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When Does Evolution Lead to Efficiency in Communication Games?*

Karl H. Schlag

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Wirtschaftstheoretische Abteilung III
Friedrich-Wilhelms-Universität Bonn
Adenauerallee 24-26
D-53113 Bonn
Germany
e-mail: schlag@glider.econ3.uni-bonn.de

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Abstract

The evolutionary selection of outcomes (modelled using the replicator dynamics) in games with costless communication depends crucially on the structural assumptions made on the underlying population. (1) In conflicts between two interacting populations, common interest implies that the set of efficient outcomes is the unique evolutionarily stable set. Lack of common interest prevents sets with minimal stability properties to exist. (2) For conflicts within one population, inefficient evolutionarily stable strategies may exist independent of whether there is common interest or not. This is no longer true when there is a dominant strategy, in this case the efficiency result of the two population setup is recovered.

Keywords: cheap talk, efficiency, common interest, evolutionarily stable set, strict equilibrium set, asymptotic stability, minimal attracting set, replicator dynamics.

JEL classification number: C79.

0. Introduction

The object of the paper will be to investigate the effect of pre-play communication on the evolution of strategies for playing a given game.

Communication is modelled as cheap talk: before the game is played, the players simultaneously exchange messages from some finite set of messages. There is no cost to exchanging these messages and hence "talk is cheap".

Evolution is a dynamic concept and as such we will explicitly specify a dynamic process and analyze dynamic stability. We select two versions of the continuous replicator dynamics (Taylor and Jonker [1978], Taylor [1979]) for our analysis, among other reasons because both of these dynamics have lately turned out to be the approximations of various individual learning models (see Binmore, Gale and Samuelson [1993], Börgers and Sarin [1993], Cabrales [1993], Schlag [1994b]).

The basic story behind these two dynamics is the same, a large number of agents are matched, receive a payoff (or fitness) according to an underlying game and then adapt their strategies (or reproduce) according to a given dynamic process in which growth rates are proportional to relative performance of a strategy. The difference lies in the population structure. In the version of Taylor and Jonker [1978] all agents belong to the same population (referred to as the one population setting) whereas the version of Taylor [1979] considers a conflict between two disjoint populations (which we refer to as the two population setting).

There are various (more or less) static models of cheap talk that each point to the fact that communication in an evolutionary environment will select against inefficient outcomes. The object of this paper will be to pursue this stylized fact in an explicit dynamic analysis. It turns out that the modelling of the population structure and the associated matching and reproduction (learning) dynamics has a drasitc influence on the results of the analysis. In the two population setting common interest among the agents that are matched is necessary and sufficient for efficient outcomes to evolve. Moreover, without common interest, sets with minimal stability properties in the dynamic process fail to exist. In the one population setting common interest only determines whether or not efficient outcomes are stable. The existence of stable sets is independent of common interest. Especially, inefficient evolutionarily stable strategies may exist in the game with cheap talk.

Due to the numerous papers in this research area we now give a brief overview of the related literature.

0.1 The literature

Matsui [1989, 1991] was the first to analyze the effect of cheap talk in a dynamic related setting. They considered games of common interest with two strategies for each player in the two population scenario described above. A game has common interest if both players only simultaneously receive their maximal outcome. Matsui [1991] shows that cheap talk leads to efficiency in cyclically stable sets, a solution concept derived from the best response dynamics.

Robson [1990] introduces a closely related model with mutants that are able to perform secret handshakes. It is assumed that a mutant can recognize when he is matched against another mutant but the rest of the population cannot distinguish them other than through the strategies they play. Robson [1990] shows for two by two unanimity games (a unanimity game is a symmetric game in which the payoffs are positive and the same for both players on the main diagonal and zero otherwise) that an evolutionarily stable strategy (ESS) must achieve the maximal payoff.

Wärneryd [1991] shows for symmetric two by two unanimity games with cheap talk that a pure strategy is a neutrally stable if and only if it achieves the maximal payoff. We will see that the restriction to pure strategies is crucial for this to be true.

Bhaskar [1992] analyzes neutral stability when the population game is modelled as a truly asymmetric contest. Here players in the population are a priori identical, however when two players are matched a random draw determines which of the two players will be in the role of player one and who will be in the role of player two. Bhaskar [1992] considers an infinite message space and perturbs the communication game.

Kim and Sobel [1991] look for sets that are equilibrium evolutionarily stable (EES) in games with cheap talk. For games that have both common interest and equilibrium common interest they essentially show that the set of efficient outcomes is the unique set that is equilibrium evolutionarily stable. They also point out that their results depend crucially on the fact that they consider the two population setting.

In a later version Kim and Sobel [1994] drastically revise this paper and consider a finite population matched to play a tournament (every one plays with everyone) and adapting according to a stochastic dynamic process. In this model cheap talk leads to efficiency in both the one and the two population settings under common interest. Here the efficiency for the one population setting relies on the fact that there are more messages than individuals in the population.

Our paper generalizes the results of an earlier version (Schlag [1993]) that analyzed the one population dynamic setting in a restricted class of games. The only other explicit dynamic analysis of cheap talk in this setup (players exchange messages simultaneously) is the recent model of Kim and Sobel [1994] mentioned above. In related models of communication, dynamics have been explicitly investigated too. Nachbar [1993] considers a situation in which sending signals is costly, Nöldeke and Samuelson [1992] consider sender-receiver games.

In section 5 the above literature is examined in more detail.

0.2 The replicator dynamics

In their biological interpretation, the replicator dynamics is the approximation of the following discrete time process defined in a large population of subjects or agents, each playing a pure strategy. In each period agents are pairwise randomly matched to play a game and receive a reproductive fitness. Agents then reproduce at a rate that is proportional to the difference between the payoff they achieved and the average payoff in the population. After that they die. These are the replicator dynamics in the one population setting (see Taylor and Jonker [1978]). This scenario can also be applied to the two population setting where the contest takes place between two disjoint populations, one associated to each player in the underlying game. As before, agents play pure strategies. In the two population setting agents of opposite populations are matched but then breeding is true, i.e. it takes place among agents of the same population. As before the reproduction (in each population) is proportional to the difference between the achieved payoff and the average payoff in the respective population. This leads to the version of the replicator dynamics for the two population setting due to Taylor [1979].

The version of the replicator dynamics we consider have independently been shown to approximate the dynamic behavior of various models of adapting individuals. In the following model of Schlag [1994b] the dynamics have unique properties since here individual behavior is determined endogenously. In the two population setting agents of opposite populations are repeatedly randomly matched. Between matching rounds each agent randomly samples the strategy and payoff of another agent in the same population. Any given agent imitates the strategy of the sampled agent only if this agent achieved a higher payoff in which case imitation occurs with probability proportional to the difference in the achieved payoffs. In large populations this behavior is approximated by the version of the replicator dynamics due to Taylor [1979]. It is easily shown that in the one population setting the dynamics are approximated by the version of Taylor and Jonker [1978].

0.3 Solution concept

We will search for a subspace in the set of all population configurations in which the population subject to very rare mutations will be absorbed. The relevant concept is that of a minimal attracting set. Starting at a population distribution in such a closed set, after a one time mutation of sufficiently small size the population will eventually evolve to a distribution close to the set again. A concept demanding stronger stability characteristics is that of an asymptotically stable set. Trajectories starting close to such a set will stay close to the initial starting point and eventually converge to an element in the set.

Various static concepts are related to dynamic stability properties of the replicator dynamics. In the one population setting, Thomas [1985] introduces the notion of an evolutionarily stable set (short, ES Set) to generalize the well known concept of an evolutionarily stable strategy (short ESS, Maynard Smith and Price [1973]). Thomas [1985] shows that each ES Set is also asymptotically stable but that the converse is not true. For the two population setting Balkenborg and Schlag [1994] show that the analog of an ES Set is a Strict Equilibrium Set (concept due to Balkenborg [1994]). Analog to the one population setting, Balkenborg [1994] shows that each Strict Equilibrium Set is an asymptotically stable set in the asymmetric replicator dynamics of Taylor [1979].

0.4 Cheap talk

The object of this paper is to analyze the effect of costless communication in the form of cheap talk on the outcomes of the evolutionary process described above. Before a given game is played, the players simultaneously exchange messages from some finite set of messages. To keep the model simple we assume that each player has the same set of messages. A strategy of the resulting communication game then consists of a message that is sent and a reaction function that, based on the message received specifies which strategy of the game is played. Object of our analysis is the replicator dynamics in which each agents is endowed with such a communication strategy.

0.5 An overview of the analysis

The intuition why communication might improve payoffs in outcomes that are selected by an evolutionary process is as follows. Consider the situation in which not all messages are sent during the communication round. Evolutionary drift in the population as to how individuals react to unsent messages can not be avoided. Especially the situation can arise in which sending a previously unsent message can strictly improve an individual's payoff. This will enable a mutant that sends this message to start to take over. Therefore, the only population that can avoid the invasion of such a mutant is one in which each individual achieves their maximal payoff. The situation in which individuals can only simultaneously achieve their maximal payoff will be called a common interest contest. Of course in order for the above scenario to be feasible there must be unused messages. Since the message set is finite, this depends on whether or not the population may drift to a state in which there are unused messages. If such a drift is not possible we speak of "lock in".

We show that "lock in" cannot occur in the two population setting. There will always be extra messages in some outcome of any minimal attracting set. This fact is derived from properties of the replicator dynamics and relies on the fact that there are no "own population effects" in the version of Taylor [1979]. Together with the arguments made above it follows that evolution will lead to efficient payoffs in common interest contests. Moreover we show that a common interest contest is necessary to ensure a very weak stability property. This is because one population may drift to an alternative state yielding maximal outcomes to agents in one population that are no longer maximal for the agents in the other population.

In the one population setting "lock in" can not always be avoided. The reason is that entering mutants will also be matched against themselves and therefore have less freedom to lead the population away from a "lock in" situation. However without the existence of unused messages, the stability and efficiency results of the two population setting no longer hold. Inefficient evolutionarily stable strategies may exist disregard of whether the contest has common interest or not. In order to recover the efficiency results of the two population setting the existence of unused messages must be forced by to the game structure. As an example we show that analog theorems to the two population setting are recovered for games with a dominant strategy.

1. Preliminaries

1.1 Notation

For a finite set A let ΔA be the set of probability distributions on A, i.e., $\Delta A=\{x\in\Re^N \text{ s.t. } x_i\geq 0 \text{ and } \sum_{i=1}^{N} x_i=1\}$. Consider a two person game in normal form $\Gamma(S_1,S_2,E_1,E_2)$ with the pure strategies $S_1=\{e^i, i=1,..,N_1\}$, $S_2=\{a^j, j=1,..,N_2\}$ and the bilinear payoff functions $E_i:\Delta S_1\times\Delta S_2\to\Re$, i=1,2. $\Gamma(S_1,S_2,E_1,E_2)$ is called symmetric if $S_1=S_2$ and $E_2(x,y)=E_1(y,x)$ for all $x,y\in\Delta S_1$ and will be denoted by $\Gamma(S,E)$. In this case we will let $S=S_1$, $E=E_1$ and $N=N_1$.

For $z\in\Delta S_1\cup\Delta S_2$ let C(z) be the support of z, i.e., $C(z)=\{e\in S_1\cup S_2 \text{ s.t. } z(e)>0\}$. For $i\in\{1,2\}$, $j\in\{1,2\}\backslash\{i\}$ and $z\in\Delta S_j$ let $BR_i(z)$ be the set of best replies of player i to the strategy z of player j, i.e., $BR_1(y)=\text{argmax}\{E_1(x',y), x'\in\Delta S_1\}$ and $BR_2(x)=\text{argmax}\{E_2(x,y'), y'\in\Delta S_2\}$ where $(x,y)\in\Delta S_1\times\Delta S_2$. To simplify notation we will not distinguish between the pure strategy $e\in S_i$ and the distribution on S_i that assigns unit probability to e (i.e., S_i is embedded in ΔS), especially, $\frac{1}{2}e^1+\frac{1}{2}e^2\in\Delta S_1$ (for $N_1>1$). The pair of strategies $(x,y)\in\Delta S_1\times\Delta S_2$ is called a Nash equilibrium if $x\in BR_1(y)$ and $y\in BR_2(x)$, it is called a strict (Nash) equilibrium if $\{x\}=BR_1(y)$ and $\{y\}=BR_2(x)$. Finally, $e\in S_1$ is called a weakly dominant strategy for player one if $E_1(e,y)\geq E_1(x,y)$ for all $x\in\Delta S_1$ and $y\in\Delta S_2$.

1.2 Dynamic stability concepts

Let X be either ΔS_1 or $\Delta S_1\times\Delta S_2$ and consider a dynamic process on X given by the solutions to the differential equation $\dot{x}=f(x)$ where $f:X\to X$ is Lipschitz continuous. A closed and non empty set $G\subseteq X$ is called attracting if there exists an open neighborhood U of G such that each trajectory starting in U converges to G ($U\subseteq X$). G is called a minimal attracting set if there is no set G' that is attracting such that $G'\subseteq G$ and $G\neq G'$. Following Zorn's lemma a minimal attracting set always exists. Notice that minimal attracting sets are candidates for the dynamics to get "caught" if mutations are very rare. A strategy $p\in\Delta S$ is called stable if for every open neighborhood U of p there exists an open neighborhood V of p s.t. the trajectories starting in V do not leave U ($U,V\subseteq X$). A set $G\subseteq X$ is called an asymptotically stable set (AS Set) if it is attracting and each $x\in G$ is stable. The element of a singleton AS Set is called an asymptotically stable strategy.

In the following we add some notes on the above definitions. A trajectory starting in W converges to L ($L,W\subseteq X$) if for any $x°\in W$ and $(t_k)_{k\in\mathbb{N}}$ such that $t_k\to\infty$ when $k\to\infty$ ($t_k\in\Re$) it follows that $\inf\{dist(x^{t_k},z), z\in L\}\to 0$ as $k\to\infty$ where x^t solves $\dot{x}=f(x)$ starting at $x^0=x°$. The above definition of asymptotic stability is slightly stronger than the classical one (see e.g. Bhatia and Szegö [1970]): in the standard definition additional to attracting the set as a whole must be stable, not necessarily each point. Finally, w.l.o.g. we also require additionally to the standard definition for an attracting set to be closed. We find it intuitive to include rest points on the border of an attracting set into the set.

Notice that a consequence of our definition of asymptotic stability is that trajectories starting sufficiently close to such a set will converge to an element of the set (this follows easily from the pointwise stability condition).

1.3 Evolution and the replicator dynamics in the one population setting

Consider an infinite population randomly matched to play a given symmetric game $\Gamma(S,E)$. Consider some dynamics in which strategies with higher expected payoffs reproduce at a higher rate. Then the strategy $p\in\Delta S$ is called evolutionarily stable (short, ESS, due to Maynard Smith and Price [1973]) if for any $q\neq p$ the mutant strategy is driven out of a monomorphic population of agents playing p after any sufficiently small mutation of agents playing q. Balkenborg and Schlag [1994] extend this notion to sets. A set $G\subseteq\Delta S$ is called an evolutionarily stable set if for sufficiently small mutations the following holds: given $q\in\Delta S$ and $p\in G$ the mutant strategy q can not spread in a population playing p and is driven out if $q\notin G$. This condition is formalized in the following definition:

DEFINITION 1.3.1:

$G \subseteq \Delta S$ is called an evolutionarily stable set (short ES set) if there exists $\epsilon > 0$ such that for all $p \in G$, $q \in \Delta S$ and $0 < \epsilon' < \epsilon$, $E(p,(1-\epsilon')p+\epsilon' q) \geq E(q,(1-\epsilon')p+\epsilon' q)$ where the inequality holds strict if $q \notin G$.

In particular, if $\{p\}$ is an ES Set for $p \in \Delta S$ then p is called an evolutionarily stable strategy.

Balkenborg and Schlag [1994] show that the above intuitive condition is equivalent to the definition of an ES set by Thomas [1985] and that it can alternatively be characterized as follows:

LEMMA 1.3.1:

$G \subseteq \Delta S$ is an evolutionarily stable set if and only if for all $p \in G$ and $q \in \Delta S$ one of the following statements holds:

i) $E(p,p) > E(q,p)$.

ii) $E(q,p) = E(p,p)$ and $E(p,q) > E(q,q)$.

iii) $E(q,p) = E(p,p)$, $E(p,q) = E(q,q)$ and $q \in G$.

The replicator dynamics is an explicit dynamic process related to the above stationary solution concepts. It describes the evolution over time of the frequencies of the pure strategies S played the population.

DEFINITION 1.3.2: (see Taylor and Jonker [1978])

The (symmetric) replicator dynamics of $\Gamma(S,E)$ on ΔS for continuous time and pure strategy types is defined as follows:

$$x^0 = x^\circ \text{ and } \dot{x}^t_i = [E(e^i, x^t) - E(x^t, x^t)]x^t_i,\ i=1,..,N,\ t \geq 0, \qquad \text{(RD)}$$

where $x^\circ \in \Delta S$ is the initial state and x^t_i is the frequency of the type using the pure strategy e^i at time t ($t \geq 0$, $e^i \in S$).

Dynamic stability of the replicator dynamics and evolutionary stability are related in the following way.

THEOREM 1.3.1: (Thomas [1985])

If $G \subseteq \Delta S$ is an evolutionarily stable set then G is an asymptotically stable set in the replicator dynamics (RD).

1.4 Evolution in the two population setting

In the asymmetric or two population setting there are two disjoint populations labelled one and two. Individuals of opposite populations are pairwise randomly matched and then breed among their own population. In contrast to the one population setting the matching situation is no longer limited to symmetric games.

Balkenborg and Schlag [1994] derive the concept of an evolutionarily stable strategy and evolutionarily stable set analogous to the one population setting. For this they consider mutants that enter both populations simultaneously in small proportions. Balkenborg and Schlag [1994] argue that the appropriate definition of an ESS in the two population setting is equivalent to that of a strict equilibrium and that of an ES set is equivalent to the following definition of a strict equilibrium set.

DEFINITION 1.4.1: (Balkenborg [1994])

$G \subseteq \Delta S_1 \times \Delta S_2$ is called a strict equilibrium set if for any $(x,y) \in G$ and $(x',y') \in \Delta S_1 \times \Delta S_2$,

i) $E_1(x',y) \leq E_1(x,y)$ where equality implies $(x',y) \in G$ and

iii) $E_2(x,y') \leq E_2(x,y')$ where equality implies $(x,y') \in G$.

The explicit dynamic process we analyze in the two population setting is the following version of the replicator dynamics, describing the evolution of the proportion of the strategies played in either population.

DEFINITION 1.4.2: (Taylor [1979])

The <u>(asymmetric) replicator dynamics</u> of $\Gamma(S_1,S_2,E_1,E_2)$ on $\Delta S_1\times\Delta S_2$ for continuous time and pure strategy types will be defined as follows:

$x^0=x°$, $y^0=y°$,

$\dot{x}^t_i=[E_1(e^i,y^t)-E_1(x^t,y^t)]x^t_i$, $i=1,..,N_1$,

$\dot{y}^t_j=[E_2(x^t,a^j)-E_2(x^t,y^t)]y^t_j$, $j=1,..,N_2$, $t\geq 0$, (RD2)

where $(x°,y°)\in\Delta S_1\times\Delta S_2$ is the initial state and x^t_i (y^t_j) is the frequency of the type using the pure strategy e^i (a^j) at time t ($e^i\in S_1$, $a^j\in S_2$, $t\geq 0$).

The following theorem is the pendant to theorem 1.3.1 and shows the connection between the static and the dynamic concepts in the two population setting.

THEOREM 1.4.1: (Balkenborg [1994])

If $G\subseteq\Delta S_1\times\Delta S_2$ is a strict equilibrium set then G is an asymptotically stable set in the asymmetric replicator dynamics (RD2).

Given (RD2) consider the evolution of a set as defined by the union of the evolution of each of its elements. Concerning this evolution of sets the asymmetric replicator dynamics have a specific property.

THEOREM 1.4.2: (Akin and Eshel [1983])

The flow of the asymmetric replicator dynamics (RD2) in the interior of $\Delta S_1\times\Delta S_2$ is incompressible, i.e., a properly defined volume is conserved over time.

It follows that (RD2) can not contain an interior asymptotically stable strategy because the volume would have to shrink. Pursuing this argument a bit further we obtain the following lemma.

LEMMA 1.4.1:

If $G \subseteq \Delta S_1 \times \Delta S_2$ is a minimal attracting set in the asymmetric replicator dynamics (RD2) then G contains a pure strategy profile, i.e., $G \cap (S_1 \times S_2) \neq \varnothing$.

PROOF:

Assume that $G \cap (S_1 \times S_2) \neq \varnothing$. Choose $S^\circ_i \subseteq S_i$, i=1,2 such that

i) $G \cap (\Delta S^\circ_1 \times \Delta S^\circ_2) \neq \varnothing$,

ii) $S'_1 \neq \varnothing$, $S'_1 \subseteq S^\circ_1$ and $G \cap (\Delta S'_1 \times \Delta S^\circ_2) \neq \varnothing$ implies $S'_1 = S^\circ_1$,

iii) $S'_2 \neq \varnothing$, $S'_2 \subseteq S^\circ_2$ and $G \cap (\Delta S^\circ_1 \times \Delta S'_2) \neq \varnothing$ implies $S'_2 = S^\circ_2$.

Let $G^\circ = G \cap (\Delta S^\circ_1 \times \Delta S^\circ_2)$. From the properties of S°_i and the fact that G is attracting it follows that there exists $U \subseteq \text{int}\{\Delta S^\circ_1 \times \Delta S^\circ_2\}$ such that trajectories starting in U converge to G°. Since the flow in the asymmetric replicator dynamics is incompressible, it follows that $|S^\circ_1| = |S^\circ_2| = 1$. Therefore $G \cap (S_1 \times S_2) \neq \varnothing$. ☐

2. Cheap Talk:

We will now introduce costless communication (commonly referred to as cheap talk, see Kim and Sobel [1991, 1994]) that will take place before the game Γ is actually played. The enlarged game now consists of two rounds, a signalling round and an action round. In the first or signalling round the two players simultaneously send a message from a finite set of messages $M=\{c^1,..,c^n\}$ to the other player. "Talk is cheap" because there is no cost of sending this message. In the second or action round each player chooses a pure strategy of the game $\Gamma(S_1,S_2,E_1,E_2)$ conditioned on the messages sent in the first round. Finally each player receives her payoff $E^c_i()$ based on the strategy combination played in the second round. Mixed strategies of the enlarged (or communication) game are just randomizations over the pure strategies described above. Although formally correct, we do not condition the strategy a player plays in the second round on the message he sent in the first round. Each player knows which message he sent and there are no mistakes. Additionally rationalization about why the individual plays a certain strategy does not arise because each individual is endowed with some fixed type. Adding these reactions leads to duplification of the present strategy which does not change the results because the notion of an evolutionarily stable set and that of an asymptotically stable set are clearly independent of spurious duplification.

DEFINITION 2.1: (Kim and Sobel [1991])

A communication game $\Gamma(S^c_1,S^c_2,E^c_1,E^c_2)$ is defined by the set of pure strategies $S^c_i=M\times S_i^M$ and the bilinear payoff function $E^c_i:\Delta S^c_1\times\Delta S^c_2\rightarrow\Re$ satisfying $E^c_i((m^1,f^1),(m^2,f^2))=E_i(f^1(m^2),f^2(m^1))$ for $(m^j,f^j)\in S^c_i$, i,j=1,2.

When using the pure strategy $(m,f)\in S^c_i$, the message $m\in M$ is sent in the signalling round and $f(m')\in S_i$ is the strategy played in the action round after receiving the message $m'\in M$ from the other player in the signalling round. A mixed strategy for player i is an element of $\Delta(M\times S_i^M)$.

Instead of (m,f) we will also write $(m;f(c^1),..,f(c^n))$ and sometimes we will write $\sigma\in S^c_i$ in the form $\sum_{j=1}^{n}\alpha_j(c^j;f^j)$ where $\alpha_j\geq 0$, $\sum_{j=1}^{n}\alpha_j=1$ and $f^j\in\{\Delta S_i\}^M$. For $\sigma\in\Delta(M\times S_i^M)$ let $BR^c_i(\sigma)$ be the set of best replies to σ in the communication game $\Gamma(S^c{}_1,S^c{}_2,E^c{}_1,E^c{}_2)$.

Our goal will be to analyze the replicator dynamics both in the one and in the two population setting where individuals are matched to play a communication game.

3. The effects of cheap talk in the two population setting

We will start out by analyzing the two population setting. At first we present an example to illustrate how cheap talk can destabilize inefficient outcomes.

Example 3.1:

Consider the asymmetric game $\Gamma(S_1,S_2,E_1,E_2)$ with $S_1=\{T,B\}$ and $S_2=\{L,R\}$ and payoffs in table I.

Table I: A pure coordination game.

	L	R
T	2,2	0,0
B	0,0	1,1

Note that (B,R) is a strict equilibrium and hence asymptotically stable in (RD2) when there is no pre-play communication. Consider now the communication game with $M=\{c^1,c^2\}$. We will now show why the play of (B,R) is unstable in the game with cheap talk. Notice that if all agents in one population play the same strategy then appropriate mutations can lead to all agents in the other population sending only one message. Moreover, mutations on reactions to messages not sent can not be punished. Therefore we may assume that the population has drifted to the state $((c^1;B,B),(c^1;R,L))$. Since $E^c((c^2;T,T),(c^1;R,L))>E^c((c^1;B,B),(c^1;R,L))$, $(c^2;T,T)$ can spread in population one which leads to the state $((c^2;T,T),(c^1;R,L))$. Notice that in this state agents in either population receive their maximal feasible payoff.

The next example illustrates how drift can cause instability when both players do not simultaneously receive their maximal payoff.

Example 3.2:

Consider the following game with $S_1=\{T\}$, $S_2=\{L,R\}$ and payoffs given in table II.

Table II: A simple game.

	L	R
T	1,1	1,0

(T,L) is a strict equilibrium and hence an asymptotically stable strategy of (RD2) in the game without communication. Moreover each individual obtains her maximal payoff. Now consider the communication game with message set $M=\{c^1,c^2\}$ and consider a state in which (T,L) is played. As argued in the above example, the population can drift to $((c^1;T,T),(c^1;L,L))$ which can drift to $((c^1;T,T),(c^1;L,R))$. Moreover since $E_1(T,L)=E_1(T,R)$, $(c^2;T,T)$ can invade population one and the population can consequently drift to $((c^2;T,T),(c^1;L,R))$. However it is easy to see that this state is unstable since $(c^1;L,L)$ can spread in population two. The relevant payoffs for this argument are given in table III, the trajectories are sketched in figure 1.

Table III: Some payoffs of the communication game of table II.

	$(c^1;L,L)$	$(c^1;L,R)$
$(c^1;T,T)$	1,1	1,1
$(c^2;T,T)$	1,1	1,0

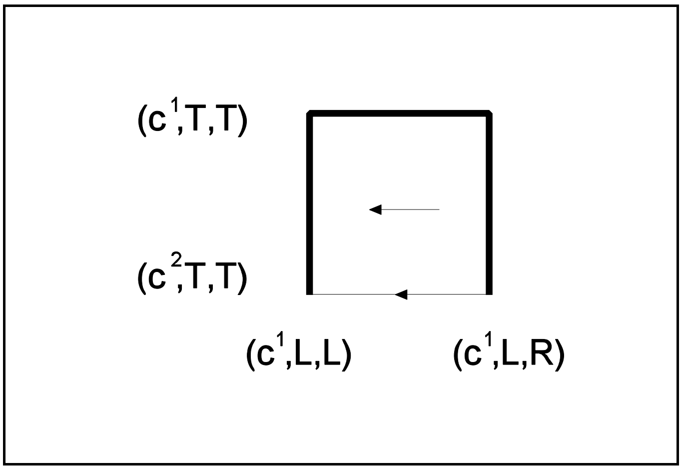

Figure 1: Trajectories associated with the strategies and payoffs in table III. Collections of rest points are marked with thick lines.

We will see that cheap talk will not lead to instability as in the above example if the individuals in each population only simultaneously achieve their maximal payoff. Such a situation will be called a common interest contest. Let π_i*=max{E_i(e,e'), e$\in S_1$, e'$\in S_2$}, i=1,2.

DEFINITION 3.1:

Given populations one and two matched to play an asymmetric game $\Gamma(S_1,S_2,E_1,E_2)$ we say that the agents are <u>involved in a common interest contest</u> if for each $e\in S_1$ and $e'\in S_2$, $E_1(e,e')=\pi_1^*$ if and only if $E_2(e,e')=\pi_2^*$.

In the literature a game satisfying the conditions in the above definition is known as a game of common interest (see Kim and Sobel [1991, 1994], Matsui [1991]). The reason why we use a different name for an established concept will become apparent in the next section where we consider the one population setting.

In the following we will present a minimal stability criterion.

DEFINITION 3.2:

A set $G\subseteq\Delta S^c_1\times\Delta S^c_2$ is said to have <u>point stability</u> if each pair of pure strategies

$(e_1,e_2)\in G\cap(S^c_1\times S^c_2)$ is stable.

The following theorem characterizes the evolutionary outcomes (modelled using the asymmetric replicator dynamics (RD2)) in communication games. We show that a common interest contest is necessary and sufficient for the dynamics to satisfy minimal stability conditions. Moreover, in a common interest the only set with such minimal stability properties is unique and contains all states in which each agent achieves his maximal payoff. Therefore the effect of cheap talk is quite drastic, either an unstable situation is created or eventually everyone gets their maximal payoff.

THEOREM 3.1:

Let $\Gamma(S_1,S_2,E_1,E_2)$ be a game and M be a finite message space. The following statements are equivalent:

i) The agents are involved in a common interest contest.

ii) There exists a strict equilibrium set G_1.

iii) There exists an asymptotically stable set G_2 of (RD2).

iv) There exists a minimal attracting set G_3 of (RD2) with point stability.

Moreover if one (all) of the above statements is (are) true then $G_1=G_2=G_3=\{(\sigma_1,\sigma_2)\in\Delta S^c_1\times\Delta S^c_2 \text{ s.t. } E^c_1(\sigma_1,\sigma_2)=\pi_1^*\}$.

PROOF:

"i) $\Rightarrow$ ii) $\Rightarrow$ iii) $\Rightarrow$ iv)"

It is easy to show that $G_1=\{(\sigma_1,\sigma_2)\in\Delta S^c_1\times\Delta S^c_2 \text{ s.t. } E^c_1(\sigma_1,\sigma_2)=\pi_1^*\}$ is a strict equilibrium set if we have a common interest contest. Following the definitions and theorem 1.4.1 the rest follows immediately.

"iv) $\Rightarrow$ i)" will be shown in two steps.

Step 1: We will show that there exists $(e_1,e_2)\in G\cap(S^c_1\times S^c_2)$ such that $E^c_i(e_1,e_2)=\pi_i^*$, i=1,2.

Following lemma 1.4.1, there exists $(e_1,e_2)\in G\cap(S^c_1\times S^c_2)$. Let $m\in M$ be such that e_1 does not send m. Let $(a,b)\in S_1\times S_2$ such that $E_1(a,b)=\pi_1^*$. Since m is not sent, player two's response to message m does not influence the payoffs obtained in population two. Therefore we may assume that each individual in population two responds to m by playing b. Let this strategy be denoted by e°_2. Let $e^\circ_1 \in S^c_1$ be the strategy of sending m and then always playing a. Given $\epsilon>0$ consider the trajectory starting in $((1-\epsilon)e_1+\epsilon e^\circ_1, e^\circ_2)$. Since G is point stable it follows that (e_1,e°_2) is stable and hence $E^c_1(e_1,e_2)=E^c_1(e_1,e^\circ_2)=\pi_1*$. Applying the same arguments to population two completes step 1.

Step 2: We will now show that the population is involved in a common interest contest. Since $E^c_1(e^\circ_1,e^\circ_2)=E^c_1(e_1,e^\circ_2)$ and G is a minimal attracting set it follows from step 1 that $(e^\circ_1,e^\circ_2)\in G$. Moreover applying step 1 to (e°_1,e°_2) it follows that $E^c_2(e^\circ_1,e^\circ_2)=E_2(a,b)=\pi_2*$. Since $(a,b)\in S_1\times S_2$ such that $E_1(a,b)=\pi_1*$ was arbitrary it follows that the population is involved in a common interest contest.

In order to show the uniqueness statement it is enough to show that if G_3 is a minimal attracting set with point stability then $G_3=G_1$ where
$G_1=\{(\sigma_1,\sigma_2)\in\Delta S^c_1\times\Delta S^c_2 \text{ s.t. } E^c_1(\sigma_1,\sigma_2)=\pi_1*\}$.

We will first show that $G_1\subseteq G_3$. From step 1 of "iv) $\Rightarrow$ i)" it follows that $(e_1,e_2)\in G_1\cap G_3$. Moreover it is easy to show that G_1 is a connected set of Nash equilibria. Since a minimal attracting set may not have Nash equilibria arbitrarily close to it (Nash equilibria are rest points of RD) that are not in it follows that $G_1\subseteq G_3$.

From the fact that G_1 is a strict equilibrium set, $G_1\subseteq G_3$ and the minimality of G it follows that $G_3=G_1$. ☐

Notice that if agents are involved in a common interest contest then $\{(x,y)\in\Delta S_1\times\Delta S_2 \text{ s.t. } E_1(x,y)=\pi_1*\}$ is a strict equilibrium set and hence an asymptotically stable set in the game without cheap talk (see theorem 1.4.1). Hence cheap talk preserves the stability of the efficient set and disrupts the stability of all other inefficient outcomes that were stable is some sense in the game without communication (see example 3.1).

The following example shows how the strength of the theorem 3.1 relies on the properties of the asymmetric replicator dynamics.

Example 3.3: (due to Kim and Sobel [1991, 1994])

Consider the game with S_1={T,M,B} and S_2={L,N,R} and payoffs given in table IV as a communication game with message set M={c^1,c^2}.

Table IV: A game.

	L	N	R
T	2,1	0,0	0,0
M	0,0	1,2	0,0
B	0,0	0,0	10,10

Following theorem 3.1, $\{(\sigma_1,\sigma_2)\in\Delta S^c_1\times\Delta S^c_2$ s.t. $E^c_1(\sigma_1,\sigma_2)=(10,10)\}$ is the unique minimal attracting set. Let p_1=½(c^1;M,T)+½(c^2;T,M) and p_2=½(c^1;N,L)+½(c^2;L,N). Kim and Sobel [1991] show that (p_1,p_2) is equilibrium evolutionarily stable (EES) because ".. players waste their words over arguing which inefficient equilibrium to play." This does not happen in the replicator dynamics (RD2) when we consider minimal attracting as a solution concept. Let us consider the replicator dynamics in this game in more detail to see why (p_1,p_2) is not contained in a minimal attracting set. Consider for the moment the case in which population i consists only of agents using strategies in C(p_i), i=1,2. The appropriate payoffs are written in table V.

Table V: Some payoffs of the communication game of table IV.

	$(c^1;N,L)$	$(c^2;L,N)$
$(c^1;M,T)$	1,2	2,1
$(c^2;T,M)$	2,1	1,2

If strategies are restricted in this way then (p_1,p_2) is stable in $\Delta C(p_1)\times\Delta C(p_2)$ and the trajectories of RD in $\Delta C(p_1)\times\Delta C(p_2)$ are closed orbits around (p_1,p_2) (see Schuster and Sigmund [1981]). Therefore the minimal attracting set in $\Delta C(p_1)\times\Delta C(p_2)$ is $\Delta C(p_1)\times\Delta C(p_2)$ itself. Especially, $((c^1;M,T),(c^1;N,L))$ is in this set. In this outcome not all messages are sent and therefore when considering the dynamics on $\Delta S^c{}_1\times\Delta S^c{}_2$ after entry of a specific sequence of mutants the dynamics will evolve to the efficient set (see proof of theorem 3.1).

4. Cheap talk in the one population setting

We will now consider the impact of cheap talk on evolution in one the one population setting. Consequently our analysis is now restricted to symmetric games. Let $E=E_1$, $E^c=E^c{}_1$, $S=S_1$ and $S^c=S^c{}_1$. It turns out that in the one population setting cheap talk is no longer able to eliminate the multiplicity of evolutionarily stable sets.

Example 4.1:

Consider the game in example 3.1 as a symmetric game with T identified with L and B with R, hence $S=\{T,B\}$. Now consider the communication with message set $M=\{c^1,c^2\}$. Straightforward calculation shows that the efficient set $G=\{\sigma\in\Delta S^c \text{ s.t. } E^c(\sigma,\sigma)=2\}$ is an ES Set. However there is also an inefficient singleton ES Set of the communication game corresponding to the ESS $p=½(c^1;B,T)+½(c^2;T,B)$. Notice that the strategy p is an ESS since $BR^c(p)=C(p)$ and all mutants with support in C(p) are driven out (see table VI).

Table VI: Some payoffs generated by the communication game of table I.

	$(c^1;B,T)$	$(c^2;T,B)$
$(c^1;B,T)$	1,1	2,2
$(c^2;T,B)$	2,2	1,1

When playing p there are no unused messages ("lock in"). Moreover, without unused messages cheap talk can not enforce efficiency. Consequently a pendant to theorem 3.1 does not exist for the one population setting and we must state a much weaker version. Especially p is mixed and hence this does not contradict the result of Wärneryd [1991] that a pure strategy

ESS must be efficient. Notice that the set G is not convex. Communication games are simple class of examples in which non convex ES sets arise quite naturally.

Agents are involved in a common interest contest if they only simultaneously achieve their maximal payoff. In the one population setting we must therefore add to the condition of common interest from the two population setting (see definition 3.1) the condition that the maximal payoff can be achieved in a symmetric outcome. Let $\pi^*=\pi_1^*=\max\{E(e,e'), e,e'\in S\}$.

DEFINITION 4.1:

Given a population of identical agents matched to play a symmetric game $\Gamma(S,E)$ we say that the agents are <u>involved in a common interest contest</u> if $\pi^*=\max\{E(e,e), e\in S\}$ and for each $e,e'\in S$, $E(e,e')=\pi^*$ implies $E(e',e)=\pi^*$.

We now come to a characterization of the relationship of cheap talk and efficiency. In contrast to the two population setting, here in the one population setting common interest is only necessary and sufficient for the set of efficient outcomes to have minimal stability properties. As seen in example 4.1 in efficient ES sets may exist, however the set can not have unused messages.

DEFINITION 4.2:

We say that the set $G\subseteq\Delta(M\times S^M)$ has <u>unused messages</u> if there exists $\sigma\in G$ and $m\in M$ such that $(m',f)\in C(\sigma)$ implies $m\neq m'$ ($m'\in M$, $f\in\Delta(M\times S^M)$).

THEOREM 4.1:

Let $\Gamma(S,E)$ be a symmetric game and let M be a finite message space. Then the following statements are equivalent.

i) The agents are involved in a common interest contest.

ii) There exists a connected ES Set G_1 with unused messages.

iii) There exists an asymptotically stable set G_2 of (RD) with unused messages.
iv) There exists a minimal attracting set G_3 of (RD) with point stability that contains an element in which all agents receive their maximal payoff, i.e., $\exists\chi\in G_3$ such that $E^c(\chi,\chi)=\pi^*$.

Moreover if one (all) of the above statements is (are) true then $G_1=G_2=G_3=\{\sigma\in\Delta(M\times S^M) \text{ s.t. } E^c(\sigma,\sigma)=\pi^*\}$.

PROOF:
"i) $\Rightarrow$ ii)" is an easy exercise, "ii) $\Rightarrow$ iii)" follows from theorem 1.3.1 and "iii) $\Rightarrow$ iv)" follows from the definitions.
"iii) $\Rightarrow$ i)" follows from the proof of theorem 3.1 except that step 1 is slightly changed.
Step 1': Assume that $\chi\in G_2$ is such that player one does not use message $m\in M$. We will show that $E^c(\chi,\chi)=\pi^*$.

Let $(a,b)\in S\times S$ such that $E(a,b)=\pi^*$. As in the proof of theorem 3.1 we may assume that χ responds to m by playing b. Let $e\in S^c$ be the strategy of sending m and then always playing a. It follows that $\pi^*=E^c(e,\chi)\geq E^c(\chi,\chi)$. On the other hand, since χ is an element of an AS Set, (χ,χ) is a Nash equilibrium (see Bomze [1986]). Hence $\pi^*=E^c(e,\chi)\leq E^c(\chi,\chi)$ and step 1' is complete.
"iv $\Rightarrow$ iii)" follows immediately since iv) implies that the statement in step 1' above is true. ☐

Notice in comparison to theorem 3.1 that part iv) of theorem 4.1 contains the additional condition that $\exists\chi\in G_3$ such that $E^c(\chi,\chi)=\pi^*$. This was not needed in theorem 3.1 because lemma 1.4.1 implies that G_3 must contain a pure strategy which has the same effect regarding the proof of the theorem as this additional condition.

Example 4.1 shows that multiple ES sets may exist in common interest contests, of course the ones containing inefficient payoffs can not have unused messages. Moreover ES Sets without unused messages can also exist when there is no common interest contest as shown in the next example.

Example 4.2:

Consider the symmetric game with $S=\{T,B\}$ and payoffs given in table VII.

Table VII: A game showing that ESS can exist without common interest.

	T	B
T	0,0	1,2
B	2,1	−1,−1

The only minimal attracting set in the above game is associated to the ESS ½T+½B which gives an expected payoff of ½ to each individual. Consider now the communication game with message set $M=\{c^1,c^2\}$. Although the population is not involved in a common interest contest, the communication game has two evolutionarily stable strategies $0.25(c^1;v,T)+0.75(c^2;B,v)$ and $0.75(c^1;v,B)+0.25(c^2;T,v)$ where $v=½T+½B$. The associated payoffs are 7/8. Especially payoffs in any ESS are higher in the communication game.

In the following we will see that the efficiency result of the two population setting can be recovered in games with a weakly dominant strategy.

COROLLARY 4.1:

Let $\Gamma(S,E)$ be a symmetric game with a weakly dominant strategy. Then there exists a minimal attracting set of (RD) with point stability if and only if the population is involved in a common interest contest.

PROOF:

The "if" statement follows directly from theorem 4.1.

"only if": Let $T\in S$ be a weakly dominant strategy. Let A be a minimal attracting set with point stability. For $c^i\in M$ the strategy $(c^i;T,T)$ dominates any other strategy $(c^i;x,y)$. Consequently the growth rate of $(c^i;T,T)$ will be larger than that of $(c^i;x,y)$. Considering a sequence of mutations of $(c^i;T,T)$ it follows that there exists $\sigma'\in A$ such that $(c^i;x,y)\notin C(\sigma')$. Repeating this argument for all $c^i\in M$ it follows that there exists $\sigma\in A$ such that $C(\sigma)\subseteq\{(c^1;T,T),..,(c^m;T,T)\}=Q$. Moreover all strategies in Q are payoff equivalent and hence any strategy with support in Q is a rest point, especially $(c^1;T,T)\in A$. Now it is easy to show that point stability implies $E(T,T)=\pi^*$ and hence the population is involved in a common interest contest (follow arguments as in the proof of theorem 3.1 and 4.1). □

We are not able to provide a complete characterization of the class of games in which cheap talk does not destroy stability. However it is easy to see that this class contains all partnership games. Partnership games are two person games in which both players necessarily receive the same payoff. Schlag [1994a] shows that each partnership game has an ES Set and since the communication game of a partnership game is again a partnership game the claim follows. Clearly the class of games in which cheap talk does not destroy stability is larger than just partnership games (see example 4.2).

5. Literature and Discussion

An alternative static solution concept to that of an ES Set (and ESS) is that of an Equilibrium Evolutionarily Stable Set (short EES set), introduced by Swinkels [1992]. Mutations are sophisticated in the sense that a mutant will only enter if it foresees that it is will be a best reply to the population mean once it has entered the population. Schlag [1994a] shows that each connected evolutionarily stable set is EES, both in the one and in the two population setting.

Kim and Sobel [1991] (abbreviated below by KS) search for EES in communication games. They essentially show that in a game with both common interest and equilibrium common interest that the only set that is EES in the communication game is the efficient set. A game has equilibrium common interest if each player has the same ranking over the Nash equilibria. KS also present an example (the same as example 4.1) showing that their result no longer holds when EES is applied to a one the one population setting.

The major difference to our analysis is that EES is an intuitive concept that is not related to any specific dynamic process. The results of our analysis differ from those of KS in various respects. KS need an extra assumption, namely equilibrium common interest, in order to ensure that cheap talk leads to efficiency (see example 3.2) in each set that is EES. On the other hand in our analysis common interest is necessary to ensure stability in the two population setting in our model. In contrast, it is easy to verify that in a prisoners' dilemma with cheap talk the set of strategies that lead to the play of ("defect","defect") is EES. Players in the model of KS are smarter in that they do not coordinate on unstable outcomes. This makes stable situations more likely. The results of our paper for two specie populations trivially fall together with those of KS if we restrict attention to partnership games. Partnership games have both equilibrium common interest and common interest. Moreover, the concept of minimal attracting set, strict equilibrium set and EES are equivalent in partnership game (see Schlag [1994a]).

How do the proofs in these two alternative models compare? As in our paper the proof of the results of KS contains two parts. First it is shown that a set that is EES will have unused messages. This they show using equilibrium common interest. We obtain the same result without additional assumptions using the fact that the flow of the asymmetric replicator dynamics is incompressible. The second step is to show that unused messages give the opportunity to coordinate on the efficient outcome. In KS, players do not necessarily punish the sending of an unused message and hence mutants can employ a secret handshake. Mutants copy the play of some other individual in the population unless they meet a mutant in which case they coordinate on the efficient outcome. In our case, the play in the population drifts to the situation where players of one population "offer" the maximal payoff to the other population upon receiving an unused message. Therefore mutants sending an unused message start to take over without using any secret handshake.

In their revised version, Kim and Sobel [1994] specify an explicit stochastic dynamic adjustment process. Each players plays against each other player and switches to strategies that performed equally well or better with positive probability. The intuition leading to the existence of unused messages now relies on individuals in one population updating while the ones in other do not. The reasons for the emergence of efficient outcomes when there are unused messages is similar to our model. Moreover, in this new framework communication strategies leading to the play of ("defect","defect") are no longer absorbing in the prisoners' dilemma.

Besides the rationality of the individuals, our model differs from the newer version of Kim and Sobel [1994] in the size of population that is analyzed. The assumption of tournament matching in Kim and Sobel [1994] is more plausible in small populations. Moreover their proof relies on events that occur with probability converging to zero as the population gets large. As the size of the population gets large, whether mutations are able to lead to efficiency seems questionable when looking at example 3.2. In contrast, our model is only applicable to large populations (see Schlag [1994b]) and the efficiency results rely on drift due to mutation. Similarly, the assumption that there are more messages than players (an assumption in Kim and Sobel [1994]) is more intuitive in small populations.

Both our model and that of Kim and Sobel [1994] show that unused messages are necessary for the population to evolve to efficient outcomes. Clearly our inefficiency result seems most intuitive when there is not a lot of time to exchange messages and the message space is restricted. The exposition of the inefficiency result is not meant to describe the most common situation but more as a reminder or caution that cheap talk with finitely many messages alone is not enough to explain the emergence of efficient outcomes.

The strong conclusions in theorem 3.1 rely on the specific version of the asymmetric replicator dynamics used. An alternative version frequent in the population genetics literature but not yet related to any individual learning model, requires for the right hand sides in definition 1.4.2 to be divided by the mean payoffs in each population (i.e., divide by $E_i(x^t,y^t)$ in the equation of population i, i=1,2). These dynamics were introduced by Maynard Smith [1982] and analyzed in Hofbauer [1985]). Example 3.2 can be used to show that theorem 3.1 can not be extended to this alternative version of the asymmetric replicator dynamics. It can be shown that the set {(p,p)}, which is EES and yields inefficient payoffs, is an asymptotically stable set in these alternative dynamics. However this in turn depends on the assumption that agents only play pure strategies. If agents are also able to play mixed strategies then clearly strict equilibrium set is a weaker concept than that of an asymptotically stable set. Especially, {(p,p)} will no longer be asymptotically stable.

Evolutionary stability in asymmetric games is often undertaken via the analysis of the (truly) asymmetric contest (e.g. Bhaskar [1992]). The asymmetric contest is a transformation of the game into a symmetric game. Each agent has a strategy he will play in the role of player one and a strategy he will play in the role of player two. Before playing the game nature determines which agent is in which role. Payoffs in the asymmetric contest are then the expected payoffs of this enlarged symmetric game. Given this symmetrization the evolutionary stability concepts of the one population setting can then be applied. Bhaskar [1992] shows that neutrally stable strategies of the asymmetric contest of the communication game must not be efficient. For example $((c^1;B,B),(c^1;R,R))$ is neutrally stable in the asymmetric contest of the communication game of table I with message set $M=\{c^1, c^2\}$. Balkenborg and Schlag [1994] show that evolutionarily stable sets in the asymmetric contest are equivalent to strict equilibrium sets. Therefore the efficiency and stability results of cheap talk apply via theorem 3.1 to the evolutionarily stable sets of the asymmetric contest. The difference to the results of Bhaskar [1994] is that an evolutionarily stable set is a set of neutrally stable strategies with additional properties. We may thus conclude this paper by noticing that whether or not costless pre-play communication leads to efficiency in evolutionarily stable sets depends on whether or not there is role identification.

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