

Entanglement distribution revealed by macroscopic observations

Johannes Kofler and Časlav Brukner

*Institut für Experimentalphysik, Universität Wien, Boltzmannngasse 5, 1090 Wien, Austria
and Institut für Quantenoptik und Quanteninformation, Österreichische Akademie
der Wissenschaften, Boltzmannngasse 3, 1090 Wien, Austria*

(Received 23 March 2006; revised manuscript received 27 July 2006; published 16 November 2006)

What can we learn about entanglement between individual particles in macroscopic samples by observing only the collective properties of the ensembles? Using only a few experimentally feasible collective properties, we establish an entanglement measure between two samples of spin-1/2 particles (as representatives of two-dimensional quantum systems). This is a tight lower bound for the average entanglement between all pairs of spins in general and is equal to the average entanglement for a certain class of systems. We compute the entanglement measures for explicit examples and show how to generalize the method to more than two samples and multipartite entanglement.

DOI: [10.1103/PhysRevA.74.050304](https://doi.org/10.1103/PhysRevA.74.050304)

PACS number(s): 03.67.Mn, 03.65.Ud, 03.65.Ta

Observation of quantum entanglement between increasingly larger objects is one of the most promising avenues of experimental quantum physics. Eventually, all these developments might lead to a full understanding of the simultaneous coexistence of a macroscopic classical world and an underlying quantum realm. Macroscopic samples typically contain $N \sim 10^{23}$ particles. Because the system's Hilbert space grows exponentially with the number of constituent particles, a complete microscopic picture of entanglement in large systems seems to be in general intractable. The question arises: What can we learn about entanglement between constituent particles of macroscopic samples, if only limited experimentally accessible knowledge about the samples is available?

There is a strong motivation in addressing this question because of recent experimental progress in creating and manipulating entangled states of increasing complexity, such as spin-squeezed states of two atomic ensembles [1]. In such experiments one typically measures only expectation values of *collective operators* of two separated samples. It is known that the two samples of spins can be characterized as either entangled or separable by measuring collective spin operators [2]. Furthermore, such measurements are shown to be sufficient to determine entanglement measures of Gaussian states [3] and of a pair of particles that are extracted from a totally symmetric spin state (invariant under exchange of particles) [4]. It appears that collective operators cannot be used to fully characterize entanglement in composite systems without strong requirements on the symmetry of the state.

Here we present a general and practical method for entanglement detection between two samples of spins. It solely employs *collective spin properties* of the samples and works irrespective of the number of spin particles constituting the samples and with no assumption about the symmetry or mixedness of the state. The method is based on an entanglement measure which is a tight lower bound for average entanglement between all pairs of spins belonging to the two samples. This measure is equal to the average entanglement for a certain class of systems which need not be totally symmetric. We generalize the method to obtain the entanglement measure between M separated spin samples based on collective measurements. The results apply for any entanglement monotone that is a convex measure on the set of density

matrices (e.g., concurrence [5], negativity [6], three-way tangle [7]).

Consider two separated ensembles A and B of spin- $\frac{1}{2}$ particles (Fig. 1). Each of them contains a large number n of spins. Because of the large dimensions ($d=2^n$) of the samples' Hilbert spaces, the structure of entanglement between the two samples is considerably more complex than between two single spins. While there are experimentally viable methods for detecting entanglement, they still require a large number of parameters to be determined (proportional to d^2 [8]). The problem simplifies in situations in which each of the ensembles of n spins can be treated as one large (total) spin of length $n/2$. This means that, within the ensembles, the individual spin- $\frac{1}{2}$ particles form symmetrized states (Dicke states). Though this reduces the dimension of the Hilbert space of A (B) to $d=n+1$, entanglement determination is still demanding for large n both experimentally and theoretically: analytical solutions exist only for pure states in general and for mixed states only for small n [9].

In this paper, we give a method to detect entanglement between large spin samples by measuring only a small number of collective spin properties (sample spin components and their correlations), which is independent of the sample size n . The collective spin operators are

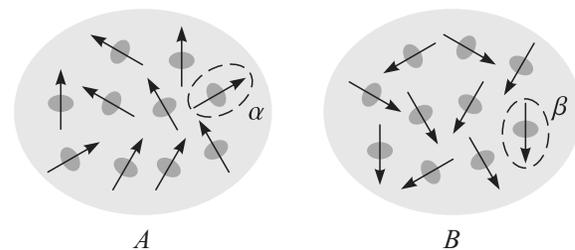


FIG. 1. Two contiguous and nonoverlapping spin subsystems A and B , each of which contains a large number n of spins. What can we learn about entanglement of a pair (α, β) of spins chosen at random where $\alpha \in A$ and $\beta \in B$, if individual spins are experimentally not accessible but only the collective properties of the samples A and B ? What can we learn about entanglement between A and B from such measurements?

$$\hat{S}_i^A \equiv \frac{\hbar}{2} \sum_{\alpha \in A} \hat{\sigma}_i^{(\alpha)}, \quad \hat{S}_i^B \equiv \frac{\hbar}{2} \sum_{\beta \in B} \hat{\sigma}_i^{(\beta)}. \quad (1)$$

The index i denotes the spatial component of the spins: $i \in \{1 \equiv x, 2 \equiv y, 3 \equiv z\}$. The Pauli matrix of the spin at site $\alpha \in A$ is given by $\hat{\sigma}_i^{(\alpha)}$, and analogously for a spin β from subsystem B . Note that the collective operators satisfy the usual commutation relations $[\hat{S}_i^A, \hat{S}_j^A] = i\hbar \varepsilon_{ijk} \hat{S}_k^A$, since $[\hat{\sigma}_i^{(\alpha)}, \hat{\sigma}_j^{(\alpha')}] = 2i\varepsilon_{ijk} \hat{\sigma}_k^{(\alpha)} \delta_{\alpha\alpha'}$.

The spin expectation values and correlations are

$$S_i^A \equiv \langle \hat{S}_i^A \rangle = \frac{\hbar}{2} \sum_{\alpha \in A} g_i(\alpha), \quad (2)$$

$$T_{ij}^{AB} \equiv \langle \hat{S}_i^A \hat{S}_j^B \rangle = \frac{\hbar^2}{4} \sum_{\alpha \in A, \beta \in B} h_{ij}(\alpha, \beta), \quad (3)$$

and analogously for $S_i^B \equiv (\hbar/2) \sum_{\beta \in B} g_i(\beta)$, where $S_i^A, S_i^B \in [-n\hbar/2, n\hbar/2]$, $T_{ij}^{AB} \in [-n^2\hbar^2/4, n^2\hbar^2/4]$. These are only 15 numbers. Here $g_i(\alpha)$, $g_i(\beta)$, and $h_{ij}(\alpha, \beta)$ are the (dimensionless) expectation values and pair correlations of two single spins (α, β) to which it is assumed there is *no experimental access*: $g_i(\alpha) \equiv \langle \hat{\sigma}_i^{(\alpha)} \rangle_{\hat{\rho}_{\alpha\beta}}$, $h_{ij}(\alpha, \beta) \equiv \langle \hat{\sigma}_i^{(\alpha)} \hat{\sigma}_j^{(\beta)} \rangle_{\hat{\rho}_{\alpha\beta}}$. They are obtained from the actual 4×4 density matrix

$$\hat{\rho}_{\alpha\beta} \equiv \frac{1}{4} \left(\mathbf{1}^{(\alpha)} \otimes \mathbf{1}^{(\beta)} + \sum_{k=1}^3 g_k(\alpha) \hat{\sigma}_k^{(\alpha)} \otimes \mathbf{1}^{(\beta)} + \sum_{l=1}^3 \mathbf{1}^{(\alpha)} \otimes g_l(\beta) \hat{\sigma}_l^{(\beta)} + \sum_{k,l=1}^3 h_{kl}(\alpha, \beta) \hat{\sigma}_k^{(\alpha)} \otimes \hat{\sigma}_l^{(\beta)} \right), \quad (4)$$

where $\mathbf{1}^{(\alpha)}$ is the 2×2 identity matrix in the Hilbert space of spin α .

Out of the experimentally accessible quantities (2) and (3) we will construct a 4×4 density matrix of two virtual qubits which describes the collective properties of the two spin sets. Its *a priori* justification is (i) that a general treatment of the problem between two large samples of spins is intractable because of the high dimensionality, and (ii) that we have a fully developed theory of entanglement for two-qubit systems. Therefore, this approach is a natural (and successful) way to say something about the entanglement between two spin systems if only collective observables are measured.

We first introduce the normalized (dimensionless) average subsystem expectation values (magnetization per particle) and correlations:

$$s_i^a \equiv \frac{1}{n} \sum_{\alpha \in A} g_i(\alpha) = \frac{2}{n\hbar} S_i^A, \quad (5)$$

$$t_{ij}^{ab} \equiv \frac{1}{n^2} \sum_{\alpha \in A, \beta \in B} h_{ij}(\alpha, \beta) = \frac{4}{n^2\hbar^2} T_{ij}^{AB}, \quad (6)$$

where $s_i^a, t_{ij}^{ab} \in [-1, 1]$. These are the coefficients of the virtual density matrix:

$$\hat{\rho}_{ab} \equiv \frac{1}{4} \left(\mathbf{1}^a \otimes \mathbf{1}^b + \sum_{k=1}^3 s_k^a \hat{\sigma}_k^a \otimes \mathbf{1}^b + \sum_{l=1}^3 \mathbf{1}^a \otimes s_l^b \hat{\sigma}_l^b + \sum_{k,l=1}^3 t_{kl}^{ab} \hat{\sigma}_k^a \otimes \hat{\sigma}_l^b \right), \quad (7)$$

with a denoting the first and b the second virtual collective qubit, associated with subsystems A and B , respectively. Here, $\mathbf{1}^a, \mathbf{1}^b, \hat{\sigma}_k^a$, and $\hat{\sigma}_l^b$ are 2×2 identity and Pauli matrices for the collective qubits a and b .

The question is whether the density matrix (7) is positive semidefinite, i.e., whether it is a physical state of two qubits. The answer is affirmative and the proof follows from consideration of an equal-weight statistical mixture of one qubit pair which can be in any of the n^2 states $\hat{\rho}_{\alpha\beta}$. The density matrix of this mixture is the mixture of density matrices of all possible pairs (α, β) : $\hat{\rho}_{\text{mix}} = (1/n^2) \sum_{\alpha, \beta} \hat{\rho}_{\alpha\beta}$. It can easily be seen that $\hat{\rho}_{\text{mix}}$ is equal to $\hat{\rho}_{ab}$ as both are uniquely determined by the same expectations and correlations: $\langle \hat{\sigma}_i^{(\alpha)} \rangle_{\hat{\rho}_{\text{mix}}} = \langle \hat{\sigma}_i^a \rangle_{\hat{\rho}_{ab}} = s_i^a = (2/n\hbar) S_i^A$ and $\langle \hat{\sigma}_i^{(\alpha)} \hat{\sigma}_j^{(\beta)} \rangle_{\hat{\rho}_{\text{mix}}} = \langle \hat{\sigma}_i^a \hat{\sigma}_j^b \rangle_{\hat{\rho}_{ab}} = t_{ij}^{ab} = (4/n^2\hbar^2) T_{ij}^{AB}$. Thus, $\hat{\rho}_{ab}$ is a density matrix. Note that without the normalizations as given in (5) and (6) the method would not work.

Encapsulated in the following two propositions, we relate the entanglement properties of the virtual qubits to those of the spin samples.

Proposition 1. For any entanglement measure E that is convex on the set of density matrices the entanglement of the virtual density matrix $E_{ab} \equiv E(\hat{\rho}_{ab})$ is a lower bound for the average entanglement between all pairs $\bar{E}_{\alpha\beta} \equiv (1/n^2) \sum_{\alpha, \beta} E(\hat{\rho}_{\alpha\beta})$. This is an immediate consequence of the average convexity of E : $E_{ab} = E((1/n^2) \sum_{\alpha, \beta} \hat{\rho}_{\alpha\beta}) \leq (1/n^2) \sum_{\alpha, \beta} E(\hat{\rho}_{\alpha\beta})$.

Remarks. First, the result holds for entanglement measures that are convex. In certain cases this is directly implied by the definition of the entanglement measure for mixed states, which involves a convex roof $E(\hat{\rho}) \equiv \min_{p_i, \psi_i} \sum_i p_i E(|\psi_i\rangle\langle\psi_i|)$, where the minimization is taken over those probabilities $\{p_i\}$ and pure states $|\psi_i\rangle$ that realize the density matrix $\hat{\rho} = \sum_i p_i |\psi_i\rangle\langle\psi_i|$ and $E(|\psi_i\rangle\langle\psi_i|)$ is the entanglement measure of the pure state $|\psi_i\rangle$. Second, the proposition implies that if $E_{ab} > 0$ then at least for one pair we must have $E(\hat{\rho}_{\alpha\beta}) > 0$. Thus, a nonzero value of E_{ab} is a sufficient condition for entanglement between the two samples A and B . Third, the maximal pairwise concurrence [5] for symmetric states is found to be $2/n$ and is achieved for the W state [10]. It is conjectured that this remains valid also when the symmetry constraint is removed. This suggests $E_{ab} \leq \bar{E}_{\alpha\beta} \leq 2/n$, if concurrence is used as an entanglement measure. The existence of this upper bound can be seen as a consequence of the monogamy of entanglement.

We refer to E_{ab} as *pairwise collective entanglement* as it is determined solely by the expectation and correlation values of the collective spin observables. The question arises: Under what conditions is E_{ab} equal to the average entanglement $\bar{E}_{\alpha\beta}$? Identifying systems for which the equality holds would

allow feasible experimental determination of the entanglement distribution in large samples by observation of their macroscopic properties only. It can easily be seen that, if the state is symmetric under exchange of particles within each of the samples, one has $E_{ab} = \bar{E}_{\alpha\beta} = E(\hat{\rho}_{\alpha\beta})$ for every pair of particles (α, β) . In what follows, we identify an important class of systems for which $E_{ab} = \bar{E}_{\alpha\beta}$ though the corresponding states need not be symmetric under exchange of particles.

Proposition 2. Consider a system (i) with $g_z(\alpha) = g_z^A$ for all $\alpha \in A$ and $g_z(\beta) = g_z^B$ for all $\beta \in B$ (translational invariance within the subsystems), (ii) with $h_{xx}(\alpha, \beta) = \varepsilon h_{yy}(\alpha, \beta)$ with $\varepsilon = \text{const} = +1$ or -1 (all pairs are in absolute value equally correlated in the x and y directions, where this correlation may be different in size for different pairs), (iii) with constant sign of the z correlations, i.e., $\text{sgn}[h_{zz}(\alpha, \beta)] = -\varepsilon$ for all (α, β) , and (iv) where all the remaining expectation values and correlations (g_x, g_y, h_{ij} with $i \neq j$) are zero. Non vanishing average entanglement $\bar{E}_{\alpha\beta}$ as measured by the negativity [6] is equal to the pairwise collective entanglement E_{ab} , if and only if the correlation functions $h_{xx}(\alpha, \beta)$ and $h_{yy}(\alpha, \beta)$ are each constant for all pairs and all pairs have a nonpositive eigenvalue of their partial transposed density matrix.

The negativity [6] of a density matrix $\hat{\rho}$ is defined as $E(\hat{\rho}) \equiv (\text{Tr}|\hat{\rho}^{\text{PT}}| - 1)/2$, where $\text{Tr}|\hat{\rho}^{\text{PT}}|$ stands for the trace norm of the partially transposed density matrix $\hat{\rho}^{\text{PT}}$. Hence the negativity is equal to the modulus of the sum of the negative eigenvalues of ρ^{PT} .

It is important to stress that Proposition 2 holds also for states that do not need to be totally symmetric, i.e., the $h_{zz}(\alpha, \beta)$ may be different for different pairs of particles. In general, under the above symmetry, the state of the virtual qubit pair is of the form

$$\hat{\rho}_{ab} = \begin{pmatrix} u_{+++} & 0 & 0 & v_- \\ 0 & u_{+--} & v_+ & 0 \\ 0 & v_+ & u_{-+-} & 0 \\ v_- & 0 & 0 & u_{---} \end{pmatrix}, \quad (8)$$

where $u_{\pm\pm\pm} \equiv \frac{1}{4}(1 \pm s_z^A \pm s_z^B \pm t_{zz}^{ab})$, $v_{\pm} \equiv \frac{1}{4}t_{xx}^{ab}(1 \pm \varepsilon)$. The state of an arbitrary pair (α, β) of particles has a similar structure. For example, the state of an arbitrary pair of particles extracted from a spin chain with xxz Heisenberg interaction has such a form. The proof of Proposition 2 is given in the Appendix.

We illustrate the method with some explicit examples.

1. Dicke states. We consider the Dicke state (generalized W state)

$$|N; k\rangle \equiv \binom{N}{k}^{1/2} \hat{P}_S | \underbrace{0 \dots 0}_{N-k} \underbrace{1 \dots 1}_k \rangle \quad (9)$$

with $N \geq 2$ spins, k excitations $|1\rangle$, and $N-k$ nonexcited spins $|0\rangle$, where $0 \leq k \leq N$. \hat{P}_S is the symmetrization operator. Within the system we consider two subsystems A and B each of size n . Because of the total symmetry of the state one has $E_{ab} = \bar{E}_{\alpha\beta} = E(\hat{\rho}_{\alpha\beta})$ for any size n . Only for the cases where just a single spin ($k=0$) or all spins are excited ($k=N$) is

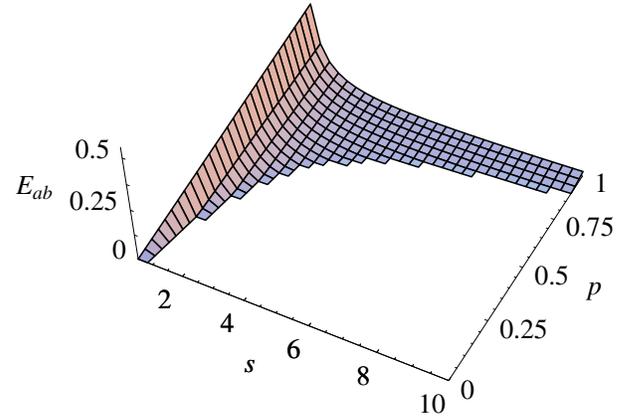


FIG. 2. (Color online) Entanglement E_{ab} between two collective spins in the generalized singlet state with an admixture of nonsymmetric noise (11) as a function of spin length s and proportion p of the singlet state in the mixture. The entanglement is nonzero for sufficiently large p and decreases inversely proportionally to s .

there no entanglement between two arbitrary pairs or arbitrary sized blocks, respectively [11]. The global maximum of the entanglement E_{ab} , measured by the negativity, is reached for $k=N/2$ and its value is $E_{ab}^{\text{max}} = \frac{1}{2}[1/(N-1)]$, for all n . It vanishes only in the limit $N \rightarrow \infty$.

2. Generalized singlet states. The two subsystems A and B , each forming a spin $s=n/2$, are in a generalized singlet state:

$$|\psi\rangle = \frac{1}{\sqrt{2s+1}} \sum_{m=-s}^s (-1)^{s-m} |m\rangle_A | -m\rangle_B, \quad (10)$$

where $|m\rangle = |2s; s+m\rangle$ denotes the eigenstates of the spin operator's z component. The collective two-qubit coefficients are $t_{ii}^{ab} = -(n+2)/3n$ and the $s_i^{a,b}$ and t_{ij}^{ab} with $i \neq j$ are all zero. The collective entanglement (negativity) is $E_{ab} = 1/2n = 1/4s$ which is nonzero for all sizes n of the subsystems and vanishes only in the limit $n \rightarrow \infty$.

3. Generalized singlet state with an admixture of nonsymmetric correlations. Consider the state

$$p|\psi\rangle\langle\psi| + (1-p) \bigotimes_{\alpha=1}^n \frac{1}{2} (|01\rangle_{\alpha, \beta=\alpha} \langle 10| + |10\rangle_{\alpha, \beta=\alpha} \langle 01|) \quad (11)$$

with $p \in [0, 1]$. This is a mixture of the generalized singlet state (10) and n perfectly z -correlated pairs $(\alpha, \beta=\alpha)$. This state is not symmetric under particle exchange in the zz correlations. The expectation values $s_i^{a,b}$ and correlations t_{ij}^{ab} with $i \neq j$ remain zero. The correlations $t_{xx} = t_{yy}$ are reduced by a factor p compared to those of the state (10). The correlations in the z direction, however, are modified and read $t_{zz}^{ab} = -p(n-1)/3n - 1/n$. Therefore, there is a critical number of particles $n_c \equiv [(1+p)/(1-p)]$ beyond which there is no collective pairwise entanglement. Only for $n < n_c$ do we have $E_{ab} = \frac{1}{4}[1+p-n(1-p)]/n > 0$. Note that (11) is in accordance with Proposition 2 and thus $E_{ab} = \bar{E}_{\alpha\beta}$. Figure 2 shows E_{ab} as

a function of the spin length $s=n/2$ and the mixing parameter p . E_{ab} is nonzero in regions where $p > (2s-1)/(2s+1)$ and decreases inversely proportionally to s .

Our method can be generalized straightforwardly to define multipartite entanglement of M collective spins belonging to M separated samples A_1, \dots, A_M , each containing a large number of spins n . Any convex multipartite entanglement measure (e.g., M -way tangle) which is applied to the corresponding collective matrix of the M virtual qubits gives a lower bound for the average multipartite entanglement, obeying the usual constraints for entanglement sharing such as the Coffman-Kundu-Wootters inequality [7].

Importantly, in the examples considered above our entanglement measure scales at most with $1/n$ and vanishes in the limit of infinitely large subsystem sizes n . This is a generic property that follows from the commutation relation for *normalized* spins in this limit. Taking $\hat{s}_i \equiv (1/n)\hat{S}_i = (\hbar/2n)\sum_{\alpha}\hat{\sigma}_i^{(\alpha)}$ one obtains $\lim_{n \rightarrow \infty} [\hat{s}_x, \hat{s}_y] = \lim_{n \rightarrow \infty} i(\hbar/2n)\hat{s}_z = 0$. This is sometimes interpreted as suggesting that averaged collective observables, like the magnetization per particle, represent “macroscopic” or classical-like, properties of samples. Note, however, that for any n there are n^2 pairs between the subsystems so that the number of pairs multiplied by the pairwise collective entanglement can scale with n , showing the existence of entanglement for arbitrarily large n .

In a recent work it was shown that macroscopic properties such as magnetic susceptibility can reveal entanglement *within* macroscopic samples [12]. The present work can be viewed as in a way complementary as it demonstrates that macroscopic properties (collective spin properties and their correlations) can reveal the entanglement distribution *between* two or more macroscopic samples. On the fundamental side, our method demonstrates that there is no reason in principle why purely quantum correlations could not have an effect on the global properties of objects. On the practical side, it enables us to characterize the structure of entanglement in large spin systems by performing only a few feasible measurements of their collective properties, independent of the symmetry and mixedness of the state.

This work was supported by the Austrian Science Foundation, Project SFB No. 1506, the European Commission Project QAP No. 015846, and the British Council in Austria.

APPENDIX: PROOF OF PROPOSITION 2

Depending on the sign ε of the zz correlations, only one eigenvalue of $\hat{\rho}_{\alpha\beta}^{\text{PT}}$ and $\hat{\rho}_{ab}^{\text{PT}}$, respectively, can be negative:

$$\mu_{\alpha\beta} = \frac{1}{4} [1 - \sqrt{(g_z^A + \varepsilon g_z^B)^2 + 4h_{xx}^2(\alpha, \beta) + \varepsilon h_{zz}(\alpha, \beta)}],$$

$$\nu_{ab} = \frac{1}{4} [1 - \sqrt{(s_z^a + \varepsilon s_z^b)^2 + 4(t_{xx}^{ab})^2 + \varepsilon t_{zz}^{ab}}]. \quad (\text{A1})$$

The corresponding negativities are given by $E_{\alpha\beta} = |\min(0, \mu_{\alpha\beta})|$ and $E_{ab} = |\min(0, \nu_{ab})|$. One can express ν_{ab} as given by

$$\nu_{ab} = \bar{\mu} + \Delta, \quad (\text{A2})$$

where $\bar{\mu} \equiv (1/n^2)\sum_{\alpha, \beta} \mu_{\alpha\beta}$ and $\Delta \equiv (1/4n^2)\sum_{\alpha, \beta} [(g_z^A + \varepsilon g_z^B)^2 + 4h_{xx}^2(\alpha, \beta)]^{1/2} - \frac{1}{4}[(s_z^a + \varepsilon s_z^b)^2 + 4(t_{xx}^{ab})^2]^{1/2}$. The quantity Δ is the difference between the entanglement measures $\bar{E}_{\alpha\beta}$ and E_{ab} , i.e., $E_{ab} = \bar{E}_{\alpha\beta} - \Delta$, for the case that $\nu_{ab} \leq 0$ and $\bar{E}_{\alpha\beta} \equiv (1/n^2)\sum_{\alpha, \beta} |\min(0, \mu_{\alpha\beta})| = |\min(0, \bar{\mu})|$. This is true if and only if $\mu_{\alpha\beta} \leq 0$ for all (α, β) , i.e., all pairs are either entangled or have eigenvalue zero. According to proposition 1, Δ is non-negative, i.e.,

$$\sqrt{c^2 + 4(t_{xx}^{ab})^2} \leq \frac{1}{n^2} \sum_{\alpha, \beta} \sqrt{c^2 + 4h_{xx}^2(\alpha, \beta)}. \quad (\text{A3})$$

Here we abbreviated $c \equiv g_z^A + \varepsilon g_z^B = s_z^a + \varepsilon s_z^b$, where the second equality is due to (5). Inequality (A3) becomes an equality, i.e., $\Delta = 0$, if and only if $h_{xx}(\alpha, \beta)$ is the same for all pairs (α, β) such that $t_{xx}^{ab} = h_{xx}$. Therefore, the pairwise collective entanglement E_{ab} equals the average entanglement $\bar{E}_{\alpha\beta}$, if and only if for all individual pairs $\mu_{\alpha\beta} \leq 0$ and $h_{xx}(\alpha, \beta) = \varepsilon, h_{yy}(\alpha, \beta) = \text{const}$ for all pairs. ■

- [1] B. Julsgaard, A. Kozhekin, and E. S. Polzik, *Nature* (London) **413**, 400 (2001).
 [2] A. Sørensen, L.-M. Duan, J. Cirac, and P. Zoller, *Nature* (London) **409**, 63 (2001).
 [3] J. Sherson and K. Mølmer, *Phys. Rev. A* **71**, 033813 (2005).
 [4] X. Wang and K. Mølmer, *Eur. Phys. J. D* **18**, 385 (2002).
 [5] S. Hill and W. K. Wootters, *Phys. Rev. Lett.* **78**, 5022 (1997).
 [6] K. Życzkowski, P. Horodecki, A. Sanpera, and M. Lewenstein, *Phys. Rev. A* **58**, 883 (1998); G. Vidal and R. F. Werner, *ibid.* **65**, 032314 (2002).

- [7] V. Coffman, J. Kundu, and W. K. Wootters, *Phys. Rev. A* **61**, 052306 (2000).
 [8] P. Horodecki and A. Ekert, *Phys. Rev. Lett.* **89**, 127902 (2002).
 [9] J. Schliemann, *Phys. Rev. A* **72**, 012307 (2005).
 [10] M. Koashi, V. Bužek, and N. Imoto, *Phys. Rev. A* **62**, 050302(R) (2000).
 [11] V. Vedral, *New J. Phys.* **6**, 102 (2004).
 [12] M. Wiesniak, V. Vedral, and Č. Brukner, *New J. Phys.* **7**, 258 (2005); Č. Brukner, V. Vedral, and A. Zeilinger, *Phys. Rev. A* **73**, 012110 (2006).