Focusing of Light in Axially Symmetric Systems

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Transparent spheres with diameterrs of the order of several wavelengths of light act as focusing lenses. If a single layer of such microspheres is put on a surface they self-organize into a close-packed structure. Due to the focusing effect one can produce millions of similar ordered structures of the size of only dozens of nanometers by a single laser pulse. This can be done by means of ablation of the surface, deposition of material or by means of surface modification. Besides material processing, spherical particles also play an important role in other areas of science, since they have the minimal surface for a given volume and originate in a natural way. Think of water droplets in air (rainbow), other aerosols and of colloids, which are used as model contaminants on sensitive surfaces (semiconductor technology, micro- and nanomechanics etc).

Therefore, the field distribution of the focused light behind such microspheres is an interesting and easily formulated question. The solution, however, cannot be described by standard methods like simple geometrical optics or weak spherical aberration. Only the theory of Mie offers an exact electrodynamical solution of Maxwell's equations. But this solution is not very illustrative, limited to the special case of the sphere and can only be found by the summation of hundreds of terms.

In fact, it is possible to understand the focusing properties not only qualitatively but also quantitatively by means of geometrical optics. Here, light is represented by rays which are refracted by the sphere (figure 1).

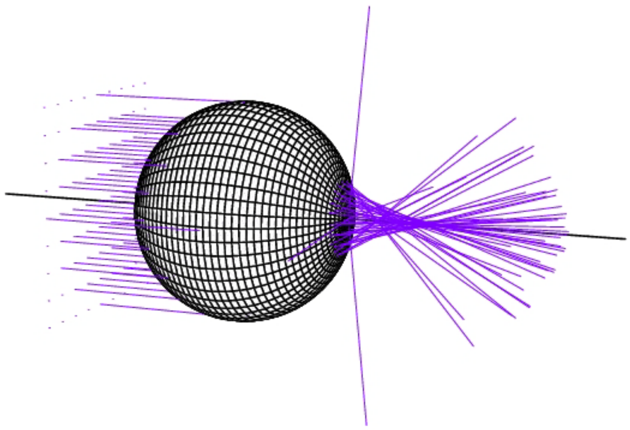

Figure 1. Rays from the left are incident parallel to the axis and refracted by a sphere. Within geometrical optics there exist regions of small and large intensity (low and high ray density) behind the sphere as well as regions with infinite intensity (caustic).

The density of rays in each arbitrary point is a measure for the light intensity there. The rays build a certain axially symmetric „skeleton“ that belongs to the general topology of spherical aberration (rays far from the axis are spherically aberrated, that means they are refracted stronger than rays near the axis). There are regions – denoted as caustic – in which the ray density becomes infinitely large. Here, geometrical optics loses its validity and it is necessary to consider the wave character of light. This is done within the frame of wave optics by means of diffraction integrals summing up the contribution of all point sources.

In the first instance, the corresponding canonical (i.e., simplest) diffraction integral for the axially symmetric geometry of spherical aberration has to be found and studied. It is referred to as Bessoid integral which does not show any infinities (figure 2).

The Bessoid integral can now be used to formulate a general approach describing the field distribution in arbitrary systems with axial symmetry and spherical aberration. The universal Bessoid wave field is „distorted“ and „bent“ in a way that it exactly reflects the field of geometrical optics with the corresponding ray skeleton while removing its infinities. Thus, the solution of arbitrary problems of geometrical optics with axial symmetry and spherical aberration can be matched with wave optics („Bessoid matching“). Mathematically this adaptation takes place through coordinate and amplitude transformations.

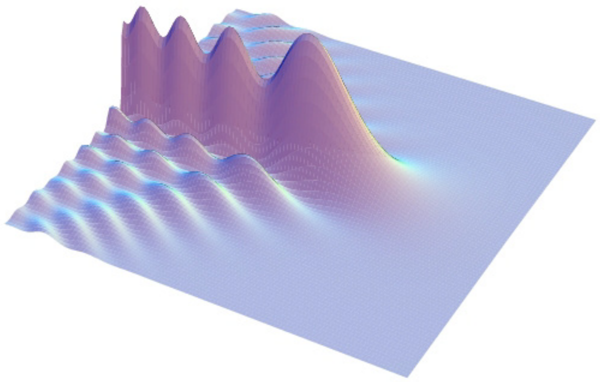

Figure 2. The canonical Bessoid integral. It represents the topology of all diffraction problems with axial symmetry and spherical aberration, especially the high intensity along the axis.

The Bessoid wave field shows a high intensity along the axis, because many rays intersect there due to symmetry. This focus region is narrower than it can be created with conventional lenses, which is of great importance for practical applications.

From the mathematical point of view, the integrand of the Bessoid integral is highly oscillatory and the integral is hard to calculate. Far from the caustic and along the axis there exist simple analytical expressions. In the other regions the Bessoid integral can be efficiently calculated by solving an ordinary differential equation.

The light field produced by a sphere is a specific problem with the corresponding topology, that is axial symmetry and spherical aberration. Hence it suffices to solve the situation within geometrical optics and substitute the result – with its caustic infinities – into the coordinate and amplitude transformations of the original wave approach. As a consequence one finds the field distribution generated by the sphere, expressed by the adapted Bessoid wave field (figure 3).

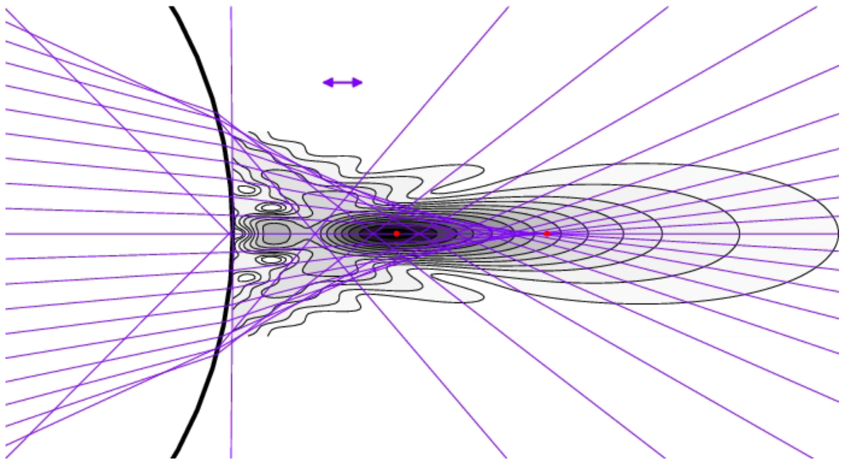

Figure 3. The field distribution behind a microsphere with a radius of three micrometers. The light is incident from the left and has a wavelength of about a quarter of a micrometer (horizontal violet arrow). White corresponds to low, black corresponds to high light intensity. For the sake of comparability the violet ray skeleton of geometrical optics is shown, which has its focus at the tip of the caustic (right red dot). The diffraction focus – the real point of maximum intensity – is significantly shifted towards the sphere (left red dot).

The results obtained by Bessoid matching agree very well with the Mie theory for spheres with radii down to 4 to 5 wavelengths. The focusing of a linearly polarized wave is a vectorial problem with angularly dependent field components. It requires the introduction of higher-order Bessoid integrals, where the transformation formulas practically remain unchanged. Furthermore, it is possible to derive simple analytical expressions for the field along the axis and for the diffraction focus (the point of maximum light intensity). The latter is universal inasmuch as it can be represented by phase differences of the geometrical rays. Moreover the two strong maxima directly behind the sphere are explained.

Besides optical waves the developed approach describes also the focusing of sound, radio and quantum mechanical matter waves.