Believing When Credible: Talking About Future Intentions and Past Actions[*]

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Abstract

In an equilibrium framework, we explore how players communicate in games with multiple Nash equilibria when messages that make sense are not ignored. Communication is about strategies and not about private information. It begins with the choice of a language, followed by a message that is allowed to be vague. We focus on equilibria where the sender is believed whenever possible, and develop a *theory of credible communication.* We show that credible communication is sensitive to changes in the timing of communication. Sufficient conditions for communication leading to efficient play are provided.

Keywords: Pre-play communication, credibility, coordination, language, multiple equilibria, virtual communication.

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1 Introduction

People who interact with each other typically also talk to each other. If they do not talk, we question why they might choose not to talk. The outcome of communication depends, of course, on how well the players know and trust each other. It also depends on the degree to which incentives are aligned in the underlying game, that is, whether they interact due to a matter of conflict or on cooperative basis. We aim to formulate a simple theory, called *credible communication*, in order to understand the impact and use of communication in games. We consider games of complete information; where communication is about strategies, not about private information.

We aim to find the Nash equilibrium outcomes of the underlying game that retain their predictive power when communication is added. Games with multiple Nash equilibria are more the rule than the exception. Such multiplicity may be viewed negatively, as, for example, when acknowledging the existence of the unintuitive mixed equilibria in pure coordination games. Multiplicity has been identified as carrying predictive power in macroeconomics (see Cooper and John (1988)). We question the intuitive content of Nash equilibria that does not persist if a little communication is added. Thus, we choose a simple yet realistic model of communication as the method for testing the robustness of Nash equilibrium outcomes to the addition of communication. We then apply our results to economic models.

Whilst there are many papers on communication under complete information, this is the first that can be used to model communication in general games at different points in time. The literature is discussed in more detail in Section 7.

The central obstacle to setting up a model with communication that has predictive power is the existence of babbling equilibria. Babbling equilibria describe the self-enforcing situation in which no information is transmitted. Broadly speaking, if players do not believe each other then there is no need to exchange any meaningful information, thus justifying why they should not believe each other in the first place. Babbling equilibria have been eliminated in the literature by selecting from amongst all equilibria those with some sort of refinement. We examine the plausibility of babbling equilibria by modeling a setting that we call credible communication – where words have meaning, and where there is a common understanding that messages are to be believed whenever possible. The sender can rely on his words being believed whenever they are believable. This is a novelty in the literature. It gives the sender a means to exit a babbling equilibrium by being able to make the listener listen and believe. The key element is the modeling of when messages can be believed. Credible communication adds structure to the communication in a realistic dimension, as opposed to refinements that are added ex post in order to select the equilibria that are most plausible. We study the degree to which the sender can take advantage of credible communication, and how this may influence the outcome in the game. We investigate which Nash equilibrium outcomes fail to persist under minimal communication, and identify cases where only efficient Nash equilibria survive.

The literature on communication in games puts the onus on the listener and on the sender to understand the content of a message from the equilibrium strategy and from out of equilibrium beliefs. However, typically during communication, we not only inform the listener about an action or intention but also about the context and what else could have been said. This allows the sender to avoid misunderstandings, and helps the sender to convince the listener that his words make sense. A further novelty of this paper is the means by which we explicitly incorporate the context into the communication protocol. We require the sender to first inform the listener about the context. The context that surrounds a given message is called the *language*, and consists of the message sent and other messages that could have been sent. The context, as defined by the language, gives the sender the possibility to truthfully communicate what he intends and does not intend to play. Communication is then considered meaningful or *credible* if all messages belonging to the chosen language can be believed. In this case, we call the language *credible*.

Strategic interaction typically takes place in a dynamic context, with a specified order of moves and associated information sets. This introduces many points in time at which communication can take place. It also reveals many different ways in which a player may talk about his play when this is not observed by others. A player can talk about past actions or commitments when these are hidden, or about future choices or intentions when these choices have not yet been made. A player can also talk to a single player or to a group of players, and in the case of a group of players, talk may be public or private. Credible communication can be considered in all of these scenarios. For expositional purposes, we present our model of credible communication in the most prominent setting, namely where one player talks to another player about his intentions. Later, we discuss how the model easily extends to the scenario, where talk is about past play.

Information about future intentions can be conveyed in many different ways. It is possible to talk about own intentions and what is expected from the other player. In order to keep the model simple, we consider talk about own intentions only. We assume that player one is communicating about his own intended choices. Messages are modeled as nonempty subsets of his set of actions. Messages that contain more than one action indicate that player one chooses to remain vague about which particular action he intends to choose from the set of actions contained in the message he sends. This could be the case, for example, when player one intends to choose a mixed action.[1] The message contains no information if it contains all possible actions of player one. In this case, it is as if player one chooses not to communicate. At the opposite extreme, the message describes the intended choice of player one if it contains only the corresponding action. A language then consists of a set of such messages and it makes sense to require that a language is a partitions of the set of actions of player one. Modeling a language as a partition can be justified as follows. It is well known that a language is conceptualized as a discrete set of arbitrary signs wherein the meaning of each sign is obtained from its opposition to other signs. Thus, experience is partitioned into discrete categories which receive arbitrary labels; a fact that is well captured by the standard system of model-theoretic semantics (Montague (1970)), wherein linguistics signs are translated into formulas of logic that get interpreted in discrete models (compare also to Blume (2002)). [2]

The rules of the game with communication about future intentions are as follows. First, player one chooses a language. Then, player one sends a message to player two. Next, player one chooses an action which is not observable by player two. Finally, player two chooses an action. We call this scenario *talk about future intentions*.

Messages that belong to a so-called *credible* language are believable. Specifically, a language L will be called credible if it satisfies the following property: There exists a perfect Bayesian equilibrium of the game in which the language L is fixed, in which only messages from L are sent with positive probability, and where all messages from L are truthful and are believed both on and off the equilibrium path. Such an equilibrium given L will be referred to as a *credible communication equilibrium under* L (CCE under L henceforth).

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[1] Note that we do not allow for the explicit communication of arbitrary mixtures of actions. The reasons are as follows. First, our aim is to formulate a model that is close to what we think happens in reality. We do not find that people communicate mixtures. Second, we wanted to formulate messages that could be verified ex post which we do not think is possible for mixtures.

[2] Andras Kornai helped us to clarify the justification of the use of partitions (via private communication).

Our solution concept for credible communication is called a *credible communication equilibrium* (CCE henceforth). It is a perfect Bayesian equilibrium where a credible language is chosen on the equilibrium path, and whenever, even off the equilibrium path, player one chooses a credible language L, then both players play a CCE under L.

To demonstrate, consider Aumann's (1990) Stag Hunt game, which can also be considered as an investment game with positive spill-overs (Baliga and Morris (2002)):

		Player two	
		S	R
Player one	S	$9,9$	$0,8$
	R	$8,0$	$7,7$

In this game, we find that talk about future intentions leads players to coordinate on the efficient outcome. If player one says, "I am going to choose R", and player two believes this, then player two plays R, and hence it is in the best interest of player one to also play R. Similarly, if player one is believed, then both players will choose S after the statement by player one, "I am going to choose S". Player one can be believed, regardless of which of these two messages he sends. The language in which player one tells player two what he will do is credible, and so player two believes player one. Player one anticipates this and informs player two that he is about to tell him which action he intends to choose (he chooses the language $\{\{S\},\{R\}\}$), and then announces that he intends to choose S (sends the message $\{S\}$). The efficient outcome emerges. As player one can always choose this language, the efficient outcome arises in any CCE.

We ask if this insight is more general, that is, if talk about future intentions leads players to coordinate on an efficient Nash equilibrium outcome or on player one's favorite Nash equilibrium. In general, the answer is no. We illustrate this in Section 5 using two examples; one of which is a game of common interest, where there is a unique efficient equilibrium; the other is a lobbying game in which all equilibria are efficient, but player one has a most preferred equilibrium. In these two games, overlap of the support of the Nash equilibria makes it impossible to communicate truthfully that player one does not intend to play the efficient Nash equilibrium in the common interest game, or his favorite Nash equilibrium in the lobbying game. As languages are only credible if each message can be believed, the only credible language in these two games is the one that involves no information transmission. Credible communication need not reduce the set of possible predictions.

However, credible communication does have predictive power in many games. These two counter examples seem to be more the exception than the rule. For instance, if the favorite Nash equilibrium of player one can be supported by pure strategies and there is some other Nash equilibrium in which different pure strategies are used, then the favorite Nash equilibrium of player one will be played in any CCE. More generally, we present necessary and sufficient conditions for the existence of a unique CCE outcome in generic games. We apply these conditions to supermodular games to uncover a multitude of applications with multiple Nash equilibria in which credible communication leads players to a unique outcome. Examples include games that involve entry, investment and production decisions.

Our model of credible communication provides new insights. In general, it is not the open talk about future intentions that leads to efficiency, rather the mere possibility of talking about them. For instance, in Aumann's Stag Hunt game there is also a CCE in which no information is revealed by the sender in equilibrium, namely, when player one chooses the language $\{\{S, R\}\}$ in equilibrium. In this equilibrium, player two knows that player one intends to choose S even though it is not explicitly communicated. The reason is that it is common knowledge that player one could have chosen to communicate explicitly and credibly that he intends to choose S (by choosing the language $\{\{S\}, \{R\}\}$ see above) and hence this information need not be conveyed via communication. More generally, we find that any outcome that can be supported by a CCE can also be supported by a CCE in which no information is transmitted in equilibrium.

Credible communication can be inserted at any point during the play of a game, requiring only small adjustments to the definitions. In particular, one could consider the scenario in which communication only occurs after player one has already chosen his action (which was not observed by player two). In Section 6, we discuss the scenario where we consider the following timing. First, player one privately chooses an action. Then, player one publicly chooses a language and sends a message. Finally, player two chooses an action. We call this scenario *talk about past play*. While the formal details change, the verbal description of when a language is credible remains as it is in the scenario – talk about future intentions. This change in timing has dramatic consequences in Aumann's Stag Hunt game, as informally argued by Farrell (1988). Player two wishes to know what player one has chosen, as she would make the same choice. However, player one always prefers player two to choose S, and hence has an incentive to lie to player two about his own move whenever he has chosen R. Hence, the language

$\{\{S\},\{R\}\}$ is not credible. The only credible language is $\{\{S,R\}\}$ under which all three Nash equilibrium outcomes of the underlying game are CCE outcomes. In this game, credible communication does not narrow down the set of possible predictions.

Aumann's Stag Hunt game belongs to the class of supermodular games that exhibit positive spill-overs. The incentives to lie in this class of games when talking about past play are so strong that the favorite CCE outcome of player one can only be supported when no communication is the equilibrium language. Yet, talk after play can be very useful in games of common interest, in contrast to our counter-example of talk about future intentions. We identify conditions that ensure that the favorite Nash equilibrium outcome is the only CCE outcome.

The literature on communication in games is extensive. For games with complete information, Aumann's Stag Hunt game is a key example. Farrell (1988) and Farrell and Rabin (1996) argue intuitively that communication plays no role in talk about past play but that it leads to efficiency when talk is about future intentions. Zultan (2013) presents a model with multiple selves in order to investigate how the timing of communication influences play in Aumann's Stag Hunt game. He focuses on communicative sequential equilibria; equilibria that carry information. In a Footnote 14 in his paper, Zultan (2013) argues in favor of using this as an equilibrium selection device. He shows that the efficient outcome can only be supported with talk about future intentions, not with talk about past play. However, in Aumann's Stag Hunt game, there is also an inefficient communicative sequential equilibrium when there is talk about future intentions.[3] So, talk about future intentions does not lead to efficiency in the model of Zultan (2013). In contrast, credible communication reveals differences in the predictions in Aumann's Stag Hunt game between these two scenarios. We find that talk about future intentions leads to efficiency while it is of no use when it is about past play, precisely mirroring the experimental findings of Charness (2000). For a more comprehensive discussion of the literature, see Section 7.

This paper is structured as follows. In Section 2, we present the preliminaries, and in Section 3, we introduce the game with talk about future intentions. Next, Section 4 contains the equilibrium concept. In Section 5, we present the two counter-examples and

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[3] Assume that there are two messages m_1 and m_2. The following constitutes a communicative sequential equilibrium. Player one sends m_1, both play the mixed equilibrium after m_1 and (R,R) after m_2. One can also find inefficient pure strategy communicative sequential equilibria in larger games. For instance, consider the symmetric pure coordination game with payoffs 10, 2 and 1 on the diagonal and 0 on the off-diagonal. Then $(2,2)$ can be supported by this concept.

a general result on supermodular games that we take to applications. Section 6 contains the alternative model with talk about past play. We summarize the related theoretical and experimental literature on communication in Section 7. Section 8 concludes. The Appendix provides some definitions and the proofs of the statements whose proofs are omitted from the body of the paper.

2 Preliminaries

We introduce the underlying game and the elements of communication. For the sake of simplicity we focus on games with two players. Let G be a two player simultaneous move game with finite action sets S_j, $S = S_1 \times S_2$, and von Neumann-Morgenstern utility functions as defined by the Bernoulli utilities $u_j : S_1 \times S_2 \to \mathbb{R}$ for player $j = 1, 2$. We refer to player one as “he” and player two as “she”. For a finite set X, let ΔX be the set of probability distributions over X and let $C(\xi) = \{x \in X : \xi(x) > 0\}$ be the support of $\xi \in \Delta X$.When $\xi(x) = 1$ for some $x \in X$ then we identify ξ with x and we write $\xi \in X$ or $\xi = x$ and vice versa. $z = (z_1, z_2) \in \mathbb{R}^2$ is called an *outcome* of G if there exists $\sigma \in \Delta(S_1 \times S_2)$ of G such that $u_j(\sigma) = z_j$ for $j = 1, 2$. z is called a Nash equilibrium outcome if the corresponding strategy profile σ is a Nash equilibrium of G. $z_j^* \in \mathbb{R}$ is called a *favorite Nash equilibrium outcome* for player j if there is no Nash equilibrium outcome z such that $z_j > z_j^*$. An outcome is called *efficient* if it is not Pareto inferior to some other outcome. A Nash equilibrium outcome that is not Pareto inferior to some other Nash equilibrium outcome is referred to as an *efficient Nash equilibrium outcome.* We say that the Nash equilibrium outcomes of (the underlying game) G are *distinct*, if for any two Nash equilibrium outcomes (y_1, y_2) and (z_1, z_2), $y_1 = z_1$ if and only if $y_2 = z_2$.

To model communication, we introduce the following terminology and notation. A message m is a nonempty subset of S_1, so $m \subseteq S_1$ and $m \neq \emptyset$. The set of messages is $M = \{m \subseteq S_1 | m \neq \emptyset\}$. A subset $L \subseteq M$ is called a *language* if it is a partition of S_1.[4] The degenerate language $\{S_1\}$ that contains a single element can be interpreted as there being no communication, while the language that consists of all singletons is also relevant. These languages will be referred to as *no communication* and *detailed communication*, respectively.

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[4] $\{m_1, .., m_k\}$ is a partition of S_1 if $\cup_{i=1}^{k} m_i = S_1$ and for all i, j in $\{1, .., k\}$ with $i \neq j$ we have that $m_i \neq \emptyset$, $m_i \subseteq S_1$ and $m_i \cap m_j = \emptyset$.

3 The Game with Pre-Play Communication

We consider a model of communication in which player one chooses a language (or any subset of messages), and sends a message to player two before either has chosen an action. Such a setting with communication before play is also referred to as pre-play communication or cheap talk (Farrell and Rabin (1996)). Let Γ be the following extensive game:

1. Player one chooses a subset $L \subseteq M$ and communicates it to the other player;
2. Player one sends a message $m \in M$ to player two;
3. Player one chooses an action s_1 which is not observed by player two;
4. Player two chooses an action s_2; and
5. Payoffs are realized, where player j receives payoff $u_j\left(s_1, s_2\right)$, $j = 1, 2$.

We call Γ the communication game with talk about future intentions. Notice that, player one is allowed to choose any subsets L of M and not only partitions, i.e. not only languages. Player one is allowed to send any messages from M and he is not restricted or committed to send only messages from L. Let us denote by $\Gamma(L)$ the game Γ in which L is fixed, so player one has no discretion in choosing L. We now introduce the notation for the strategies used in Γ. Let $L_1 \in \Delta\left(2^M\right)$ be the mixed subset chosen by player one in stage 1. Let $m_1^L \in \Delta M$ be the mixed message sent by player one in stage 2 after subset L has been chosen in stage 1. Let $m_1 = (m_1^L)_{L \subseteq M}$. Let $\sigma_1^L\left(m\right)$ be the mixed action of player one in stage 3 after subset L has been chosen in stage 1 and message $m \in M$ has been sent in stage 2, so $\sigma_1^L : M \to \Delta S_1$. Similarly, let $\sigma_2^L\left(m\right)$ be the mixed action of player two in stage 4 after L has been chosen in stage 1 and message m has been sent in stage 2, so $\sigma_2^L : M \to \Delta S_2$. We write $\sigma_j = (\sigma_j^L)_{L \subseteq M}$ for $j = 1, 2$.

Hence, a strategy profile in the game Γ is a tuple $(L_1, m_1, \sigma_1, \sigma_2)$.

4 The Solution Concept

First, we define the notion of credible languages in terms of the game $\Gamma(L)$. Next, we define our solution concept for Γ using credible languages. Credible languages are those in which it is conceivable – in the sense that it cannot be ruled out – that player one can be believed. For language L to be credible means that there are beliefs that make player one believable when player one uses messages from L. Let $\mu_2^L(m) \in \Delta S_1$ indicate player two's belief about player one's action after $L \subseteq M$ and message $m \in M$. Let $\mu_2^L = (\mu_2^L(m))_{m \in M}$ and $\mu_2 = (\mu_2^L)_{L \subseteq M}$. Notice that in $\Gamma(L)$ player one is still allowed to send any message $m \in M$.

Definition 1 *We say that a language L is* ***credible*** *if there is a perfect Bayesian equilibrium* $(m_1^L, \sigma_1^L, \sigma_2^L, \mu_2^L)$ *of* $\Gamma(L)$ *in which in equilibrium player one sends a message from L, and whenever he sends a message from L player one tells the truth and player two believes it and correctly anticipates player one's action. Formally, we have the following conditions:*

1. $m_1^L \in \Delta L$ *(using the language L);*
2. *for all* $m \in L$, $C(\sigma_1^L(m)) \subseteq m$ *(truth telling);*
3. *for all* $m \in L$, $C(\mu_2^L(m)) \subseteq m$ *(believing); and*
4. *for all* $m \in L$, $\mu_2^L(m) = \sigma_1^L(m)$ *(correctly believing).*

Such an equilibrium is called a credible communication equilibrium under L (CCE under L), and the outcome in the underlying game G corresponding to $\sigma^L(m_1^L) \in \Delta S$ *is called a CCE outcome under L. We denote the set of credible languages by* $\boldsymbol{\mathcal{L}}$.

Remark 1 *It is easy to see that the following statements are true (details are given in the Appendix):*

1. *(a) It follows from the definition that no communication is always a credible language. (b) At the opposite end, detailed communication need not be credible (see examples in Section 5). (c) L is credible if and only if for each* $m \in L$ *there is a Nash equilibrium* $(\sigma_1^L(m), \sigma_2^L(m))$ *of G for which* $C(\sigma_1^L(m)) \subseteq m$. *It simply follows that, if L is credible and L′ is a coarsening of L then L′ is also credible. For example, if detailed communication is credible, then any other language is also credible.*

2. *L is credible if and only if there is a subgame perfect equilibrium* $(m_1^L, \sigma_1^L, \sigma_2^L)$ *of* $\Gamma(L)$ *in which player one sends a message from L and where player one tells the truth whenever his message is from L.*

3. *Note that conditions 3 and 4 are superfluous and follow from perfection and condition 2, however we keep them to clarify the role of condition 2 and perfection. Specifically, we require in addition to telling the truth, and whenever the message is from L and not just on the equilibrium path of $\Gamma(L)$, that player two* always *believes the message of player one (condition* 3*) and correctly anticipates player one's action (condition* 4*). For a detailed discussion of weaker definitions of credibility, in terms of weak perfect Bayesian equilibria and without condition 4, see Schlag and Vida (2013).*

We now present our model of communication in which messages are believed whenever they are believable. The notion of being believable means that they come from a credible language. Specifically, we apply this notion to identify equilibria in the communication game with talk about future intentions Γ. We consider the following equilibrium concept for Γ. We search for perfect Bayesian equilibria of Γ in which communication is truthful and believed when the language is credible.

Definition 2 (CCE) $(L_1, m_1, \sigma_1, \sigma_2, \mu_2)$ *is called a* ***credible communication equilibrium*** (CCE) *of* Γ *if it is a perfect Bayesian equilibrium of* Γ, *and:*

1. $L_1 \in \Delta\mathcal{L}$;
2. *for all* $L \in \mathcal{L}$: $(m_1^L, \sigma_1^L, \sigma_2^L, \mu_2^L)$ *is a CCE under* L *of* $\Gamma(L)$.

The outcome in the underlying game G *corresponding to* $\sigma^{L_1}(m_1^{L_1}) \in \Delta S$ *is called a CCE* outcome.

We start with a simple yet insightful result.

Proposition 1 *If the Nash equilibrium outcomes of* G *are distinct then any CCE outcome is a Nash equilibrium outcome of* G *and can be attained with no communication as the equilibrium language.*[5]

Remark 2 *It is easy to see that the following statements are true (details are given in the Appendix):*

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[5] Along the lines of this result, an alternative and equivalent approach would be to drop the explicit choice of a language, and instead define the notion of a communication-proof equilibrium in the spirit of Zapater (1997).

1. *Player one's favorite Nash equilibrium outcome is always a CCE outcome. It can be supported by no communication as the equilibrium language and beliefs that the corresponding Nash equilibrium is played after this language is chosen;*
2. *There always exists a CCE (see point 1);*
3. *If no communication is the only credible language then the set of CCE outcomes coincides with the set of Nash equilbrium outcomes;*
4. *It is easy to construct examples of G such that there is an $L \in \mathcal{L}$ which is not played in any CCE;*
5. *Every CCE outcome is in the convex hull of the Nash equilibrium outcomes of G. If the Nash equilibrium outcomes of G are distinct then any CCE outcome is a Nash equilibrium outcome of G;*
6. *z is a CCE outcome if and only if there is an $L \in \mathcal{L}$ such that z is a CCE outcome under L, and there is no $L' \in \mathcal{L}$ such that* all *CCE outcomes under L' are strictly preferred by player one to z;*
7. *If G is a two by two game with distinct Nash equilibrium outcomes then there is a unique CCE outcome. This unique CCE outcome is player one's favorite Nash equilibrium outcome, and hence it is an efficient Nash equilibrium outcome.*

We establish necessary and sufficient conditions that characterize the underlying games in which in all CCE player one gets his favorite Nash equilibrium outcome.

Proposition 2 *The following three statements are equivalent:*

1. *Player one gets his favorite Nash equilibrium outcome z_1^* in all CCE, and hence all CCE outcomes are efficient Nash equilibrium outcomes;*
2. *There is an $L \in \mathcal{L}$, such that in* all *CCE outcomes under L player one gets z_1^*;*
3. *There is a unique Nash equilibrium outcome for player one of G or there are Nash equilibria σ, σ' of G such that:*
   - *(a) $u_1(\sigma) = z_1^*$;*
   - *(b) $C(\sigma_1) \cap C(\sigma_1') = \emptyset$; and*
   - *(c) there is no σ'' Nash equilibrium of G such that $C(\sigma_1'') \subseteq C(\sigma_1)$ yielding an outcome different from z_1^* for player one.*

5 Examples and a Sufficient Condition for Efficiency

In this section, we present further insights on the connection between credible communication and efficiency. First, we present two examples to show that CCE outcomes need not be efficient Nash equilibrium outcomes (first example) or need not be the favorite Nash equilibrium outcome of player one (second example). The first example is a game of common interest, while the second is symmetric, and which we relate to an application. We next show that in supermodular games (a rich set of games often appearing in applications), CCE outcomes are efficient Nash equilibrium outcomes.

5.1 A Common Interest Game

Communication seems simplest when preferences are aligned, as in common interest games. Here one expects that communication leads to efficiency. Formally, G is a game of *common interest* if for all $(s_1, s_2), (s'_1, s'_2) \in S$, $u_1(s_1, s_2) \geq u_1(s'_1, s'_2)$ holds if and only if $u_2(s_1, s_2) \geq u_2(s'_1, s'_2)$.[6] Consider the following representative:

Player two

		L	N	R
	T	5,5	0,0	-3,-3
Player one	M	-1,-1	1,1	2,2
	B	4,4	-2,-2	3,3

(1)

The efficient pure Nash equilibrium is (T, L). However, there are two other mixed Nash equilibria, $\left(\left(\frac{2}{7}, \frac{5}{7}, 0\right), \left(\frac{1}{7}, \frac{6}{7}, 0\right)\right)$ and $\left(\left(\frac{4}{15}, \frac{43}{60}, \frac{1}{60}\right), \left(\frac{4}{15}, \frac{31}{60}, \frac{13}{60}\right)\right)$, with corresponding equilibrium payoffs 5/7 and 41/60.[7] The important feature of this game is that T belongs to the support of the strategy of player one in any of the Nash equilibria. For credible communication, each message belonging to a credible language must lead to a Nash equilibrium in which player one chooses actions within this message. Hence, disjoint messages can only be associated with Nash equilibria where the supports of the associated strategies of player one are disjoint. This is, however, not possible in this game. Thus, no credible language can have disjoint messages. According to our defi-

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[6] Generically, in any common interest game there is a unique efficient outcome – a Nash equilibrium outcome – which can be attained by a pure strategy profile. In particular, this outcome is the favorite Nash equilibrium outcome of both players.

[7] In our notation $((p_1, p_2, p_3), (q_1, q_2, q_3))$, p_1, p_2 and p_3 denote the probability of choosing T, M, and B, respectively. Similarly, q_1, q_2 and q_3 denote the probability of choosing L, N, and R, respectively.

nition of languages, this means that only no communication is credible. Consequently, all three Nash equilibrium outcomes are CCE outcomes.

5.2 The Lobbying Game

Consider two lobbyists who can try to change the status quo by making a bid belonging to $\{0, 1, 2\}$. The player who bids strictly more than the other is able to shift the outcome in his favor. The status quo, however, remains if both bid the same amount. The outcome of bidding is worth w for the winner, and l for the loser, $w > x > l$. The status quo is worth x to each of them in the case of a tie. All bids are paid. This simultaneous move game is represented by the following matrix:

		Player two		
		0	1	2
	0	x, x	$l, w-1$	$l, w-2$
Player one	1	$w-1, l$	$x-1, x-1$	$l-1, w-2$
	2	$w-2, l$	$w-2, l-1$	$x-2, x-2$

(2)

For $x = 6/5, w = 14/5$, and $l = 3/5$ the game has three Nash equilibria, none of which are pure or completely mixed. These are $\left(\left(\frac{1}{3}, \frac{2}{3}, 0\right), \left(\frac{5}{8}, 0, \frac{3}{8}\right)\right)$, $\left(\left(\frac{5}{8}, 0, \frac{3}{8}\right), \left(\frac{1}{3}, \frac{2}{3}, 0\right)\right)$ and $\left(\left(\frac{2}{5}, \frac{3}{5}, 0\right), \left(\frac{2}{5}, \frac{3}{5}, 0\right)\right)$.[8] All three outcomes are efficient Nash equilibrium outcomes. Note also that there is extensive lobbying as both players would be better off if they could both commit not to bid (that is, to bid 0).

As the support of player one's strategies overlap in any two Nash equilibria, we obtain that only no communication is credible. All three Nash equilibrium outcomes are CCE outcomes as credible communication is not able to narrow down the set of possible predictions in this example. Hence, player one is not able to guarantee his favorite Nash equilibrium outcome.

5.3 Efficiency in Supermodular Games and Applications

The result below identifies an important class of games in which all CCE outcomes are efficient Nash equilibrium outcomes. This class of games includes Aumann's Stag Hunt game.

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[8] In our notation $((p_1, p_2, p_3), (q_1, q_2, q_3))$, p_1, p_2 and p_3 denote the probability of player one bidding 0,1, and 2, respectively. Similarly, q_1, q_2 and q_3 denote the probability of player two bidding 0, 1, and 2, respectively.

Proposition 3 *Assume that G is supermodular and has a pure strategy Nash equilibrium that yields player one's favorite Nash equilibrium outcome. Then player one's favorite Nash equilibrium is the unique CCE outcome, and hence all CCE outcomes are efficient Nash equilibrium outcomes.*[9]

Proof: The proof is straightforward, along the lines of Milgrom and Roberts (1990) and Shannon (1990). All that is necessary to consider is the case where the game has at least two Nash equilibria. It is sufficient to show that the language $\{\{k\}, S_1 \setminus \{k\}\}$ is credible where k is the action of player one used in a pure strategy Nash equilibrium that supports his favorite Nash equilibrium outcome. This follows as $S_1 \setminus \{k\}$ contains the pure action of player one associated with any of the (remaining) extreme (pure) equilibria. ■

Remark 3

1. *Following Theorem 7 of Milgrom and Roberts (1990), a sufficient condition for Proposition 3 is that the supermodular game has positive spill-overs (as defined in the Appendix). In that case the highest equilibrium is in pure strategies and it Pareto dominates all the other equilibria.*
2. *The proof of Proposition 3 reveals that whenever the favorite Nash equilibrium outcome of player one is in pure strategies, in supermodular games there is a CCE in which player one tells player two whether or not he intends to choose the action corresponding to his favorite Nash equilibrium outcome.*

Proposition 3 applies to many economic situations, as supermodularity is a very common property in applications (see for example Milgrom and Roberts (1990) and Cooper and John (1988)). We particularly mention here three examples from Cooper and John (1988) (Examples A - C). The first example is an input game with coordination among input suppliers towards a shared production process. The second concerns trading externalities when agents facing uncertain cost search for a trading partner. The third example deals with demand externalities in an economy with many sectors and demand linkages across sectors. To fit them into our context, consider their discretized versions with finite sets of actions.[10] In each of these examples there are positive spill-overs. Consequently, following Proposition 3 and Remark 3 player one's favorite Nash

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[9] For the definition of supermodularity, see the Appendix.

[10] In fact, credible communication can easily be defined also for infinite action sets. However, one then needs to add some technical qualifications to be able to properly model mixed strategies and beliefs.

equilibrium is the unique CCE outcome. Hence, we obtain in these three economic applications that adding a little communication in the form of credible communication before the actual game starts eliminates the multiplicity of Nash equilibria.

6 A Variant of the Model

Credible communication is a flexible concept that can be applied to many different scenarios. Thus far, we have considered talk about future intentions where player one chooses a language, then sends a message about his intended action to player two, after which both players choose an action. In this section, we show how credible communication can be used equally as well to investigate an alternative communication scenario. We consider what happens if player one only talks after he has already privately chosen his action; namely, player one is talking about past actions or about past irrevocable commitments. When considering this variation, we demonstrate that the building blocks and definitions introduced for talk about future intentions can be easily adapted to the situation to be modeled. Credible communication becomes a tool that is both general and easy to apply in order to understand the different forms of communication in games.[11]

Hence, let us change the timing of the communication game with talk about future intentions Γ, as defined in Section 3. We move the action choice of player one to the beginning and obtain the following different order of the first three stages. First, player one chooses an action s_1 which is not observed by player two (stage 1). Next, player one chooses a subset $L \subseteq M$ and communicates it to the other player (stage 2). Then, player one sends a message $m \in M$ to player two (stage 3). Player two then chooses an action (stage 4).[12] We call this the *communication game with talk about past play* Γ. Let us maintain the definition that $\Gamma(L)$ denotes the game Γ in which L is fixed and player one has no discretion in choosing L.

The strategies in the communication game with talk about past play are given as follows. In stage 1, player one chooses a mixed action $\sigma_1 \in \Delta S_1$. In stage 2,

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[11] Other variants of the model such as for example extending credible communication to more general languages, letting player two choose the language or when player one can talk, publicly or privately, to multiple audiences is investigated in the working paper version Schlag and Vida (2019).

[12] In our model, the language explains the context of the message that is being sent. Hence we assume that the language is chosen before sending the message and after the action has been chosen. However, in some settings, the alternative timing in which the language is chosen before the action is chosen may be more appropriate.

player one chooses a subset $L_1(s_1) \subseteq M$ after action s_1 has been chosen in stage 1, so $L_1 : S_1 \to \Delta(2^M)$. In stage 3, player one chooses a message $m_1^L(s_1)$ after action s_1 has been chosen in stage 1 and subset L has been chosen in stage 2, so $m_1^L : S_1 \to \Delta M$. The strategy space of player two in stage 4 remains unchanged; she chooses an action $\sigma_2^L(m)$ after subset L has been chosen in stage 2 and message m has been sent in stage 3, so $\sigma_2^L : M \to \Delta S_2$.

The verbal description of a credible language remains unchanged. However, due to a different order of moves, we need to present a new formal definition.

Definition 3 *We say that a language L is **credible** if there is a perfect Bayesian equilibrium $(\sigma_1, m_1^L, \sigma_2^L, \mu_2^L)$ of $\Gamma(L)$, in which no matter which action player one has chosen, player one sends a message from L, and whenever he sends a message from L player one tells the truth and player two believes it. Formally, we have the following conditions:*

1. *for all s_1, $m_1^L(s_1) \in \Delta L$ (using the language L);*
2. *for all $s_1 \in S_1$, for all $m \in C(m_1^L(s_1)), s_1 \subseteq m$ (truth telling); and*
3. *for all $m \in L$, $C(\mu_2^L(m)) \subseteq m$ (believing).*

We say that $(\sigma_1, m_1^L, \sigma_2^L, \mu_2^L)$ is a CCE under L and let $\mathcal{L}$ denote the set of credible languages.

In contrast to the setting with talk about future intentions, there is no condition of correctly believing (compared to point 4 in Definition 1), as player one has already chosen an action when selecting the language.

Consider now the communication game with talk about past play Γ. The definition of CCE is adapted to once again accommodate the different order of moves. As before, we write $m_1 = (m_1^L)_{L\subseteq M}, \sigma_2 = (\sigma_2^L)_{L\subseteq M}$ and $\mu_2 = (\mu_2^L)_{L\subseteq M}$. Remember σ_1 now is simply an element of ΔS_1.

Definition 4 (CCE) *$(\sigma_1, L_1, m_1, \sigma_2, \mu_2)$ is called a **credible communication equilibrium** (CCE) of Γ if it is a perfect Bayesian equilibrium of Γ, and:*

1. *for all $s_1 \in C(\sigma_1) : L_1(s_1) \in \Delta\mathcal{L}$; and*
2. *for all $L \in \mathcal{L}$: $(m_1^L, \sigma_2^L, \mu_2^L)$ is part of a CCE under L of $\Gamma(L)$.*

The outcome in the underlying game G corresponding to a CCE is called a CCE outcome.

Note that a CCE need not have a unique equilibrium language when player one chooses a mixed action in stage 1, as player one's choice of the subset of messages in stage 2 may depend on which action he has chosen in stage 1. Technically speaking, the choice of language could be used as a signal to indicate what happened in stage 1. However, in the results that follow, this possibility is not used.

Remark 4 *It is easy to see that the following statements remain true (details are given in the Appendix):*

1. *(a),(b) of point 1 of Remark 1,*
2. *Proposition 1,*
3. *Remark 2 points 1 to 6,*
4. *Proposition 2 points 1 and 2.*

On the other hand, point 7 in Remark 2 and Proposition 3 does not remain true as we demonstrate in the sequel.[13]

We present positive and negative results on credible communication with talk about past play and contrast them with our results on talk about future intentions. We begin with negative results and consider the Aumann's Stag Hunt game as described in the Introduction. It is easy to see that only no communication is credible (see the arguments made in the Introduction). Consequently, all three Nash equilibrium outcomes are CCE outcomes. Credible communication about past play cannot help players in this game to select from amongst the Nash equilibria in Aumann's Stag Hunt game.

Aumann's Stag Hunt game is an example of a supermodular game that exhibits positive spill-overs. In Proposition 3, we have seen that credible communication with talk about future intentions is very effective in these games, as it selects a unique outcome. Yet for the same class of games, our next result shows that it is difficult to transfer information with credible communication when talk is after play. This result can also be considered as the counterpart of Proposition 10 in Baliga and Morris (2002).

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[13] The rest of Remark 1 does not remain true, we provide a counter example for (c) of point 1 in the Appendix, while the rest is obvious. Obviously, point 3 of Proposition 2 fails to be equivalent with points 1 and 2.

Proposition 4 *Assume that G is strictly supermodular and exhibits positive spill-overs.*[14] *The unique efficient (Nash equilibrium) outcome is a CCE outcome if and only if no communication is the equilibrium language and for all $s_1 \in S_1, L_1(s_1) = \{S_1\}$.*

We now discuss the positive results of the effectiveness of talk about past play. Specifically, we provide conditions for when player one can guarantee his favorite outcome under talk about play. We focus on understanding when this is possible using detailed communication. Assume that player two has a unique best response to each pure action of player one. When player one chooses some s_1 player two will react by choosing $b_2(s_1)$. In order for detailed communication to be credible, player one has no incentive to lie about the action chosen, thus that $u_1(s_1, b_2(s_1)) \geq u_1(s_1, b_2(s_1'))$ holds for all $s_1, s_1' \in S_1$. When this condition holds, we call the game *self-choosing*. Note that common interest games such as the game in Figure 1 are self-choosing. Recall that credible communication about future intentions is useless in this game, as shown in Section 5.1. Note also that our self-choosing definition is weaker than Baliga and Morris's (2002) notion of *self-signalling* that requires $u_1(s_1, b_2(s_1)) \geq u_1(s_1, s_2)$ to hold for all $(s_1, s_2) \in S$. Thus, in self-signalling games, player one wants to inform player two about which action he has taken. In self-choosing games, player one has no incentive to act as if he had chosen a different action.

This leads to the following result which can be considered to be a counterpart of Proposition 7 in Baliga and Morris (2002). In this result, we identify conditions under which the mere existence of credible communication leads to a unique outcome, regardless of which credible language is chosen in equilibrium.

Proposition 5 *Assume that no two pure strategy outcomes yield the same payoff for player two, and:*

1. *Player one has a favorite Nash equilibrium outcome in pure strategies and the game is self-choosing; or*

2. *The game is self-signalling.*

Then in all CCE outcomes, player one gets his favorite Nash equilibrium outcome.

Proof: In self-choosing games, detailed communication is, by definition, credible. Thus, detailed communication is a language under which player one's favorite outcome

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[14] For the definitions of strict supermodularity and positive spill-overs, see the Appendix.

is the only CCE outcome if player one has a favorite Nash equilibrium outcome in pure strategies. Given that $b_2(.)$ is a function, player one's favorite Nash equilibrium outcome is the unique CCE outcome under detailed communication. Then, using Remark 4 and points 1 and 2 of Proposition 2 completes the proof of point 1. For point 2, note that self-signalling games are also self-choosing in which player one has a favorite Nash equilibrium outcome in pure strategies. To see this, given any mixed equilibrium σ, choose any $s_1 \in C(\sigma_1)$ and find $b_2(s_1)$. By self-signaling $u_1(s_1, b_2(s_1)) \geq u_1(\sigma)$. Then calculate $b_1(b_2(s_1)), b_2(b_1(b_2(s_1)))$... which converges to a pure equilibrium, as players' payoffs are weakly increasing in this sequence and player one's payoff in this equilibrium is clearly weakly larger than in the mixed equilibrium σ. $\blacksquare$

Finally, there are some games in which neither talk about future intentions nor talk about past play is useful.

Remark 5 *It is easy to see that the following statement is true (details are given in the Appendix): In the lobbyist game of Section 5.2, only no communication is credible and all Nash equilibrium outcomes are CCE outcomes.*

7 Related Literature

In this section, we position our paper in the context of the literature on communication in games.

There is extensive research on pre-play communication under complete information where communication occurs before the underlying game starts. However, the papers involved are more distant than our paper from a basic game theoretic analysis of the original game; they build on elaborate models of language or apply sophisticated solution concepts. There is also literature on sender-receiver games in which communication is about exogenously determined private information. However, we do not learn how to model communication about endogenously determined private information; generated by unobservable own previous choices. Rabin (1990, p. 166) calls for an explicit modeling of communication when asymmetric information results from unobservable actions. Yet to date – apart from Zultan (2013) that is discussed in detail further below – there has been no paper that provides a uniform framework for analyzing the role of communication surrounding the play of a game.

The following are the most closely related papers. When formally modeling preplay communication, Farrell (1986) suggests neologisms, while Farrell (1988) considers rationalizability after imposing some plausibility requirements that are far from being intuitive. Baliga and Morris (2002, p. 457) argue that incomplete information must be introduced in order to be able to define credible communication; a statement that our paper finds to be false. As a consequence our model is very different from that of Baliga and Morris (2002) and almost impossible to compare. Lo (2007) proposes the elimination of weakly dominated strategies for a particular class of messages, but imposes intricate conditions which are hard to interpret. Rabin (1994) creates a specific protocol for how players talk to each other over time, and investigates whether or not this leads to efficiency in the long run. There is also a behavioral model with level k reasoning (Ellingsen and Östling (2010)), and evolutionary models of pre-play communication (e.g., see Demichelis and Weibull (2008) and Hurkens and Schlag (2003)). While there exists a long standing literature on pre-play communication, the only previous paper – apart from Zultan (2013) – that is close to the original game in its formulation and analysis is Lo (2007). The quality of our paper by design is that the description and execution of our solution concept is simple, which adds to its plausibility for making predictions. In contrast, the modeling of language by Lo (2007) is far more complicated, and this moves her paper (like many others) to a more abstract level. Note that it is straightforward to prove one of Lo's (2007) main results (Proposition 1 in Lo (2007)) in our setup.[15]

The literature on sender-receiver games is significantly larger. In an early and influential paper, Crawford and Sobel (1982) show how partial information can be transmitted in a game in which the sender does not have the incentive to reveal all information. In the literature on neologisms (e.g., see Farrell (1986) and Farrell (1993)), unexpected messages are checked in terms of their credibility. Reasoning becomes more involved when more than one message passes this test (e.g., see Matthews et al. (1991)). Baliga and Morris (2002) conduct a formal game theoretical analysis, thus avoiding plausibility checks. However, they only identify circumstances in which all the information is transmitted.

Zultan (2013) is the paper most closely related to ours. Indeed, it is the only paper in which talk about both past play and future intentions is modeled, and to date, there

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[15] It is easy to see that in our setup, player one gets his favorite Nash equilibrium outcome under talk about intentions, if (a) each action is part of a unique pure strategy Nash equilibrium and (b) his favorite Nash equilbrium outcome is attained by one of them. Lo states that self-committing (which implies (a)), together with self-signaling (which implies (b)), implies that the sender gets his favorite equilibrium.

is no other paper that considers communication about past unobserved play. We now discuss Zultan (2013) in more detail. Zultan (2013) is able to explain the intuitions in the discussions surrounding Aumann's Stag Hunt game (see Charness (2000) for a summary) by imposing the following three conditions. First, the sender is not a single player, but is modeled by two players, separating the choice of the message from the action. Second, only communicative sequential equilibria are considered, and third, only pure strategies are considered. As argued in the Introduction (see Footnote 3), without the third condition, an inefficient outcome can be predicted in Aumann's Stag Hunt game. Note further that the notion of communicative sequential equilibrium is only defined for the Stag Hunt game. It is based on reacting to differences in strategies, not in payoffs. Hence, it is not immune to adding redundant strategies. For instance, the inefficient outcome can be supported in the Stag Hunt game with talk about future intentions if one considers two different but redundant ways to hunt a rabbit. Zultan (2013) contains no formal definition of communicative sequential equilibrium and no results for general games with talk about past play. For general games and talk about future intentions, Zultan (2013) shows the existence of a communicative equilibrium that supports the favorite Nash equilibrium outcome of the sender. The paper contains no results on the possible existence of other outcomes that can be supported. In fact, as pointed out in Footnote 3, inefficient pure strategy outcomes exist when the underlying game has more than two Nash equilibrium outcomes.

In the view of the literature outlined here, we now discuss the different elements of our model and put these into context.

In formal terms, it might be possible to argue that credible communication is yet another equilibrium selection criterion. However, its intention is different. Equilibrium refinements – as the term indicates – are typically initiated after the game has been analyzed, checking whether or not each equilibrium satisfies the refinement. The reason for the refinement is often taken more for granted than justified, such as when looking only for equilibria in which all information is transmitted (see Baliga and Morris (2002) and Zultan (2013)). Our notion of credibility is applied during the game at the moment communication is initiated, as an attempt to introduce a natural form of communication.

We define messages in terms of the information they contain. The set of messages is determined by the set of actions, unlike most other models in which messages are abstract symbols and given exogenously. We explicitly allow for vagueness, where the degree of vagueness is chosen by the sender. In contrast, the degree of vagueness in Crawford and Sobel (1982) is driven by mutual beliefs and is not purposely chosen by any of the players.

We require that the sender – when delivering a message – includes the context of the message in the form of a language. To model the choice of the context formally is novel in the literature. Without the inclusion of language as part of the communication in our model, the listener would not know how to interpret non-equilibrium messages. They could be mistakes, and they could be purposeful deviations. In particular, it makes no sense for the sender to make an alternative suggestion if the listener stubbornly believes – when receiving any other message – that it has been mistakenly sent by the sender. In contrast, by choosing an alternative language in our model, the sender can actively nudge (similar to forwards induction) the listener away from her current beliefs. The listener is forced to adapt her beliefs – even out of equilibrium – whenever this can be done in a way that is consistent with truth-telling. Note that neologisms are an alternative way to introduce the context surrounding a message.

Truth and credibility play a central role in our solution concepts, and also appear in various forms in other papers. Neologisms build on informal plausibility arguments which are believed if they satisfy postulated plausibility checks. Other approaches assume that senders tell the truth with positive probability (Chen (2004)), or whenever indifferent (Ellingsen and Östling (2010)), or when they add an explicit cost of telling a lie (Kartik et al. (2007), Kartik (2009), and Serra et al. (2013)).

We test which Nash equilibrium outcomes persist if only a little communication is added. We obtain that it is typically not the message sending per se that drives the results, rather the mere possibility of sending messages. Specifically, we find that any CCE outcome can be obtained by the language no communication (Proposition 1). This is close to postulating in a game without communication that the players form beliefs as if communication would be allowed (as we learn from Proposition 1, explicit message sending is not needed). The notion of virtual communication is born. Any Nash equilibrium outcome that is also a CCE outcome can be qualified as being consistent with virtual communication (compared with the notion of virtual observability of Amershi et al. (1989)).

Some experiments are closely related. There is experimental evidence to suggest that adding one-sided pre-play communication increases efficiency (see Cooper et al. (1989), Cooper et al. (1992), and Blume and Ortmann (2007)). The findings of Cooper et al.

(1992) are slightly contradictory to those of Charness (2000). In a Stag Hunt game with a constant payoff action, Cooper et al. (1992) find a significant number of coordination failures under one-way communication. Only simultaneous communication between the two players induces subjects to coordinate on the efficient outcome. Weber et al. (2004) shows that being an unobservable first mover can influence outcomes in favor of this player. This is explained by virtual observability. We posit that *virtual communication* is a natural alternative that is closer to the true environment.

Further experiments are required. Under traditional implementations it is particularly difficult to identify preferences from monetary outcomes, as subjects need not be selfish. Moreover, the experiments mentioned above are implemented in an anonymous environment. Yet a willingness to understand and believe others – as postulated under credible communication – seems most natural when communication occurs among people who know each other.

The explicit modeling of communication within a game, the use of a language with messages that have a meaning and can be believed, the choice of Perfect Bayesian equilibrium as the solution concept; all of these elements make this a successful model for teaching, and relevant to applied researchers who are concerned with the multiplicity of Nash equilibria in their models.

8 Conclusion

We present a uniform framework for modeling communication surrounding the play of a game with complete information. We introduce our approach in a simple, clear and salient model in which one player speaks to another about his future intentions. We show how to extend it to situations where talk is about past unobserved play. More players and more complicated communication scenarios may be readily incorporated. However, our objective is not to show how complicated communication can be. Rather, we aim to show how easy it is to destabilize some Nash equilibria with the simplest communication. We suggest that it is difficult to imagine that all forms of communication are ruled out.

Our model contains two important elements. First, we assert a common understanding between the players that messages that can be believed will be believed. Second, we consider messages with meaning and allow the sender, by choosing the language, to explain the context surrounding the message. We abstract from a separate, more theoretical research agenda that aims to understand when and why credibility should play a role. Our messages are limited to identifying sets that contain the support of a strategy. Explicit talk about mixed strategies could change results in Sections 5.1 and 5.2 but would also add an artificial element, that is, the explicit use of probabilities in conversations about choices. Modeling messages as subsets allows for vagueness. A player may find it necessary to remain vague in the situation when adding more details would not be credible. Recall that any CCE outcome can be supported by choosing no communication as the equilibrium language. Hence, whenever a Nash equilibrium outcome is not a CCE outcome (which is often the case), we obtain that the predictive value of this Nash equilibrium outcome is eliminated by the mere possibility of credible communication. In fact, it may be argued that such Nash equilibria are less plausible even when communication is not allowed, as messages do not have to be sent in order to eliminate it. These arguments can be used to develop a model of virtual communication (see Section 7).

A final plausibility test of our model of credible communication is to see whether or not it can also be used in a game of incomplete information. The treatment of incomplete information is discussed in Schlag and Vida (2013), while further investigations are left for future research. Note that communication in sender-receiver games is very similar to talk after past play. In fact, definitions are more straightforward; simply treat the distribution of the state of nature as an exogenously determined mixed action of player one. Thus, we find that our model of credible communication passes this plausibility test.

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Appendices

A Some Definitions

Consider G with $S_1 = \{1, 2, \ldots, p\}$ and $S_2 = \{1, 2, \ldots, q\}$.

Definition 5 *We say that G is supermodular if for all $k, k' \in S_1$ with $k \leq k'$ and for all $l, l' \in S_2$ with $l \leq l'$ we have that:*

$$u_1(k', l') - u_1(k, l') \geq u_1(k', l) - u_1(k, l) \text{ and } u_2(k', l') - u_2(k', l) \geq u_2(k, l') - u_2(k, l).$$

Definition 6 *We say that the game G exhibits positive spill-overs if for all $l, l' \in S_2$ with $l < l'$ and for all $k \in S_1$ we have that $u_1(k, l) < u_1(k, l')$, and for all $k, k' \in S_2$ with $k < k'$ and for all $l \in S_2$ we have that $u_2(k, l) < u_2(k', l)$.*

Let us denote by $b_i(s_j) = argmax_{s_i \in S_i} u_i(s_i, s_j)$.

Definition 7 *We say that the game G has (strictly) increasing best responses if $b_1(.), b_2(.)$ are functions such that for all $l, l' \in S_2$ with $l < l'$ we have that $b_1(l) \leq (<) b_1(l')$ and for all $k, k' \in S_1 : k < k'$ we have that $b_2(k) \leq (<) b_2(k')$.*

It is easy to see that supermodular games have increasing best responses.

Definition 8 *We say that the game G is strictly supermodular if it is supermodular, and in addition, has strictly increasing best responses.*

A.1 Omitted Proofs of Propositions

A.1.1 Proof of Proposition 1

Proof: Consider a CCE $(L_1, m_1, \sigma_1, \sigma_2, \mu_2)$ with outcome (z_1, z_2). Consider any $L \in C(L_1)$ and any $m \in C(m_1^L)$ then $(\sigma_1^L(m), \sigma_2^L(m))$ is a Nash equilibrium of G. Player one must be indifferent between the different Nash equilibria played with positive probability on the equilibrium path. By distinctness, player two will also be indifferent between these Nash equilibria. It follows that the convex combination of such Nash equilibrium outcomes is a Nash equilibrium outcome, that of $\sigma^L(m)$. Finally, we show that no communication can be chosen to be the equilibrium language resulting in the outcome (z_1, z_2). First, keeping everything else unchanged as in the original equilibrium, set the beliefs after the language no communication such that play results in the Nash equilibrium with outcome (z_1, z_2), namely set $\mu_2^{\{S_1\}}(S_1) = \mu_2^L(m) = \sigma_1^{\{S_1\}}(S_1) = \sigma_1^L(m)$ and $\sigma_2^{\{S_1\}}(S_1) = \sigma_2^L(m)$. This is still a CCE. Next, change the equilibrium language choice to no communication, namely set $L_1 = \{S_1\}$. Again, this is a CCE with no communication on the equilibrium path resulting in the Nash outcome (z_1, z_2). ■

A.1.2 Proof of Proposition 2

Proof: Point 2 follows from point 1, as otherwise for all $L \in \mathcal{L}$ we have $(m_1^L, \sigma_1^L, \sigma_2^L, \mu_2^L)$ which is a CCE under L resulting in $z_1^L < z_1^*$ for player (for $L \notin \mathcal{L}$ one can choose $m_1^L, \sigma_1^L = \mu_2^L, \sigma_2^L$ in a way that $\sigma^L(m)$ is a Nash equilibrium for all $m \in M$ resulting in the worst Nash outcome for player one). One can then choose $L_1 \in argmax_{L \in \mathcal{L}} z_1^L$ to be the equilibrium language. Completing the strategy profile with $(m_1, \sigma_1, \sigma_2, \mu_2)$ one obtains a CCE resulting in $z_1^{L_1} < z_1^*$ (this is basically an implication of point 6 in Remark 2). Point 1 follows from point 2 because player one can choose the language which gives him his favorite outcome in any CCE under this language. Point 2 follows from point 3, as L can be chosen to be $\{C(\sigma_1), S_1 \setminus C(\sigma_1)\}$ in case (a),(b) and (c) holds or to be $\{S_1\}$ when there is a unique equilibrium outcome for player one of G. Finally point 3 follows from point 2. Suppose the language satisfying point 2 can be chosen to be $\{S_1\}$. Then G must have a unique equilibrium outcome for player one, as any Nash equilibrium outcome is a CCE outcome under $\{S_1\}$. Suppose now the language satisfying point 2 can be chosen to be $L \neq \{S_1\}$. It is clear that (a) and (b) must hold because there are at least two different (disjoint) messages in L after which Nash equilibria of G must be played in any CCE under L (see points 1 and 2 in Remark 1). Suppose (c) is not satisfied. We can assume that for all $m \in L$ there is a σ^m Nash equilibrium such that $C(\sigma_1^m) \subseteq m$ yielding an outcome different from z_1^* for player one, as otherwise there is $m \in L$ such that (a), (b) and (c) holds choosing σ to be any Nash equilibrium for which $C(\sigma_1) \subseteq m$. Now, it is easy to construct a CCE under L with an outcome different from z_1^* for player one by setting $\sigma^L(m) = \sigma^m$ and choosing $m_1^L \in argmax_{m \in L} u_1(\sigma^m), \mu_2^L(m) = \sigma_1^m$ which is a contradiction. ■

A.1.3 Proof of Proposition 4

Proof: Consider G with $S_1 = \{1, 2, \ldots, p\}$ and $S_2 = \{1, 2, \ldots, q\}$. Given that strict supermodularity implies strictly increasing best responses (see the definition in the Appendix) it must be that $p = q$ and the pure strategy Nash equilibria are $(k, l) \in S_1 \times S_2$, where $k = l$. Given supermodularity and positive spill-overs, $(k, l) = (p, p)$ results in the unique efficient Nash equilibrium outcome $z^* = (z_1^*, z_2^*)$ which also Pareto dominates all the other, possibly non-Nash equilibrium outcomes. Players get their overall best payoffs.

To prove the "if" statement it is easy to check that $\sigma_1 = p$, for all $k \in S_1$, $L_1(k) = \{S_1\}, m_1^{\{S_1\}}(k) = S_1$ and $\sigma_2^{\{S_1\}}(S_1) = \mu_2^{\{S_1\}}(S_1) = p$ is part of a CCE where the rest of the strategy profile can be chosen arbitrarily satisfying the conditions for CCE. After the choice of any other credible language player one can be just worse off no matter the choice of k. After non-credible languages play and beliefs can be set arbitrarily respecting sequential rationality.

The "only if" statement is proven by contradiction. It is enough to prove that $L = \{S_1\}$ is the unique credible language for which z^* is a CCE outcome under L. It then follows that for all $k \in S_1$ player 1 must choose $L_1(k) = \{S_1\}$ in any CCE resulting in z^*. Hence, suppose to the contrary that there is a credible language L, different from no communication, such that z^* is a credible outcome under L. Suppose that the equilibrium message is $m_1^L(p) = m \neq S_1$, which follows from the fact that $L \neq \{S_1\}$ is a partition, $p \in m$ and it induces player two to play p, i.e. $\sigma_2^L(m) = p, \mu_2^L(m) = p$. We show now that for any other $m' \in L$ for which $p \notin m'$, necessarily by the partition structure of L, we have that for all $\mu_2^L(m')$ supported within m', $p-1$ is a better response of player two than p. Hence, whenever $k \in m'$, player one will deviate and send the message m resulting in the highest action of player 2, in p, and by positive spill-overs, in a higher utility for player one compared to sending m'. We prove that:

$$p \notin argmax_{l \in S_2} \sum_{k \in S_1 \setminus \{p\}} \mu_2^L(m')(k) u_2(k, l),$$

where $C(\mu_2^L(m')) \subseteq m'$. It is enough to prove that $u_2(k, p-1) > u_2(k, p)$ for all $k \neq p$ in S_1. This follows from the fact that

$$0 > u_2(p-1, p) - u_2(p-1, p-1) \geq u_2(k, p) - u_2(k, p-1)$$

for every $k \leq p-1$, where the first inequality follows from the fact that $(p-1, p-1)$ is a Nash equilibrium (and that the best responses are unique) and the second inequality follows from supermodularity. ■

B Details for Remarks 1,2,4 and 5: Not for Publication

B.1 Remark 1

1. (a) Pick any Nash equilibrium σ and for all $m \in M$ set $m_1^{\{S_1\}}(\{S_1\}) = S_1, \sigma_1^{\{S_1\}}(m) = \mu_2^{\{S_1\}}(m) = \sigma_1$ and $\sigma_2^{\{S_1\}}(m) = \sigma_2$. These constitute a perfect Bayesian equilibrium of $\Gamma(\{S_1\})$ satisfying points 1-4 in Definition 1.

(c) For each $m \in L$ set $\mu_2^{\{S_1\}}(m) = \sigma_1^L(m)$ and choose $m_1^L \in argmax_{m \in L} u_1(\sigma^L(m))$. For any $m \notin L$ choose $\sigma^L(m) = \sigma^L(m')$, where $m' \in argmin_{m \in L} u_1(\sigma(m))$.

2. If L is a credible language then the corresponding strategy profile is a subgame perfect equilibrium of $\Gamma(L)$ by perfection. If there is a subgame perfect equilibrium in which player one tells the truth for any message $m \in L$ can be used to form a perfect Bayesian equilibrium just as in point 1 (c) above.

B.2 Remark 2

1. See the proof of point 1 (a) of Remark 1 and notice that player one can then choose no communication as the equilibrium language, given that choosing any other credible language or any $L \notin \mathcal{L}$ result in a payoff less than his favorite Nash outcome.

2. See the equilibrium in point 1.

3. See the proof of point 1 (a) of Remark 1 and choose $L_1 = \{S_1\}$ and for any other L and m set $\sigma^L(m)$ to be the same equilibrium as on the equilibrium path.

4. Consider the following game:

Player two

		L	N	R
Player one	T	1,-1	-1,1	-1,2
	M	-1,1	1,-1	-2,-1
	B	1,-1	-1, 1	-2,-1
	D	-1, 1	1,-1	-1,2

(3)

The language $L = \{\{T, M\}, \{B, D\}\}$ is credible, but in any CCE under L player one obtains -1. On the other hand, the language $L' = \{\{T, D\}, \{M, B\}\}$ is also credible and in any CCE under L' player one obtains 0. Hence, in no CCE player one ever chooses L with positive probability as the equilibrium language.

5. This is a restatement of Proposition 1.

6. (If:) For each $L' \in \mathcal{L}$ different from L and for any $m \in L'$ set $m_1^{L'}, \sigma^{L'}, \mu_2^{L'}$ be the CCE under L' yielding outcome $z_1^{L'} < z_1$ for player one and similarly for L set these values to yield the outcome z . Then setting $L_1 = L$ and for any m and for

any $L' \notin \mathcal{L}$ pick an arbitrary Nash equilibrium of G with an outcome (weakly) less than z_1 for player one. This completes a strategy profile which is a CCE with outcome z. (Only if:) Clearly, z must be a CCE outcome under some L. Suppose now, that there is an $L' \in \mathcal{L}$ such that in all CCE under L' player one's payoff is strictly higher than z_1. Choosing L' is a strictly profitable deviation from the putative CCE with outcome z.

7. Consider the following parametrization of 2 by 2 bimatrix games:

$$\begin{array}{cc|c|c|} & & \multicolumn{2}{c}{\text{Player two}} \\ & & \multicolumn{1}{c}{L} & \multicolumn{1}{c}{R} \\ \cline{3-4} \text{Player one} & U & a, x & b, y \\ \cline{3-4} & D & c, z & d, w \\ \cline{3-4} \end{array}$$

Suppose that z^*, player one's favorite Nash outcome, is attainable in a pure Nash equilibrium w.l.o.g. in U,L (that is $a \geq c, x \geq y$). It must be that playing D is never part of a Nash equilibrium otherwise detailed communication is credible and z^* is the unique CCE outcome and the proof is completed. Hence if $z \geq w$ then $a > c$, if $z \leq w$ then $b > d$. It must be that only one of them holds otherwise U strictly dominates D and by distinctness z^* is the unique Nash outcome and the proof is completed. Consider the case when $z > w$ and $b \leq d$ and $a > c$, i.e. L weakly dominates R. Consider the case when $z < w$ and $a = c$ and $b > d$, i.e. U weakly dominates D. In both cases it is clear that D is never part of a Nash equilibrium and the unique equilibrium outcome is z^* by distinctness.

Suppose now that z^* is attainable only in a mixed Nash equilibrium, where player one is mixing. By distinctness it must be that player two is also mixing. We claim that this is the unique Nash equilibrium of the game. We can assume w.l.o.g. that $a > c$. If $a = c$ then $b = d$ and player one can play U or D. Now if playing U two can still mix then $x = y$ and hence $a = b$ and we have z^* in a pure Nash equilibrium. Otherwise, suppose that if player one plays U then two plays L ($x > y$) but then $z < w$ and hence both U,L and D,R are Nash equilibria one of which is better for player one than z^*. Hence, assume that w.l.o.g. that $a > c$ but then also $b < d$. If $b < z_1^* < a$ then it must be that $x < y$ and hence $z > w$ otherwise U,L is better than the mixed equilibrium for player one. But then there are no more Nash equilibrium outcomes than z^*. Hence, it must be that $a < z_1^* < b$, but then $z > w$ (to avoid D,R as Nash resulting in b) and hence $x < z$ and again there are no more Nash equilibrium outcomes than z^*.

B.3 Remark 4

1. Remark 1 point 1 (a): Pick any Nash equilibrium σ of G. Then σ_1, $\forall s_1 \in S_1, L_1(s_1) = \{S_1\}, m_1^{\{S_1\}}(s_1) = S_1$, $\mu_2^{\{S_1\}}(m) = \sigma_1$ and $\sigma_2^{\{S_1\}}(m) = \sigma_2$ for all $m \in M$ forms a CCE under $\{S_1\}$. (b): We have seen the Stag Hunt game as an example.

2. Proposition 1: Again, start from a CCE $(\sigma_1, L_1, m_1, \sigma_2, \mu_2)$ and pick any L and an $m \in L$ which has positive probability under the equilibrium. Then $\mu_2^L(m), \sigma_2^L(m)$ is a Nash equilibrium of G. Keeping everything else unchanged, set $\sigma_1 = \mu_2^L(m), L_1 = L$ is also a CCE. Finally, change the equilibrium language to $L_1(s_1) = \{S_1\}$ for all $s_1 \in C(\sigma_1)$ and set $\mu_2^{\{S_1\}}(S_1) = \sigma_1$ and $\sigma_2^{\{S_1\}}(S_1) = \sigma_2^L(m)$.

3. The proofs of Remark 2 points 1,2,3,5 and 6 are simple modifications of the original proofs. We must construct a different example for point 4 which is inspired by an example in Sobel (2012):

   Player two

		L	N	R
Player one	T	10,0	7,5	6,-10
	M	10,0	6,-10	7,5
	B	5,10	6,5	5,-10
	D	5,10	5,-10	6,5

   (4)

   There is no CCE in which the credible language $L = \{\{T, B\}, \{M, D\}\}$ is played on the equilibrium path. The reason is that there is another credible language $L' = \{\{T, M\}, \{B, D\}\}$ such that in any CCE under L' player one obtains 10, whereas there is no CCE under L in which player one obtains 10. First we investigate the CCE-s under L. L can be made credible with the strategy profile $\sigma_1 = T, m_1^L(T) = m_1^L(B) = \{T, B\}, m_1^L(M) = m_1^L(D) = \{M, D\}, \forall m \neq \{M, D\}$: $\mu_2^L(m) = T, \sigma_2^L(m) = N$ and $\mu_2^L(\{M, D\}) = M, \sigma_2^L(\{M, D\}) = R$ resulting in payoff 7 for player 1. Clearly, player one cannot get 10 in some CCE under L because then player one would reveal whether he has chosen T or M and then player 2 would never choose L. Next we investigate the CCE-s under L'. L'

can be made credible with the strategy profile $\sigma_1(T) = \sigma_1(M) = 1/2, m_1^{L'}(T) = m_1^{L'}(M) = \{T, M\}, m_1^{L'}(B) = m_1^{L'}(D) = \{B, D\}, \forall m \neq \{B, D\} : \mu_2^{L'}(m)(T) = \mu_2^{L'}(m)(M) = 1/2, \sigma_2^{L'}(m) = L$ and $\mu_2^{L'}(\{B, D\}) \in \Delta\{B, D\}, \sigma_2^{L'}(\{B, D\}) = L$. Clearly, in any CCE under L' it must be that $\sigma_2^{L'}(\{B, D\}) = L$, hence σ_1 will be supported on $\{T, M\}$. $\sigma_2^{L'}(\{T, M\})$ must be L, otherwise player one would lie when his action is B or D. It follows that player one obtains 10 in any CCE under L'.

4. Proposition 2 points 1 and 2: The equivalence of these points follow in a very similar way as in the proof of Proposition 2. Clearly, point 3 does not apply.

Finally we demonstrate a counter example which shows that point (c) of point 1 of remark 1 does not hold. Consider the following game:

		Player two	
		L	R
Player one	T	$1, 1$	$0, 0$
	M	$0, 0$	$1, 1$
	N	$1, 1$	$0, 0$
	B	$0, 0$	$1, 1$

(5)

In this game detailed communication is credible but the coarsere language $\{\{T\}, \{M\}, \{N, B\}\}$ is not credible because either player one wants to send T when he chooses N or he wants to send the message M when he chooses B.

B.4 Remark 5

Detailed communication is not credible because once player two believes the message $\{2\}$ and player one's action is 1 then player one would send $\{2\}$ and induce player two to play 0 obtaining $w - 1$ instead of $l - 1$ which he would obtain when telling the truth $\{1\}$ inducing player two to play 2 (as she believes his messages) as it would be required from a CCE under detailed communication.

The language $\{\{0, 1\}, \{2\}\}$ is not credible. It is clear that in any CCE under this language player two should play 0 after the message $\{0, 1\}$, otherwise, as before player one can induce player two to play 0. But then player one's equilibrium action is 1 and hence player two should play 2 after the message $\{0, 1\}$.

The language $\{\{0, 2\}, \{1\}\}$ is not credible. In a CCE under this language player two should play 2 after the message $\{1\}$ and should play 2 after the message $\{0, 2\}$ as well otherwise after action 1 player one would send the message $\{0, 2\}$ and obtain something strictly better than $l - 1$. But for any belief over $\{0, 2\}$ 0 is a better action of player one than 2.

Finally, the language $\{\{1, 2\}, \{0\}\}$ is also not credible. In any CCE after message $\{0\}$ 1 is played by player two. 0 cannot be played by player two with positive probability after $\{1, 2\}$, as otherwise player one would send this message instead of $\{0\}$ once he played 0. Also, 2 cannot be played after $\{1, 2\}$ because player one would send $\{0\}$ and induce player two to play 1 once he has chosen action 1 or 2. Hence, player two always plays 1, but then player one's equilibrium action is 2, the equilibrium message is $\{1, 2\}$ after which player two should play 0.