Elliptic Fibonomial numbers

An *elliptic function* is a function defined over the complex numbers that is meromorphic and doubly periodic. Elliptic functions can be obtained as quotients of *modified Jacobi theta functions*. These are defined as

$$\theta(x;p) := \prod_{j\geq 0}\left((1-p^j x)(1-\frac{p^{j+1}}{x})\right), \qquad \theta(x_1,\ldots,x_\ell;p) = \prod_{k=1}^{\ell}\theta(x_k;p),$$

where $x, x_1, \ldots, x_\ell \neq 0$ and $|p| < 1$.

The *elliptic analog of the Fibonacci number* is

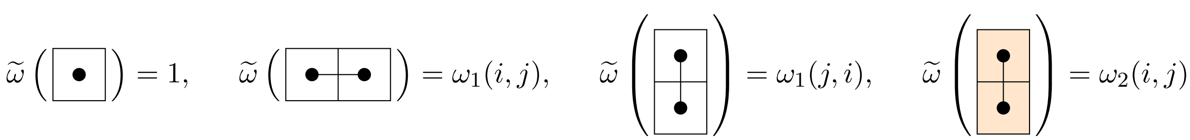
$$[F_n]_{a,b;q,p} := \frac{\theta(q^{F_n}, aq^{F_n}, bq, \frac{a}{b}q; p)}{\theta(q, aq, bq^{F_n}, \frac{a}{b}q^{F_n}; p)}$$

For $m, n \in \mathbb{N}$, the *elliptic analog of the Fibonomial number* is defined as

$$\begin{bmatrix} m+n \\ n \end{bmatrix}_{\mathcal{F}_{a,b;q,p}} := \frac{[F_{m+n}]^!_{a,b;q,p}}{[F_m]^!_{a,b;q,p}\cdot[F_n]^!_{a,b;q,p}}, \tag{1}$$

where $[F_n]^!_{a,b;q,p} := \prod_{k=1}^{n}[F_k]_{a,b;q,p}$ is the elliptic Fibonacci analog of $n!$.

Similarly as before, the elliptic Fibonomial number counts path-domino tilings of an $m \times n$ rectangle according to certain elliptic weights. For $T \in \mathcal{T}_{m,n}$, the *elliptic weights* of the possible tiles in T are defined as follows:



where (i, j) denotes the coordinate of the top-right corner of the tile, the shaded vertical domino represents a special vertical domino touching the path from below, and

$$\omega_1(i,j) := v_{a,b;q^{F_j},p}(F_i, F_{i-1}), \tag{2}$$
$$\omega_2(i,j) := v_{a,b;q,p}(F_{i+1}F_j, F_iF_{j-1}), \tag{3}$$

are defined in terms of the following expression:

$$v_{a,b;q,p}(m,n) := \frac{\theta(aq^{2m+n}, b, bq^n, \frac{a}{b}q^n, \frac{a}{b}; p)}{\theta(aq^n, bq^m, bq^{m+n}, \frac{a}{b}q^m, \frac{a}{b}q^{m+n}; p)}q^m. \tag{4}$$