Job Separation under Uncertainty and the Wage Distribution.*

Julien Prat†


Abstract

This paper examines a search-matching model in which match specific output follows a geometric Brownian motion. As opposed to Poisson Processes, Brownian motions generate a negative correlation between job output and the likelihood of separation. A parametrized version of the model shows that it fits the observed pattern of worker turnover and wage dispersion more accurately than the standard model, without taking from its relevance at the macro level. Firstly, the proposed set-up does not require learning about match quality in order to yield a hump-shaped hazard rate of job separation. Secondly, the aggregate wage distribution is unimodal and its right tail belongs to the Pareto family, so it satisfies the “fat-tail” property that is commonly observed in the data.

THEME : Macroeconomics of unemployment and inequality.

KEYWORDS : matching model, uncertainty, turnover, wage dispersion.

JEL-CODE : J31, J63, J64.

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1 Introduction

Since its publication in 1994, the model proposed by Mortensen and Pissarides (1994) (hereafter termed M/P) has established itself as the standard framework for the analysis of equilibrium unemployment. The main contribution of Mortensen and Pissarides was to endogenize job separation through the introduction of idiosyncratic uncertainty. This hypothesis has immediate implications for worker turnover, tenure-earnings profile and wage dispersion. Unfortunately, they are at odds with the data since the model predicts that the hazard rate of job separation is constant through time (Pries, 2004) and that wages are negatively correlated with job spells. These shortcomings explain why the burgeoning literature inspired by M/P has remained separated from the growing body of research on wage dispersion\(^1\) and individual worker turnover. This paper proposes to bridge the gap by investigating the idea that the stochastic evolution of productivity in a standard search-matching model can account for many observed features of both worker turnover and wage dispersion. We find that the discrepancy between the search-matching model and the observations mostly arises for technical reasons and that it can be solved by modifying the stochastic process that drives jobs output.

In M/P, jobs output follow a Poisson process. When a job is hit by a shock, its new productivity is drawn from an ergodic distribution with a fixed support. While being easy to manipulate, Poisson processes have the major drawback of being memory-less. Instead, we propose to use geometric Brownian motions. As opposed to Poisson processes, the sample paths of geometric Brownian are continuous. This distinctive property generates a negative correlation between output and the likelihood of separation. We find that the selection of matches with a low output enables us to accurately approximates the empirical hazard rate of job separation as well as the qualitative features of the wage distribution. A calibrated version of the model yields a hump-shaped hazard rate of job separation that fits reasonably well the estimations reported in Farber (1994). Furthermore, we prove that the equilibrium wage density is always unimodal with a right tail of Pareto functional form, as is commonly observed in the data.

\(^1\)A notable exception is the recent paper by Wong (2003) on wage inequality. However, its methodology is very different from the one proposed in this paper, since the model in Wong (2003) does not endogenize job destruction.
Evidence on rising inequality in the United States over the last twenty years has spurred a revival of interest in estimating the structural components of the wage distribution. It has incited researchers to devote a great deal of effort to render equilibrium models of wage dispersion tractable. The common methodology of this burgeoning literature is to explore the implications of between-employer competition due to on-the-job search. In his latest book, Mortensen (2003) summarizes and extends the main results of this research program.

One of the most salient empirical facts about wage distributions is that they have a “fat” right tail. Although the wage-posting approach was precisely devised in order to explain wage dispersion, the basic specification laid out in Burdett and Mortensen (1998) predicts that the distribution of wages in a cross-section of workers has an increasing density. Thus, one needs additional assumptions. As explained by Mortensen (2003), the theory is consistent with empirical evidence given a distribution of employers productivity with a fat right tail.

This paper follows a completely different approach since we exclude on-the-job search. Still, we find that the model does not require any particular assumption in order to explain the fat-tail property. On the contrary, the underlying process that drives job fluctuation within our set-up is Gaussian, so its right tail is thin. The fat tail property is always satisfied because it arises endogenously from the selection process. Optimal job separation weeds out solely the bad jobs. The asymmetric nature of job separation yields an unbalanced steady-state distribution with a more than proportional share of highly productive jobs.

Given that the separation process is the driving force that shapes the wage distribution, it matters to know whether or not the model fits the observed pattern of worker turnover. Jovanovic (1979) provides the benchmark framework for the analysis of worker turnover. It emphasizes that the quality of a match remains largely unknown after the matching decision. The worker and the firm face a signal extraction problem: as tenure accumulates, they infer the true quality of the match. The learning mechanism reproduces the hump-shaped hazard rate of job separation that is well documented (Farber, 1994). Conversely, M/P assumes that job separation occurs at a constant rate. It was precisely in order to solve this discrepancy that researchers (Pries, 2004; Moscarini, 2003) originally embedded a signal extraction problem into the framework of M/P.
The model proposed in this paper provides another explanation based on the interaction between the separation process and the diffusion of job output. Jobs always start at a higher output than the one at which it is optimal to split-up the match. Since output evolves continuously, it is quite unlikely that shocks will be bad enough to induce separation within a short period of time. As time elapses, the unlucky jobs are weed out so that the distribution of the surviving sample becomes more and more right-skewed. As a result, the hazard rate of separation eventually decreases with tenure. It turns out that such a simple mechanism convincingly approximates the empirical estimation reported in Farber (1994).

The paper is organized as follows. All the proofs of the propositions are collected in Appendix. In the next section, we define the set up of the model. Following that, the equilibrium of the economy is derived and analyzed in section 3. In section 4 the model is parametrized and its simulated hazard rate of job separation is compared to empirical estimates. Section 5 proposes a closed-form expression for the aggregate distribution of wages. Section 6 concludes.

2 Set-up of the model

In order to highlight the key role of the separation process, we adopt a set-up that is similar to the one proposed in M/P except that the stochastic process for output is a geometric Brownian motion instead of a Poisson process.

Hence, we assume that each firm has one job which can be either filled or vacant. When an unemployed worker meets a firm with a vacant job, they form a match and start producing. Once a job is created, its output starts to fluctuate so that jobs’ values among a given cohort drift away as time elapses. The stochastic process which changes the job’s output is a geometric Brownian motion. Its law of motion is given by

\[
\frac{dP^i_t}{P^i_t} = \sigma dB^i_t
\]  

(1)

where \(P^i_t\) denotes the output of job \(i\) at time \(t\) and \(dB^i_t\) is the increment of a standard
Brownian motion. In the remainder of the text we will neglect the subscript $i$ when not necessary. The parameter $\sigma$ captures the standard deviation of job-specific shocks. We exclude stochastic matching so that jobs have the same initial value. Hence, if job $i$ is created at time $t$, then $B^i_t = 0$.

Equation (1) implies three important divergences from M/P. First of all, future outputs are drawn from log-normal distributions with a constant mean equals to the initial output. Hence, the job’s output follows a martingale, whereas in M/P the expected worker’s output is a negative function of tenure since the starting value is also the upper support of the distribution. Secondly, shocks are geometric and so proportional to current output. It is a standard assumption to consider geometric Brownian motions in order to exclude negative values from the set of potential realizations. But most importantly, the sample paths of geometric Brownian motions are continuous. On the other hand, Poisson processes are memory-less since the new output is drawn from an ergodic distribution, independently of its pre-shock level. This is why they are also called jump processes. As it will be shown latter, continuity is the crucial factor which allows to generate a realistic mechanism of selection.

The second source of jobs selection is a death shock that forces jobs out of business by hitting them randomly at the Poisson arrival rate $\delta$. It captures separations that occur

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2Notice that we do not consider aggregate shocks so that changes in productivity are uncorrelated across jobs.

3Of course, this is at odds with the positive empirical correlation between worker tenure and wage. It is easy to solve this problem by introducing a deterministic trend in (1), as in the companion paper Prat (2005a). We have chosen not do so in order to avoid diverting the attention of the reader from the focus of this paper.

4The geometric specification distinguishes our approach from the one of Moscarini (2005) who considers standard Brownian motions. Whereas negative outputs would never be observed in Moscarini’s set-up because of the selection process and the continuity of Brownian motions, our approach rule them out from first principle, as it is common in quantitative analyses of search-matching models.

5Considering more general Markov processes which combine Brownian motions with jumps in productivity could also be an interesting avenue of research. Intuitively, it seems reasonable to assume that the sample paths are not always continuous. Actually, the death shocks can be interpreted as discrete jumps from current to very low productivity. Adding to the model positive and stochastic jumps is not difficult at the match level, but it would complicate a lot the aggregation procedure. Since this extension does not promise to add any important insights, we find it more parsimonious and instructive to restrict our attention to geometric Brownian motions.
for exogenous reasons such as retirement, death, partner reallocation...

We exclude on-the-job search so that trade in the labor market is completely separate from production. Firms post vacancies that are randomly matched. This process is captured by the matching function which gives the number of job matches per unit of time

\[ m = m(u, v) \]  

where the size of the labor force has been normalized to 1. \( u \) denotes the unemployment rate and \( v \) the number of vacant jobs as a fraction of the labor force. In this respect, the set-up is completely standard with a matching function increasing in both its arguments, concave and homogenous of degree 1. The tightness parameter \( \theta \) denotes the ratio of vacancies per unemployed worker. The homogeneity of the matching function implies that the Poisson rate at which vacant jobs become filled is given by

\[ q(\theta) = m \left( \frac{u}{v}, 1 \right) = \frac{m(u, v)}{v} \]  

with \( q'(\theta) \leq 0 \). Accordingly, unemployed workers move into employment at the rate

\[ \theta q(\theta) = \frac{m(u, v)}{u} \]  

Let \( J(P_t) \) denote the asset value for the firm of an occupied job with output \( P_t \) and \( V_t \) the asset value of a vacancy. Normalizing the initial output to 1 implies that the capital gain from filling a vacancy is equal to \( J(1) - V_t \). So the Bellman equation for \( V_t \) reads

\[ rV_t = -c + q(\theta)(J(1) - V_t) + \mathbb{E}_t \frac{dV_t}{dt} \]  

where it is assumed that the vacant job costs \( c \) per unit of time and \( r \) denotes the rate at which agents discount the future.

To the extent that vacancies can be freely posted, the exhaustion of arbitrage opportunities implies that rents from vacant jobs are driven down to zero. Thus the equilibrium condition for the supply of vacancies is simply \( V_t = 0 \). Replacing it in (5), we finally obtain

\[ J(1) = \frac{c}{q(\theta)} \]  

Employees earn an output-contingent wage \( w(P_t) \). They also face the risk of becoming unemployed because their job can be hit by a death shock. Therefore, the flow value of
working in a match with idiosyncratic productivity $P_t$ is equal to
\[ rW(P_t) = w(P_t) + \delta (U - W(P_t)) + \frac{E_t}{dt} [dW(P_t)] \] (7)
where $U$ denotes the asset value of being unemployed.

As it is standard in the literature, we assume that wages are set by surplus-splitting, so that the rent of the worker remains proportional to that of the firm. The rent is defined as the difference between the asset value obtained by participating in the match and the outside option of the agent. Hence, wages ensure that
\[ W(P_t) - U = \left( \frac{\beta}{1 - \beta} \right) (J(P_t) - V_t) \] (8)
where $\beta$ belongs to $[0, 1]$. The left hand side of (8) defines the rent of the worker. Let $S(P_t)$ denote the total surplus of the match. It obviously corresponds to the sum of the two rents, so that
\[ S(P_t) = W(P_t) - U + J(P_t) - V_t = \frac{W(P_t) - U}{\beta} \] (9)

It appears that $\beta$ gives the share of the surplus that goes to the workers or, in other words, their bargaining power.

### 3 The equilibrium

#### 3.1 Solving for the equilibrium

While being unemployed, workers benefit from unemployment income $b$. Unemployed workers also evaluate the expected gain from search. They anticipate that their instantaneous probability of receiving an acceptable job offer is $\theta q(\theta)$. So the flow value of being unemployed can be decomposed in the following way
\[ rU = b + \theta q(\theta) (W(1) - U) + \frac{E_t}{dt} [dU(t)] \] (10)
Inserting (6) in (10) yields
\[ rU = \left( b + \left( \frac{\beta}{1 - \beta} \right) \theta c \right) \] (11)
The right hand side gives the opportunity cost of employment. It is the flow income to which workers renounce while being on the job. Solving for the value of the surplus is not that straightforward. It can be done noticing that agents face an optimal stopping problem. Since the surplus is increasing in $P_t$ whereas the opportunity cost of employment is independent of $P_t$, there exists a unique reservation output. Applying standard methods of stochastic calculus enables us to derive the surplus of the match in closed form.

**Proposition 1** The expected surplus of a match with current output $P_t$ is equal to

$$S(P_t) = \frac{P_t}{r + \delta} - \left(\frac{1}{r + \delta}\right) \left(b + \left(\frac{\beta}{1 - \beta}\right) \theta c\right) - \left[\frac{R}{r + \delta} - \left(\frac{1}{r + \delta}\right) \left(b + \left(\frac{\beta}{1 - \beta}\right) \theta c\right)\right] \left(P_t^R\right)^{\alpha}$$

(12)

where $R$ is the reservation output and $\alpha$ is the negative root of the following quadratic equation

$$Q(\alpha) \equiv \frac{\sigma^2}{2} \alpha (\alpha - 1) - r - \delta = 0$$

(13)

As one can see, the surplus is made of two terms. The first one is simply the expected value of future profits if the job could not be destroyed. It would give the value of the surplus if the matching decision were irreversible. The second term is the value of the option to destroy the job.

From the expected stream of revenues, the option deducts the realizations which are below reservation output. Since these realizations have to be discounted, the option itself is composed of two terms. The first one gives the expected value of future revenues when output is at its reservation level, while the second term discounts the probability of separation. Maximizing the surplus with respect to $R$ yields$^6$

$$R = \left(\frac{\alpha}{\alpha - 1}\right) \left(b + \left(\frac{\beta}{1 - \beta}\right) \theta c\right)$$

(14)

$^6$The concavity of the surplus with respect to $R$, as depicted in Figure 1, suggests that the optimal reservation output can be derived using standard first order conditions. The correct reasoning is more complex because Figure 1 displays the surplus when current output is equal to one. So it is also necessary to be sure that the optimal reservation output is state independent. Fortunately, it has been demonstrated by McDonald and Siegel (1985) that the exercise locus of parabolic differential equations, such as the one satisfied by $S(P_t)$, is homogenous of degree zero. So the intuition behind Figure 1 actually holds: the optimal reservation output is pinned down using standard first order condition.
We will refer henceforth to (14) as the *Optimal Separation Rule*. It defines a linear and upward sloping function of $\theta$. It slopes up because at higher $\theta$, workers’ outside opportunities are better and wages go up. This increase in the cost of labor implies that matches are less profitable so that jobs are destroyed more frequently.

The first term on the right hand side of (14) is always inferior to one. So the optimal reservation productivity is lower than the opportunity cost of employment, indicating the existence of *labor hoarding*. The option drives a positive wedge between the discounted value of future revenues, as given by the first component of (12), and the expected surplus of the match. The *Optimal Separation Rule* is an inverted image of the rule for irreversible investment, where the discounted value of profits has to exceed the cost of investment. It also shares with it the analytical form of the multiplicative factor $(\alpha / (\alpha - 1))$. Intuitively, destroying a job means that you renounce definitively to a profit opportunity. Because of the irreversibility of the separation, the firm will procrastinate up to the point where the value of waiting equals operational losses.

To solve for the equilibrium, the *Optimal Separation Rule* has to be interacted with the *Free Entry Condition*. Figure 1 plots the flow value of posting a vacancy as a function of $R$. Parameters reported in Table 1.

Figure 1: Flow value of posting a vacancy as a function of $R$. Parameters reported in Table 1.
of the reservation productivity for different values of $\theta$.\footnote{Which is equal to $q(\theta)(1 - \beta)S(1) - c.$} It has been explained in the previous section why free trade in the labor market implies that vacancy posting yields zero profit. Consequently, the \textit{Free Entry Condition} is satisfied when the curves cross the zero-axis.

But the reservation output is a control variable. Hence, firms always choose it so as to set to zero the slope of the surplus. Consider the curve associated to the lowest value of $\theta$. At the optimal reservation productivity, the value of posting a vacancy is positive, so more firms will enter and $\theta$ will increase. On the other hand when $\theta$ is relatively high, as in the lowest curve of the graph, firms will close unprofitable vacancies and $\theta$ will go down. As a matter of fact, the only stable point is the one where the curve is tangent to the zero-axis. It specifies the equilibrium values of the two endogenous variables $\theta$ and $R$.

\textbf{Proposition 2} \textit{The labor market is in equilibrium when both}

$$ R = \left(\frac{\alpha}{\alpha - 1}\right) \left[b + \left(\frac{\beta}{1 - \beta}\right) \theta c\right] $$(Optimal Separation Rule)

and

$$ q(\theta)(1 - \beta)S(1; \theta, R) = c $$ (Free Entry Condition)

\textit{are simultaneously satisfied. This system of two equations determines the equilibrium values of the two endogenous variables $\{\theta, R\}$. When the matching function is of the Cobb-Douglas type and unemployment income is inferior to the job initial output, the model has a unique equilibrium.}

\section*{3.2 The equilibrium rate of unemployment}

To derive the rate of unemployment $u$ as a function of the two endogenous variables $R$ and $\theta$, we have to characterize further the separation process. Fortunately, the proposed set-up allows to express in closed-form the probability of job separation as a function of tenure.
Proposition 3 The probability for a given match of being operational at tenure a is given by

\[ 1 - H(a) = e^{-\delta a} \left( \Phi \left( \frac{-\ln R + \mu a}{\sigma \sqrt{a}} \right) - \Phi \left( \frac{\ln R + \mu a}{\sigma \sqrt{a}} \right) \right) \]  (15)

where \( \mu = -\frac{\sigma^2}{2} \) is the trend of \( \ln(P_t) \) and \( \Phi(.) \) is the standard normal cumulative distribution function.

By aggregation, the individual probabilities also characterize the cross sectional distribution of matches.\(^8\) This property follows from the law of large numbers, where convergence is not a function of time but results from the infinite dimension of the economy. Therefore, it holds almost surely at every point in time. So we can use the expression of \( H(\cdot) \) to equate the flows in and out of the employment pool. On the one hand, we can compute the level of employment as a function of the labor market tightness and of the job creation rate. On the other hand, the rate of job creation is an outcome of the matching process which ultimately depends on the number of unemployed workers. Being the only free variable, the equilibrium rate of unemployment must adjust to balance the outcomes of both production and matching processes. Proposition 4 gives the level of employment for a given rate of job creation.

Proposition 4 The size of the employment pool \( N \) is equal to

\[ N = m(u, v) \int_0^{+\infty} (1 - H(a))da = \frac{(m(v, u)}{\delta} \left( 1 - R^{\mu+d} \right) \]  (16)

where \( d = \sqrt{\mu^2 + 2\delta \sigma^2} \).

The first term on the right hand side of (16) is the level of employment when idiosyncratic outputs do not fluctuate. The second term is the number of matches that did not split up because of endogenous job separation. Hence, \( R^{\mu+d} \) measures the impact on employment of optimal separation. As one might expect, employment is a negative function of the reservation productivity.

From the definition of the matching function, we know that the normalized rate of job creation is a function of labor market tightness and of unemployment given by

\[ m(u, v) = \theta q(\theta) u \]  (17)

\(^8\)See Judd (1985) and Uhlig (1996).
Since $N$ is equal to $(1 - u)$, inserting equation (17) into (16) yields

$$u = \frac{\delta}{\theta q(\theta) \left(1 - R \frac{\mu + d}{\sigma^2}\right) + \delta}$$

Unemployment is decreasing in the tightness parameter $\theta$. The homogeneity of the matching function ensures that the Beveridge curve is convex and downward slopping in the vacancy-unemployment space, as in the data. This result stands in contrast to M/P where the Beveridge curve may slope down or up according to the choice of the parameters.

Unemployment is an increasing function of $R$ because a higher reservation output intensifies the rate of job separation. It leads to the congestion of the labor market which is ultimately offset by an increase in the equilibrium rate of unemployment. When $\sigma$ goes to zero, the exponent $(\mu + d)/\sigma^2$ diverges to infinity. As $R$ is inferior to one, it follows that (18) converges to the standard expression under certainty.

4 The hazard rate of job separation

In this section, we show that the model approximates the hazard rate of job separation more accurately than the standard model without taking from its relevance at the macro level. As we do not want to abandon the aggregate perspective, the model is parametrized with the aim of fitting macroeconomic moments. Then the shape of the hazard rate is compared to its empirical estimation in Farber (1994).

We calibrate the model using statistics of the U.S. labor market from the 1980s.\textsuperscript{9} The parameters choices are summarized in Table 1. The first set of parameters are standard

\textsuperscript{9}The model could easily be parameterized using more recent data. We have chosen to focus on the 1980s for two reasons. Firstly, M/P was calibrated for this period, so it allows a more direct comparison between the two models. Secondly, the estimations reported in Farber (1994) are also based on data from the 1980s. In any case, this choice should not raise doubt on the capacity of the model to fit the current pattern of match separation because it is well documented that worker turnover in the US has remained remarkably stable over the last two decades (Gottschalk and Moffitt, 1999). Hence, if the model can fit the hazard rate of job separation observed in the 1980s, it can certainly do so for the 1990s, as shown in Prat (2005b).
so that their values can be found in the literature. The discount rate \( r \) is set equal to 5%, as it is common in real-business models (Cooley, 1995). Following Shimer (2003), equal bargaining power \( \beta \) has been assumed. We consider a Cobb-Douglas matching function, so that

\[
q(\theta) = A\theta^{-\eta}
\]  

(19)

According to the evidence collected in Pertrongolo and Pissarides (2001), the mid-range of estimated values for the elasticity of the matching function with respect to unemployment is close to 0.5.

### TABLE I

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Moment to match</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r = 0.05 )</td>
<td>Interest rate</td>
<td>Cooley (1995)</td>
</tr>
<tr>
<td>( \eta = 0.5 )</td>
<td>Elasticity of the matching function</td>
<td>Petrongolo et Pissarides (2001)</td>
</tr>
<tr>
<td>( \beta = 0.5 )</td>
<td>Optimality of the decentralized equilibrium</td>
<td>Shimer (2003)</td>
</tr>
<tr>
<td>( b = 0.5 )</td>
<td>Unemployment benefit=26% average wage</td>
<td>OECD (1996)</td>
</tr>
<tr>
<td>( A = 4 )</td>
<td>Mean Unemployment Duration=12 weeks</td>
<td>Abraham et Shimer (2001)</td>
</tr>
<tr>
<td>( \delta = 0.045 )</td>
<td>Mean Unemployment Rate= 7.3%</td>
<td>Economic Report of the President (1997)</td>
</tr>
<tr>
<td>( c = 1 )</td>
<td>Recruiting costs =7 weeks of worker’s wage</td>
<td>Ljungqvist (2002)</td>
</tr>
<tr>
<td>( \sigma = 0.3 )</td>
<td>( v/u =1 )</td>
<td>Shimer (2003)</td>
</tr>
</tbody>
</table>

The remaining parameters are parameterized to fit the moments reported in Table 1. The following reasoning is used in order to pin down the scale factor \( A \) of the matching function. First of all, we refer to the evidence reported in Shimer (2003) to set to 1 the steady-state ratio of vacancies to unemployed worker. Then we match an average unemployment duration of 12 weeks, an approximation that is consistent with the estimates reported in Abraham and Shimer (2001). Given that \( \theta \) has been set to one, unemployment duration in year units is given by the inverse of the scale factor. Hence, we can immediately infer that \( A \) has to be equal to 4, which is the factor used in M/P.

Unemployment income \( b \) is set to 0.5 in order to obtain a ratio of unemployment income to wages of 26%, which is in the range of its sample mean in the US (Pissarides, 1996).
The cost of posting a vacancy is set to 1, which yields a recruiting cost equal to seven weeks of the average worker’s income (Ljungqvist, 2002).

We are left with two latent parameters: the rate of arrival of the death shocks $\delta$ and the variance of the idiosyncratic shocks $\sigma$. We use the following statistics to pin them down. As explained before, we normalize $\theta$ to 1. Then we approximate an unemployment rate of 7.3%, which is the mean U.S. civilian unemployment rate in the 1980s (Economic Report of the President, 1997).

Figure 2 reports the estimations obtained by Farber (1994) using the NLSY dataset over the 1979-1988 period. Each panel depicts the empirical hazard rate of job separation estimated at different frequencies. One can see that for low frequencies, such as quarters or years, the hazard rate is decreasing. However, when the intervals are shortened to months or weeks, the initial period of employment is characterized by an increasing rate of job separation. One of the key findings of Farber is that the hazard rate peaks at the third month of employment.

Figure 3 reports the simulated outcome of the model. One can see that it reproduces the humped-shape observed in high frequency data. Moreover, the mode of the simulated hazard rate is also situated at the third month of tenure and is of similar magnitude than the empirical one. The model predicts a slightly smoother evolution of the hazard rate since it tends to underestimate the rate of separation in the early periods and to overestimate it for long job spells. Yet, when compared to the flat line implied by $M/P$, the two panels highlight the accuracy of the proposed model.

The shape of the hazard rate predicted by the simulation is not specific to the choice of parameters. On the contrary, it is easy to prove that the hazard rate always start to decrease after a certain tenure because job censoring is such that the average output

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10 In the absence of on-the-job search, worker flows and job flows are identical. However, it is well known that in reality many workers move from job to job so that worker flows are higher than job flows. Deciding which of the two statistics should be used to calibrate the model depends on the objective of the analysis. As Mortensen and Pissarides were primarily interested in explaining the aggregate dynamics of job destruction and job creation, they compared their model to evidence on job flows collected by Davis and Haltiwanger (1994). Our motivation is different since we are trying to understand the distribution of workers’ wages. Hence, it is more consistent with our purpose to consider worker flows because they occur due to match-specific factors which also determine the dispersion of wages.

11 Proof available upon request.
Figure 2: Hazard rate of job separation estimated in Farber (1994).

Figure 3: Simulated hazard rate of job separation.
Parameters of the simulation are reported in Table 1.
Figure 4: Sensitivity analysis with respect to $\sigma$.
All parameters except $\sigma$ are reported in Table 1.

of surviving jobs increases. This is due to the reflection principle which states that the diffusion paths of standard Brownian motions are symmetric. Since all the paths that hit $R$ are taken out of the sample, the output distribution becomes more and more right-skewed as time elapses. Thus, the matches that have survived are more likely to be far from the reservation productivity.$^{12}$

The hazard rate of separation initially increases before decreasing because jobs start with a productivity that is higher than $R$. Hence, the continuity of the stochastic process implies that endogenous separation is very unlikely to happen within a short period. Surprisingly enough, the model reproduces the finding of Jovanovic (1979) without the learning mechanism.

To ascertain the robustness of the parametrization, we report in Figure 5 a sensitivity analysis of the model with respect to the volatility parameter $\sigma$. An increase in $\sigma$ does not modify the expected values of future outputs. Given that the option to separate puts a floor on unlucky realizations, $\sigma$ augments the value of the option through its effect on

$^{12}$The fact that the hazard rate of separation decreases with tenure when output follows a Brownian motion was first noticed by Shimer (1998).
Therefore, the value of posting a vacancy is higher and $\theta$ increases.

The impact on $R$ follows from two opposite effects. On the one hand, waiting becomes more valuable when the variance increases. On the other hand, given that $\theta$ has increased, the opportunity cost of employment is higher. As a result, the total impact of $\sigma$ on $R$ is ambiguous. For the parameters of the simulation, the positive effect on the value of waiting dominates, so that $R$ decreases.

To obtain the impact on unemployment, we need first of all to characterize job separation. A higher variance leads to more separation. This effect is counteracted by the decrease of the reservation output. For the parameters of the simulation, the direct effect of $\sigma$ prevails and job separation is intensified, as can been seen from the figure in the lower-left corner of the panel.

As in M/P, the eventual effect on unemployment is ambiguous. On the one hand, jobs are destroyed more frequently. On the other hand, firms post more vacancies. Therefore, an increases in idiosyncratic variance leads to bigger flows in and out of the unemployment pool. Which of the two effects will dominate will depend on the parameters of the model. Simulations show that in most cases the rate of unemployment and $\sigma$ are positively correlated. In terms of sensitivity analysis, these offsetting effects imply that the elasticity of the unemployment rate with respect to $\sigma$ is relatively low.

5 The wage distribution

Wage dispersion results from the diffusion of jobs’ output. Since the shocks have no behavioral content, our approach belongs to what Neal and Rosen (2000) call the ”stochastic process theories” of earnings distribution. A classical difficulty of these approaches is that the variance of wages is an ever growing function of tenure. Hence, it is necessary to introduce offsetting mechanisms in order to ensure that the aggregate distribution converges to a steady-state distribution. In our case, the stabilizing force is job separation. Although the variance of wages within a given cohort of workers tends to infinity, the cohort’s mass converges even faster to zero. As a result, the aggregate distribution is well defined.

13 Applying standard comparative static to the characteristic equation (13) shows that $\partial \alpha/\partial \sigma > 0$. For more details on this argument, see Dixit and Pindyck (1994), p.144.
Proposition 5  The aggregate density of output is piecewise and given by

\[
\Psi(x) = \begin{cases} 
  x^{\mu - d - 1} \left( x^{\frac{2d}{\sigma^2}} - \frac{R^2}{\sigma^2} \right) \left( \frac{y}{\sigma} \right) & \text{if } x \in [R, 1] \\
  x^{\frac{\mu - d}{\sigma^2} - 1} \left( \frac{1 - R^2}{\sigma^2} \right) \left( \frac{y}{\sigma} \right) & \text{if } x \geq 1 
\end{cases}
\]

(20)

The distribution of wages follows directly since

\[
w(x) = \beta x + \beta \theta c + (1 - \beta) b
\]

Although the underlying process that drives job fluctuations is Gaussian, inspection of the aggregate density shows that its right tail belongs to the Pareto family. The fat tail property arises endogenously because of the asymmetric nature of jobs censoring. As explained before, the output distribution as a function of tenure becomes more and more skewed to the right. Therefore, the wage distribution aggregates layers with a more than proportional share of highly productive jobs.

The aggregate density also has an increasing left tail. Due to the infinite-variation property of Brownian motions, job cannot survive in the neighborhood of the reservation output. This is why the density converges to zero as it approaches its lower bound. Put together, these two components give to the theoretical wage distribution a shape that captures the main features commonly observed in the data.\(^\text{14}\)

The aggregate wage density is plotted against its empirical counterpart in Figure 5.\(^\text{15}\)

The empirical CDF is computed using data from the January 1983 Current Population Survey (hereafter CPS). If the individuals are employed and paid an hourly wage, the CPS

\(^{14}\text{Notice that } \Psi(\cdot) \text{ is not differentiable at the initial productivity. This peculiarity is easily resolved through the introduction of stochastic matching. However, to keep the analysis as clear as possible, we have chosen to consider the simplest case where initial output is the same across jobs.}

\(^{15}\text{A long list of observable characteristics (such as education, experience...) contribute to the wage distribution whereas the proposed model is based on the premise that workers are homogenous. So, a more refined procedure would compare the model to the empirical wage distribution conditional on observable workers and firms characteristics. However, it is not clear how these characteristics impact the unemployment rate and so such an approach would not be consistent with the derivations in section 3. As we want to devise a tractable macro-framework, we consider our approach as a first approximation and leave these refinements to further research. This methodology mirrors the standard assumption according to which the matching function captures the heterogeneities which determine the trading process without the need to introduce them explicitly in the model.}
observations correspond to the hourly wage rate they report. In the case where the worker is not paid on an hourly basis, we have divided the gross weekly wage by the usual hours of work per week in order to impute the hourly wage. After excluding observations with missing data or extreme reported wages,\textsuperscript{16} we obtained a sample of 11707 individuals. To make the two curves comparable, we have normalized the simulated distribution so that the empirical and simulated medians are equal.

Figure 5 shows that the simulated CDF differs substantially from the one observed in the data. Most of the discrepancy is due to the left tail of the distribution which does not display enough dispersion. The density is too concentrated around its mode because the reservation output is not far enough from the starting productivity. Nevertheless, the simulated distribution converges to the empirical one at high quantiles. Hence the model has no difficulty to fit the right tail of the density. For example, it predicts that the ninth decile is equal to 13 dollars per hour whereas its value in the data is equal to 13.5 dollars.

Although the model can account for the fact that many workers earn high wages, the match-specific component of productivity is not sufficient to explain the overall dispersion observed in the data. One way to fix the discrepancy is to extend the model by assuming that initial productivities are drawn from a non-degenerate distribution. The companion paper Prat (2005b) proposes to estimate such a model by maximum likelihood and shows that it fits almost perfectly the entrants and the aggregate wage distributions.

With respect to other stochastic process theories of earnings distribution, our approach has the advantage of identifying the stabilizing mechanism. So we can interpret the economic forces that shape the wage distribution. For example, we can predict the effect on wage dispersion of a change in idiosyncratic variance.

As shown in Figure 6, the equilibrium density of wages becomes flatter when $\sigma$ increases. We also observe that the reservation wage is lower. Given that such an increase in idiosyncratic variance is well documented (Comin and Mulani, 2004), the model suggests that it might not only help to explain the rise in inequality but also the loss of incomes among the poorest workers. However, it cannot justify the negative correlation between unemployment and inequality which has been observed in the last two decades.

\textsuperscript{16}In practice, we have excluded all workers with a reported wage below the first or above the last percentile of the wage distribution.
Figure 5: Empirical and Simulated CDF.
Parameters of the simulation are reported in Table 1.

Figure 6: Equilibrium probability density of wages.
All Parameters except $\sigma$ are reported in Table 1.
that the diffusion of new technologies to ongoing job relationships provides a potential explanation.

6 Directions for further research

The model proposed by Mortensen and Pissarides (1994) was primarily designed to explain the dynamics of job separation and job creation. This is probably why idiosyncratic uncertainty is not often considered as an important factor for the explanation of wage dispersion. Actually, most of the recent papers dedicated to the analysis of wage inequality have neglected it in favor of between-employers competition. Of course the point of this paper is not to divert the attention of the reader from this promising line of research, but to underline that the role of job separation as a shaping force is probably insufficiently appreciated.

When the data-generating process for output is realistic, a standard matching model delivers accurate predictions about job separation and the main features of the wage distribution. This paper also identifies the limits of the basic model since we find that the match-specific component of productivity cannot explain on his own the actual dispersion of wages. A satisfactory explanation would require to incorporate the mechanisms described in this paper into a more general set-up. The most obvious adjustment is to extend the model so that initial productivities differ across jobs. The companion paper Prat (2005b) proposes an estimation procedure which shows that such a model can capture the connection between the wage distribution among job entrants and the wage distribution in the whole population.

With respect to models of on-the-job search, the proposed set-up has the advantage of requiring solely equilibrium existence to reproduce the fat-tail property. Given that models of employers competition are better designed to explain other phenomena, there would probably be great explanatory gains in combining both approaches within a unified framework.

The model could also be compared to the approach proposed by Moscarini (2005). In concurrent research, Moscarini (2005) has introduced standard Brownian motions in a search-matching model. His set-up is different since it nests a learning model about match
quality a la Jovanovic (1979) into the framework of M/P. Given that the wage distribution implied by the resulting model and the one derived in this paper share similar properties, it appears that learning about match quality is not required to reproduce the main features observed in the data.

A way to compare the two approaches would be to assess whether idiosyncratic uncertainty increases wage dispersion, as predicted by our model, or decreases it, as predicted by Moscarini’s model. To do so, one would have to design an estimation procedure based on wage data which disentangles endogenous from exogenous separations in order to identify the value of the variance parameter. Hence, deciding whether signal extraction or idiosyncratic uncertainty determines the dispersion of wages is ultimately an empirical question. Fortunately, the proposed model is well suited for empirical purposes. Since the equilibrium characteristics have been solved analytically, we can estimate the values of the parameters by likelihood maximization as in the companion paper Prat (2005b).

Apart from its empirical implementation, the proposed model lays out a tractable framework for further theoretical inquiries. Given that the set-up has been kept as basic as possible in order to focus on the role of job separation, several extensions should yield interesting avenues of research. For example, in another companion paper Prat (2005a), technological progress is introduced in order to characterize its impact on unemployment and inequality. We find that uncertainty modifies some of the conclusions previously obtained in deterministic settings.

APPENDIX

Proof of proposition 1: We derive the surplus of the job by direct calculation of its discounted cash flows. We define the stopping time $\tau_1$ as the first time of arrival of the death shock and $T$ as the first time at which the job would have been endogenously destroyed, so that

$$T = \min \{ s > v_i : P^i_s = R_s \}$$

where $v_i$ is the vintage of job $i$. Accordingly, the actual time at which the job is destroyed $\tau = \min \{ \tau_1; T \}$. Combining the three flow equations described in Section 2 and 3, we obtain

$$(r + \delta)S (P^i_t) = P^i_t - b + \left( \frac{\beta}{1 - \beta} \right) \theta c + \frac{E}{dt} [dS (P^i_t)]$$

(21)
Equation (21) can be solved directly, calculating the expected revenues generated by the job.

\[ S(P_t^i) = E_{P_t^i} \left[ \int_t^T e^{-(r+\delta)(s-t)} \left( P_s^i - b - \left( \frac{\beta}{1-\beta} \right) \theta c \right) \, ds \right] \]

where \( E_{P_t^i} [\cdot] \) is the expectation operator conditional on the information that output at time \( t \) equals \( P_t^i \). The previous expression can be rearranged in the following way

\[ S(P_t^i) = E_{P_t^i} \left[ \int_t^\infty e^{-(r+\delta)(s-t)} \left( P_s^i - b - \left( \frac{\beta}{1-\beta} \right) \theta c \right) \, ds \right] - E_{P_t^i} \left[ \int_T^\infty e^{-(r+\delta)(s-t)} \left( P_s^i - b - \left( \frac{\beta}{1-\beta} \right) \theta c \right) \, ds \right] \]

The first term on the right hand side is obviously equal to

\[ \frac{P_t^i}{r+\delta} - \left( \frac{1}{r+\delta} \right) \left( b + \left( \frac{\beta}{1-\beta} \right) c \right) \]

We introduce the time dependent distribution \( F_{P_t^i} \) such that

\[ B_t^i(\omega) \text{ is a } (0, \sigma) \text{ geometric Brownian motion on the probability space } (\Omega, \mathcal{F}, F_{P_t^i}) \]

and

\[ F_{P_t^i} \left\{ \omega \in \Omega : \sigma B_t^i(\omega) = \ln \left( P_t^i e^{-\left( \frac{\sigma^2}{2} \right) (t-u_i)} \right) \right\} = 1 \]

To ease notations we introduce the flow surplus \( D_s^i = P_s^i - b - \left( \frac{\beta}{1-\beta} \right) \theta c \). By definition, \( P_T^i \) is equal to \( R \) at time \( T \). So the second integral can be expressed and solved in the following way

\[ E_{P_t^i} \left[ \int_T^\infty e^{-(r+\delta)(s-t)} D_s^i \, ds \right] = \int_{\{T<\infty\}} e^{-(r+\delta)(T-t)} E_R \left[ \int_t^\infty e^{-(r+\delta)(s-t)} D_s^i \, ds \right] \, dF_{P_t^i} \]

\[ = E_R \left[ \int_t^\infty e^{-(r+\delta)(s-t)} D_s^i \, ds \right] \int_{\{T<\infty\}} e^{-(r+\delta)(T-t)} \, dF_{P_t^i} \]

\[ = \left( \frac{R}{r+\delta} - \left( \frac{1}{r+\delta} \right) \left( b + \left( \frac{\beta}{1-\beta} \right) \theta c \right) \right) \left( \frac{P_t^i}{R} \right)^\alpha \]

where the definition of \( \alpha \) is given in Proposition 1. By definition, \( P_T^i \) is equal to \( R \) at time \( T \), which explains the first equality. The second equality is true because the expectation of \( D_s^i \) is
constant with respect to the probability measure $F_{P_i}$, The last equality follows from standard calculations on hitting times probability (See Harrison (1985), p.42).

**Proof of proposition 2:** The Optimal separation Rule allows to express $R$ as a function of $\theta$ so that the two conditions are equivalent to

$$0 = q(\theta^*)(1 - \beta) \left[ -\left( \frac{1}{r + \delta} - \left( \frac{1}{r + \delta} \right) \left( b + \left( \frac{1}{1 - \beta} \right) \theta^* c \right) \right) - c \right]$$

(22)

Differentiating the right hand side shows that it is monotonously decreasing in $\theta$ as long as $R(\theta) \leq 1$. Therefore, if $R(0) < 1$ and the Free Entry Condition changes of sign within the interval where $R(\theta)$ belongs to $(R(0), 1)$, the system admits a unique solution with reasonable parameters values. Let’s assume that $R(0) < 1$. Observe that when $R(\theta) = 1$, the firm expects zero surplus from the match so that the right hand side of (22) is equal to $-c$. Moreover, when the matching function is of the Cobb-Douglas type, $q(\theta)$ diverges to infinity when $\theta$ goes to zero. Since $\lim_{\theta \to 0} R(\theta) = \left( \frac{\alpha}{\alpha - 1} \right) b$, it follows that (22) also diverges to infinity. Therefore the Free Entry Condition always change of sign. We still have to check that $R(0) < 1$, which is true if $\left( \frac{\alpha}{\alpha - 1} \right) b < 1$. As $\left( \frac{\alpha}{\alpha - 1} \right) < 1$, it is sufficient but not necessary that $b < 1$.

**Proof of proposition 3:** By complementarity and independence between $\tau_1$ and $T$

$$\Pr \{ P_a \in dx \cap M_a \geq R \cap \tau_1 > a \}$$

$$= (\Pr \{ P_a \in dx \} - \Pr \{ P_a \in dx \cap M_a < R \}) * \Pr \{ \tau_1 > a \}$$

where $a$ is the tenure on the job, $P_a$ is the productivity of the job, $M_a = \min \{ P_s, 0 \leq s \leq a \}$ and $\tau_1$ is the time of arrival of the first death shock. The equality follows because death shocks are independent from output fluctuations. To compute these probabilities, one uses the logarithmic values of the variables in order to consider standard Brownian motion. The expression of the second term on the right-hand side is easily obtained from the reflection principle when the Brownian motion has no trend. The general expression is derived in Harrison (1985) through a change of measure and reads

$$\Pr \{ \ln(P_a) = x \} = e^{-\delta a} \left( \frac{e^{-\frac{1}{2} \left( \frac{\ln(x - \ln\mu)}{\sigma \sqrt{2 \pi a \delta} \right)^2}}}{\sigma \sqrt{2 \pi a}} - R^{2\sigma} \frac{e^{-\frac{1}{2} \left( \frac{\ln(x) - 2 \ln(R) - \ln(\mu)}{\sigma \sqrt{2 \pi a \delta} \right)^2}}}{\sigma \sqrt{2 \pi a}} \right)$$

$$= e^{-\delta a} g(\ln(x), a)$$

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where \( \mu = -\frac{\sigma^2}{2} \) is the trend of \( \ln(P_a) \). The surviving probability for a given match follows directly

\[
1 - H(a) = e^{-\delta a} \int_{\ln(R)}^{+\infty} g(\ln(x), a) d\ln(x)
\]

Applying the change of variable formula yields (15).

**Proof of proposition 4:** Total employment \( N \) is equal to

\[
N = m(u, v) \int_0^{+\infty} (1 - H(a)) da
\]

\[
= m(u, v) \int_0^{+\infty} e^{-\delta a} \left( \int_{\ln(R)}^{+\infty} g(\ln(x), a) d\ln(x) \right) da
\]

\[
= m(u, v) \int_0^{+\infty} e^{-\delta a} (1 - F(a)) da
\]

where \( F(a) \) is equal to

\[
1 - F(a) = \Phi\left(\frac{-\ln R + \mu a}{\sigma \sqrt{a}}\right) - R^{\frac{\sigma^2}{2}} \Phi\left(\frac{\ln R + \mu a}{\sigma \sqrt{a}}\right)
\]

Differentiation of \( F(a) \) yields

\[
f(a) = \frac{dF(a)}{da} = -\frac{\ln(R) e^{-\frac{1}{2} \left(\frac{-\ln(R) + \mu a}{\sigma \sqrt{a}}\right)^2}}{a \sigma \sqrt{2 \pi a}}
\]

It is shown in Rubinstein and Reiner (1991) that

\[
- \int_0^t e^{-\delta a} f(a) da = R^{\frac{\mu + d}{\sigma^2}} \Phi\left(\frac{\ln(R) - at}{\sigma \sqrt{t}}\right) + R^{\frac{\mu + d}{\sigma^2}} \Phi\left(\frac{\ln(R) + at}{\sigma \sqrt{t}}\right)
\]

where \( d = \sqrt{\mu^2 + 2\delta \sigma^2} \). Given that \( a \) is always positive it follows that

\[
\lim_{t \to +\infty} \int_0^t e^{-\delta a} f(a) da = -R^{\frac{\mu + d}{\sigma^2}}
\]

Integrating \( N \) by parts yields

\[
\frac{N}{m(u, v)} = \left(-\frac{e^{-\delta a}}{\delta} (1 - F(a))\right)_{+\infty}^{+\infty} + \left(\frac{1}{\delta}\right) \int_0^{+\infty} e^{-\delta a} f(a) da
\]

\[
= \frac{1}{\delta} \left(1 - R^{\frac{\mu + d}{\sigma^2}}\right)
\]

The result follows because \( F(0) = 1 \) and \( F(a) \) converges to 1 as \( a \) goes to infinity. Since \( d \geq \mu \), it is clear that \( \mu + d \geq 0 \). Hence \( R^{\frac{\mu + d}{\sigma^2}} \leq 1 \) so employment is always positive, as desired.
Proof of Proposition 5: The aggregate wage distribution is also obtained through the calculation of the productivity distribution. Consider the mass of jobs with a given output

$$\psi(P_a = x; \ a \in [0, +\infty)) = \int_0^{+\infty} e^{-\delta_a g(\ln(x), a)} da$$

where $x \geq R$. Calculations show that

$$e^{-\delta_a g(\ln(x), a)} = x^{\mu_d - \frac{d}{\sigma^2}} - x^{\mu_d - \frac{d}{\sigma^2}} R^{\frac{2d}{\sigma^2}}$$

It is shown in Leland and Toft (1997) that when $x$ is positive

$$\int_0^A e^{-\frac{1}{2} \left( \frac{\ln(x) + da}{\sigma \sqrt{2\pi}} \right)^2} da = \left( \frac{1}{d} \right) \left( -\Phi \left( \frac{-\ln(x) - dA}{\sigma \sqrt{A}} \right) + x^{-\frac{2d}{\sigma^2}} \Phi \left( \frac{-\ln(x) + dA}{\sigma \sqrt{A}} \right) \right)$$

So

$$\lim_{A \to +\infty} \int_0^A e^{-\frac{1}{2} \left( \frac{\ln(x) + da}{\sigma \sqrt{2\pi}} \right)^2} da = \frac{x^{-\frac{2d}{\sigma^2}}}{d}$$

Expressing the mass in terms of $x$ instead of $\ln(x)$ yields

$$\psi(x) = \begin{cases} x^{\mu_d - \frac{d}{\sigma^2}} \left( \frac{1 - R^{\frac{2d}{\sigma^2}}}{d} \right) & \text{if } x \geq 1 \\ x^{\mu_d - \frac{d}{\sigma^2}} \left( \frac{x^{\frac{2d}{\sigma^2}} - R^{\frac{2d}{\sigma^2}}}{d} \right) & \text{if } x \in [R, 1] \end{cases}$$

One can check that integrating the mass over all $x \geq R$ gives the expression of $N$. Finally, normalizing $\psi(\cdot)$ by $N$ yields (20).

References


