Bargaining and Coalition Formation

Dr James Tremewan (james.tremewan@univie.ac.at)

Experimental tests of the Nash bargaining solution
Why laboratory experiments?

- Important features of the Nash bargaining model cannot be identified or controlled outside the lab:
  - The preferences of each player over outcomes.
  - Common knowledge of preferences: each player must know the preference ordering of the other etc.

- In the laboratory we can control for things like appearance, reputation, etc, which may be important in the field.

- We can alter one factor at a time to identify causality.

- Laboratory experiments allow for precise replication of results to ensure outcomes are regular.

- Problem: External validity: experiments may fail to include some element that is important in "real world" bargaining (also true of theory).
Experimental tests of the Nash bargaining solution

- Two-Person Bargaining: An Experimental Test of the Nash Axioms, Nydegger and Owen (1974)
  - Tests each of Nash’s four axioms.

- Game-Theoretic Models and the Role of Information in Bargaining, Roth and Malouf (1979)
  - Looks at the effect of what should be irrelevant information.

- How sensitive are bargaining outcomes to changes in disagreement payoffs?, Anbarci and Feltovich (2011)
  - Tests the prediction that players with higher disagreement payoffs gain a larger share.

  - Tests the predictions about the effect of player’s risk aversion on outcomes.
Two-Person Bargaining: An Experimental Test of the Nash Axioms, Nydegger and Owen (1974)

- An early experiment. Experimental methodology not well developed, and computers unavailable.
- All bargaining face-to-face across table. All rules were common knowledge. $1 show-up fee.
- Treatment 1: Bargaining over $1. In case of disagreement, the dollar is lost.
- Treatment 2: As Treatment 1, but player 2 could receive no more than 60 cents (to test IIA).
- Treatment 3: Bargaining over 60 poker chips. Player 1 could cash them in for 2 cents/chip, Player 2 for 1 cent/chip (to test INV).
- Subjects: 20 male undergraduate students per treatment.
Results

- Treatment 1: All 10 pairs split money equally (consistent with SYM, PAR).
- Treatment 2: All 10 pairs split money equally (consistent with IIA).
- Treatment 3: All 10 pairs divided the chips to equalize monetary payoffs (contradicting INV which predicts there should be no difference from Treatment 1).
Shortcomings

- Assumes EV maximization, whereas people tend to be risk-averse.
- Lack of anonymity.
- Weak tests of the theory:
  - Many possible explanations for equal split in symmetric game (weak test of SYM).
  - With equal split so salient (no other reasonable outcome) disagreement unlikely (weak test of PAR).
  - Only one of many ways of constraining the set of bargaining outcomes (weak test of INV).
Roth and Malouf (1979)

Game-Theoretic Models and the Role of Information in Bargaining, Roth and Malouf (1979)

- Similar to previous study but:
  - Does not assume risk-neutrality.
  - Anonymous (messages sent between computers, partner not identified).
  - More subtle treatment of information.
Binary lottery experiments

- Instead of bargaining over money, subjects bargain over lottery tickets: if a player gets 40% of the tickets, they have a 40% chance of winning a prize $M$, and a 60% chance of winning nothing in a personal lottery.

- Consider any set of preferences over outcomes satisfying vNM assumptions: WOLOG can be represented by a utility function $u(x)$ where $u(0) = 0$ and $u(M) = 1$.

- Expected utility maximization $\Rightarrow$ utility of obtaining $p\%$ of the lottery tickets is $pu(M) + (1 - p)u(0) = p$

- We can view a subject’s utility of an outcome as being the same as the percentage of tickets they receive.
Binary lottery experiments: notes

- This is now a common way of accounting for deviations from risk-neutrality in experiments.
- But assumes expected utility maximisers (whereas ambiguity aversion may be important).
- And it is not clear that subjects really treat lottery tickets differently from points (at least I am not aware of any experiment that tests this).
Experiment

- Game 1: bargaining over lottery tickets, $M_1 = M_2 = $1.
- Game 2: as Game 1 but player 2 can receive at most 60%.
- Game 3: as Game 1 but $M_1 = $1.75, $M_2 = $3.75.
- Game 4: as Game 2 but $M_1 = $1.75, $M_2 = $3.75.
- Games played under two conditions:
  - Full information: prizes are common knowledge.
  - Partial information: subjects only know own payoff.
- Subjects seated at isolated computer terminals and communicate by "teletype." Any messages allowed that did not identify subject, or give information about prize in partial information condition. If no agreement after 12 minutes, both players receive nothing.
Predictions of Nash bargaining solution

- Game 1 is symmetric (in terms of preferences, not just payoffs). $\text{SYM+PAR} \Rightarrow 50-50$ split.
- $\text{IIA} \Rightarrow$ same outcome in Game 2 as Game 1.
- $\text{INV} \Rightarrow$ same outcome in Game 3 as Game 1.
- $\text{INV} \Rightarrow$ same outcome in Game 3 as Game 2.
- Number of lottery tickets equivalent to utility, so knowledge of partner’s prize should not be important: 50-50 split predicted in both information conditions.
- Reported statistic: Number of tickets obtained by player 2 minus tickets obtained by player 1 (predicted to be 0).
Results: differences in lottery tickets (P2-P1)

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Game 1</th>
<th>Game 2</th>
<th>Game 3</th>
<th>Game 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Std. dev</td>
<td>Mean</td>
<td>Std. dev</td>
</tr>
<tr>
<td>Full information</td>
<td>0.0</td>
<td>0.0</td>
<td>-1.9</td>
<td>12.2</td>
</tr>
<tr>
<td>Mean</td>
<td>-34.6</td>
<td>19.3</td>
<td>22.5</td>
<td>22.5</td>
</tr>
<tr>
<td></td>
<td>0.0</td>
<td>8.3</td>
<td>4.6</td>
<td>4.1</td>
</tr>
<tr>
<td>Partial information</td>
<td>0.0</td>
<td>0.0</td>
<td>1.3</td>
<td>4.6</td>
</tr>
<tr>
<td>Mean</td>
<td>2.5</td>
<td>4.1</td>
<td>-2.5</td>
<td>4.1</td>
</tr>
<tr>
<td>Std. dev</td>
<td>19.3</td>
<td>4.6</td>
<td>22.5</td>
<td>4.1</td>
</tr>
</tbody>
</table>
Conclusions

• Predictions of NBS perform well in “partial information” games (remember the information in these games is complete in the sense that it is theoretically all that is required for players to identify the NBS.)

• Adding (theoretically irrelevant) information allows players to compare expected payoffs, and subjects often try to equalize these rather than choose the NBS.

• Two focal points: equal tickets and equal money; outcomes tend to be one of these, or somewhere in between.

• Does this ”invalidate” the NBS?
  • Not necessarily... perhaps the utility function should include a preference for fairness in outcomes?
How sensitive are bargaining outcomes to changes in disagreement payoffs?, Anbarci and Feltovich (2011)

- Subjects bargain over a fixed sum. Disagreement payoffs vary and are asymmetric.
- (Assuming risk-neutrality) NBS predicts

\[
\left| \frac{\delta x_1}{\delta d_1} \right| = \left| \frac{\delta x_2}{\delta d_2} \right| = \frac{1}{2} \quad \text{and} \quad \left| \frac{\delta x_1}{\delta d_2} \right| = \left| \frac{\delta x_2}{\delta d_1} \right| = -\frac{1}{2}
\]

and

\[
\left| \frac{\delta x_1}{\delta d_1} \right| + \left| \frac{\delta x_2}{\delta d_2} \right| = 1
\]

(see previous set of slides)
Conclusions

- Effects are in the correct direction, but too small (around 0.25, and significantly less than 0.5).

- Can this be explained by risk-aversion? No. Authors show that with risk-aversion, NBS implies:
  \[
  \left| \frac{\delta x_1}{\delta d_1} \right| + \left| \frac{\delta x_2}{\delta d_2} \right| > 1
  \]

- However, authors show that with a utility function including fairness concerns, NBS implies:
  \[
  \left| \frac{\delta x_1}{\delta d_1} \right| + \left| \frac{\delta x_2}{\delta d_2} \right| < 1
  \]

- As shown in the previous set of slides, in a simple divide the dollar game with zero disagreement payoffs, NBS predicts the less risk-averse player will gain more.
- This paper identifies two bargaining games, one where increased risk-aversion should lead to lower shares and one higher shares.
- Risk preferences of subjects are elicited, and high risk-aversion subjects bargain with low risk-aversion subjects.
- Some support is found for the risk-aversion hypothesis, but not so strong.
- Authors hypothesize that bigger stakes may increase effect, and also that any risk-aversion effect is dominated by "focal-point" effect.
- Some evidence that risk-aversion weakens bargaining position also found in Dickinson, Theory and Decision (2009).
Nash bargaining solution: pros and cons

• Pros:
  • It is general, in the sense that it does not relate only to a particular bargaining process. Can be widely applied.
  • Captures some key features of bargaining, such as importance of disagreement payoffs and risk preferences.
  • Easily calculated, so widely used as a component in bigger models without adding much complexity.

• Cons:
  • People do appear to make inter-personal comparisons of utility, which violates INV.
  • Does not account for focal points which may exist outside the formal strategic structure of the game.
  • In some cases precise features of the bargaining process may be important (possibly violating IIA).