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Catching hipos: screening, wages, and competing for a job

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In this paper, I study the wage a firm sets to attract high ability workers (hipos) in situations where people compete for a job. I show that the more people compete, the larger a firm's incentives to sort high and low ability workers. Moreover, workers will signal their (high) ability in situations with many competitors only if a job offers a high enough wage. The main result, therefore, is that a firm sets higher wages, when more people compete.

1. Introduction

Since the pioneering work of Spence (1973) many economists have investigated the consequences of asymmetric information in the labour market. Although Spence studied the consequences of signalling in a competitive market, a large part of the subsequent literature is game theoretic in nature.[1] In the strategic literature the typical model has two firms competing for one worker whose ability is unknown. The reason that a worker may signal his ability is due to the fact that firms are willing to offer higher wages to more able workers (who have signalled their ability). In this type of models there is no role for competition among potential employees to get a (well-paid) job.

It is quite plausible that signalling is observed more in periods or sectors where there is substantial competition to get jobs. The main reason workers (students) may signal their (high) ability is that they want to increase their chances of getting a (well-paid) job. In order to study the impact of competition between workers on their signalling activity I look at a situation in which N workers compete for one job in one firm.[2] The ability of each worker is modelled as an independent draw from a common pool of abilities. Each individual worker knows his own ability, but does not know the abilities of the other workers. The firm only knows the characteristics of the common pool from which abilities are drawn. In the model, the firm sets wages before the workers signal and firms are committed to pay the

[1] See, e.g., Cho and Kreps (1987) for the first complete game theoretic analysis of the Spence model, and Fudenberg and Tirole (1991) and Rasmusen (1989) for textbook presentations.

[2] For a survey of the way monopsony models have been used in studies of the labour market, see Boal and Ransom (1997).

wages they announced.[3] Once the firm has observed the signals, it will choose one of the workers.

The model is intended to describe certain segments of the labour market, especially the market for hipos, i.e. high potential (hipo) students who just graduated and apply for 'big boys in the city jobs', including jobs in investment banking, consultancy, and law firms. Firms in these market segments have a reputation for offering high wages and students in universities and MBA programs know this.[4] The commitment to paying high wages that I assume is made credible through this reputation mechanism. Students compete to get jobs for these highly paid jobs by trying to get high grades, doing many extra curricular activities, and so on. Accordingly, the signals that are modelled here are not just education, but activities students pursue during or instead of their studies (cv-building). As long as these signals are positively correlated with ability, firms are willing to offer higher wages, because it increases the chance of attracting high ability workers.[5]

The main result of the model says that for a wide range of parameter values there is a positive relation between the number of potential employees, measured by N, and the relative wage rate. In other words, the monopsonist firm sets higher relative wages, the larger the number of workers looking for a job.[6] The intuition for the result can be explained in two steps. First, if the firm wants high ability workers to signal their ability it should set higher wages, the more people are competing for a job. To understand why this must be so, it is important to realise that as the firm is a monopsonist in the labour market, it will set the lowest wage possible such that high ability workers will signal their ability. Moreover, workers will signal only if the expected benefits of signalling are larger than the signalling cost. Each worker realises that as there are more people competing for a job, there may also be more highly qualified people so that the chance of getting a job in case of signalling decreases in N. This implies, of course, that the expected returns of signalling are decreasing in N and therefore, people are less willing to signal. To induce the high ability workers to signal, the firm has to set higher wages when N gets larger. Second, when the incentives for firms to make workers signal their ability are stronger, the larger the level of unemployment. If no worker signals, the firm has

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[3] Hence, our model falls in the category of screening models (see Weiss, 1995). The reason for the assumption is that in a signalling model in which workers signal before the firm sets its wages, the signalling costs of the worker are sunk and the firm does not have an incentive to set a wage above the reservation wage of the worker.

[4] It can be argued that these firms commit themselves to setting high wages by organizing information gatherings for the students in which they inform them about their career perspectives.

[5] Alternatively, it makes the selection costs of the firms lower, because part of the selection is done in the form of self-selection on the part of the students.

[6] For simplicity, we keep the reservation wage constant in our model. The wage rate that appears in the model should be interpreted as a level of wages relative to the reservation wage. To the extent that the reservation wage depends on N, our result basically says that the gap between the 'haves' and the 'have nots' increases with the level of competition between workers. In this way, we explain the empirical fact alluded to above.

to choose one of the workers at random. However, if the number of applicants increases relative to the number of vacancies, the chances of getting really good people increases. Thus, it may be the case that the firm does not induce workers to signal their ability when there are few people competing for a job, but that it does induce them to signal when this number gets larger. This explains that at a certain number of competing workers, the wage rate may jump upwards.

It is difficult to verify whether there is some empirical evidence that sheds some light on this somewhat surprising result. In practice it may be difficult to demarcate workers in groups competing for the same jobs. There is, however, some macroeconomic evidence suggesting that the difference between low and high income groups becomes larger if unemployment increases. Blinder and Esaki (1978, p. 607), for example, find that 'each one percent point rise in the unemployment rate takes about 0.26%–0.30% of the national income away from the lowest 40% of the income distribution and gives it to the richest 20%'. Especially, high income groups apparently benefit from high levels of unemployment. Similar results are also found in other studies, see e.g. Gramlich (1974). Our microeconomic model has a similar flavour as this macroeconomic evidence: if more people are competing for jobs, the wage of the people who get the job may increase relative to the reservation wage of people who do not get a job.

Information asymmetries have been used in many different ways in studies of the labor market; see Weiss (1995) for a survey. Traditionally, the focus was on asymmetric information between a worker and a firm about the worker's ability level. More recently, a number of papers have shifted attention to an employer's private information (vis-à-vis the market) about an employee's ability (see, e.g. Waldman, 1984, and Gibbons and Katz, 1991). In contrast, I stay within the framework set out by Spence (1973). Dynamic issues concerning signalling and education have been studied in Nöldeke and van Damme (1990).

In the efficiency wage literature (see Weiss, 1990, for a survey) a firm chooses to set relatively high wages as this yields higher labour productivity. Two mechanisms are distinguished, a selection effect and an incentive effect. Our paper can be considered as giving an alternative explanation for the selection mechanism based on asymmetric information. At low wages, firms have to select a worker at random, whereas at higher wages, workers selects themselves out through signalling and the firm can select from a smaller pool of high quality workers.

Another related paper is that of Lazear and Rosen (1981). They show that it may be optimal for a firm to set wages based on rank, and to pay high salaries to executives, as this provides incentives for all other individuals in the firm to work hard in order to 'win' one of the executive positions in the future. Our model shows that the idea of competitive lotteries can also be applied in the labor market, a well-paid job being the prize and fellow students being the competitors.

The paper is organised as follows. Section 2 describes the model. In Section 3 I analyse the model and characterise the equilibrium properties and depend on the parameters. Section 4 concludes.

2. The model

The model has N workers and one firm with one job vacancy.[7,8] In stage 0, Nature decides about the type of each worker. Workers can be of different abilities (productivities); the ability of agent i is denoted by θ_i. Abilities are uniformly distributed on the interval [0,1]. The ability level is private information to the worker, i.e. neither the firm nor the other workers know the ability of a worker. The timing of events is as follows. First, in stage 0, workers receive a private signal about their ability level. In stage 1 the firm offers wage contracts. It sets two wages, $w(1)$ and $w(0)$, where $w(1)$ denotes the wage for a worker who signals and $w(0)$ denotes the wage for a worker who does not signal. As it is optimal for the firm to set $w(0) = 0$, we will concentrate on $w(1)$ and when there is no room for confusion, we drop the dependence of the wage rate on the signal. In stage 2, knowing the wages that are offered, workers simultaneously decide whether or not to signal. For simplicity, I assume that the signal s can take on two values, 0 (no signal) or 1 (signal). Finally, in the last stage of the game (stage 3) the firm selects one worker possibly based on the signal it observes. If none of the workers signals or if they all signal, the firm randomly selects a worker. If some workers signal and the others do not, it may choose randomly among the workers that signalled or among those that did not.

The pay-off to the firm depends on the worker it hires and the wages it offers. The pay-off to the worker if he is hired depends on the wages that are offered and the signalling cost. Denote the ability (or labour productivity) of individual i by θ_i and the signalling cost of individual i by $c(\theta_i)$. I assume the signalling cost of an individual with ability level θ to be given by $c(\theta) = k/\theta$, i.e. it is more costly to signal for a low ability worker than for a high ability one. In order to have non-trivial solutions, I assume $k < \frac{1}{2}$.[9] The pay-off to the firm when hiring worker i with signal s is then given by $\pi_f = \theta_i - w(s)$ and the pay-off to the worker who is hired by $\pi_i = w(s) - c(\theta_i)$. The workers that signalled and are not hired get a negative pay-off equal to their signalling cost.[10] Both the firm and the workers are assumed to be risk neutral and maximise their pay-offs given the strategies of the others.

A strategy for a worker is a decision whether or not to signal depending on his ability level and the wage that is offered. Such a strategy can be denoted by $s(\theta, w)$. A strategy for the firm is to set a wage rate $w(1)$ and to select a worker in stage 3 depending on the signalling activities of workers. The equilibrium we seek is a symmetric subgame perfect Nash equilibrium in pure strategies. To this end, we

[7] In other words, the marginal productivity of labour of a second worker is very low.

[8] In a discrete version of the model (see, Janssen *et al.*, 1998) we analyze a duopsony extension with two firms hiring workers. The analysis is quite messy, but for some parameter values the equilibrium exhibits similar properties as the equilibrium of the present model.

[9] When k is larger, the signalling cost are so high that workers only signal at wages the firm is never willing to offer.

[10] Hence, we assume that the worker's reservation.

use Backward Induction where, in stage 2, we analyze the symmetric equilibrium decisions of workers given the wage rate that is set by the firm.

2.1 Equilibrium properties

In this section I will analyse the equilibrium properties of the model. From the set-up described in the previous section, it is clear that the firm can set $w(1)$ in such a way that workers with ability levels larger than some critical level θ^*, depending on $w(1)$, will signal and workers with an ability level lower than θ^* do not signal. What the firm is in fact doing is dichotomising the continuous type space. Given this behaviour, it is clear that it can never be optimal for the firm to induce some workers to signal and then to choose among those workers that did not signal. This is not optimal as the firm would be better off not offering any wage and just to select a worker at random: the average ability of workers he would attract in this way is higher than when high quality workers signal and then not considered for hiring.

Knowing that one of the workers who signalled will be hired (if one signals at all), a worker with ability θ will signal if the wage multiplied by the probability of getting the job is larger than or equal to his cost of signalling. The probability of getting the job depends, of course, on the signalling behaviour of the others and not on the others' ability level. For any arbitrary level of w let us look for an equilibrium in stage 2 of the game where people who have an ability level larger than θ^* and only those will signal. It can be shown (see Appendix 1) that the probability of getting the job for worker i is then given by

$$\sum_{k=0}^{N-1} \binom{N-1}{k} (1-\theta^*)^{N-k-1} (\theta^*)^k \frac{1}{N-k} = \frac{(1-\theta^*)^N)}{N(1-\theta^*)} \tag{1}$$

Hence, in the N worker case, worker i with ability level θ will signal if, and only if

$$\pi_i(\theta;\theta^*) = \frac{(1-\theta^{*N})}{N(1-\theta^*)}\, w(1) - \frac{k}{\theta} \geqslant 0$$

Given the signalling behaviour of others and the wage rate offered by the firm, worker i will signal if his ability level is at least equal to $kN(1-\theta^*)/(1-\theta^{*N})w(1)$. As in a symmetric equilibrium $\theta = \theta^*$, it follows that θ^* is implicitly given by

$$w(1) = \frac{kN(1-\theta^*)}{\theta^*(1-\theta^{*N})} \tag{2}$$

Workers with ability levels smaller than θ^* will not signal at this wage rate. By using (2) the maximisation problem of the firm can be written as if the firm chooses θ^*

$$\max \pi_f = (1-\theta^{*N})\left(\int_{\theta^*}^{1} \theta\, \mathrm{d}\theta - w(\theta^*)\right) + \theta^{*N}\left(\int_{0}^{\theta^*} \theta\, \mathrm{d}\theta\right). \tag{3}$$

Equation (3) has a straightforward interpretation: given a certain critical value θ^*, there is a probability $(1-\theta^{*N})$ that at least one worker signals and the expected pay-off to the firm is then given by the expected ability level conditional on the worker signalling minus the wage he has to pay. There is also a remaining probability that no worker signalled in which case the firm selects one worker at random knowing that all workers have an ability level below θ^*. Substituting the separating wage given in (2) in (3) I get

$$\pi_f = (1-\theta^{*N})\left(\frac{1}{2}(1+\theta^*) - \frac{kN(1-\theta^*)}{\theta^*(1-\theta^{*N})}\right) + \frac{1}{2}(\theta^*)^{N+1} \tag{4}$$

which reduces to

$$\pi_f = \frac{1}{2}(1-\theta^{*N}) + \frac{1}{2}\theta^* - \frac{kN(1-\theta^*)}{\theta^*} \tag{5}$$

Maximising this expression with respect to ability yields an optimal θ^*. This is the demarcation ability level chosen by the firm in the separating equilibrium. The optimal θ^* is

$$-\frac{N}{2}\theta^{*N-1} + \frac{1}{2} + \frac{kN}{\theta^{*2}} = 0 \tag{6}$$

From the θ^* that solves equation (6) the wage $w(\theta^*)$ that the firm offers follows by virtue of (2). In order to emphasize the fact that the optimal wage rate and the demarcation value of θ depend on the exogenous parameters N and k, we also write $w_N(k)$ and $\theta_N(k)$, respectively. The rest of the section is devoted to investigate how $w_N(k)$and $\theta_N(k)$depend on N and k.

First, note that the second-order derivative of the profit function with respect to θ is strictly negative and that the first-order derivative at $\theta = 0$ is strictly positive. This implies that for each value of N and k there is a unique optimal value of θ^* in [0,1], possibly 1 itself. In order to have an interior solution the first-order derivative at $\theta = 1$ must be strictly negative. This is the case if

$$(\tfrac{1}{2} - k)N > \tfrac{1}{2}$$

When this condition is violated, it is optimal for the firm to set wages in such a way that no agent signals his ability and the firm randomly selects one of the agents. I will define the corresponding wage rate to be equal to 0. Figure 1 indicates the

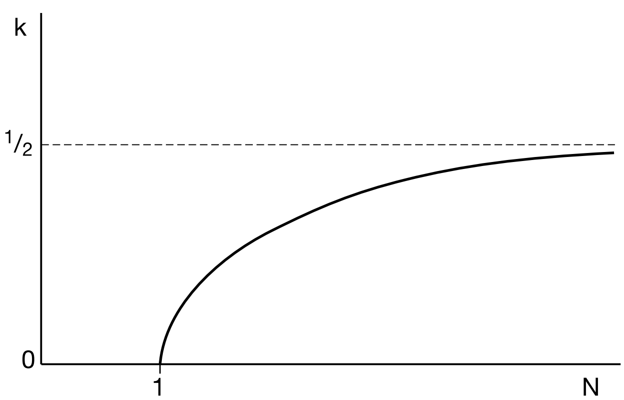

Fig. 1. For a given level of k competition for the job has to be strong enough to have the firm induce agents to signal their ability.

region of N and k where the firm chooses to induce agents to signal. From the figure it is clear that for a given value of k, there must be more than a critical number of workers competing for a job to make it worthwhile for the firm to induce workers to signal. Basically, the idea here is that if the number of applicants is large relative to the number of vacancies, the chances of getting really good people becomes larger and it pays to have them self-select. The firm has to offer a strictly positive wage to induce signalling. Hence, when competition for the job rises above a critical value, the optimal wage rate jumps from zero upwards. This is the second effect mentioned in the Introduction.

In case of an interior solution, we can also ask the question how the optimal wage rate $w_N(k)$ depends on the unemployment level, measured by N. It turns out that the dependence is unambiguously positive: the larger N, the larger $w_N(k)$. The result is formally stated in Proposition 1:

Proposition 1 If $(k - \frac{1}{2})N + \frac{1}{2} < 0$, then $w_N(k)$ is positively related to N.

The proof is given in Appendix 2. The standard way of proving a result like the one of Proposition 1 is to use the envelope theorem. Unfortunately, as we cannot explicitly write θ as a function of w, it is difficult to assess $\partial^2\pi_f/\partial w\,\partial N$. Instead, the proof compares the optimal value of $\theta_{N+1}(k)$ with a demarcation value of θ in case the firm keeps its wage rate at the level of $w_N(k)$. It is shown that the optimal value of $\theta_{N+1}(k)$ is smaller and that therefore the firm has to adjust its wage rate upward. The main economic intuition is that as there is more competition for the job, the chance of getting a job in case of signalling decreases and so do the expected returns of signalling. Therefore, the more competition among workers, the higher the wage the firm has to set in order to induce the high ability workers to signal.

The impact of N on $\theta_N(k)$ is less clear. When the cost of signalling is very low, i.e. k is close to 0, it follows from (6) that $\theta_N(k)$ is approximately equal to $\sqrt[N-1]{1/N}$. This expression is increasing in N. If, on the other hand, k is such that for a certain N $\theta_N(k)$ is close to 1, then it follows from (6) that $\theta_{N+1}(k) < \theta_N(k)$. There are two opposing forces at work. A lower value of $\theta_N(k)$ increases the chances of having at least one individual signal. This is beneficial to the firm as it can select a high ability worker. On the other hand, the firm has to pay higher wages if it wants to induce more signalling. When $\theta_N(k)$ is close to 1, the second effect is negligible as the probability is small that there is another agent with an ability level close to 1.

Figures 2 and 3 illustrate how for different values of k the optimal value of θ^* and $w(\theta^*)$ varies with N. When the signalling cost is small, the firm induces agents to signal for all values of $N > 1$. Figure 2 shows for $k = 0.2$ that when the number of competitors increases beyond a critical value, firms become more and more selective. The relation between the optimal wage w and N is also depicted in Fig. 2. One can see that wages are smoothly increasing in N and that they increase by around 40% from $N = 2$ to $N = 10$.

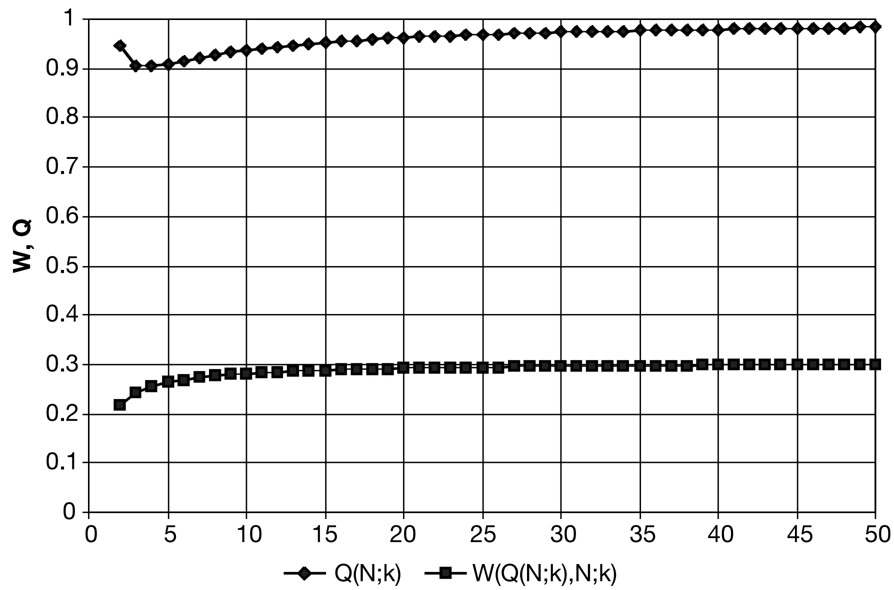

Fig. 2. θ^* and $w(\theta^*)$ as a function of N for $k = 0.2$.

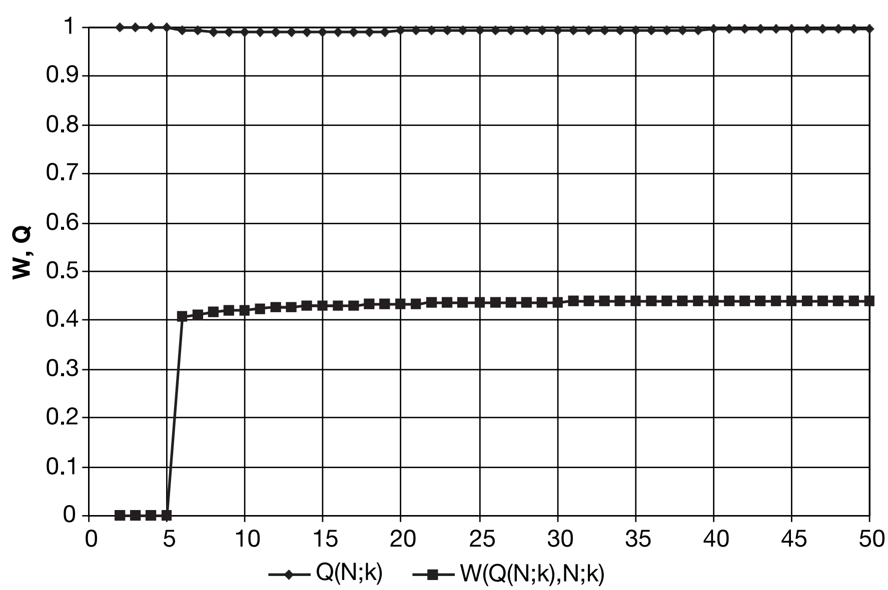

Fig. 3. θ^* and $w(\theta^*)$ as a function of N for $k = 0.4$.

When k is larger and signalling more costly, the firm does not induce the agents to signal for small values of N. This is depicted in Fig. 3 for $k = 0.4$. the figure shows that wages jump upwards at $k = 5$ and increase smoothly from that moment onwards. The corresponding values of θ^* are very close 1 and show a similar pattern as in Fig. 2.

Finally, I briefly discuss the impact of the cost of signalling measured by k $w_N(k)$ and $\theta_N(k)$. From eq. (6) it is clear that the impact on $\theta_N(k)$ is positive as long as there is an interior solution, i.e. the higher the cost of signalling the higher the demarcation value chosen by the firm. Also, the impact of a higher k on $w_N(k)$ is, as is to be expected, positive.

4. Discussion and conclusion

In this paper, I have analysed the consequences of having more people competing for any given job on the relative wage that is set by firms to sort out workers (students) with different ability levels. It has been shown that in a variety of settings this relation is positive, i.e. the more people are competing, the higher the (relative) wage set by the firm(s). This indicates that the gap between the rich and the poor, i.e. between those that have a well-paid job and those that have not, may become larger in situations where the model applies. I think that this is the case in 'big boys in the city jobs' in the sphere of investment banking, consultancy, and law firms.

It is important to understand the driving forces behind the result. The main ingredient of the model is that ability is private information, i.e. the firm and the workers do not know the ability levels of others. This implies that each worker has to conjecture what are his chances of getting the job if he signals. If there are more competitors, it is also more likely that there are a certain number of other people that find it profitable to signal. Hence, the expected benefits of signalling decrease when there are more potential competitors. If the firm wants to induce workers to signal their ability, then it has to pay a higher wage. Moreover, when there are more competitors for the job, it also becomes more profitable for the firm to have workers sort themselves out. This is because there is a greater chance that some very able people are competing and it is better to pay a bit more and hire one of them than to hire someone at random.

If, in contrast, the realisations of ability were common knowledge, but not who had what realisation, the firm could simply offer a wage such that just the individual with the highest ability finds it worthwhile to signal if the chance of getting the job equals 1. Nobody apart from the individual with the highest ability would find signalling worthwhile. The wage the firm has to offer is then equal to the signalling cost of the individual with the highest ability. As, in expected terms, this signalling cost decreases with the number of competitors, the optimal wage would decrease with the number of competitors in this alternative setting. Hence, the private information aspect of our model is crucial to get the result described here.

Another assumption concerns the crudity of the signalling space: individuals can only decide whether to signal or to abstain from it. There is no choice in signals.

The main result, however, also holds true in a slightly generalised world where individuals can choose a variety of signals that 'build on each other' in the sense that everyone can decide how many signals to choose, that more signals are considered to be better than fewer signals, and that individuals who find it more costly to signal in one dimension will also find it more costly to signal in another dimension. Whether the result also holds true in other signalling set-ups remains an issue for further research.

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Appendix 1

Equation (1) can be arrived at in the following way. Using the fact that

$$\binom{N-1}{k} = \frac{N-k}{N}\binom{N}{k},$$

the LHS of (1) can be written as

$$\frac{1}{N}\sum_{k=0}^{N-1}\binom{N}{k}(1-\theta^*)^{N-1-k}(\theta^*)^k.$$

Taking out $(1-\theta^*)$ and moving the summation up to N and correcting by subtracting the N^{th} term one gets:

$$\frac{1}{N(1-\theta^*)}\left(\sum_{k=0}^{N}\binom{N}{k}(1-\theta^*)^{N-k}(\theta^*)^k - (\theta^*)^N\right).$$

The binomial term in the left of the brackets is equal to 1, so that the whole expression equals

$$\frac{1}{N(1-\theta^*)}\,(1-(\theta^*)^N).$$

Appendix 2

Proof of Proposition 1

We will prove that for any k and N such that $(k-\frac{1}{2})N + \frac{1}{2} < N + \frac{1}{2} < 0$, $w(N) < w(N+1)$. The proof is in several steps. We first look for a fixed N at the first order condition for profit maximisation. From (7) it follows that the optimal θ^* has to satisfy

$$-\frac{N}{2}\,\theta_N^{N-1} + \frac{1}{2} + \frac{kN}{\theta_N^2} = 0 \tag{A.1}$$

where θ_N is the optimal value of θ for a given N. Note that for a fixed value of N, θ_N is increasing in k. This in turn implies that the smallest value of θ_N is given by $\sqrt[N-1]{1/N}$, which is larger than $\frac{1}{2}$. From (A.1) it follows that for any k

$$\theta_N^{N+1} - \frac{\theta_N^2}{N} = \theta_{N+1}^{N+2} - \frac{\theta_{N+1}^2}{N+1} \tag{A.2}$$

From eq. (2), we can also derive a relation between different values of θ in case the firm does not adjust w when N increases. I denote by $\tilde{\theta}_{N+1}$ the value of the demarcation value of θ_{N+1} when the firm does not adjust the wage rate from its optimal value in case of N unemployed agents. For each value of θ_N, it follows that

$$(N+1)(\theta_N + \cdots + \theta_N^N) = N(\tilde{\theta}_{N+1} + \cdots + \tilde{\theta}_{N+1}^{N+1}) \tag{A.3}$$

From eq. (2) it easily follows that for a fixed N, w is decreasing in θ for all $\theta < 1$. The rest of the proof concentrates on the relation between θ_{N+1} and $\tilde{\theta}_{N+1}$ and we show that $\theta_{N+1} < \tilde{\theta}_{N+1}$. This implies that the optimal value of θ is smaller than the value in case the firm does not adjust w. This together with the fact that w is decreasing in θ implies that if N increases, the firm should adjust its wage upwards.

From (A.3) it follows that $\theta_N < \tilde{\theta}_{N+1}$. On the other hand, from (A.1) it does not follow that $\theta_N < \theta_{N+1}$. In case $\theta_{N+1} < \theta_N$, it follows trivially that $\theta_{N+1} < \tilde{\theta}_{N+1}$. Hence, in the rest of the proof we concentrate on the case where $\theta_N < \theta_{N+1}$.

For notational simplicity, we will use in the rest of the proof x and y instead θ_N and θ_{N+1}, respectively. Using this notation, we rewrite (A.2) as

$$x^{N+1} - \frac{x^2}{N} = y^{N+2} - \frac{y^2}{N+1}$$

For each x let us define y_1 as the solution to

$$x^{N-1} - \frac{1}{N} = y_1^N - \frac{1}{N+1}.$$

For all x such that $x \leqslant y$ we know that $y \leqslant y_1$. We can rewrite the equation defining y_1 as

$$x = \sqrt[N-1]{y_1^N + \frac{1}{N(N+1)}}. \tag{A.4}$$

Next, let us rewrite (A.3) $(N+1)(x + \cdots + x^N) = N(\tilde{y} + \cdots + \tilde{y}^{N+1})$. For each x let us define y_2 implicitly as the solution to $(N+1)x = Ny_2 + y_2^2$. I will argue that $y_2 \leqslant \tilde{y}$. The proof consists of two steps. The first step argues that if $(N+1)x = Ny_2 + y_2^2$, then $(N+1)x^k \geqslant (N+1-k)y_2^k + ky_2^{k+1}$ for all k. Details of this first step are given below. The second step argues that if for all k $(N+1)x^k \geqslant (N+1-k)y_2^k + ky_2^{k+1}$, then it follows by simple summation that

$$(N+1)\sum_{j=1}^{N} x^j \geqslant N \sum_{j=1}^{N+1} y_2^j.$$

As $\tilde{y}$ is implicitly defined by

$$(N+1)\sum_{j=1}^{N} x^j = N \sum_{j=1}^{N+1} \tilde{y}^j.$$

it follows that $y_2 \leqslant \tilde{y}$.

The proof of the first part is by induction on k. It is clear that the statement holds for $k=1$. So, suppose then that the statement holds true up to a certain k, $2 \leqslant k < N$, i.e. if $(N+1)y_2 = Ny_2 + y_2^2$, then $(N+1)x^k \geqslant (N+1-k)y_2^k + ky_2^{k+1}$. We will then show that also if $(N+1)x = Ny_2 + y_2^2$, then $(N+1)x^{k+1} \geqslant (N-k)y_2^{k+1} + (k+1)y_2^{k+2}$. This is the case if (using expressions for x and x^{k+1})

$$\begin{aligned}
&\frac{(Ny_2 + y_2^2)[(N+1-k)y_2^k + ky_2^{k+1}]}{(N+1)} \geqslant (N-k)y_2^{k+1} + (k+1)y_2^{k+2} \\
&\quad\Leftrightarrow (N+y_2)[N+1-k+ky_2] \geqslant (N-k)(N+1) + (N+1)(k+1)y_2 \\
&\quad\Leftrightarrow k - 2ky_2 + y_2^2 \geqslant 0 \\
&\quad\Leftrightarrow k(1-y_2)^2 \geqslant 0.
\end{aligned}$$

Hence $(N+1)x = Ny_2 + y_2^2$ then $(N+1)x^k \geqslant (N+1-k)y_2^k + ky_2^{k+1}$ for all $k \leqslant N$.

We can rewrite the equation defining y_2 as

$$x = \frac{Ny_2 + y_2^2}{(N+1)} \tag{A.5}$$

Finally, we prove that $y_1 < y_2$. Using (A.4) and (A.5) and the fact that in both equations x is a monotonically increasing function of y_1 and y_2, respectively, it suffices to show that for the some arbitrary value of $y_1 = y_2 = z$

$$\frac{Nz + z^2}{N+1} < \sqrt[N-1]{z^N + \frac{1}{N(N+1)}}$$

$$\Leftrightarrow z^N + \frac{1}{N(N+1)} > \frac{z^{N-1}(N+z)^{N-1}}{(N+1)^{N-1}}. \tag{A.6}$$

For $N = 2$, (A.6) reduces to $(N+1)z^2 + \frac{1}{2} > (N+z)y$, or, $2z^2 - 2z + \frac{1}{2} > 0$. It is easy to see that this holds if $z > \frac{1}{2}$, which is true given the discussion just below (A.1). For $N > 2$, the inequality is more difficult to prove. The interested reader is referred to the working paper (see Janssen, 2001).