatever be the distribution of θ , Condition C is always satisfied if agents are sufficiently impatient. More specifically, if $\delta \leq \delta_0$, then Condition C is always satisfied (as $\alpha(\theta) = \underline{\theta}$ for all θ) and so there is an equilibrium with no break in trading. This is in conformity with our general argument that for low δ the *relative* incentive to wait for sellers with lower quality is low and so it is easier to separate the types. If δ is large, whether or not Condition C can be satisfied depends on the distribution of θ . It is, however, possible to identify a large class of distributions for which Condition C holds for all $\delta < 1$.

Suppose that θ is uniformly distributed on $[\underline{\theta}, \bar{\theta}]$. If $v \geq 2$, then $vE(\theta) > \bar{\theta}$ so that all goods can be traded in period 1. Therefore, consider $v < 2$. In this case, Condition C is satisfied for all $\delta \in [0, 1)$. To see this consider any $\delta \in (\delta_0, 1]$ where δ_0 (as defined earlier) is given by

$$\delta_0 = [2(v - 1)\underline{\theta}] / [(v - 2)\underline{\theta} + v\bar{\theta}]$$

For such δ , the critical values z^0 and z^1 (as defined earlier) are given by

$$z^1 = [(v/2)\underline{\theta}] / [\delta(v - 1) + (1 - (v/2))]$$

$$z^0 = [2/(\delta v)] [(v - 1) + \delta(1 - (v/2))]$$

It can be verified that $z^1 < z^0$ if and only if

$$(v - 1)(v - 2)(1 - \delta)^2 < 0$$

which is true for $1 < v < 2$. Now, consider any $y \in (z^0, \bar{\theta}]$. Then, $y > z^1$. It is sufficient to show that $\alpha(y) < \beta(y)$. From (4.1) and (4.2) we have

$$\alpha(y) = vy / [2\{(v - 1)/\delta\} + 1] - v$$

$$\beta(y) = [(2/v)(\delta(v - 1) + 1) - 1]y$$

Again, after simplification it can be checked that $\alpha(y) - \beta(y) < 0$ if and only if

$$(2/\delta)(v - 1)(v - 2)(1 - \delta)^2 < 0$$

which is true for $1 < v < 2$.

Thus, *if the distribution of quality is uniform, then for all $\delta \in [0, 1)$, Condition C is always satisfied.*

More generally, suppose that the distribution of θ has a density function that is decreasing on $[\underline{\theta}, \bar{\theta}]$. Then,

$$E(\theta | a \leq \theta \leq b) \leq (a + b)/2$$

Using this, similar arguments (as above) show that Condition C holds for $1 < v < 2$. To summarize:

PROPOSITION 4.2. *Suppose that the distribution of θ has a (weakly) decreasing density function on $[\underline{\theta}, \bar{\theta}]$ and that $v < 2$. Then, for any $\delta \in [0, 1)$, there exists a dynamic equilibrium where strictly positive measure of trade occurs every period until all goods are sold.*

5. THE NECESSITY OF BREAK IN TRADING ALONG THE EQUILIBRIUM PATH:
AN EXAMPLE

In this section, we outline an example where the dynamic equilibrium path is *necessarily* characterized by intermediate periods of no trade. The crucial characteristic of this distribution is that there is an interval on which the density function is steeply increasing such that if trade occurs in consecutive periods, then the average quality (and hence the prices) increase too rapidly to make any seller indifferent between selling in two consecutive periods. Consider an initial distribution of quality whose support is the interval $[10, 20.1]$ and whose density function $g(\theta)$ is given by

$$\begin{aligned} g(\theta) &= p/(0.1), & 10 \leq \theta < 10.1 \\ &= (1 - p)(1/8.9) [1/(1 + k + \lambda k)], & 10.1 \leq \theta < 19 \\ &= (1 - p)[k/(1 + k + \lambda k)], & 19 \leq \theta < 20 \\ &= (1 - p)(1/0.1)[\lambda k/(1 + k + \lambda k)], & 20 \leq \theta \leq 20.1 \end{aligned}$$

Here, $p = 0.5$ and we set $\lambda = k = 100$ (λ and k are relatively large numbers). The density function is depicted in Figure 1. Let $\epsilon = [1/(1 + k + \lambda k)]$; ϵ is a very small number. The distribution is piece-wise uniform on the intervals $[10, 10.1]$, $[10.1, 19]$, $[19, 20]$, and $[20, 20.1]$. The total probability mass on each of these intervals is as follows: $\mu[10, 10.1] = 0.5$, $\mu[10.1, 19] = 0.5\epsilon$, $\mu[19, 20] = 0.5k\epsilon$, and $\mu[20, 20.1] = 0.5\lambda k\epsilon$. Finally, we choose $\nu = 1.2$ and $\delta = 0.9$.

Suppose that there is a dynamic equilibrium where the market clears in $T > 1$ periods of consecutive trading. It follows that there exists $z_0, \dots, z_T, z_0 = 20.1, z_T = 10, z_t < z_{t-1}$ such that

$$(5.1) \quad \begin{aligned} [v\eta(z_{t+1}, z_t) - z_t] &= \delta[v\eta(z_t, z_{t-1}) - z_t], & t = 1, 2, \dots, T - 1 \\ v\eta(z_{t+1}, z_t) &\geq z_t, & t = 1, \dots, T - 1 \end{aligned}$$

Following our discussion in the previous section, having positive trade in every period would mean

$$(5.2) \quad z_t \geq \alpha(z_{t-1}), \quad t = 1, \dots, T - 1$$

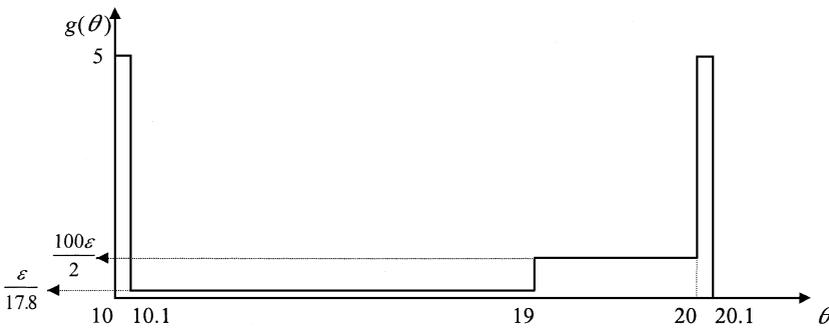


FIGURE 1

THE DENSITY FUNCTION $g(\theta)$ OF QUALITY FOR WHICH EVERY DYNAMIC EQUILIBRIUM EXHIBITS INTERMEDIATE BREAK IN TRADING

In particular, z_1 must lie between $\alpha(20.1)$ and 20.1 . We divide the interval $[\alpha(20.1), 20.1]$ into different subintervals and argue that there is a contradiction if z_1 lies in any of these intervals. The key argument is as follows: For $z_t < 19 < z_{t-1}$ or for $z_t < 20 < z_{t-1}$, the chosen density function is such that $\eta(z_t, z_{t-1})$, the average quality in the interval (z_t, z_{t-1}) , is actually very close to z_{t-1} . This means that a seller with quality z_{t-1} is indifferent between selling now and selling in the next period only if z_t is sufficiently below z_{t-1} ; otherwise, he prefers to sell now. Now, as δ is relatively large, $\eta(z_t, z_{t-1})$ being very close to z_{t-1} implies that the surplus the seller with quality z_t gets by selling in the next period, that is, $\delta[v\eta(z_t, z_{t-1}) - z_t]$, is relatively large and, in particular, may exceed $(v - 1)z_t$, which, in turn, means that there does not exist a z_{t+1} such that the sellers with qualities z_t and z_{t-1} are indifferent between selling in two consecutive periods. The formal proof involves many small steps and gets somewhat complicated as z_{t-1} gets close to 20. The details are contained in Janssen and Roy (1998).

6. STRATEGIC VERSIONS OF THE MODEL: COMPARISON OF EQUILIBRIA

In this section, we compare the notion of equilibrium and the nature of equilibria in our Walrasian model to that in closely related strategic models. In particular, we relate the restrictions on buyers' beliefs about quality in periods of no trade and the restrictions implied by the concept of Walrasian equilibrium to some well-known refinements of Bayes–Nash equilibria. The analysis also clarifies the robustness of our results to settings where there is an explicit story of price formation. We discuss two strategic models of dynamic trading: a dynamic signaling game that is a direct strategic version of our model with sellers setting prices and a dynamic auction game (analyzed by Vincent, 1990) where there is one seller and it is the buyers who set prices.

6.1. *A Signaling Game: Sellers Set Prices.* Consider the following strategic version of our model. The specifications of the set of traders, their endowments, the initial information structure, and the ex ante distribution of quality are identical to that in our Walrasian framework as outlined in Section 2. The difference is that in every period, each potential seller (who has not yet traded) announces a price at which he is willing to sell in that period. In each period, buyers observe the price announcements made by sellers and then decide whether or not to buy in that period and if so, from which seller. Price announcements are binding; that is, a seller has to sell at his quoted price if a buyer wishes to buy at that price (if multiple buyers wish to buy from a seller, he sells to any one of them randomly). In conformity with the idea of an anonymous market, we assume that while buyers observe the current price announcements made by each seller and recall perfectly the distribution of announced prices in previous periods, they are not able to associate the identity of any specific seller with the prices announced by *this* same seller in the past. Thus, the beliefs of buyers about the quality of the good owned by a particular seller cannot be conditioned on prices charged by him in *previous periods*. The payoffs are analogous to that in the Walrasian market model. Observe that this is a dynamic signaling game, where the signal chosen by an informed player is his pricing strategy.

Condition (iv) of the definition of dynamic equilibrium in the Walrasian market requires that as long as all goods are not traded, in any period in which trade does not occur, a buyer's expected quality is at least as high as the "lowest unsold quality" of that period (which can be inferred from the equilibrium path). It is easy to verify that this restriction on "expected quality" is always satisfied in any perfect Bayesian equilibrium (*pbe*) of the dynamic signaling game. On the equilibrium path of any *pbe*, buyers use their initial priors and the equilibrium strategies to infer that the sellers with certain types of quality have already traded. As long as there is a positive measure of unsold goods (i.e., a positive measure of sellers making price announcements), the updated equilibrium beliefs of buyers must assign probability one to the event that all price announcements in the current period come from sellers with quality above the "lowest unsold quality."

We say that a dynamic equilibrium of our Walrasian market model and an equilibrium of the dynamic signaling game are *outcome equivalent* if the time periods in which positive trades occur, the set of agents who trade, the qualities traded in such periods, and the prices at which trades occur in such periods are identical. Note that this notion of outcome equivalence allows for the possibility that in periods in which no trade occurs, there may be a difference between the price in the Walrasian market and prices quoted by sellers in the dynamic signaling game. The main reason behind this is that when sellers set prices, no trading on the equilibrium path is consistent with sellers setting prices arbitrarily high so that no buyer wishes to buy. On the other hand, if the Walrasian market price is that high, price-taking sellers may wish to trade, leading to excess supply.

It is easy to check that every dynamic equilibrium of our model can be shown to be outcome equivalent to a *pbe* of the signaling game. To construct such an equilibrium, set the equilibrium strategies as follows: sellers charge the same price as in the Walrasian market in the period in which they trade, and in other periods, they charge very high prices at which (with appropriately defined beliefs of buyers) no one buys from them. The Appendix contains details of this argument.

However, not every *pbe* is outcome equivalent to a dynamic equilibrium of the Walrasian market. The concept of Walrasian equilibrium implies certain kinds of restrictions that do not have to be satisfied in the dynamic signaling game. One such restriction is the requirement about market clearing in every period. It is possible that there are *pbe* of the dynamic signaling game where buyers earn strictly positive expected surplus so that some buyers are actually rationed. The underlying process that could lead to such an equilibrium is as follows: sellers who sell to such buyers do not dare to raise the price at which they sell as the off-equilibrium beliefs of buyers associate much lower quality with higher prices; further, as buyers do not set prices, they cannot compete among themselves and bid up the price. It is difficult to rule out such off-equilibrium beliefs using any of the standard refinements.

Even if we confine attention to equilibria where all buyers earn zero surplus, not every *pbe* of the signaling game is outcome equivalent to a dynamic equilibrium of our model. Here is a striking example. Recall that θ^s is the highest quality that would be sold in the one-period version of our model, $\theta^s < \bar{\theta}$. An outcome where sellers with quality lying in $[\underline{\theta}, \theta^s]$ trade at price θ^s in period 1 (the marginal seller in period 1 makes zero surplus), and trade never occurs in any

subsequent period, is actually sustainable as a *pbe* outcome of the signaling game. All we need is that sellers with quality larger than θ^s set their prices equal to $v\bar{\theta}$ in all time periods after period 1 and that buyers have sufficiently pessimistic out-of-equilibrium beliefs.¹⁵ Note that the notion of *pbe* does not impose any restriction on out-of-equilibrium beliefs. However, if we impose a well-known refinement of *pbe* such as the intuitive criterion (Cho and Kreps, 1987), then any such outcome can be ruled out. The main argument is as follows: In any equilibrium where trade stops after period 1, marginal quality must be sold at zero surplus (must equal θ^s) and it must be true that in period 2 sellers do not quote a price very close to θ^s for in that case, buyers would want to buy (because $v > 1$ and buyers' Bayesian updated belief tells them that remaining sellers are almost surely those with quality above θ^s). Now, suppose the marginal seller of period 1 deviates and quotes a price in period 2 that is just a bit higher than θ^s . Given the price at which trade occurs in period 1 and the valuations of sellers, it is easy to see that only sellers with quality extremely close to θ^s could have gained by quoting such a price. The intuitive criterion says that buyers should infer this and therefore be willing to buy at that price (they would be willing to pay almost $v\theta^s$). So, the deviation would be gainful. The same argument can be stretched to show that every *pbe* of the signaling game that meets the intuitive criterion and where all buyers earn zero surplus is outcome equivalent to a dynamic equilibrium of our model. Details of the argument are contained in the Appendix. We sum up the discussion in the following proposition:

PROPOSITION 6.1. (i) *Every dynamic equilibrium of the competitive model is outcome equivalent to a perfect Bayes–Nash equilibrium of the signaling model; the converse is not necessarily true.* (ii) *Any perfect Bayes–Nash equilibrium of the signaling model that satisfies the intuitive criterion and in which all buyers earn zero surplus is outcome equivalent to a dynamic equilibrium of the Walrasian model.*

There is one respect in which the intuitive criterion is more restrictive than the notion of dynamic Walrasian equilibria. It imposes restrictions on *out-of-equilibrium* beliefs even in periods after all goods are sold. As a result, the price in the last period of positive trading has to be large enough so that the seller with quality $\bar{\theta}$ has no incentive to deviate and sell in the next period.¹⁶

6.2. Dynamic Auction. Next, we consider the dynamic auction game analyzed by Vincent (1990) in which two uninformed buyers compete in prices for a unit of a good held by one seller with private information about the quality of the good. It is assumed that quality is uniformly distributed. Here, prices are set by buyers. On any *pbe*, competition between buyers ensures that they earn zero expected net

¹⁵ For example, if $p_t < v\bar{\theta}$ for $t > 1$, then the expected quality is $\bar{\theta}$.

¹⁶ However, the equilibrium constructed in Proposition 3.2 is such that it can be supported as a *pbe* satisfying the intuitive criterion.

surplus and further, the *skimming* property holds—higher quality is traded in later periods. It can be shown that every dynamic equilibrium of our Walrasian model can be sustained as a *pbe* of the Vincent model. However, Vincent focuses on the concept of perfect sequential equilibrium (*pse*), a stronger refinement of Bayes–Nash equilibrium, and he shows that there is a unique *pse*. As there can be multiple dynamic equilibrium outcomes in our model, it follows that not all dynamic equilibrium outcomes can be sustainable as *pse*.¹⁷ One way to understand this is to observe that the restrictions on out-of-equilibrium beliefs implied by the concept of *pse* are actually stronger than the intuitive criterion.¹⁸ However, as we have noted above, the notion of intuitive criterion is, in some respects, more restrictive than the concept of dynamic Walrasian equilibrium.

Finally, it should be observed that in the Vincent model, trading occurs with positive probability in every period until all types of sellers have traded. Our analysis in Sections 4 and 5 indicates that this is related to the assumption that quality is uniformly distributed.

7. CONCLUSION

In markets for durable goods with asymmetric information, the possibility of dynamic trading implies that, in contrast to a static framework, goods of all qualities can be traded and, when the support of quality is bounded, all goods are traded within finite time, regardless of the distribution of quality. Sellers with higher-quality goods have greater incentive to wait for higher prices. Given time, the market price mechanism allows all gainful trade to take place but agents have to incur the cost of waiting. Our analysis sheds an alternative light on the problem of adverse selection in markets, viz, in certain classes of situations the problem is not so much that better qualities cannot be sold at all, but rather that it may take time to separate out good and bad qualities and better quality owners may have to wait for lower-quality goods to leave the market. The gains from eventual trading are offset by this waiting cost and the time preference parameter plays a crucial role in determining the net social surplus.

APPENDIX

A.1. *Proof of Proposition 3.1.* Observe that there is no equilibrium where trade never occurs as under conditions (ii) and (iv) of equilibrium $p_1 \geq v\bar{\theta} > \underline{\theta}$. First, we claim that there is no equilibrium where (strictly positive measure of) trade occurs for a finite number of time periods and the highest quality sold is strictly below $\bar{\theta}$. Suppose that there is such an equilibrium and no trade occurs after period T ; let $\theta^0 = \sup\{\theta(i): \tau(i, \mathbf{p}) < \infty\}$. Then, $\max\{p_t; t = 1, \dots, T\} \geq \theta^0$. If

¹⁷ The article contains an example where the buyers' valuation of the average quality equals the valuation of a seller with the highest quality. In the unique *pse* of the dynamic auction game, trade never occurs in period 1. However, if the market is Walrasian, all goods (qualities) could be traded even in a static market. Naturally, one of the dynamic equilibrium outcomes of our model is one where all goods are traded in period 1.

¹⁸ Vincent provides an example where there are two equilibria satisfying the intuitive criterion.

$\max\{p_t; t = 1, \dots, T\} > \theta^0$, sellers with quality just above θ^0 will choose to sell in one of the first T periods, rather than not sell. So, $\max\{p_t; t = 1, \dots, T\} = \theta^0$. Conditions (ii) and (iv) of the definition of equilibrium imply that $p_{T+1} \geq v\theta^0$. As $v > 1$, there is a strictly positive measure of sellers with quality just below θ^0 who strictly prefer to sell in period $T + 1$, a contradiction. Next, observe that if a seller with quality $\theta^* > \underline{\theta}$ sells in period t^* , then

$$p_{t^*} - \theta^* \geq \delta^k [p_{t^*+k} - \theta^*], \quad k = 1, 2, \dots, p_{t^*} \geq \theta^*$$

so that for all $\theta < \theta^*$

$$p_{t^*} - \theta \geq \delta^k [p_{t^*+k} - \theta], \quad k = 1, 2, \dots, p_{t^*} \geq \theta$$

so all sellers with $\theta < \theta^*$ sell in some period $t \leq t^*$. Similarly, if a seller with quality $\theta^* < \bar{\theta}$ sells in period t^* , then all sellers with quality $\theta > \theta^*$ will never sell their goods in a time period earlier than t^* . It follows that if $\theta(i) < \theta(j)$, then $\tau(i, \mathbf{p}) \leq \tau(\mathbf{p})$; further, if $\tau(i, \mathbf{p}) \leq \tau(\mathbf{p})$, then $\theta(i) \leq \theta(j)$. Therefore, if t and t' are two periods in which strictly positive measure of trade takes place, $t < t'$, and no trade takes place in any period lying between t and t' , then (the closure of) the sets of quality traded in these periods are two intervals $[\gamma_{t-1}, \gamma_t]$ and $[\gamma_{t'}, \gamma_{t'+1}]$ where $\gamma_{t-1} < \gamma_t = \gamma_{t'} < \gamma_{t'+1}$; sellers of quality γ_t must be indifferent between selling in period t and period t' . In any period t'' between t and t' , $p_{t''} \geq v\gamma_t > v\eta(\gamma_{t-1}, \gamma_t) = p_t$. Let $\{t_i\}$ be the sequence of time periods in which strictly positive measure of trade occurs. As stated earlier, trade must occur in period 1 so that $t_1 = 1$. Then, there exist constants $\{\gamma_n\}$, $n = 0, 1, \dots, N$ (N can be $+\infty$) where $\gamma_n < \gamma_{n+1}$, $\gamma_0 = \underline{\theta}$, $\gamma_N = \bar{\theta}$ if N is finite, such that $\tau(i, \mathbf{p}) = t_n$ if $\theta(i) \in (\gamma_{n-1}, \gamma_n)$, all sellers endowed with quality lying in the open segment (γ_n, γ_{n+1}) strictly prefer to sell in period t_n while sellers with quality γ_n are indifferent between selling in period t_{n-1} and period t_n , which gives (3.2); note that the definition of equilibrium implies that

$$\mathbf{p}_{t_n} = v\eta(\gamma_n, \gamma_{t+1}), \quad \mathbf{p}_{t_{n-1}} = v\eta(\gamma_{n-1}, \gamma_n)$$

To see that $N < \infty$, suppose not. From (3.2) we have $v\eta(\gamma_{n-1}, \gamma_n) - \gamma_n \leq \delta[v\eta(\gamma_n, \gamma_{n+1}) - \gamma_n]$, which implies that

$$(A.1) \quad v\gamma_{n-1} - \gamma_n \leq \delta[v\gamma_{n+1} - \gamma_n]$$

Observe that $\{\gamma_n\}$ is a bounded monotonic sequence and hence converges to some $\gamma^* > \underline{\theta} > 0$. Taking limits on both sides of (A.1), we obtain a contradiction. Thus, $N < \infty$ and $\gamma_N = \bar{\theta}$, because there cannot be any positive measure of unsold goods left after the last period of trading. This completes the proof of (i)–(iii). A necessary condition for no trade to occur is that the seller with the marginal quality must (weakly) prefer to not sell in period t . This, with condition (iv) of the definition of equilibrium, yields part (iv) of the proposition. ■

A.2. Proof of Proposition 3.2. Suppose that $\liminf_{\delta \uparrow 1} \tau(\delta) < \infty$. Let $p_1(\delta)$ and $p_{\tau(\delta)}(\delta)$ be the prices in the first and the $\tau(\delta)$ -th period of trading, respectively. Let $\theta_1(\delta)$ be the highest quality traded in the first period. Obviously, $\theta_1(\delta) \in [\underline{\theta}, \theta^*]$ for all δ . Then, $p_1(\delta) = v\eta(\underline{\theta}, \theta_1(\delta)) < v\eta(\underline{\theta}, \theta^*) = \theta^*$, so that

$$(A.2) \quad p_1(\delta) - \theta_1(\delta) < \theta^s - \theta_1(\delta)$$

As quality θ^* is traded in period $\tau(\delta)$,

$$(A.3) \quad p_{\tau(\delta)}(\delta) \geq \theta^*$$

Consider a sequence $\{\delta_n\} \uparrow 1$ such that $\{\tau(\delta_n)\}$ is bounded above, say, by $K < \infty$. For each n , let $p_n = p_{\tau(\delta)}(\delta)$ when $\delta = \delta_n$. From (A.3)

$$(A.4) \quad p_n \geq \theta^* \quad \text{for all } n$$

The seller with quality $\theta_1(\delta_n)$ prefers to sell in period 1 rather than in period $\tau(\delta_n)$ and as $\tau(\delta_n) \leq K$ we have

$$p_1(\delta_n) - \theta_1(\delta_n) \geq (\delta_n^K)(p_n - \theta_1(\delta_n))$$

Using (A.2) and (A.4) we have

$$\theta^s - \theta_1(\delta_n) \geq \delta_n^K(\theta^* - \theta_1(\delta_n))$$

that is, $\theta^s - \delta_n^K \theta^* \geq (1 - \delta_n^K)\theta_1(\delta_n)$.

As $\theta_1(\delta_n) \in [\underline{\theta}, \theta^s]$ for all n , taking the \liminf as $n \rightarrow \infty$ on both sides of the above inequality, we have $\theta^s - \theta^* \geq 0$, that is, $\theta^* \leq \theta^s$, a contradiction. ■

A.3. *Proof of Proposition 3.3.* Let β be defined by

$$(A.5) \quad \beta = \sup\{y: v\eta(y, \bar{\theta}) - \bar{\theta} < \delta(v - 1)\bar{\theta}\}$$

The fact that $v\eta(\underline{\theta}, \bar{\theta}) - \bar{\theta} < 0$ and $\lim_{y \rightarrow \bar{\theta}} (v\eta(y, \bar{\theta}) - \bar{\theta}) = (v - 1)\bar{\theta} > \delta(v - 1)\bar{\theta}$ along with the continuity of η ensures that β is well defined and that it is the unique solution to

$$(A.6) \quad v\eta(\beta, \bar{\theta}) - \bar{\theta} = \delta(v - 1)\bar{\theta}$$

Obviously,

$$(A.7) \quad \underline{\theta} < \beta < \bar{\theta}$$

For each $y \in [\beta, \bar{\theta}]$, we can define a finite positive integer $T(y) \geq 1$ and a set of numbers $\{\psi_t(y)\}_{t=0,1,\dots,T(y)-1}$ in the following manner: set $\psi_0(y) = \bar{\theta}$, $\psi_1(y) = y$. Define: $\tau_0(y) = 1$, $\tau_1(y) = \min\{\tau \geq 1: (v - 1)y \geq \delta^\tau[v\eta(y, \bar{\theta}) - y]\}$. Set $\psi_t(y) = \psi_1(y) = y$, $t = 1, \dots, \tau_1(y) - 1$, if $\tau_1(y) > 1$.

There are two possibilities:

- (i) $v\eta(\underline{\theta}, y) - y > \delta^{\tau_1(y)}[v\eta(y, \underline{\theta}) - y]$,
in which case set $T(y) = 1 + \tau_1(y)$, that is, $(\tau_0(y) + \tau_1(y))$.
- (ii) $v\eta(\underline{\theta}, y) - y \leq \delta^{\tau_1(y)}[v\eta(y, \bar{\theta}) - y]$
in which case let $\psi_{\tau_0(y)+\tau_1(y)}(y) \in [\underline{\theta}, y]$ be defined by

$$(A.8) \quad v\eta(\psi_{\tau_0(y)+\tau_1(y)}(y), y) - y = \delta^{\tau_1} [v\eta(y, \bar{\theta}) - y]$$

Note that as $\eta(x, y)$ is continuous and strictly increasing in x for $\underline{\theta} \leq x < y \leq \bar{\theta}$, $\psi_{\tau_0(y)+\tau_1(y)}(y)$ is well defined and is the unique solution to (A.8). Equation (A.8) can be written as

$$(A.9) \quad v\eta(\psi_{\tau_0(y)+\tau_1(y)}(y), \psi_1(y)) - \psi_1(y) = \delta^{\tau_1(y)}[v\eta(\psi_1(y), \psi_0(y)) - \psi_1(y)]$$

In general, given $\psi_t(y)$, $t = 0, 1, \dots, \sum_{i=0,1,\dots,k-1} \tau_i(y)$, we can set

$$(A.10) \quad \tau_k(y) = \min\{\tau \geq 1: (v - 1)\psi_{\hat{\tau}}(y) \geq \delta^{\tau}[v\eta(\psi_{\hat{\tau}}(y), \psi_{\hat{\tau}}(y)) - \psi_{\hat{\tau}}(y)]\}$$

where $\hat{\tau} = \sum_{i=0,1,\dots,k-1} \tau_i(y)$ and $\tilde{\tau} = \sum_{i=0,1,\dots,k-2} \tau_i(y)$.

Set $\psi_t(y) = \psi_{\hat{\tau}}(y)$, $t = \hat{\tau}, \dots, (\hat{\tau} + \tau_k(y) - 1)$, if $\tau_k(y) > 1$.

There are two possibilities:

- (i) $v\eta(\underline{\theta}, \psi_{\hat{\tau}}(y)) - \psi_{\hat{\tau}}(y) > \delta^{\tau_k(y)}[v\eta(\psi_{\hat{\tau}}(y), \psi_{\hat{\tau}}(y)) - \psi_{\hat{\tau}}(y)]$
in which case set $T(y) = (\hat{\tau} + \tau_k(y)) = \sum_{i=0,1,\dots,k} \tau_i(y)$
- (ii) $v\eta(\underline{\theta}, \psi_{\hat{\tau}}(y)) - \psi_{\hat{\tau}}(y) \leq \delta^{\tau_k(y)}[v\eta(\psi_{\hat{\tau}}(y), \psi_{\hat{\tau}}(y)) - \psi_{\hat{\tau}}(y)]$
in which case $\psi_{\hat{\tau}+\tau_k(y)}(y) \in [\underline{\theta}, \psi_{\hat{\tau}}(y)]$ is (uniquely) defined by

$$(A.11) \quad v\eta(\psi_{\hat{\tau}+\tau_k(y)}(y), \psi_{\hat{\tau}}(y)) - \psi_{\hat{\tau}}(y) = \delta^{\tau_k(y)}[v\eta(\psi_{\hat{\tau}}(y), \psi_{\hat{\tau}}(y)) - \psi_{\hat{\tau}}(y)]$$

Thus, one can inductively define for each $y \in [\beta, \bar{\theta}]$ a set of numbers $\{\psi_t(y)\}_{t=0,1,\dots,T(y)-1}$

To see that $T(y) < \infty$, note that if $T(y) = \infty$, then one can define a decreasing (infinite) sequence of points $\{z_k\}$, $z_0 = \psi_0(y)$, $z_k = \psi_t(y)$ where $t = \sum_{i=0,1,\dots,k} \tau_i(y)$. As z_k is bounded below by $\underline{\theta}$, it converges to some $z^* \geq \underline{\theta}$. Rewriting (A.11), we have for $k > 1$

$$(A.12) \quad v\eta(z_{k+1}, z_k) - z_k = \delta^{\tau_k(y)}[v\eta(z_k, z_{k-1}) - z_k] \leq \delta[v\eta(z_k, z_{k-1}) - z_k]$$

Taking limit as $k \rightarrow \infty$ in the inequality (A.12) and using continuity of η , we obtain $(v - 1)z^* \leq \delta(v - 1)z^*$, a contradiction

Let $T = \min\{T(y): y \in [\beta, \bar{\theta}]\}$. It can be shown that

R.1. For $t = 0, 1, \dots, T - 1$, $\psi_t(y)$ is continuous on $[\beta, \bar{\theta}]$.¹⁹

Let $A = \{y \in [\beta, \bar{\theta}]: T(y) = T\}$ and let $w = \sup\{y: y \in A\}$. For each $y \in A$, $T = T(y) = \sum_{i=0,1,\dots,n} \tau_i(y)$ for some n (depending on y); let $k(y) = \tau_n(y)$

$$(A.13) \quad v\eta(\underline{\theta}, \psi_{T-1}(y)) - \psi_{T-1}(y) > \delta^{k(y)}[v\eta(\psi_{T-1}(y), \psi_{T-k(y)}(y)) - \psi_{T-1}(y)]$$

First, observe that $\bar{\theta}$ does not lie in the set A . This is because, by definition, $\psi_t(\bar{\theta}) = \psi_{t-1}(\beta)$, $t = 1, \dots, T - 1$, so that $T(\bar{\theta}) = T(\beta) - 1$, which, in turn, would contradict the fact that $T = \min\{T(y): y \in [\beta, \bar{\theta}]\}$. Consider any sequence $\{y_i\} \uparrow w$, $y_i \in A$ and the associated sequence $\{k(y_i)\}$. Let \hat{T} be a large enough positive integer such that

$$(v - 1)\underline{\theta} > \delta^{\hat{T}}(v\bar{\theta} - \underline{\theta})$$

¹⁹ The proof of R.1, which is somewhat long, is contained in Janssen and Roy (1998).

It is easy to see that $k(y_n) \leq \hat{T}$. Therefore $\{k(y_i)\}$ has a convergent subsequence $\{k(y_{i_j})\}$, converging to some k^* . Further, since $k(y_j)$ lies in a finite set $\{1, 2, \dots, \hat{T}\}$ it follows that for j large enough, $k(y_j) = k^*$ and

$$v\eta(\underline{\theta}, \psi_{T-1}(y_j)) - \psi_{T-1}(y_j) > \delta^{k^*} [v\eta(\psi_{T-1}(y_j), \psi_{T-k^*}(y_j)) - \psi_{T-1}(y_j)]$$

so that using continuity of η, ψ_t for $t < T$, it follows that

$$(A.14) \quad v\eta(\underline{\theta}, \psi_{T-1}(w)) - \psi_{T-1}(w) \geq \delta^{k^*} [v\eta(\psi_{T-1}(w), \psi_{T-k^*}(w)) - \psi_{T-1}(w)]$$

If (A.14) holds as strict inequality, then $w \in A$ and hence $w < \bar{\theta}$ so that using continuity of η and ψ_t for $t < T$, there would be an open interval $N(w)$ around w so that for all $y \in N(w)$, (A.14) holds, which contradicts the definition of w . Hence, (A.14) holds with an equality; that is,

$$v\eta(\underline{\theta}, \psi_{T-1}(w)) - \psi_{T-1}(w) = \delta^{k^*} [v\eta(\psi_{T-1}(w), \psi_{T-k^*}(w)) - \psi_{T-1}(w)]$$

By construction, we now have a T -period equilibrium.²⁰ The proof is complete. ■

A.4. *Proof of Proposition 4.1.* The proof relies on the construction of dynamic equilibrium in the proof of Proposition 3.3. There we had shown the existence of a dynamic equilibrium where the lowest quality sold in the last period (with strictly positive measure) of trading lies in $[\beta, \bar{\theta}]$. It is easy to check that β , as defined in the proof of Proposition 3.3, is identical to $\beta(\bar{\theta})$. Suppose T is the last period in which strictly positive measure of goods is traded and suppose the interval of quality traded in period T is $(\gamma_{T-1}, \bar{\theta})$. As $\gamma_{T-1} \geq \beta(\bar{\theta})$ and $\beta(\bar{\theta}) > \underline{\theta}$, then Condition C implies that $\gamma_{T-1} > \alpha(\bar{\theta})$, which implies that

$$(A.15) \quad (v - 1)\gamma_{T-1} > \delta[v\eta(\gamma_{T-1}, \bar{\theta}) - \gamma_{T-1}]$$

If the measure of trade occurring in period $(T - 1)$ is zero, then by definition of equilibrium, $p_{T-1} \geq v\gamma_{T-1}$ so that (A.15) would imply

$$p_{T-1} - \gamma_{T-1} > \delta[v\eta(\gamma_{T-1}, \bar{\theta}) - \gamma_{T-1}] = \delta[p_T - \gamma_{T-1}]$$

which would imply that a strictly positive measure of sellers with quality above γ_{T-1} would be better off selling in period $(T - 1)$ instead of T , a contradiction. Hence, positive trading must occur in period $(T - 1)$. More generally, suppose strictly positive measure of trade occurs in the last k periods, $k = 0, 1, \dots, T - 2$ and suppose $[\gamma_{T-i}, \gamma_{T-i+1}]$ is the (closure) of the quality interval sold in period $(T - i)$, $i = 1, 2, \dots, k$. Then,

$$(A.16) \quad [v\eta(\gamma_{T-k}, \gamma_{T-k+1}) - \gamma_{T-k+1}] = \delta[v\eta(\gamma_{T-k+1}, \gamma_{T-k+2}) - \gamma_{T-k+1}]$$

From (A.16) and the fact that $\gamma_{T-k+1} < \gamma_{T-k+2}$ (strictly positive measure of trade occurs in period $(T - k + 1)$), it follows that

²⁰ Observe that in this equilibrium, the range of quality traded in period T is $[w, \bar{\theta}]$ where $w \geq \beta$, which implies that $(v\eta(w, \bar{\theta}) - \bar{\theta}) \geq \delta(v - 1)\bar{\theta}$. Further, if $w = \bar{\theta}$, then almost all goods are traded by period $(T - 1)$, which is effectively the last period of trading; observe that, by definition, $\psi_2(\bar{\theta}) = \beta$ so we can choose $(\gamma_{N-1}, \gamma_N) = (\beta, \bar{\theta})$.

$$[v\eta(\gamma_{T-k}, \gamma_{T-k+1}) - \gamma_{T-k+1}] > \delta(v - 1)\gamma_{T-k+1}$$

which, using the definition of $\beta(\cdot)$, implies

$$\gamma_{T-k} > \beta(\gamma_{T-k+1})$$

which, in turn, implies (using Condition C and the fact that $\gamma_{T-k} > \underline{\varrho}$) that

$$\gamma_{T-k} > \alpha(\gamma_{T-k+1})$$

so that

$$(A.17) \quad (v - 1)\gamma_{T-k} > \delta[v\eta(\gamma_{T-k}, \gamma_{T-k+1}) - \gamma_{T-k}]$$

If the measure of trade occurring in period $(T - k - 1)$ is zero, then by definition of equilibrium $p_{T-k-1} \geq v\gamma_{T-k}$ so that (A.17) would imply

$$p_{T-k-1} - \gamma_{T-k} > \delta[v\eta(\gamma_{T-k}, \gamma_{T-k+1}) - \gamma_{T-k}] = \delta[p_{T-k} - \gamma_{T-k-1}]$$

which, in turn, would imply that a strictly positive measure of sellers with quality above γ_{T-k} would be better off selling in period $(T - k - 1)$ instead of $(T - k)$, a contradiction. Hence, strictly positive measure of trade must occur in period $(T - k - 1)$. The proposition follows by induction. ■

A.5. *Proof of Proposition 6.1.* (i) Consider any dynamic (Walrasian) equilibrium. The following strategies and out-of-equilibrium beliefs form an outcome equivalent *pbe* of the signaling game. If a seller i with quality $\theta(i)$ sells in period t at price $p_t(c)$ in the dynamic equilibrium of the Walrasian market, then in the signaling game, he sets price equal to $p_t(c)$ in period t and price equal to $v\bar{\theta}$ in all other periods. Buyers follow the following strategy: in periods in which trade occurs in the dynamic equilibrium of the Walrasian market, they randomize between buying and not buying from a seller who charges a price equal to $p_t(c)$; they randomize in such a way that the market clears in all such periods. Otherwise, they do not buy. The out-of-equilibrium beliefs of buyers are such that the expected quality (conditional on observing an out-of-equilibrium price) is exactly equal to $\underline{\varrho}$. Given their beliefs, the buyers' strategy is optimal. Similarly, if an individual seller charges a higher price than $p_t(c)$ in any period t in which trade occurs in our dynamic equilibrium, then no buyer would buy. On the other hand, if a seller tries to sell at any price in a period where no trade occurs in our dynamic equilibrium, the expectation about his quality ensures that no one buys, unless he sets a price exactly equal to $v\underline{\varrho}$. From the definition of dynamic equilibrium it follows that sellers cannot gain from any such deviation.²¹ This establishes our claim.

(ii) The main arguments are as follows. First, observe that in any *pbe*, sellers with lower qualities will not sell later than sellers with higher qualities (if they sell at all). Also, if positive trading occurs in any period, then all trade must take place at the

²¹ Note that our definition of dynamic equilibrium in Section 2 ensures that sellers find it optimal to sell in periods in which they are supposed to sell even when prices are at least as large as buyers' valuation of the highest unsold quality (which is greater than or equal to $v\underline{\varrho}$) in periods of no trade.

same price. This implies that every *pbe* is characterized by a partition of the support of quality with higher intervals traded in later periods and sellers with quality in the same interval trading at the same price, if they trade at all. The seller with the marginal quality traded in any intermediate period with positive trading must earn the same discounted surplus if he sells in the next period in which positive trade occurs (at the price at which other sellers sell in that period). Next, observe that if the highest quality traded (over all periods) is less than $\bar{\theta}$, then the marginal seller selling the highest quality traded must earn zero surplus. If we now impose the requirement that the equilibrium satisfies the intuitive criterion, then an identical argument as the one outlined in Section 6 (just before Proposition 6.1) shows that this marginal seller can deviate and sell at a price slightly higher than his reservation value in the next period. Hence, the highest quality traded cannot be below $\bar{\theta}$. Finally, a limiting argument along the lines of that used in the proof of Proposition 3.3 shows that all goods must be traded in finite time.

All that remains to be done is to specify the Walrasian prices. In periods where trade occurs, we can set the Walrasian price exactly equal to the price at which trade takes place in the equilibrium of the signaling game. In periods where no trade occurs in the signaling game, we cannot readily imitate the prices chosen by sellers (with no intention of selling). In fact, such prices should meet condition (iv) of the definition of dynamic equilibrium and at the same time, no seller should sell in those periods. Consider any period $t < T$, such that trade does not occur in that period on the equilibrium path of the signaling game, though trade occurs in an earlier and in a later period. Let $(t - k)$ be the last period before t in which trade occurred, say at price p_{t-k} , and let θ^* be the marginal quality sold in that period. Let $(t + h)$ be the next period in which trade occurs—at price p_{t+h} . Then,

$$(A.18) \quad \delta^{k+h}(p_{t+h} - \theta^*) = (p_{t-k} - \theta^*)$$

It is sufficient to show that

$$(A.19) \quad \delta^k(v\theta^* - \theta^*) \leq [p_{t-k} - \theta^*] = \delta^{k+h}(p_{t+h} - \theta^*)$$

for in that case, we can set the Walrasian price in period t equal to $v\theta^*$ and no seller would gain by selling in that period in the competitive market model. Now, suppose the equilibrium of the signaling games violates (A.19). Then

$$(A.20) \quad \delta^k(v\theta^* - \theta^*) > [p_{t-k} - \theta^*] = \delta^{k+h}(p_{t+h} - \theta^*)$$

so that there exists $p^* < v\theta^*$ such that

$$(A.21) \quad \delta^k(p^* - \theta^*) = [p_{t-k} - \theta^*] = \delta^{k+h}(p_{t+h} - \theta^*)$$

and

$$(A.22) \quad (p^* - \theta) < \delta^h(p_{t+h} - \theta) \quad \text{if and only if } \theta > \theta^*$$

$$(A.23) \quad \delta^k(p^* - \theta) < [p_{t-k} - \theta] \quad \text{if and only if } \theta < \theta^*$$

Consider a deviation by a seller with quality θ^* such that he charges a price *slightly* above p^* in period t (in particular, less than $v\theta^*$). From (A.21), it is easy to see that this would be gainful provided he could trade at such a price. From (A.22) and

(A.23), it follows that only sellers with quality very close to θ^* could have the incentive to charge such a price. The intuitive criterion then says that if such a price is observed, buyers should infer that this price quote comes from a seller with a quality close to θ^* , and hence their willingness to pay would be close to $v\theta^*$. Therefore, the seller would gain from such a deviation. A slight modification of this argument can show that if no trade occurs before period t and $(t+h)$ is the first period after t in which trade occurs, then $(v\theta^* - \theta^*) \leq \delta^h(p_{t+h} - \theta^*)$. This completes the proof. ■

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