Measuring the absolute photodetection efficiency using photon number correlations

Michael Lindenthal and Johannes Kofler

We present two methods for determining the absolute detection efficiency of photon-counting detectors directly from their singles rates under illumination from a nonclassical light source. One method is based on a continuous variable analog to coincidence counting in discrete photon experiments, but it does not actually rely on high detector time resolutions. The second method is based on difference detection, which is a typical detection scheme in continuous variable quantum optics experiments. Since no coincidence detection is required with either method, they are useful for detection efficiency measurements of photodetectors with detector time resolutions far too low to resolve coincidence events. © 2006 Optical Society of America

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1. Introduction

High detection efficiency is crucial in many contexts within quantum optics. In particular, recent linear optics quantum computation methods have been shown to require detection efficiencies higher than 90% to be scalable.[1–4] Another well known example comes from Bell inequality experiments where a detection efficiency of at least 2/3 is required to close the detection loophole.[5] Thus higher detector efficiencies not only have the obvious advantage of providing more signal, but are also of great relevance for both current quantum information science and fundamental quantum physics. Therefore building new high efficiency detectors is a crucial area of research. In this paper we present methods to evaluate their absolute detection efficiency. The typical way to measure detection efficiency is to use a calibrated reference detector and to compare its response to an incident light beam of constant intensity with the response of the detector under test. It has been shown that quantum mechanics itself provides a way to measure the absolute detection efficiency of light detectors without the need for a reference detector. By using nonclassical photon statistics such as those produced in spontaneous parametric downconversion,[6] both theory and experiment show that the absolute detection efficiency can be determined by the ratio of the coincidence rate to the singles rate.[7–9] However, this method requires the detectors to have a high enough time resolution for coincidence counting. We provide workaround schemes that overcome this limitation. Specifically, we have developed two related detector efficiency calibration methods that use quantum correlated light but do not rely on high time resolution coincidence counting.[10] This might be of particular interest for novel detectors and prototypes that are not yet capable of coincidence counting, such as electron multiplying CCD cameras.

The first method is based on a continuous variable analog of coincidence counting in discrete photon experiments. Recall that coincidence counting is essentially bitwise multiplication (i.e., an AND gate) of single counts within a small coincidence window. In the continuous variable limit, this is achieved by measuring the mean product of the detected photon numbers in two beams generated by spontaneous parametric downconversion. Since downconversion photons are emitted in pairs, the normalized mean product of the photon numbers measured in the two beams during a specific time is maximal in the case of perfect detection efficiencies when all photons are detected. On the other hand, in the limit of small detection efficiencies, the normalized mean product of the detected photon numbers has its minimum. For the general case an expression can be derived that allows calculation of detection efficiencies from the

M. Lindenthal (michael.lindenthal@quantum.at) and J. Kofler are with the Institut für Experimentalphysik, Universität Wien, 1090 Wien, Austria, and also with the Institut für Quantenoptik und Quanteninformation, Österreichische Akademie der Wissenschaften, 1090 Wien, Austria.

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measured mean product of the detected photon numbers in the two downconversion beams.

The second method uses the variance of the detected photon number differences in two beams generated by spontaneous parametric downconversion. Such difference detection is a typical detection scheme in continuous variable quantum optics experiments.[11,12] Since the photons in the two beams are perfectly correlated, the difference in the photon numbers measured in the two beams is zero in the case of perfect detection efficiencies when all photons are detected. Uncorrelated loss in these two beams diminishes those perfect correlations. Therefore in the limit of small detection efficiencies, the normalized variance of the differences in the detected photon numbers has its maximum. For the general case, again an expression can be derived that allows us to calculate the detection efficiencies from the measured variance of the detected photon number differences in the two downconversion beams.

The paper is structured as follows: First we derive these relations mentioned above disregarding background. Then we generalize these results to include two different background levels in the two downconversion beams, since background light is a very significant contribution in single photon counting experiments.

2. Theory

A. Product Detection Method

We begin by deriving a relationship between the mean product of the singles rates and the detection efficiencies in each downconversion beam, η_1 and η_2, in the absence of background. We assume that parametric downconversion emits light beams described by a general distribution $G_N(k)$ of the number of photon pairs k with mean value $\langle k\rangle_G = N$ and the second moment $\langle k^2\rangle_G$.[13] The probability of detecting l out of k photons in each of the two downconversion beams is given by the binomial distribution $B_{k,\eta_i}(l) = (k\ l)\eta_i^{\,l}(1-\eta_i)^{k-l}$, with mean value $\langle l\rangle_B = \eta_i k$ and the second moment $\langle l^2\rangle_B = \eta_i k - \eta_i^{\,2}k + \eta_i^{\,2}k^2$ ($i = 1, 2$). Thus the mean product of the detected photon numbers in the two downconversion beams is given by

$$\langle lm\rangle = \sum_{k=0}^{\infty}\sum_{l=0}^{k}\sum_{m=0}^{k} G_N(k)B_{k,\eta_1}(l)B_{k,\eta_2}(m)lm = \eta_1\eta_2\langle k^2\rangle_G = \frac{\eta_2}{\eta_1}\left(\langle l^2\rangle - \langle l\rangle + \eta_1\langle l\rangle\right), \tag{1}$$

where l and m are the detected photon numbers in each downconversion beam, $\langle l\rangle = \eta_1\langle k\rangle_G$ and $\langle l^2\rangle = \eta_1\langle k\rangle_G - \eta_1^{\,2}\langle k\rangle_G + \eta_1^{\,2}\langle k^2\rangle_G$.

Together with the expression $\langle l\rangle/\langle m\rangle = \eta_1/\eta_2$, the detection efficiency η_1 follows as

$$\eta_1 = \frac{\langle lm\rangle}{\langle m\rangle} - \frac{\langle l^2\rangle}{\langle l\rangle} + 1. \tag{2}$$

The corresponding result for η_2 is

$$\eta_2 = \frac{\langle lm\rangle}{\langle l\rangle} - \frac{\langle m^2\rangle}{\langle m\rangle} + 1. \tag{3}$$

Note that neither formula depends on the coincidence rate. However, the quantum statistics of the light enters the expressions in the mean product of the singles rates.

In a former method[7] the absolute detection efficiency is determined from the ratio of the mean coincidence rate $\langle c\rangle$ to the mean singles rate $\langle l\rangle$ or $\langle m\rangle$, $\eta_1 = \langle c\rangle/\langle m\rangle$ and $\eta_2 = \langle c\rangle/\langle l\rangle$. Together with Eqs. (2) or (3) one obtains for $\eta = \eta_1 = \eta_2$, and hence $\langle s\rangle = \langle l\rangle = \langle m\rangle$ and $\langle s^2\rangle = \langle l^2\rangle = \langle m^2\rangle$,

$$\langle s\rangle - \langle c\rangle = \langle s^2\rangle - \langle lm\rangle = \frac{\langle (l-m)^2\rangle}{2}. \tag{4}$$

This simple expression relates the difference between the mean singles rate and the mean coincidence rate to the variance of the detected photon number differences in the two downconversion beams. This motivates our second approach for determining the detection efficiencies from the variance of the detected photon number differences in the two downconversion beams described in Subsection 2.C.

B. Product Detection Method: General Approach Including Background

We now extend this theory to cover more realistic experimental conditions and correct the measurements for possibly different backgrounds in the two detectors. The averaged quantities contained in Eqs. (2) and (3) have to be extracted from experimentally accessible quantities that include background. We do so by splitting up the measured photon numbers (subscript M) into the photon numbers corresponding to the signal (no subscript) and into photon numbers corresponding to the background (subscript B), where $l = l_M - l_B$ and $m = m_M - m_B$. The background can be estimated experimentally from a separate configuration. We get

$$\langle l\rangle = \langle l_M\rangle - \langle l_B\rangle, \tag{5}$$

$$\langle m\rangle = \langle m_M\rangle - \langle m_B\rangle, \tag{6}$$

$$\langle l^2\rangle = \langle l_M^{\,2}\rangle - \langle l_B^{\,2}\rangle - 2\langle l_M\rangle\langle l_B\rangle + 2\langle l_B\rangle^2, \tag{7}$$

$$\langle m^2\rangle = \langle m_M^{\,2}\rangle - \langle m_B^{\,2}\rangle - 2\langle m_M\rangle\langle m_B\rangle + 2\langle m_B\rangle^2, \tag{8}$$

$$\langle lm\rangle = \langle l_M m_M\rangle - \langle l_M\rangle\langle m_B\rangle - \langle l_B\rangle\langle m_M\rangle + \langle l_B\rangle\langle m_B\rangle. \tag{9}$$

Here we used the statistical independence of l and l_B and m and m_B, respectively. By inserting these expressions into Eqs. (2) and (3), the detection efficiencies can be determined from the data directly measurable in an experiment:

$$\eta_1 = \frac{\langle l_M m_M\rangle - \langle l_M\rangle\langle m_B\rangle - \langle l_B\rangle\langle m_M\rangle + \langle l_B\rangle\langle m_B\rangle}{\langle m_M\rangle - \langle m_B\rangle} - \frac{\langle {l_M}^2\rangle - \langle {l_B}^2\rangle - 2\langle l_M\rangle\langle l_B\rangle + 2\langle l_B\rangle^2}{\langle l_M\rangle - \langle l_B\rangle} + 1, \quad (10)$$

$$\eta_2 = \frac{\langle l_M m_M\rangle - \langle l_M\rangle\langle m_B\rangle - \langle l_B\rangle\langle m_M\rangle + \langle l_B\rangle\langle m_B\rangle}{\langle l_M\rangle - \langle l_B\rangle} - \frac{\langle {m_M}^2\rangle - \langle {m_B}^2\rangle - 2\langle m_M\rangle\langle m_B\rangle + 2\langle m_B\rangle^2}{\langle m_M\rangle - \langle m_B\rangle} + 1. \quad (11)$$

C. Difference Detection Method

Our second approach for determining the absolute detection efficiencies in each downconversion beam, η_1 and η_2, relies on measuring the variance of the differences in the singles rates $\langle(l - m)^2\rangle$. This method is closely related to the approach described in Subsection 2.A, since $\langle(l - m)^2\rangle = \langle l^2\rangle + \langle m^2\rangle - 2\langle lm\rangle$, and both $\langle(l - m)^2\rangle$ and $\langle lm\rangle$ depend on the degree of second-order coherence which is affected by uncorrelated loss. However, the two methods may be useful under different circumstances, especially since difference detection is a typical detection scheme in continuous variable quantum optics experiments.

The variance of the detected photon number differences in the two downconversion beams is given by

$$\begin{aligned}\langle(l - m)^2\rangle &= \sum_{k=0}^{\infty}\sum_{l=0}^{k}\sum_{m=0}^{k} G_N(k) B_{k,\eta_1}(l) B_{k,\eta_2}(m)(l - m)^2 \\ &= \langle l\rangle + \langle m\rangle - (\eta_1\langle l\rangle + \eta_2\langle m\rangle) + (\eta_1 - \eta_2)^2 \\ &\quad\times \frac{\langle l^2\rangle - \langle l\rangle + \eta_1\langle l\rangle}{{\eta_1}^2},\end{aligned} \quad (12)$$

where l and m are the detected photon numbers in each downconversion beam.

Together with the expression $\langle l\rangle/\langle m\rangle = \eta_1/\eta_2$, the detection efficiency η_1 follows as

$$\eta_1 = \frac{3\langle m\rangle - \dfrac{\langle m\rangle^2}{\langle l\rangle} + \langle l^2\rangle\left(1 - \dfrac{\langle m\rangle}{\langle l\rangle}\right)^2 - \langle(l - m)^2\rangle}{2\langle m\rangle}. \quad (13)$$

Correspondingly,

$$\eta_2 = \frac{3\langle l\rangle - \dfrac{\langle l\rangle^2}{\langle m\rangle} + \langle m^2\rangle\left(1 - \dfrac{\langle l\rangle}{\langle m\rangle}\right)^2 - \langle(l - m)^2\rangle}{2\langle l\rangle}. \quad (14)$$

For $\eta = \eta_1 = \eta_2$, and hence $\langle s\rangle = \langle l\rangle = \langle m\rangle$, Eq. (12) becomes

$$\langle(l - m)^2\rangle = 2(1 - \eta)\langle s\rangle. \quad (15)$$

For perfect detection efficiencies ($\eta = 1$), $\langle(l - m)^2\rangle = 0$. Since the downconverted photons are always created in pairs, the difference in the detected photon number, and hence the variance of that quantity, is exactly zero if all of the pairs are detected. In the limit of very small detection efficiencies ($\eta \ll 1$), $\langle(l - m)^2\rangle \approx 2\langle s\rangle$, which corresponds to the variance of two independent Poissonian light beams of equal intensities. The non-Poissonian contributions in $G_N(k)$ cancel out, showing that quantum statistics strongly depends on the detection efficiency of the detectors.

Expression (15) can be rewritten as

$$\eta = 1 - \frac{\langle(l - m)^2\rangle}{2\langle s\rangle}. \quad (16)$$

A similar expression was stated without explicit derivation previously.[11,12]

With $\langle s\rangle = \eta N$, the normalized expression (15) reads

$$\frac{\langle(l - m)^2\rangle}{\langle s\rangle^2} = \frac{2}{N}\left(\frac{1}{\eta} - 1\right),$$

which verifies that the normalized variance of the detected photon number differences in the two downconversion beams diverges for $\eta \to 0$ and goes to zero for $\eta \to 1$. Using Eq. (4), an analogous treatment can be performed for the normalized mean product of the detected photon numbers in the two beams.

D. Difference Detection Method: General Approach Including Background

As in Subsection 2.B, we extend this theory to cover the more realistic experimental conditions including background. The quantities in Eq. (13) and (14) have to be extracted from quantities that are directly accessible to measurement. We get Eqs. (5)–(8) and

$$\begin{aligned}\langle(l - m)^2\rangle &= \langle(l_M - m_M)^2\rangle + 2(\langle l_B\rangle - \langle m_B\rangle)^2 \\ &\quad - 2(\langle l_M\rangle - \langle m_M\rangle)(\langle l_B\rangle - \langle m_B\rangle) \\ &\quad + 2\langle l_B\rangle\langle m_B\rangle - \langle {l_B}^2\rangle - \langle {m_B}^2\rangle\end{aligned} \quad (17)$$

for the background-corrected difference term.

By inserting Eqs. (5)–(8) and (17) into Eqs. (13) and

(14), the detection efficiencies can be determined from the data directly measurable in an experiment,

$$\eta_1 = \frac{3(\langle m_M\rangle - \langle m_B\rangle) - \dfrac{(\langle m_M\rangle - \langle m_B\rangle)^2}{\langle l_M\rangle - \langle l_B\rangle}}{2(\langle m_M\rangle - \langle m_B\rangle)} + \frac{\langle l_M{}^2\rangle - \langle l_B{}^2\rangle - 2\langle l_M\rangle\langle l_B\rangle + 2\langle l_B\rangle^2}{2(\langle m_M\rangle - \langle m_B\rangle)} \times\left(1 - \frac{\langle m_M\rangle - \langle m_B\rangle}{\langle l_M\rangle - \langle l_B\rangle}\right)^2 - \frac{\langle (l_M - m_M)^2\rangle + 2(\langle l_B\rangle - \langle m_B\rangle)^2}{2(\langle m_M\rangle - \langle m_B\rangle)} - \frac{2(\langle l_M\rangle - \langle m_M\rangle)(\langle l_B\rangle - \langle m_B\rangle)}{2(\langle m_M\rangle - \langle m_B\rangle)} + \frac{2\langle l_B\rangle\langle m_B\rangle - \langle l_B{}^2\rangle - \langle m_B{}^2\rangle}{2(\langle m_M\rangle - \langle m_B\rangle)}, \qquad (18)$$

$$\eta_2 = \frac{3(\langle l_M\rangle - \langle l_B\rangle) - \dfrac{(\langle l_M\rangle - \langle l_B\rangle)^2}{\langle m_M\rangle - \langle m_B\rangle}}{2(\langle l_M\rangle - \langle l_B\rangle)} + \frac{\langle m_M{}^2\rangle - \langle m_B{}^2\rangle - 2\langle m_M\rangle\langle m_B\rangle + 2\langle m_B\rangle^2}{2(\langle l_M\rangle - \langle l_B\rangle)} \times\left(1 - \frac{\langle l_M\rangle - \langle l_B\rangle}{\langle m_M\rangle - \langle m_B\rangle}\right)^2 - \frac{\langle (l_M - m_M)^2\rangle + 2(\langle l_B\rangle - \langle m_B\rangle)^2}{2(\langle l_M\rangle - \langle l_B\rangle)} - \frac{2(\langle l_M\rangle - \langle m_M\rangle)(\langle l_B\rangle - \langle m_B\rangle)}{2(\langle l_M\rangle - \langle l_B\rangle)} + \frac{2\langle l_B\rangle\langle m_B\rangle - \langle l_B{}^2\rangle - \langle m_B{}^2\rangle}{2(\langle l_M\rangle - \langle l_B\rangle)}. \qquad (19)$$

E. Error Estimates

Finally, we want to derive the statistical errors of the detection efficiencies (without background), i.e., (A) for the product detection, Eq. (2), (B) for the difference detection, Eq. (13), and (C) for the coincidence method,[7] described above Eq. (4):

$$\eta_1{}^{(A)} = \frac{\langle lm\rangle}{\langle m\rangle} - \frac{\langle l^2\rangle}{\langle l\rangle} + 1, \qquad (20)$$

$$\eta_1{}^{(B)} = \frac{3\langle m\rangle - \dfrac{\langle m\rangle^2}{\langle l\rangle} + \langle l^2\rangle\left(1 - \dfrac{\langle m\rangle}{\langle l\rangle}\right)^2 - \langle (l-m)^2\rangle}{2\langle m\rangle}, \qquad (21)$$

$$\eta_1{}^{(C)} = \frac{\langle c\rangle}{\langle m\rangle}. \qquad (22)$$

In each case η_1 is a function of several mean values, i.e., $\eta_1 = \eta_1(\langle u\rangle, \langle v\rangle, \ldots)$ where $u, v, \ldots \in \{l, m, l^2, lm, (l-m)^2, c\}$. The sample variance of η_1 is defined as

$$\sigma^2(\eta_1) \equiv \sigma_{\langle u\rangle}{}^2\left(\frac{\partial\eta_1}{\partial\langle u\rangle}\right)^2 + \sigma_{\langle v\rangle}{}^2\left(\frac{\partial\eta_1}{\partial\langle v\rangle}\right)^2 + 2\sigma_{\langle u\rangle\langle v\rangle}\frac{\partial\eta_1}{\partial\langle u\rangle}\frac{\partial\eta_1}{\partial\langle v\rangle} + \cdots, \qquad (23)$$

where

$$\sigma_{\langle u\rangle}{}^2 \equiv \frac{\langle u^2\rangle - \langle u\rangle^2}{M}, \qquad (24)$$

$$\sigma_{\langle u\rangle\langle v\rangle} \equiv \frac{\langle uv\rangle - \langle u\rangle\langle v\rangle}{M} \qquad (25)$$

are the variances and covariances of the sample means with sample size M. It has to be stressed that in experiments the time interval chosen for accumulating the individual measurements needs to be much larger than the resolving time of the detector under test. The mean values are given by

$$\langle x\rangle = \sum_{k=0}^{\infty}\sum_{l=0}^{k}\sum_{m=0}^{k} G_N(k)B_{k,\eta_1}(l)B_{k,\eta_2}(m)x. \qquad (26)$$

Due to the perfect correlations, the first and second moment of the coincidences can be computed by applying the binomial distribution to one downconversion arm twice:

$$\langle c^p\rangle = \sum_{k=0}^{\infty}\sum_{l=0}^{k}\sum_{m=0}^{l} G_N(k)B_{k,\eta_1}(l)B_{k,\eta_2}(m)m^p, \qquad (27)$$

where $p = 1, 2$.

For a Poissonian downconversion distribution $G_N(k) = N^k \exp(-N)/k!$, with N the expected mean photon number in one sample measurement, the efficiency sample variances in the three cases are

$$\sigma^2(\eta_1{}^{(A)}) = \frac{\eta_1(1 + N - \eta_1) + N\eta_2[2 + \eta_1(\eta_1 - 4)]}{MN\eta_2}, \qquad (28)$$

$$\sigma^2(\eta_1{}^{(B)}) = \frac{1}{2MN\eta_1{}^2\eta_2}\{2\eta_1{}^4(N\eta_2 - 1) + 2\eta_1{}^3 \times [1 + N(1 + 2\eta_2(\eta_2 - 3))] + N\eta_1{}^2\eta_2 \times [5 - 2\eta_2(\eta_2 - 2)] - 4N\eta_1\eta_2{}^2 + N\eta_2{}^3\}, \qquad (29)$$

$$\sigma^2(\eta_1{}^{(C)}) = \frac{\eta_1(1 + \eta_1 - 2\eta_1\eta_2)}{MN\eta_2}. \qquad (30)$$

In the limit $N \gg 1$ the variance for the coincidence method vanishes: $\sigma^2(\eta_1{}^{(C)}) \to 0$. For the product and

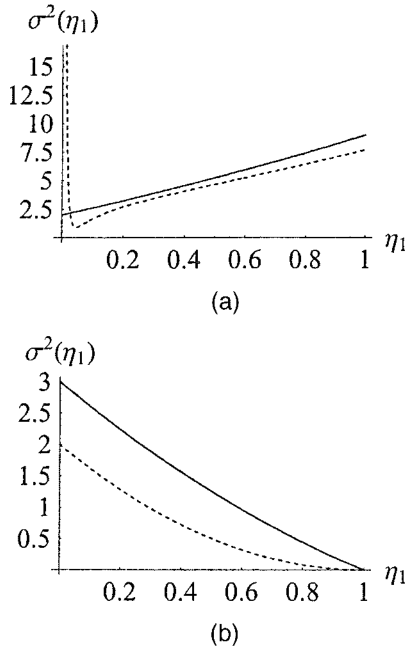

Fig. 1. Sample variances $\sigma^2(\eta_1{}^{(A)})$ (product method, solid curve) and $\sigma^2(\eta_1{}^{(B)})$ (difference method, dashed curve) as a function of η_1 for a Poissonian distribution in the limit $N \gg 1$. For the sake of generality the sample size is $M = 1$, where all variances scale with $1/M$. (a) The second efficiency is constant: $\eta_2 = 0.1$. The variance $\sigma^2(\eta_1{}^{(B)})$ diverges for $\eta_1 \to 0$. (b) Equal efficiencies: $\eta_2 = \eta_1$.

difference detection the variances approach constant values that depend on the efficiencies η_1 and η_2 as well as the sample size M. Figure 1 shows these two variances as a function of η_1 for $M = 1$, where all sample variances scale inversely with the sample size. In general, the difference method is more accurate than the product method, except in the case of fixed η_2 and vanishing η_1. In the special case of equal efficiencies $\eta_1 = \eta_2$, the expression for the difference method simplifies tremendously:

$$\sigma^2(\eta_1{}^{(B)})|_{\eta_2=\eta_1} = \frac{2(1-\eta_1)^2}{M}. \tag{31}$$

For a thermal distribution $G_N(k) \propto \exp(-k/N)$ the first two variances are more cumbersome, and we do not write them here. In the limit of increasing N the variance for the coincidence method $\sigma^2(\eta_1{}^{(C)})$ vanishes again, while the variances for the product and difference detection method linearly diverge in the limit $N \gg 1$, still also scaling with $1/M$. Only in the special case of equal efficiencies $\eta_1 = \eta_2$ does the variance for the difference method $\sigma^2(\eta_1{}^{(B)})$ become independent of N. Therefore, if equal detectors are used, the difference method is more favorable. In this special case and for $N \gg 1$ we have the simple expression

$$\sigma^2(\eta_1{}^{(B)})|_{\eta_2=\eta_1} = \frac{4(1-\eta_1)^2}{M}. \tag{32}$$

Hence, in this case, the variance for the thermal distribution has the same form as expression (31) for the Poissonian distribution.

3. Conclusions and Outlook

We have presented two methods for determining the absolute detection efficiency of photodetectors. Since they are applicable to detectors with low time resolution, they overcome the limitations typical for absolute detection efficiency measurements. The first is based on measuring the mean product of the detected singles rates in two beams generated by spontaneous parametric downconversion. The second method uses the variance measurements of the differences in the detected singles rates in the two downconversion beams. The two methods correspond to the different detection methods typically used in either the discrete photon or continuous variable communities, respectively. Both procedures could be used for measuring the absolute detection efficiency of photo detectors that do not provide the appropriate time resolution for coincidence counting.

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