q-Fibonacci analog of the Coxeter Catalan numbers

Given a crystallographic Coxeter group W with Coxeter exponents $e_1 < e_2 < \ldots < e_n$, the rational W-Catalan number is defined as $C_W(a) = \prod_{i=1}^{n} \frac{a+e_i}{e_i+1}$, and this is an integer when a is relatively prime to $e_n + 1$. The Coxeter exponents for the crystallographic Coxeter groups are:

type of W	$e_1, e_2, \ldots, e_n$
A_n	$1, 2, 3, \ldots, n$
B_n	$1, 3, 5, \ldots, 2n-1$
D_n	$n-1, 1, 3, 5, \ldots, 2n-3$
E_6	$1, 4, 5, 7, 8, 11$
E_7	$1, 5, 7, 9, 11, 13, 17$
E_8	$1, 7, 11, 13, 17, 19, 23, 29$
F_4	$1, 5, 7, 11$
G_2	$1, 5$

The classical Catalan number corresponds to type A_n. We can now define a q-Fibonacci analog as follows:

$$C_{W,\mathcal{F}}(a) = \prod_{i=1}^{n} \frac{[F_{a+e_i}]}{[F_{e_i+1}]}.$$

We have computationally checked that this is a polynomial with positive integer coefficients when a and e_n+1 are relatively prime, for each type and various values of a. It is interesting to note that although in type A_n we have shown that it is a polynomial as long as $(F_a, F_{e_n+1}) = 1$, for other types we must have the stronger condition $(a, e_n + 1) = 1$. For example $C_{F_4,\mathcal{F}}(2)$ is not a polynomial.