Pension Systems and their Influence on Fertility and Growth

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Jannuary 2009

Abstract

In this article we study the implications of different pension systems on fertility and economic growth. We show that the introduction of a public pension system to a developing economy with informally financed pension benefits reduces fertility and stimulates economic growth. Additionally, we highlight that in a framework of fully crowded out gifts a fully funded public pension system results in higher economic growth than a pay-as-you-go public pension system. This is the case because the growth enhancing effect of higher fully funded savings is dominating the growth decreasing effect of higher fully funded fertility.

JEL classification: H55, J13, O41

Key words: Public and informal pension systems, endogenous growth, fertility

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1 Introduction

Developing economies are usually facing a whole bundle of obstacles on their way to development. Overpopulation, political instability and insecure property rights are only some of the problems. Our work tries to contribute to the topic by analyzing the role of pension systems on fertility and economic growth. In particular, we are interested in the question whether the introduction of a public pension system in a developing economy has the potential to promote economic growth.

Throughout the literature a variety of articles analyzes population and economic growth implications connected to pension systems (Zhang (1995), Wigger (1999), Boldrin, De Nardi, Jones (2005)). Different to these positive works our main interest lies in the comparison of outcomes connected to informal and public pension systems. Informal pension systems are chosen to capture the situation of developing countries. Empirical findings by Holzmann (2005) support this modeling choice. He observes that only 1/5th of the working population in Sub-Saharan Africa is covered by a public pension system and that the biggest part of the contributions is used for the inefficient and bureaucratic organizational structure of the systems. In this context the paper by Zhang and Zhang (1995) is closest to our work. They show that a pay-as-you-go public pension system economy supports higher economic growth than an economy with a fertility dependent pension system because in the former fertility is lower and savings are higher. Different to their approach we use the insight, that the existence of a public pension system fully crowds out the old-age security motive.[1] Therefore we exclude the voluntary, non mandatory intergenerational transfers from adult children to their parents if a working public pension system is present. This modification results in a reduction of economic growth connected to the pay-as-you-go pension system because the negative effect of pay-as-you-go contributions on capital accumulation can not be offset by lower gifts.

The intuition behind the result of fully crowded out gifts in the light of public pension systems can be described by Caldwells theory of intergenerational wealth flows (Caldwell (1982)) which defines two different types of societies. The

[1] See Hansson and Stuart (1989). For a technical proof of the argument consult Holler (2009).

"traditional/rural" society is characterized by low retirement income. Children are therefore expected to directly contribute to the parental retirement budget. Translating this into our framework means that private intrafamily gifts take place, or in other words developing societies value the old-age security motive of fertility. The second so called "modern" society covers the situation of developed countries which are in our work represented by a working public pension system. In these economies children are not expected to contribute to the retirement income of the parents because private savings and public pension payments are high enough to offer a sufficient level of retirement consumption. This implies that the public pension system takes over the role of the private intrafamily transfers and reduces procreation benefits.

Within the pension system and endogenous fertility literature two motives of having children are prominent. The first motive captures the fact that individuals are expected to procreate because they expect their children to contribute to their retirement budget. Due to its insurance character, this motive is known as the "old-age security motive" of fertility (Leibenstein (1957)). The second so called "consumption good motive" of fertility states that parents simply enjoy the fact to have a successor and see children as a durable consumption good (Dasgupta (1993) and Zhang (1995)). Under this motive, children provide utility by their mere existence. Our work picks up the idea of a mixed fertility motive first introduced by Wigger (1999) where the demand for children is simultaneously determined by the insurance and consumption good motive. Due to fully crowded out gifts in the public pension system case only developing economies value the old-age security motive. The consumption good motive is incorporated by all pension systems because procreation is assumed to be a basic need of human beings. As already described, publicly financed pension benefits are independent of individual fertility. Models of the old-age security motive kind that examine public pension systems therefore have to include positive intrafamily transfers in the form of gifts from adult children to their parents (Bental (1989), Zhang and Nishimura (1993)) to treat fertility endogenously. Through the use of the mixed procreation motive our paper is able to cover the case of fully crowded out gifts while still keeping fertility endogenously.

Following a zero gift version of a model by Wigger (1999) we formulate the different pension system economies inside a Diamond type overlapping generations model where the engine of growth is captured by labor productivity. In the first part we focus towards the developing economy situation where pension systems are informally financed. After the derivation of the corresponding fertility and growth rates, we examine pay-as-you-go and fully funded public pension systems. The following comparison of the different outcomes enables us to show the impacts of a public pension system introduction to a developing economy. In the last part, we calibrate our model using observed total fertility and production growth data for the OECD and Sub-Saharan Africa. The derived results are used to produce new insights in the observed fertility and growth differences for the United States and Europe.

2 Model

We consider a Diamond type OLG model economy populated by finitely living agents belonging to three generations. Each individual lives for three periods: childhood, adulthood and retirement. During childhood individuals consume θW_t, where θ is the fraction of adult income needed to rear one child.[2] During the sole productive period, adulthood, individuals born at time t earn working income W_t. Moreover in this period households decide about fertility n_t, adult consumption c_t, and future retirement consumption c_{t+1}. The population dynamics for the productive adult population are described by $N_{t+1} = N_t n_t$. Retired people only consume and have no influence on household optimization. We further expect them to consume their whole savings plus pension benefits during their third period of life. Bequests are therefore excluded from the model.

Following Zhang (1995) and Doepke, De La Croix (2003) individuals preferences include a descending "altruistic" part capturing the consumption good motive of fertility.[3] This approach can be seen as a modification of the Barro and Becker (1988) dynastic utility function. In contrast to their idea that adults incorporate the whole utility of their offsprings in the utility function we assume that parents value only the number of children. In other words we exclude the dynastic component of the Barro-Becker descending altruism.

Next to the consumption good motive of fertility we additionally model the old-age security motive of fertility by incorporating ascending altruism. Inspired by Caldwells intergenerational flow theory ascending altruism is only present for

[2] The assumption of income dependent child raising cost is following an approach by Zhang and Zhang (1995). An alternative to our simple child raising cost formulation would be to introduce nonlinearities. We abstract from this more realistic "complication" since the qualitative results would be unchanged.

[3] While the word altruism implies that actions are made despite any own utility considerations (this is not the case in our model) we stick to the word to pay tribute to earlier work we are building on.

countries without a mandatory public pension system. Following Morand (1996) the ascending altruism of individual's preferences is captured through gifts from the own adult children to their retired parents. The ascending altruistic part of preferences is therefore captured in the composition of pension payments π_{t+1}.

We assume that individuals utility is represented by the following logarithmic additive separable function:

$$V_t = \log(c_t) + \beta \log(c_{t+1}) + \gamma \log(n_t) \quad \text{where } \beta, \gamma < 1 \tag{1}$$

Utility is dependent on adult consumption c_t, retirement consumption c_{t+1}, discount factor β, descending altruism factor γ and the number of children n_t.

The household budget constraint is represented by adult consumption c_t and retirement consumption c_{t+1}. Adult consumption is dependent on wage, child rearing cost θn_t, pension contributions $W_t \tau$ and savings s_t. Pension contributions are income dependent with a tax rate τ. We additionally assume perfect foresight implying that individuals exactly know the future gross interest rate R_{t+1}. Old-age consumption is financed through the yield of savings and pension benefits. Notice that the pension system is assumed to be always budget balanced.

$$c_t = W_t(1 - \theta n_t - \tau) - s_t \tag{2}$$

$$c_{t+1} = s_t R_{t+1} + \pi_{t+1} \tag{3}$$

The economy is populated by one representative firm that uses the production factors capital K_t and effective labour $A_t L_t$ to produce a single homogeneous good at time t. A_t determines labour productivity at time t which is assumed to be driven through a Romer type positive spillover. In equilibrium labor demand L_t equals labor supply which is equivalent to adult population N_t. The aggregate production function is determined by

$$Y_t = F(K_t, A_t N_t) \tag{4}$$

Following Grossman and Yanagawa (1993) the technological spillover is dependent on the fraction of capital per worker and the parameter m which is a positive technology parameter controlling for the influence of capital intensity on labor productivity. The lower m the higher is the productivity of labour.

$$A_t = \frac{K_t}{mN_t} \tag{5}$$

Now define capital per effective unit of labour with k_t. It follows that k_t is constant and capital per unit of labour $\widehat{k}_t$ grows at the rate of A_t.

$$k_t = \frac{K_t}{A_t N_t} = m \tag{6}$$

$$\widehat{k}_t = A_t m \tag{7}$$

Profit maximization of the firm implies that production factors are paid by their marginal products.

$$F'_L(K_t, A_t N_t) = W_t = \left[f(k_t) - f'(k_t)k_t\right] A_t \tag{8}$$

$$F'_K(K_t, A_t N_t) = f'(k_t) = R_t \tag{9}$$

Since firm profits are distributed to capital owners, cleared capital markets imply that the return on savings is equal to the marginal product of capital. Equation (8) and (9) imply that labor- and capital market are cleared. Due to Walras' law this further implies a cleared goods market.

Now use capital and labor market clearing conditions and the fact that capital per efficient unit of labor is constant over time (see equation (6)) to state that gross interest rate R_t and wage per efficient unit $w_t = \frac{W_t}{A_t}$ are constant.

$$R_t = f'(m) = R \tag{10}$$

$$w_t = \left[f(m) - f'(m)m\right] = w \tag{11}$$

From the labor market clearing condition we can furthermore see that wage is growing with the level of labor productivity A_t. This enables us to describe economic growth by the growth rate of technological spill over g.

$$\frac{\widehat{k}_{t+1}}{\widehat{k}_t} = \frac{W_{t+1}}{W_t} = \frac{A_{t+1}}{A_t} \equiv g \tag{12}$$

2.1 Informal pension arrangement

This section focuses towards the situation of a developing country. In reality developing and developed countries differ in a lot of economic factors. We are only focusing on the variations of retirement income composition. Developing countries are mainly represented by a not existent or unreliable public pension system that only covers a small part of the population resulting in a level of retirement budget that is below a minimum.[4] This implies that savings alone are not enough to finance a sufficient level of retirement consumption. For this reason we assume children to finance the pensions of their parents through private contributions resulting in ascending altruism. Caldwell motivated developing societies high fertility levels exactly by these informal intrafamily retirement age income contributions. Additional support for the absence of working public pension systems in developing countries can be found in the empirical literature. Due to the World Bank 70% of the old throughout the world rely exclusively on informal pension arrangements. International pension coverage data mentioned in the World Bank report additionally shows that low public pension system coverage rates correlate with a high percentage of inhabitants being supported by their own family while for high coverage rates the opposite is true.[5] Further evidence from developing country surveys (Arnold et. al. (1975), Kagitvibasi (1982)) also indicates that old-age security certainly is a fertility motive in developing countries.[6]

Although private intrafamily transfers are usually freely chosen by the family members we abstract from heterogeneity in contributions assuming that developing country contributions are socially mandatory.[7] This means that individuals are forced to contribute by the threat of punishment which can take the form of exclusion from social village life. The contribution rate τ is therefore not a decision variable but socially determined and constant over time.

Retirement consumption before gifts which is assumed to be below a sufficient level leads to positive ascending altruistic transfers in the form of children contributing to their parents retirement budget. These gifts can be seen as an informal intrafamily pension system implying that gifts are equivalent to

[4] WB. "Averting the Old Age Security Crises". p.63 and p.192.

[5] Nigeria and Kenya show the highest percentage (95% and 88%) of population over 60 covered by family transfers, while public pension coverage rates for these countries are almost zero. WB."Averting the Old Age Security Crises". p. 57, Table 2.1.

[6] 40% of respondents to the surveys indicated that old age help is a very important motive for having children.

[7] In the absence of legal rules, social norms can evolve self-enforcing (Cigno (1993)).

pension contributions τW_{t+1}.

A balanced budget pension system demands that benefits equal contributions at every point in time. This is implying that the number of own children n_t times the part of children's adult income offered as a gift to their parents τW_{t+1} has to equal the pension benefits π_{t+1}.

$$\pi_{t+1} = \tau n_t W_{t+1} \tag{13}$$

(1), (2), (3) and (13) describe the optimization problem of a representative developing country household. Optimal fertility and savings decision are covered by the first order conditions:

$$c_t = \frac{c_{t+1}}{\beta R} \tag{14}$$

$$\frac{\theta W_t}{c_t} = \frac{\gamma}{n_t} + \frac{\beta \tau W_{t+1}}{c_{t+1}} \tag{15}$$

Adults have to decide whether to spend their money in the first or second period. The optimal decision of splitting overall consumption between the two periods is represented by equation (14), which states that marginal utility of adult consumption has to equal marginal utility of retirement consumption. An increase in interest rates or a higher discount factor imply that consumption today will be skipped for consumption tomorrow.

Equation (15) deals with cost and benefit of having children. It states that at an optimum the marginal cost of child rearing must equal the present value of marginal benefit gained through the birth of a child. Marginal benefit of having a child (the right hand side of equation (15)) consists of two parts reflecting the mixed procreation motive. The first part ($\frac{\gamma}{n_t}$) represents the consumption good value while the second part ($\frac{\beta \tau W_{t+1}}{c_{t+1}}$), measuring the present value of marginal benefit of child investments arising in period $t+1$, captures the security value.

Solving the two equations for fertility and savings leads to optimal household decisions (16) and (17).

$$s_t = \frac{(1-\tau)W_t((\beta+\gamma)\tau W_{t+1} - \beta\theta R W_t)}{(1+\beta+\gamma)(\tau W_{t+1} - \theta R W_t)} \tag{16}$$

$$n_t = \frac{\gamma R(1-\tau)W_t}{(1+\beta+\gamma)(\theta R W_t - \tau W_{t+1})} \tag{17}$$

Our assumption of homogeneous agents implies that aggregate savings can not be negative. We use this fact to obtain the result that the opportunity cost of having a child have to be equal or larger than the marginal benefit of having a child.

$$\theta R W_t \geqslant \tau(1+\frac{\gamma}{\beta})W_{t+1}$$

Through the use of $W_{t+1} = gW_t$ we can rewrite the equation in constant terms:

$$\beta\theta R \geqslant \beta\tau g + \gamma\tau g \tag{18}$$

Equation (18) shows that the benefits of the two different types of intertemporal transfers of income from period t to period $t+1$ (savings and fertility) are weighted against each other and secures that fertility rates are well behaved (positive and finite)[8]. $\beta\tau g$ represents the insurance value, $\gamma\tau g$ the consumption good value of children. Households optimally choose zero or negative savings if child investments pay equal or more than saving investments. If the left hand side is smaller than the right hand side agents are willing to borrow money in order to get more kids. In this case retirement consumption is solely financed by the yield on fertility investments and optimal fertility becomes infinite. If the equality sign in (18) holds households are indifferent between the two investment opportunities and savings could become 0. To rule out the case of zero savings we abstract from the equality sign in (18). This further

$$\beta\theta R > \beta\tau g + \gamma\tau g$$

This condition further implies that the pension contribution tax rate τ is limited by:

$$\tau^{\max} = \frac{\theta R}{(1+\frac{\gamma}{\beta})g} \tag{19}$$

Capital market equilibrium demands that future capital is equal to actual aggregate savings. In our model only old people, who do not leave any bequests and totally use up their savings are holding capital. Capital market equilibrium is therefore described by:

$$K_{t+1} = s_t N_t$$

Production-, household optimization and capital market equilibrium define a competitive equilibrium with intergenerational transfers. From equation (12) we already know that per capita output growth is solely defined by labor pro-

[8] Fertility is positive and finite if $\theta R > \tau g$. This parameter assumption is clearly weaker and already included in (18).

ductivity growth g. Equation (5) and the capital market equilibrium imply that savings are growth enhancing and fertility is growth diminishing reproducing a well known feature of endogenous growth models (Grossmann, Yanagawa (1993)).

$$g = \frac{A_{t+1}}{A_t} = \frac{\frac{K_{t+1}}{mN_{t+1}}}{\frac{K_t}{mN_t}} = \frac{K_{t+1}N_t}{K_tN_{t+1}} = \frac{s_t}{n_tA_tm}$$

(5), (12) and the optimal savings and fertility decisions lead to:

$$g = \frac{\beta\theta Rw}{\beta\tau w + \gamma\tau w + m\gamma R} \tag{20}$$

Constant per capita production growth also implies constant fertility.

$$n = \frac{\gamma R(1-\tau)}{(1+\beta+\gamma)(\theta R - \tau g)}$$

Use the already obtained result for g to rewrite fertility:

$$n = \frac{(1-\tau)(w(\beta+\gamma)\tau + m\gamma R)}{(1+\beta+\gamma)\theta(Rm + \tau w)} \tag{21}$$

Because fertility and consumption grow at a constant rate the equilibrium is describing the situation of a balanced growth path.

To capture the impact of intrafamily pension system contributions on the equilibrium values of growth we examine the partial derivatives. From equation (20) we follow that pension contributions financed through gifts from children to the old are growth diminishing.

$$\frac{\partial g}{\partial \tau} < 0 \tag{22}$$

Proposition 1 *Informally financed pension system contributions in the form of gifts from adult children to their parents lead to decreasing per capita production growth.*

To understand the underlying dynamics the growth determining variables savings and fertility have to be examined. Use optimal savings and equation (20) to get:

$$s_t = \frac{m\beta(1-\tau)RW_t}{(1+\beta+\gamma)(w\tau + mR)}$$

Now take the partial derivative of savings with respect to τ to see that positive pension contributions $\tau > 0$ are crowding out savings.

$$\frac{\partial s_t}{\partial \tau} = -\frac{m\beta R(w + mR)W_t}{(1 + \beta + \gamma)(w\tau + mR)^2}$$

$$\frac{\partial s}{\partial \tau} < 0$$

Proposition 2 *Informal, gift based pension contributions are crowding out savings.*

The intuition behind this result is simply the fact that intergenerational transfers always crowd out savings. In contrast to savings the effect on the second growth determining variable fertility is ambiguous.

$$\frac{\partial n}{\partial \tau} = \frac{m(\beta - 2\beta\tau - 2\gamma\tau)Rw - m^2\gamma R^2 - (\beta + \gamma)w^2\tau^2}{(1 + \beta + \gamma)\theta(Rm + \tau w)^2} \tag{23}$$

The partial derivative of fertility with respect to τ shows, that informally financed contributions can lead to higher or lower lifetime income. If lifetime income goes down the demand for children goes down and vice versa. The income effect is dependent on the adult and retirement budget effects. While positive contributions decrease the adult budget, the effect on the retirement budget is twofold. For the case of lump-sum pension contributions the retirement budget effect would be clearly positive. In our framework of wage dependent contributions where growth is determining future adult income the effect is not that clear. Increasing pension contributions similar to the lump-sum case increase the base of payments but also decrease their internal interest rate captured by the rate of economic growth (see equation (22)). The crucial parameter deciding about the strength of the underlying income effects is the pension contribution rate τ.

Proposition 3 *Depending on whether τ is smaller (bigger) than $\widetilde{\tau}^{inf}$ informal pension system contributions increase (decrease) fertility. At the contribution level $\widetilde{\tau}^{inf}$ a fertility maximum is reached.*

Proof. From (23) we follow that $\frac{\partial n}{\partial \tau} > 0$ if $m\beta Rw < (2\beta\tau + 2\gamma\tau)Rw + m^2\gamma R^2 + (\beta + \gamma)w^2\tau^2$. Rearranging gives us $\tau < \frac{-mRw(\beta+\gamma)+\sqrt{mRw^2(mR+w)\beta(\beta+\gamma)}}{w^2(\beta+\gamma)} = \widetilde{\tau}^{inf}$. Contribution levels above this threshold ($\tau > \widetilde{\tau}^{inf}$) decrease fertility ($\frac{\partial n}{\partial \tau} < 0$). ■

Different pension contribution levels can increase or decrease lifetime income because the negative effect on retirement budget can be positive or negative. From equation (22) we know that the internal pension interest is decreasing in the contribution rate. The fact that the decrease takes place at an increasing rate[9] highlights that higher contribution rates lead to even stronger marginal interest decreases. This is the reason why for relatively low contribution levels adult budget decreases can be compensated by retirement budget increases because the negative effect on growth is very small. As τ is increasing the negative effect on the internal interest rate becomes stronger while the positive effect on the base of payments stays constant decreasing the positive effect on retirement budget. From a certain contribution threshold $\widetilde{\tau}^{inf}$ onward, adult budget decreases can not be compensated. Lifetime income decreases and fertility goes down.

Independent of contribution rates, $\tau > 0$ always leads to lower economic growth (22). Therefore the case where $\tau > \widetilde{\tau}^{inf}$ (informal pension contributions decrease fertility) implies that the growth decreasing effect of lower savings is dominant. If $\tau < \widetilde{\tau}^{inf}$, fertility and savings effects are both growth decreasing (fertility increases and savings decrease).

2.2 Pay-as-you-go public pension system

In this subsection we focus on fertility and growth implications caused by a pay-as-you-go pension scheme. In reality children support their parents for a variety of reasons. Ascending transfers can be motivated by altruism, taking place only because parents are in need. Alternatively one could assume that the transfers are part of an intergenerational exchange incorporating a connection between transfers and bequests. Since we exclude bequest from the analysis our model only captures the altruistic transfer motive. In this framework the introduction of a public pension system, guaranteeing a minimum retirement wage, fully crowds out transfers (gifts) from adult children to their parents. this is the case because donors (adult children) no longer see the need to provide transfers. In contrast to developing economies with informal pension systems, pension benefits are not dependent on own fertility decisions n_t but on average fertility of the whole economy $\bar{n}_t$. Furthermore pensions are also independent of individual future adult income of children. In a public pension system the average future income $\bar{W}_{t+1}$ of children instead of W_{t+1} enters the pension benefit formula. A

[9] $\frac{\partial^2 g}{\partial \tau^2} > 0$

balanced budget pay-as-you-go public pension system demands:

$$\pi_{t+1} = \tau \bar{n}_t \bar{W}_{t+1}$$

While production and capital market stay the same the described change in the pension system funding changes the household optimization problem. The crowding out of private intergenerational transfers through a public pension system has a big influence on the value of a child. Pension benefits are now independent of own fertility decisions and agents do not incorporate the old-age security motive of fertility in their fertility decisions. This change is represented by the new retirement budget constraint:

$$c_{t+1} = s_t R + \tau \bar{n}_t \bar{W}_{t+1}$$

Notice that τ is now a policy decision variable instead of a socially determined rate.

Household optimization leads to the following first order conditions:

$$\frac{1}{c_t} = \frac{\beta R}{c_{t+1}}$$

$$\frac{\theta W_t}{c_t} = \frac{\gamma}{n_t}$$

While the first equation handling the optimal split between present and future consumption is the same as in the informal pension contribution scenario, the second equation dealing with cost and benefit of having children changes. This fact is due to the change in marginal benefit of having children which now only reflects the consumption good motive. The insurance motive of fertility becomes obsolete.

Because our model economy assumes homogeneous agents one can set $\bar{n}_t$ and $\bar{W}_{t+1}$ equal to n_t and W_{t+1} after the optimization. Solving the two equations for fertility and savings gives us the optimal household decisions for fertility and savings:

$$s_t = \frac{(1-\tau)W_t(\gamma\tau W_{t+1} - \beta\theta R W_t)}{\tau\gamma W_{t+1} - (1+\beta+\gamma)\theta R W_t}$$

$$n_t = \frac{\gamma R(1-\tau)W_t}{(1+\beta+\gamma)\theta R W_t - \gamma\tau W_{t+1}}$$

Similar to the informal pension system model we assume positive aggregate savings. This implies that the marginal opportunity cost of having a child $\beta\theta R$ have to be higher than marginal benefit of procreation $\gamma\tau g$.

$$\beta\theta R > \gamma\tau g \tag{24}$$

The condition securing positive and finite fertility[10] is like in the previous model weaker and included in the condition for positive savings. Pension contribution tax rate τ for the pay-as-you-go pension system is again limited by $\tau^{\max}$. Contribution rates above this maximum harm the non-negativity assumption of savings because investments in savings would pay less than investments in fertility.

$$\tau^{\max} = \frac{\beta\theta R}{\gamma g}$$

The relationship between savings, fertility and labor productivity ($g = \frac{s_t}{n_t A_t m}$) resulting from capital market equilibrium and input prices together with optimal savings and fertility decisions define constant per capita production growth.

$$g = \frac{\beta\theta R w}{\gamma(mR + \tau w)} \tag{25}$$

Use the result for g to show that also optimal fertility is constant.

$$n = \frac{\gamma(1-\tau)(Rm + \tau w)}{\theta((1+\beta+\gamma)mR + (1+\gamma)\tau w)} \tag{26}$$

After the equilibrium description which represents the situation of a balanced growth bath we focus towards the influence of pay-as-you-go pension contributions on per capita growth. Use the first derivative of equation (25) with respect to τ to show that contributions act growth diminishing.

$$\frac{\partial g}{\partial \tau} < 0$$

Proposition 4 *A pay-as-you-go pension system decreases economic growth.*

The reason for this negative impact lies again in the behavior of fertility and savings. Use $W_t g = W_{t+1}$ and equation (25) to reformulate optimal savings.

[10] $(1+\beta+\gamma)\theta R > \gamma\tau g$

$$s_t = \frac{m\beta RW_t(1-\tau)}{mR(1+\beta+\gamma)+\tau w(1+\gamma)}$$

Now derive s_t with respect to τ to see that a pay-as-you-go pension system reduces savings.

$$\frac{\partial s}{\partial \tau} = -\frac{m\beta RW_t(w(1+\gamma)+m(1+\beta+\gamma)R)}{(mR(1+\beta+\gamma)+\tau w(1+\gamma))^2}$$

$$\frac{\partial s}{\partial \tau} < 0$$

Proposition 5 *A pay-as-you-go pension system intergenerationally redistributes resources from young to old. This crowds out private savings and reduces capital accumulation.*

Our assumption that a public pension system fully crowds out private intrafamily gifts is the reason why the savings reducing effect of pay-as-you-go contributions can not be offset by reduced gifts like in Yoon, Talmain (2001).

To see the effect of a pay-as-you-go pension system on fertility, we built the derivative of (26) with respect to τ.

$$\frac{\partial n}{\partial \tau} = -\frac{\gamma(m^2R^2(1+\beta+\gamma)+mRw(2\tau(1+\gamma)+\beta(2\tau-1))+\tau^2w^2(1+\gamma))}{\theta(mR(1+\beta+\gamma)+\tau w(1+\gamma))^2} \tag{27}$$

Similar to the informal pension system, pay-as-you-go pension contributions imply ambiguous fertility effects. Pay-as-you-go pension contributions decrease the adult budget while the retirement budget can increase or decrease. The retirement budget effect is again dependent on a trade-off between the base of pension payments and the internal interest of the pension scheme. Higher contributions increase the base of pension payments but decrease the internal interest because future income goes down. Pension contribution rate τ again decides about whether the fertility increasing effect of higher pension payments base or the fertility decreasing effects of lower pension interest and lower adult budget are dominant.

Proposition 6 *Depending on whether τ is smaller (bigger) than $\widetilde{\tau}^{pay}$, a pay-as-you-go pension system increases (decreases) fertility. At the contribution level $\widetilde{\tau}^{pay}$ a fertility maximum is reached.*

Proof. From (27) we follow that $\frac{\partial n}{\partial \tau} > 0$ only if $-\gamma(m^2(1+\beta+\gamma)R^2 + m(2(1+\gamma)\tau + \beta(2\tau - 1))Rw + (1+\gamma)\tau^2 w^2) > 0$. Now solve this expression for τ and rearrange it to get $\tau < \frac{-mRw(1+\beta+\gamma)+\sqrt{mRw^2\beta(w(1+\gamma)+mR(1+\beta+\gamma))}}{w^2(1+\gamma)} = \widetilde{\tau}^{pay}$. Contribution levels above this threshold ($\tau > \widetilde{\tau}^{pay}$) decrease fertility ($\frac{\partial n}{\partial \tau} < 0$). ■

Notice that if $\tau > \widetilde{\tau}^{pay}$ implying that $\frac{\partial n}{\partial \tau} < 0$ the savings reducing effect of a pay-as-you-go pension system is stronger than the fertility decreasing effect because the overall growth effect is always negative ($\frac{\partial g}{\partial \tau} < 0$).

2.3 Fully funded public pension system

Following the already stressed argument that intrafamily gifts are not considered in the household optimization if a public pension system is present, we assume fully crowded out private intergenerational transfers. Compared to the previous subchapter only the retirement budget constraint and the capital market equilibrium change. Pension benefits are now financed through own contributions during adulthood which are invested in the capital market, paying the gross interest rate R_{t+1}. Because no transfers from children to their parents are taking place fertility completely exits the retirement budget constraint. The balanced budget pension system constraint changes to:

$$\pi_{t+1} = \tau \bar{W}_t R_{t+1}$$

This clearly also changes the capital market equilibrium because the additional investments have to be considered. Notice that we again assume perfect foresight. Capital market equilibrium is represented through:

$$K_{t+1} = N_t(s_t + \tau W_t)$$

The first order conditions again control the equalization between marginal benefit over time and between the two different investment opportunities.

$$\frac{1}{c_t} = \frac{\beta R}{c_{t+1}}$$

$$\frac{\theta W_t}{c_t} = \frac{\gamma}{n_t}$$

Solving the equations for s_t and n_t gives us the following optimal household decisions :

$$s_t = \frac{\beta W_t}{(1+\beta+\gamma)} - \tau W_t$$

$$n = \frac{\gamma}{(1+\beta+\gamma)\theta}$$

Aggregate savings are positive as long as $\beta > \tau(1+\beta+\gamma)$. Therefore the maximum pension contribution tax is determined by:

$$\tau^{\max} = \frac{\beta}{(1+\beta+\gamma)}$$

Input prices and the new capital market equilibrium condition define economic growth g.

$$g = \frac{\widehat{k}_{t+1}}{\widehat{k}_t} = \frac{K_{t+1}N_t}{K_t N_{t+1}} = \frac{s_t + \tau W_t}{n_t A_t m}$$

Use optimal fertility and savings decision to show that g is constant. This implies together with constant fertility that the equilibrium defines a balanced growth path equilibrium.

$$g = \frac{\beta\theta w}{\gamma m}$$

Growth and fertility are independent of τ, implying that a fully funded pension system has no influence on equilibrium per capita growth. The only effect of the funded pension system is the reduction of savings which is equivalent to the amount of pension contributions. Pension contributions, invested in the capital market exactly work like savings offsetting the impact of fully funded pension contributions on capital accumulation. Consumers anticipate additional future payments and therefore reduce savings exactly by the same amount reproducing the Ricardian equivalence theorem which states that economic growth is neutral towards fully funded pension contributions.

$$\frac{\partial n_t}{\partial \tau} = 0; \frac{\partial g}{\partial \tau} = 0$$

Proposition 7 *A fully funded pension system has no impact on economic growth and fertility.*

3 Public pension system implementation

This section highlights the per capita growth and fertility impacts of a public pension system introduction to a developing economy. In the first step an informally organized pension system is compared to a pay-as-you-go public pension system. Variables with indices *inf* and *pay* respectively indicate the informal and pay-as-you-go case. For a direct comparison of the results one has to assume that the part of income used for private intergenerational gifts τ of the informal system is equal to the pension contribution tax rate τ of the pay-as-you-go system. This implies an equal level of adult pension contributions for both pension systems. All other variables are assumed to be independent of the pension system. To analyze growth implications, one has to start by examining the effects on fertility and savings. Fertility of the two pension systems is represented by:

$$\begin{aligned} n^{inf} &= \frac{(1-\tau)(w(\beta+\gamma)\tau + m\gamma R)}{\theta(1+\beta+\gamma)(Rm+\tau w)} \\ n^{pay} &= \frac{\gamma(1-\tau)(Rm+\tau w)}{\theta((1+\beta+\gamma)mR+(1+\gamma)\tau w)} \end{aligned}$$

Proposition 8 *An introduction of a pay-as-you-go pension system to an economy with informal pension system leads to lower population growth.*

This is the case since the fertility increasing old-age security motive is completely crowded out by the public pension system.

Proof. Rewrite optimal informal fertility to get:

$$n^{inf} = \underbrace{\frac{1+\frac{\beta\tau w}{\gamma(Rm+\tau w)}}{1+\frac{\beta\tau w}{(1+\beta+\gamma)mR+(1+\gamma)\tau w}}}_{>1} \cdot \underbrace{\frac{\gamma(1-\tau)(Rm+\tau w)}{\theta((1+\beta+\gamma)mR+(1+\gamma)\tau w)}}_{n_t^{pay}}$$

Since the first term is bigger than 1 informal fertility is higher than pay-as-you-go fertility ($n^{inf} > n^{pay}$). ■

In an economy without a public pension system own children are financing the pensions of their parents. Individual fertility decision has therefore a direct influence on retirement consumption which is considered in the optimization process. A pay-as-you-go public pension system finances pensions through the average number of children. Therefore instead of own the average number of fertility enters the retirement budget constraint neglecting the security motive of fertility in the household optimization. In other words, economies with a pay-as-you-go public pension system are represented by households which do not expect own fertility decisions to have an influence on their pension benefits. Households living in an economy with informal pension system clearly do so because their pension benefits are paid directly by their own children. This leads to the feature of our model that marginal benefits of procreation are decreasing if a public pension system is introduced because the security value of fertility cancels out.

Now compare savings s^{inf} and s_t^{pay} to see that the introduction of a pay-as-you-go pension system increases savings.

$$\begin{aligned} s^{inf} &= \frac{m\beta RW_t(1-\tau)}{(1+\beta+\gamma)(w\tau+mR)} \\ s_t^{pay} &= \frac{m\beta RW_t(1-\tau)}{mR(1+\beta+\gamma)+\tau w(1+\gamma)} \end{aligned}$$

Proposition 9 *An introduction of a pay-as-you-go pension system to an economy with informal pension system acts savings increasing ($s^{inf} < s_t^{pay}$).*

The positive change in savings is due to the fact that the public pension system reduces the crowding out effect of intergenerational transfers on savings. This is the case because the decreasing effect of the public system on the value of a child transfers income from procreation to savings.

After the examination of savings and fertility we are in the position to analyze impacts on economic growth. The informal and pay-as-you-go cases are represented by the following growth rates:

$$\begin{aligned} g^{inf} &= \frac{\beta\theta Rw}{\beta\tau w+\gamma(mR+\tau w)} \\ g^{pay} &= \frac{\beta\theta Rw}{\gamma(mR+\tau w)} \end{aligned}$$

Proposition 10 *The introduction of a pay-as-you-go pension system to an economy with informal pension system increases economic growth since fertility decreases and savings increase.*

The absence of private altruistic transfers from children to the old leads to positive impacts on both growth determining effects. Savings go up and fertility goes down. Our outcomes are closely connected to the results derived by Zhang and Zhang (1995) who show that a pay-as-you-go public pension system increases per capita output growth and reduces fertility compared to a fertility related security system. In contrast to Zhang and Zhang we do not only model ascending transfers from adult to old but also descending transfers from adult to the young by including the consumption good motive of fertility. This enables us to study the pay-as-you-go public pension system with endogenous fertility in the framework of fully crowded out private transfers. Because Zhang and Zhang are only modeling the security value of children they can not cover this case because the marginal benefit of procreation would become zero. Their model can be seen as covering the transition period between an informal system and a public pension system where private gifts are still positive nevertheless a public social security system is already present. Our model is focusing on the final period when the transition is already finished. The different periods could be reasoned by different levels of trust in the public pension system. People do not fully trust the public system during the adjustment period and therefore still support their parents with private gifts. The final period is characterized by zero gifts because the households have already adjusted their behavior. Our assumption of fully crowded out private gifts does not change the direction of the growth effect but changes its level. A pay-as-you-go pension system introduction leads in our model to lower future capital and lower per capita growth compared to the Zhang and Zhang approach. This is the case because the Zhang and Zhang model compensates the negative effect of pension contributions on capital accumulation through a decrease of gifts. Our assumption of fully crowded out gifts omits this reaction.

After clarifying the growth and fertility impacts caused by a pay-as-you-go public pension system introduction we focus towards the impacts of the introduction of a fully funded (superscript *ff*) public pension system which is the most prominent alternative to a pay-as-you-go public pension system in reality. Use the results from the pervious chapters to describe informal and fully funded fertility:

$$\begin{aligned} n^{inf} &= \frac{(1-\tau)(w(\beta+\gamma)\tau + m\gamma R)}{\theta(1+\beta+\gamma)(Rm+\tau w)} \\ n^{ff} &= \frac{\gamma}{(1+\beta+\gamma)\theta} \end{aligned}$$

Proposition 11 *The level of the pension system contribution tax τ decides about whether the introduction of a fully funded system to an economy without working pension scheme leads to lower or higher population growth. While for positive $\tau < \frac{\beta}{\beta+\gamma} - \frac{Rm\gamma}{w(\beta+\gamma)}$ informal fertility is higher than fully funded fertility, $\tau > \frac{\beta}{\beta+\gamma} - \frac{Rm\gamma}{w(\beta+\gamma)}$ or $\frac{\beta}{\beta+\gamma} < \frac{Rm\gamma}{w(\beta+\gamma)}$ result in lower informal fertility than fully funded fertility. Informal and fully funded fertility are identical if $\tau =$*

$\frac{\beta}{\beta+\gamma} - \frac{Rm\gamma}{w(\beta+\gamma)}$.

Proof. Rewrite informal fertility to get:

$$n^{inf} = \underbrace{\frac{\gamma}{\theta(1+\beta+\gamma)}}_{n^{ff}} \cdot \frac{(1-\tau)(w\beta\tau\frac{1}{\gamma} + w\tau + mR)}{(Rm+\tau w)}$$

For positive pension contributions three cases are observable:

- Case 1: if $\tau < \frac{\beta}{\beta+\gamma} - \frac{Rm\gamma}{w(\beta+\gamma)}$, the second term of n^{inf} is bigger than 1 and $n^{inf} > n^{ff}$.
- Case 2: if $\tau = \frac{\beta}{\beta+\gamma} - \frac{Rm\gamma}{w(\beta+\gamma)}$, the second term of n^{inf} cancels out and $n^{inf} = n^{ff}$.
- Case 3: if $\tau > \frac{\beta}{\beta+\gamma} - \frac{Rm\gamma}{w(\beta+\gamma)}$ or $\frac{\beta}{\beta+\gamma} < \frac{Rm\gamma}{w(\beta+\gamma)}$, the second term of n^{inf} is smaller than 1 and $n^{inf} < n^{ff}$.

■

The different cases are showing that the amount of income contributed to the pension system decides whether fertility is higher or lower. This is the case because informal fertility can decrease or increase depending on whether the decreasing effect on informal growth and available adult income or the increasing effect on pension payments' base is stronger. The contrary effects are exactly offset if $\tau = \frac{\beta}{\beta+\gamma} - \frac{Rm\gamma}{w(\beta+\gamma)}$. In this case informal and fully funded fertility are equal and independent of contribution payments. If $\tau < \frac{\beta}{\beta+\gamma} - \frac{Rm\gamma}{w(\beta+\gamma)}$ the effect of lower informal growth and lower available adult income is weaker than the effect due to increasing pension payments base and informal fertility is higher than fully funded fertility. $\tau > \frac{\beta}{\beta+\gamma} - \frac{Rm\gamma}{w(\beta+\gamma)}$ implies exactly the opposite leading to lower informal fertility than fully funded fertility.

After the description of the fertility effect we focus towards the capital accumulation effect to fully understand the overall growth effect. A fully funded pension system invests the whole part of income reserved for retirement consumption in the capital market and therefore reaches the same capital stock than without a pension system. Capital holdings are clearly higher than in the informal case leading to a growth enhancing effect since in the Grossmann Yanagawa endogenous growth model growth is driven by labor productivity determined by capital intensity.

$$
\begin{aligned}
g^{inf} &= \frac{\beta\theta R w}{\beta\tau w + \gamma\tau w + m\gamma R} \\
g^{ff} &= \frac{\beta\theta w}{\gamma m}
\end{aligned}
$$

$$g^{inf} < g^{ff}$$

Proposition 12 *The introduction of a fully funded public pension system to an economy with informal pension system increases economic growth.*

This implies that even for case 2 where fully funded fertility is higher, the growth increasing effect of higher capital accumulation is dominant.

Now we are in the position to state that countries aiming to increase per capita production growth should introduce a public pension system no matter whether the system is funded or unfunded. If the main goal is to decrease population growth only the pay-as-you-go pension system is useful for all contribution levels. To draw light on the question whether it is preferable to introduce a funded or unfunded system we now focus on the comparison of the two public pension systems.

Pay-as-you-go fertility and fully funded fertility are represented through:

$$
\begin{aligned}
n^{pay} &= \frac{\gamma(1-\tau)(Rm + \tau w)}{\theta((1+\beta+\gamma)mR + (1+\gamma)\tau w)} \\
n^{ff} &= \frac{\gamma}{(1+\beta+\gamma)\theta}
\end{aligned}
$$

Proposition 13 *The tax rate level τ decides about whether pay-as-you-go fertility is higher or lower than fully funded fertility. If $0 < \tau < \frac{\beta}{(1+\beta+\gamma)} - \frac{Rm}{w}$ fertility is higher in the pay-as-you-go system ($n^{pay} > n^{ff}$). If $\tau > \frac{\beta}{(1+\beta+\gamma)} - \frac{Rm}{w}$ or $\frac{\beta}{(1+\beta+\gamma)} < \frac{Rm}{w}$ fertility is lower in the pay-as-you-go system ($n^{pay} < n^{ff}$). For the case where $\tau = \frac{\beta}{(1+\beta+\gamma)} - \frac{Rm}{w}$ both systems lead to identical fertility decisions.*

Proof. Reformulate pay-as-you-go fertility to get:

$$n^{pay} = \underbrace{\frac{\gamma}{(1+\beta+\gamma)\theta}}_{n^{ff}} \cdot \frac{(1-\tau)(Rm + \tau w)}{mR + (1+\gamma)\tau w \frac{1}{(1+\beta+\gamma)}}$$

Now check if the second term on the right side is smaller, bigger or equal to 1. Therefore analyze if the nominator $(1-\tau)(Rm+\tau w)$ is bigger or smaller than the denominator $(mR+(1+\gamma)\tau w\frac{1}{(1+\beta+\gamma)})$.

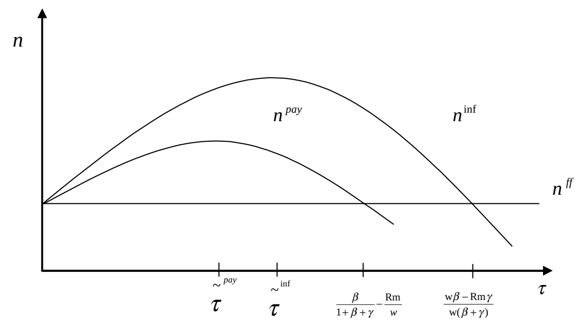
- Case 1: if $\tau < \frac{\beta}{(1+\beta+\gamma)} - \frac{Rm}{w}$ the second term is bigger than 1 and $n^{pay} > n^{ff}$.
- Case 2: if $\tau = \frac{\beta}{(1+\beta+\gamma)} - \frac{Rm}{w}$ the second term is equal to 1 implying that $n^{pay} = n^{ff}$.
- Case 3: if $\tau > \frac{\beta}{(1+\beta+\gamma)} - \frac{Rm}{w}$ or $\frac{\beta}{(1+\beta+\gamma)} < \frac{Rm}{w}$ the second term is smaller than 1 and $n^{pay} < n^{ff}$.

■

The three cases are corresponding to the variable dependent strength of the pay-as-you-go contribution effects on fertility. If contributions are low enough (Case 1) the fertility increasing effect of higher pension payments base dominates the fertility diminishing effect of lower growth and lower available adult income for the pay-as-you-go pension system. This is supporting higher fertility in the pay-as-you-go pension system. At a certain contribution level (Case 2) the contrary fertility effects are exactly offset implying equal fertility for both pension schemes. Taxation above this critical level (Case 3) leads to negative growth and adult budget effects that are larger than the positive effect of higher pension payments base. In this case fertility in the pay-as-you-go pension system becomes lower than in the fully funded system.

Figure 1 summarizes the already obtained fertility insights for the three pension system types.

Figure 1: Fertility comparison for different contribution levels



Now assume a Cobb-Douglas production function of the form:

$$F(A_tL_t, K_t) = K_t^{\alpha}(A_tL_t)^{1-\alpha}$$

Use the results for factor prices $w = m^{\alpha}(1-\alpha)$ and $R = \alpha m^{\alpha-1}$ to reformulate the threshold contribution level for the three above mentioned cases:

$$\tau \gtreqless \frac{\beta}{(1+\beta+\gamma)} - \frac{\alpha}{1-\alpha}$$

If we set α equal to 1/3 which is standard in the literature case 2 is true because τ, β and γ are positive and smaller than 1.

Proposition 14 *If the production function is Cobb-Douglas and $\alpha = 1/3$ pay-as-you-go fertility is lower than fully funded fertility ($n^{pay} < n^{ff}$).*

While the general result for fertility is case dependent, the result for growth is not. Per capita production growth corresponding to the funded pension system is always higher than growth for the pay-as-you-go scheme.

$$g^{pay} = \frac{\beta\theta R w}{\gamma(mR + \tau w)}$$
$$g^{ff} = \frac{\beta\theta w}{\gamma m}$$

Proposition 15 *A pay-as-you-go pension system leads to lower economic growth than a fully funded pension system ($g^{pay} < g^{ff}$).*

This highlights that our model reproduces the classical result for exogenous fertility models by Feldstein (1974). From the derivation of growth we know that the model exhibits two growth effects. Capital accumulation is growth enhancing and fertility is growth diminishing. If fully funded growth is always higher than pay-as-you-go growth despite higher fully funded fertility for the Cobb-Douglas case with $\alpha = 1/3$, fully funded capital holdings have to be higher than the pay-as-you-go ones. We follow that savings plus pension contributions corresponding to a funded pension system are higher than pay-as-you-go savings ($s^{ff} + \tau \bar{W}_t > s_t^{pay}$). To understand the result one has to examine the different effects on capital accumulation. While all pension contributions are always savings reducing because they transfer income to the future and reduce uncertainty, the type of the system decides about the impact on capital accumulation. Fully funded pension contributions exactly act like savings because they are invested in the capital market and therefore do not change capital accumulation. In contrast pay-as-you-go contributions which go directly from the adults to the old reduce future capital despite the fact that pay-as-you-go savings can be higher than fully funded savings. This is the case because contributions are not invested in the capital market and the savings reducing effect of pension contributions can not be offset.

The result that pay-as-you-go-growth is always lower than fully funded growth further implies that the growth enhancing effect of lower pay-as-you-go fertility can not compensate the growth decreasing effect of lower pay-as-you-go future capital. This is contrary to the findings of Yoon, Talmain (2001) who study exactly the same question similar to the already mentioned Zhang and Zhang model in a positive private transfer framework without descending altruism. The different result is again driven by the assumption of zero interfamilial intergenerational transfers which omits the growth increasing effect of gift reductions.

4 Calibration

The theoretical results obtained in the previous sections show that pension systems influence growth through impacts on fertility and capital accumulation. While the growth impacts of the different pension systems can clearly be ranked, the variable values of R, m, w, γ, β and τ decide whether fully funded fertility is higher or smaller than informal fertility. In order to clarify the fertility ranking, we calibrate our model for an average OECD as well as for an average Sub-Saharan country inside a Cobb-Douglas production function economy.

The parameters are chosen such that the balanced growth path equilibrium matches the empirical features of an average OECD country with a pay-as-you-go pension system. Periods have a length of 30 years implying a life expectancy of 90 years. Due to empirical findings we set capital productivity α equal to 1/3. The discount factor β is assumed to be 0.99 per quarter of a year corresponding to the standard real-business-cycle literature. In our 30 years per adult period framework this corresponds to 0.99^{120}. Following Doepke and De La Croix (2003) child rearing cost, measured through the time parameter θ, corresponds to 15% of adult working time. This cost only arises if children are still living with their parents. We assume that this is the case for half a period setting θ equal to 0.075. Pension contribution rate τ is chosen to be equal to the OECD average of 30%. This number together with the child rearing cost limits maximum fertility to 5.7 children per person. We further choose the descending altruism factor γ to be 0.142 and the technology parameter m controlling the influence of capital intensity equal to 0.0069 because these variable reproduce a steady state fertility rate at the reproduction level $n_t = 1$ and a steady state per capita output growth rate of 2% per year. The values of m and α are further implying an interest rate of 7.67%.

The chosen variable values reproduce our theoretical result that the pay-as-you-go system leads to lowest fertility. Additionally we show that for the observed contribution rate informal fertility is clearly higher than fully funded fertility. This is the case since the fertility increasing effect of higher pension payments base is dominating the fertility decreasing effects of reduced growth and adult budget. Only if pension contributions are unrealistically larger than 51.6% of adult income the fully funded system produces higher fertility than the informal one (see table 1). Since aggregate savings can not be negative these cases can be excluded (see equation (19)) and we follow that informal fertility is higher than fully funded fertility.

Table 1: Fertility dependence on τ	$\tau = 0.3$		$\tau = 0.516$		$\tau = 0.7$	
	n_t	g_t	n_t	g_t	n_t	g_t
Informal Pension System	1.65	1.01	1.31	0.69	0.88	0.54
Pay-as-you-go Pension System	1	1.81	0.71	1.43	0.45	1.21
Fully Funded Pension System	1.32	2.9	1.32	2.9	1.32	2.9

Graphical examination of the results (see figure 3) shows that informal fertility creates a hump shaped curve in a fertility and pension contribution rate plane. The behavior of the curve is reflecting the strength of the underlying effects which are dependent on the level of pension contributions τ. Hump shaped behavior can only be observed for the informal pension system where the old-age security motive is still present. As the contribution payments per child increase the insurance motive becomes less important while the negative growth effect becomes stronger. At the fertility maximum the effects are offset. A further increase of τ leads to decreasing fertility. Despite the narrow scope of our simple analysis the comparison of fully funded and pay-as-you-go fertility suggests that fertility differences between the US and Europe can partly be explained by the different types and not only by the different contribution levels (Boldrin, De Nardi and Jones (2005)) of the pension systems. The US, where pensions are mainly financed through a funded system show a Total Fertility Rate (TFR) of 2, while Europe, represented through mainly pay-as-you-go pension systems, shows a TFR of 1.4.

Figure 3: Fertility OECD

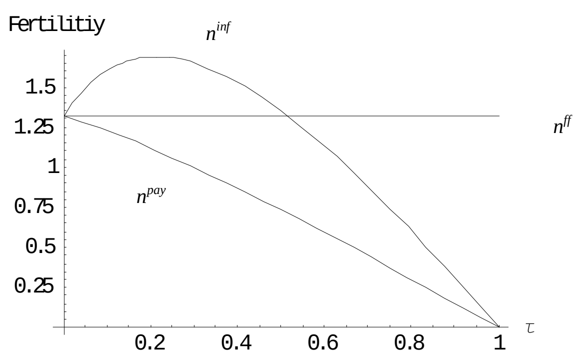
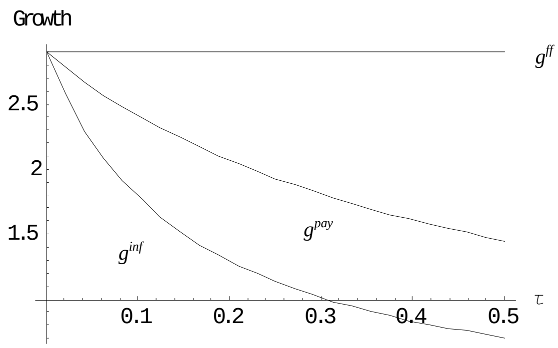


Figure 4 shows the growth diminishing effect of an increase in the pension contribution rate for an informal and a pay-as-you-go pension system. Like in the theoretical results, the pay-as-you-go growth level is always higher than the informal one since growth reducing fertility is lower and growth increasing savings are higher. If the contribution rate is too high the informal as well as the pay-as-you-go pension system could lead to negative growth. Fully funded growth is graphically represented by a horizontal line since it is independent on the pension contribution rate.

This suggests that also growth differences are dependent on the type of pension system. Higher US growth compared to European growth can therefore partly be explained by the regions differences in pension funding.

Figure 4: Per capita growth OECD



The second numerical example is dealing with developing countries. Therefore the parameters are chosen such that the balanced growth path equilibrium matches the empirical features of an average Sub-Saharan country. Periods now have a length of 15 years implying a life expectancy of 45 years. Capital productivity and the discount rate per quarter are equal to the OECD case ($\alpha = 1/3$ and $\beta = 0.99$). In this framework the discount rate corresponds to the value $\beta = 0.99^{60}$. Child cost measured through the parameter θ are expected to be lower than for the OECD case since in informally organized societies children are looked after by a broader sense of the family which can even take the form of a village unity. Taking the above 7.5% of working time for OECD Countries into account we choose child raring cost for developing countries to be equal to 0.042. This number leads together with the observed fertility rate of 2.75 [11] to a descending altruism factor γ equal to 0.117 which is only slightly smaller than the value for the OECD case. This is creating additional support for our child rearing cost choice since we can not see any reason why descending altruism representing the genetic imprint to procreate should be much different for developing countries. The parameterization of τ for the developing country case is quite tricky since no data about social mandatory contribution is available. Therefore we again use the observed average benefits for OECD countries which

[11] World Population Data Sheet 2006.

are around 30% of working income and divide them through the steady state level of fertility to get $\tau = 0.11$. We implicitly assume that 30% of adult working income plus the own fruit of savings are high enough to finance a sufficient level of retirement consumption. We further use the growth rate of 0.6%[12] per year to set the technology parameter m, controlling the influence of capital intensity on labor productivity, equal to 0.012. The technology parameter which is governing the transition of capital intensity to labor productivity m is higher than the OECD one, reflecting the lower technological standard. Our numerical developing country example implies an interest rate of close to 13% what can partly be justified by existing risk prime.

Our variable values again result in lowest fertility for the fully funded system (see table 2). An unrealistically high contribution rate of $\tau = 73.5\%$ is needed to equal fertility levels for the informal and fully funded pension system. Positive aggregate savings again exclude these high levels of the contribution rate. Our example therefore implies that the informal pension system leads to lower fertility than the fully funded one.

Table 2: Fertility dependence on τ	$\tau = 0.11$		$\tau = 73.5$		$\tau = 0.8$	
	n_t	g_t	n_t	g_t	n_t	g_t
Informal Pension System	2.75	1.09	1.68	0.26	1.3	0.24
Pay-as-you-go Pension System	1.59	2.01	0.55	0.99	0.42	0.95
Fully Funded Pension System	1.68	2.46	1.68	2.46	1.68	2.46

Graphical examination of the outcomes (see figure 5) shows that pay-as-you-go fertility and informal fertility, drawn in a fertility and pension contribution rate plane, create a hump shaped curve. Increasing pension contribution rates are leading to increasing fertility as long as the positive utility effect through higher retirement budget is dominant. At the maximum the increasing effects are offset by the decreasing growth and adult budget effects. From this level of τ onwards fertility is decreasing.

Figure 5: Fertility Sub-Saharan Africa

[12] Sub-Saharan average for 1990-2004.

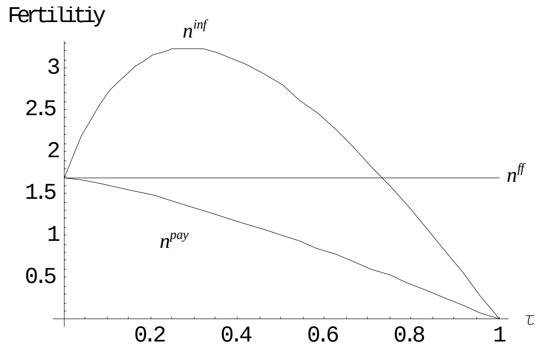
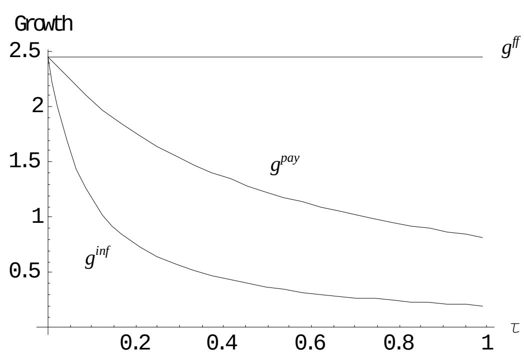


Figure 6 shows the growth diminishing effect of an increase in the pension contribution rate for an informal and a pay-as-you-go system. Like in the theoretical results prompted the pay-as-you-go growth level is always higher than the informal one since growth reducing fertility is lower and growth enhancing savings are higher. Fully funded growth is graphically represented by a horizontal line since it is independent on the pension contribution rate.

Figure 6: Per capita growth Sub-Saharan Africa



Now we are in the position to give a full description of the impacts corresponding to the introduction of a public pension to a developing economy. The fully funded Pension system clearly leads to the highest per capita growth while the pay-as-you-go one produces the lowest fertility rates. Dependent on whether the reduction in fertility or the increase of per capita growth is the main task of the governmental program the pay-as-you-go or fully funded system should be introduced. Independent on this question any of the two described public pension systems lead to a preferable outcome compared to an informal pension system .

5 Conclusion

This paper analyzes the growth promoting potential of a public pension system introduction to a developing economy. We show that no matter if the introduced public pension scheme is funded or pay-as-you-go the consequences on economic growth are positive.

A pay-as-you-go pension system introduction increases per capita growth through higher capital accumulation and lower fertility. Introducing a funded pension scheme also increases capital accumulation while the demand for children can increase or decrease depending on the level of pension contribution τ. The overall effect on economic growth is nevertheless again positive. This is the case because the growth enhancing effect of higher future capital is dominating the possible growth diminishing effect of higher fertility. The calibration of the model further shows that realistic contribution levels exclude the case of increasing fertility.

Within the debate about the impact of different public pension systems on growth, works incorporating endogenous determined fertility (Zhang and Zhang (1995), Yoon and Talmain (2001)) usually produce the result that a pay-as-you-go public pension system implies higher growth than a fully funded one. We show that this result is crucially dependent on the existence of intrafamily intergenerational gifts which can be assumed to be fully crowded out by the existence of a public pension system. Fully crowded out gifts imply higher economic growth for an economy with a fully funded public pension system than for an economy with an unfunded public pension system. The result is driven by the fact that the growth decreasing effect of lower pay-as-you-go capital accumulation outweighs the growth enhancing effect of lower pay-as-you-go fertility. A public pension system framework with fully crowded out gifts is therefore reestablishing the conventional "exogenous fertility view" of growth diminishing pay-as-you-go pension schemes.

Acknowledgements

I would like to thank Gerhard Sorger and Alexia Fürnkranz-Prskawetz for their support, ideas and helpful comments. The work further heavily profited from comments and discussion by Ben Heijdra, Lex Meijdam and participants at the NETSPAR pension conference in Utrecht.

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